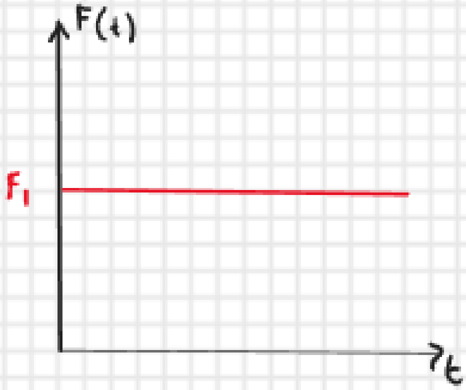
Titolo:

$m\ddot{x} + b\dot{x} + kx = F(t)$ Funzione qualsiasi

$= -m\ddot{y}(t)$

Integrale di Duhamel



- $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F_1$

$F(0) = 0$

$F(t) = F_1 \quad t > 0$

Condizioni iniziali

$x(0) = 0 \qquad \dot{x}(0) = 0$

Soluzione eq. moto

$$x(t) = e^{-\nu\omega t}\left[A\,\mathrm{sen}\,\Omega t + B\cos\Omega t\right] + \frac{F_1}{k}$$

Condizioni al contorno A e B

- $x(0) = 0 \Rightarrow B = -\frac{F_1}{k}$
- $\dot{x}(0) = 0 \Rightarrow A = -\frac{F_1}{k}\frac{\nu\omega}{\Omega}$

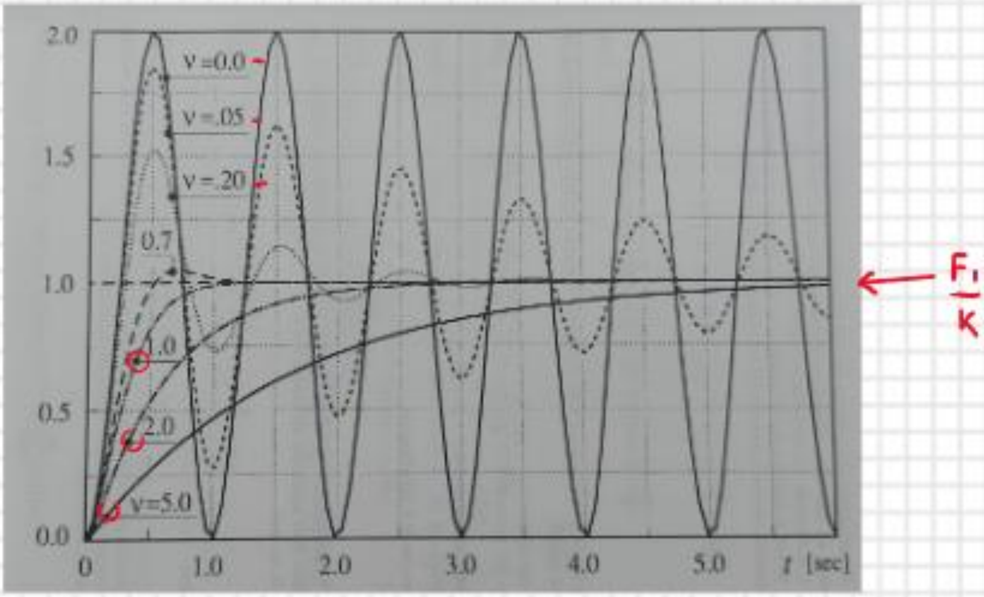
$$\dot{x}(t) = -\nu\omega e^{-\nu\omega t}\left(A\,\mathrm{sen}\,\Omega t + B\cos\Omega t\right) + e^{-\nu\omega t}\left(A\Omega\cos\Omega t - \Omega B\,\mathrm{sen}\,\Omega t\right)$$

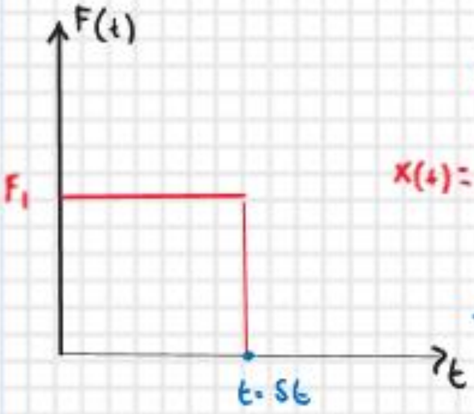
$$x(t) = \frac{F_1}{k}\left[1 - e^{-\nu\omega t}\left(\frac{\nu\omega}{\Omega}\sin\Omega t + \cos\Omega t\right)\right]$$

$t \to \infty$

$x(t) \to \frac{F_1}{k}$

Titolo:





$$t = \delta t \longrightarrow -F_1$$

$$x(t) = \frac{F_1}{k}\left[1 - e^{-\nu\omega t}\left(\frac{\nu\omega}{\Omega}\sin\Omega t + \cos\Omega t\right)\right] +$$

$$-\frac{F_1}{k}\left[1 - e^{-\nu\omega(t-\delta t)}\left(\frac{\nu\omega}{\Omega}\,\mathrm{sen}\,\Omega(t-\delta t) + \cos\Omega(t-\delta t)\right)\right]$$

$$t > \delta t$$

Risposta all'impulso $F_1\,\delta t$

$$g(t) = \frac{F_1}{k}\left[1 - e^{-\nu\omega t}\left(\frac{\nu\omega}{\Omega}\sin\Omega t + \cos\Omega t\right)\right]$$

$$x(t) = g(t) - g(t-\delta t) \qquad \delta t \rightarrow \text{infinitesimo}$$

$$x(t) = \frac{dg(t)}{dt}\,\delta t$$

$$\frac{dg(t)}{dt} = -\frac{F_1}{k}\left[-\nu\omega e^{-\nu\omega\cdot}\left(\frac{\nu\omega}{\Omega}\sin\Omega t + \cancel{\cos\Omega t}\right) + e^{-\nu\omega t}\left(\nu\omega\cancel{\cos\Omega t} - \Omega\sin\Omega t\right)\right]$$

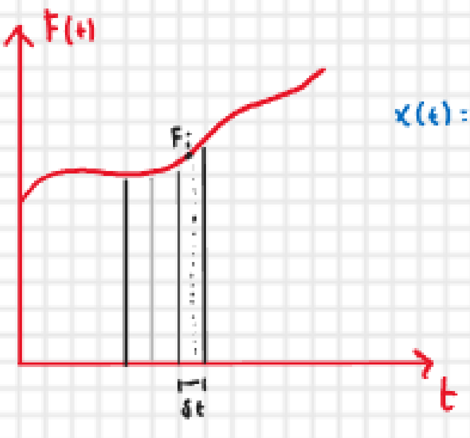
Titolo:

$$X(t) = \frac{F_1}{K}\left(\Omega + \frac{\nu^2\omega^2}{\Omega}\right) e^{-\nu\omega t} \operatorname{sen}\Omega t \ \delta t = \frac{F_1}{m\Omega} e^{-\nu\omega t} \operatorname{sen}\Omega t \ \delta t$$

Spostamento dovuto all'impulso applicato a $t=0$

Impulso agisca a $t = t_1$

$$x(t) = \frac{F_1}{m\Omega} e^{-\nu\omega(t-t_1)} \operatorname{sen}\Omega(t-t_1)\, dt_1 \qquad t > t_1$$



$$x(t) = \int_0^t \frac{F(t)}{m\Omega} e^{-\nu\omega(t-t_1)} \operatorname{sen}\Omega(t-t_1)\, dt_1$$

Integrale di Duhamel

Risposta del nostro sistema ad una forzante $F(t)$

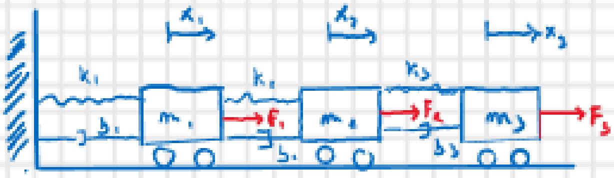
- Esempio semplice $\quad F(t) = p_0 \quad \nu = 0 \quad \omega = \Omega$

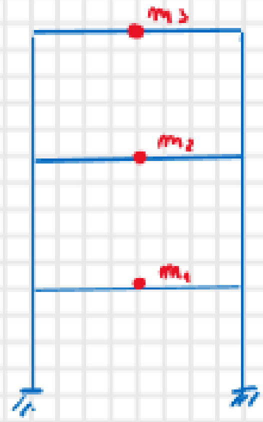
$$x(t) = \frac{1}{m\omega}\int_0^t p_0 \operatorname{sen}\omega(t-t_1)\, dt_1 = \frac{p_0}{m\omega^2} \cos\omega(t-t_1)\Big|_0^t = \frac{p_0}{m\Omega}\left(1-\cos\omega t\right)$$

Integriamo rispetto a t_1

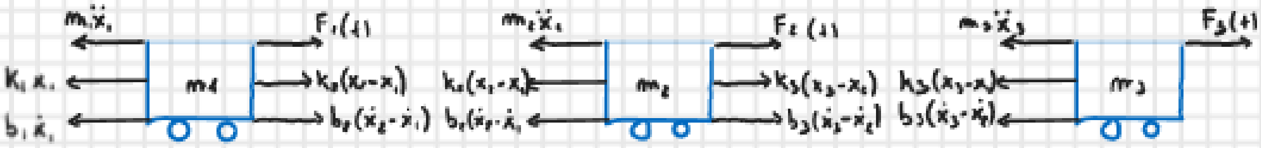
Titolo:

Sistemi a n gradi di libertà





$$k = \frac{24EJ}{h^3}$$



$$-m_1\ddot{x}_1 - k_1x_1 - b_1\dot{x}_1 + k_2(x_2-x_1) + b_2(\dot{x}_2-\dot{x}_1) + F_1 = 0$$

- $m_1\ddot{x}_1(t) + (b_1+b_2)\dot{x}_1(t) - b_2\dot{x}_2(t) + (k_1+k_2)x_1(t) - k_2x_2(t) = F_1(t)$
- $m_2\ddot{x}_2(t) - b_2\dot{x}_1(t) + (b_2+b_3)\dot{x}_2(t) - b_3\dot{x}_3(t) - k_2x_1 + (k_2+k_3)x_2 - k_3x_3 = F_2(t)$
- $m_3\ddot{x}_3(t) - b_3\dot{x}_2(t) + b_3\dot{x}_3 - k_3x_2 + k_3x_3 = F_3(t)$

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} b_1+b_2 & -b_2 & 0 \\ -b_2 & b_2+b_3 & -b_3 \\ 0 & -b_3 & b_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\underset{\text{Matrice massa}}{[M]} \{\ddot{x}(t)\} + \underset{\text{smorzamento}}{[b]} \{\dot{x}(t)\} + \underset{\text{rigidezza}}{[k]} \{x(t)\} = \{F(t)\}$$

Titolo:

$[M]\{\ddot{x}(t)\}$ = Forze d'inerzia

$[b]\{\dot{x}(t)\}$ = Forze viscose

$\{F\}$ = forzanti

$[k]\{x(t)\}$ = Forze elastiche

Studi per oscillazioni libere

$$[M]\{\ddot{x}(t)\} + [k]\{x(t)\} = \{0\}$$

La soluzione

$$\{x(t)\} = \sum_{i=1}^{n} \{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i)$$

C_i e φ_i => condizioni iniziali

$$\{x(t)\} = \{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i)$$

$$\{\ddot{x}(t)\} = -\{\psi^i\} \omega_i^2 C_i \operatorname{sen}(\omega_i t + \varphi_i) = -\omega_i^2 x(t)$$

$$-\omega_i^2 [M]\{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i) + [K]\{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i) = 0$$

$$-\omega_i^2 [M]\{\psi^i\} + [k]\{\psi^i\} = \{0\}$$

Titolo:

$$\bullet \quad \left(-\omega_i^2 [M] + [K]\right)\{\psi^i\} = \{0\}$$

incognite

Affinché si abbiano soluzioni non banali del sistema omogeneo

$$\det \left| -\omega_i^2 [M] + [K] \right| = 0$$ eq. caratteristica di n ω_i^2

$\omega_i^2 \Rightarrow$ reali e positive

$\omega_1^2 \leq \omega_2^2 < \dots < \omega_n^2$ $\quad$ ω_1 (la più piccola) sarà la pulsazione fondamentale

$T_1' > T_1 > \dots T_n$

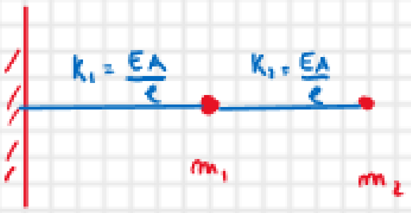
$$\omega_i \Rightarrow \{\psi^i\}$$

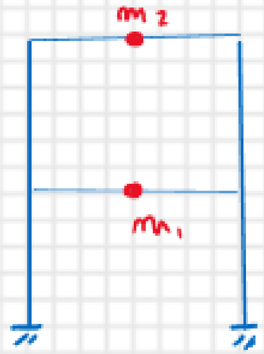
$$\{x(t)\} = \sum_{i=1}^{n} \{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i)$$

$$\{x(t)\} = \{\psi^i\} C_i \operatorname{sen}(\omega_i t + \varphi_i)$$

Se il sistema vibra secondo un modo di vibrare i valori max e nulli dello spostamento si verificano nello stesso istante

Titolo:





$$K_i = \frac{24EJ}{h^3}$$

$$k_1 = k_2 = k$$

$$m_1 = 2m_2 = 2m$$

- $m_1 \ddot{x}_1(t) + (k_1+k_2)x_1(t) - k_2 x_2(t) = 0$
- $m_2 \ddot{x}_2(t) - k_2 x_1 + k_2 x_2 = 0$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\left(-\omega_i^2 [M] + [K]\right) \{\psi^i\} = \{0\}$$

$$\left(-\omega_i^2 m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\right) \{\psi^i\} = \{0\}$$

$$\begin{pmatrix} -2\omega_i^2 m + 2k & -k \\ -k & -\omega_i^2 m + k \end{pmatrix} \{\psi_i\} = \{0\} \qquad \det\begin{pmatrix} -2\omega_i^2 m + 2k & -k \\ -k & -\omega_i^2 m + k \end{pmatrix} = 0$$

$$2\omega_i^4 m^2 - 4\omega^2 m k + k^2 = 0 \qquad \text{eq. caratteristica}$$

$$\omega_i^2 = \frac{k}{m}\,\frac{2 \pm \sqrt{2}}{2} \qquad \omega_1^2 = \frac{k}{m}\,\frac{2-\sqrt{2}}{2} \qquad \omega_2^2 = \frac{k}{m}\,\frac{2+\sqrt{2}}{2}$$

Titolo:

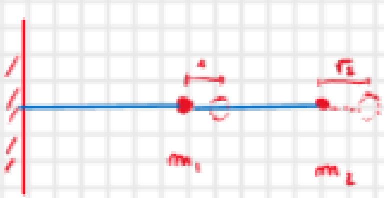
• $\omega_1^2 = \frac{K}{m}\,\frac{2-\sqrt{2}}{2}$ Lo inseriamo nel sistema e calcoliamo gli autovettori

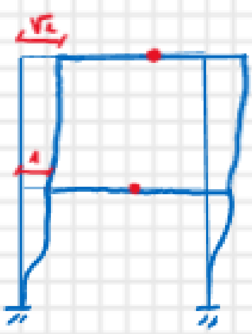
$$\begin{cases} \left(-2\omega_1^2 m + 2K\right)\psi_1^{(1)} - K\psi_2^{(1)} \\ -K\psi_1^{(1)} + \left(-\omega_1^2 m + K\right)\psi_2^{(1)} = 0 \end{cases} \Rightarrow \psi_1^{(1)} = \frac{\psi_2^{(1)}}{\sqrt{2}}$$

$$\left\{\psi^{(1)}\right\} = \begin{Bmatrix} 1 \\ \sqrt{2} \end{Bmatrix} \Rightarrow \begin{Bmatrix} 1/\sqrt{3} \\ \sqrt{2/3} \end{Bmatrix}$$ 1° modo di vibrare

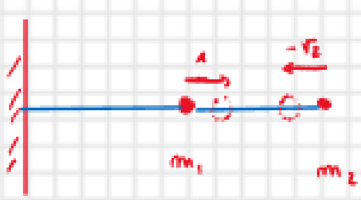
• $\omega_2^2 = \frac{K}{m}\,\frac{2+\sqrt{2}}{2}$ $\quad \left\{\psi^{(2)}\right\} = \begin{Bmatrix} 1 \\ -\sqrt{2} \end{Bmatrix}$

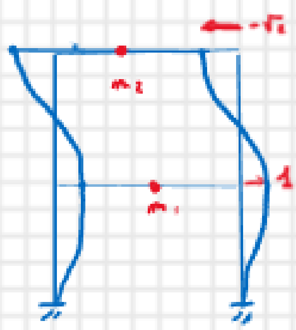
$$\left\{x(t)\right\} = \sum_{i=1}^{n} \left\{\psi^{i}\right\} C_i \,\text{sen}(\omega_i t + \varphi_i)$$ Soluzione (calcolare condizioni iniziali)





1° modo di vibrare $\begin{Bmatrix} 1 \\ \sqrt{2} \end{Bmatrix}$





2° modo di vibrare $\begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix}$