Image Processing for Physicists

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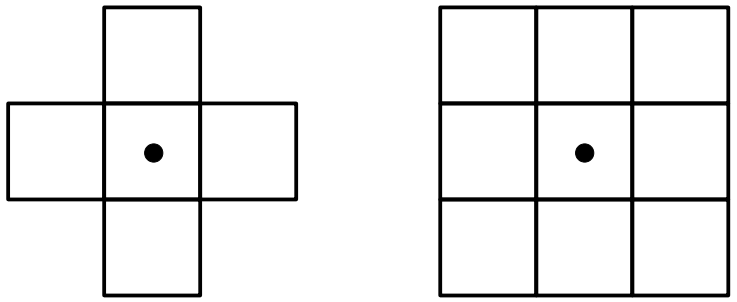
Image manipulation in the spatial domain

Overview

- coordinate transformations
  - translation, rotation, shear, ...
- intensity transformations
  - normalization, gamma, thresholding, ...
- image analysis using morphological operations
  - dilation, erosion, opening, closing, ...
- image segmentation
  - by morphology, intensity, region, ...

General image transformations

- coordinate transformations → change pixel positions
  - ↳ interpolation
- intensity transformations
- pixel-wise transformations → change pixel values
- neighborhood transformations → morphology → convolutions, ...



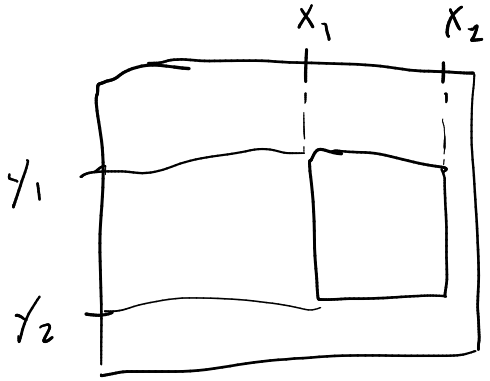
4-neighborhood 8-neighborhood

General image transformations

- images as an array

$M \times N$

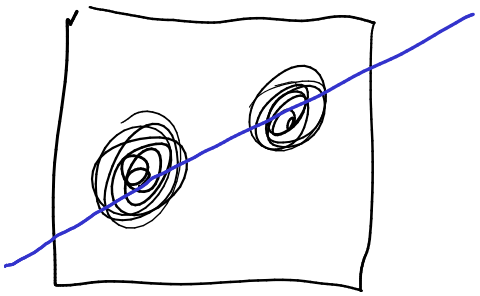
- sub array operations

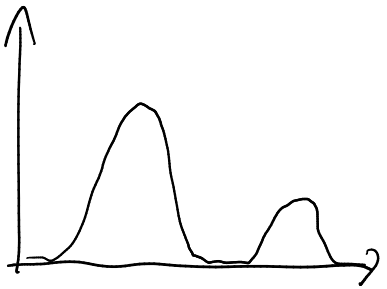


region of interest

ROI

- line extractions





General image transformations

- element wise addition

$$I = I_1 + I_2$$

$$I(i,j) = I_1(i,j) + I_2(i,j) \quad \text{for all } (i,j)$$

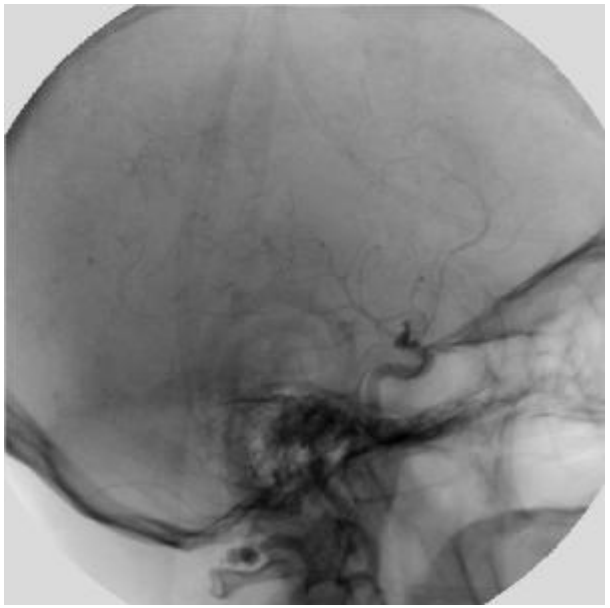
- element wise multiplication

$$I = I_1 \times I_2$$

$$= I_1(i,j) \times I_2(i,j)$$

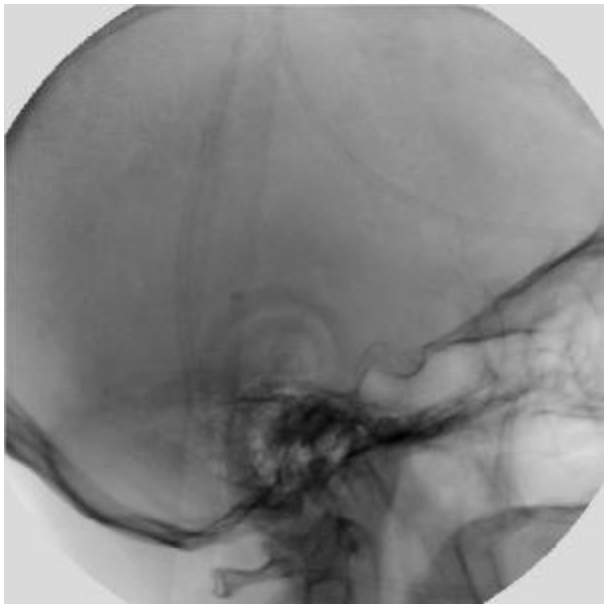
Image Subtraction Example

- Digital Subtraction Angiography
- Xray images before/after contrast agent



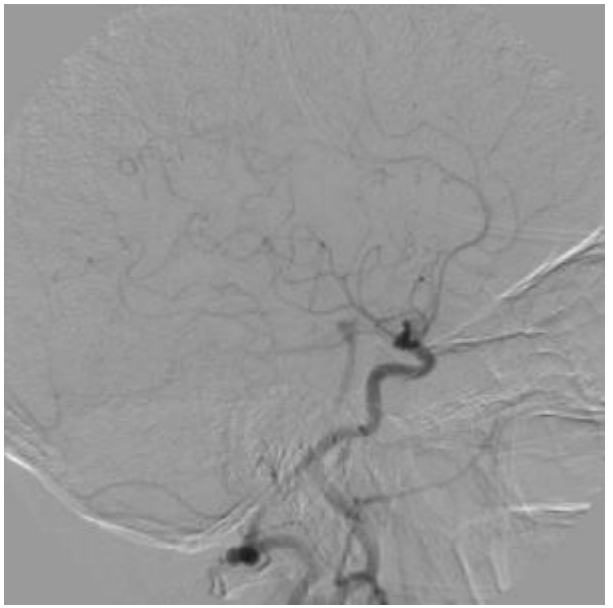
Live or contrast image

-



Mask image

=



DSA image

Source: Gonzales, Digital Image Processing

Image Addition Example

- Add multiple noisy images of same object
- (More on noise in later lectures)

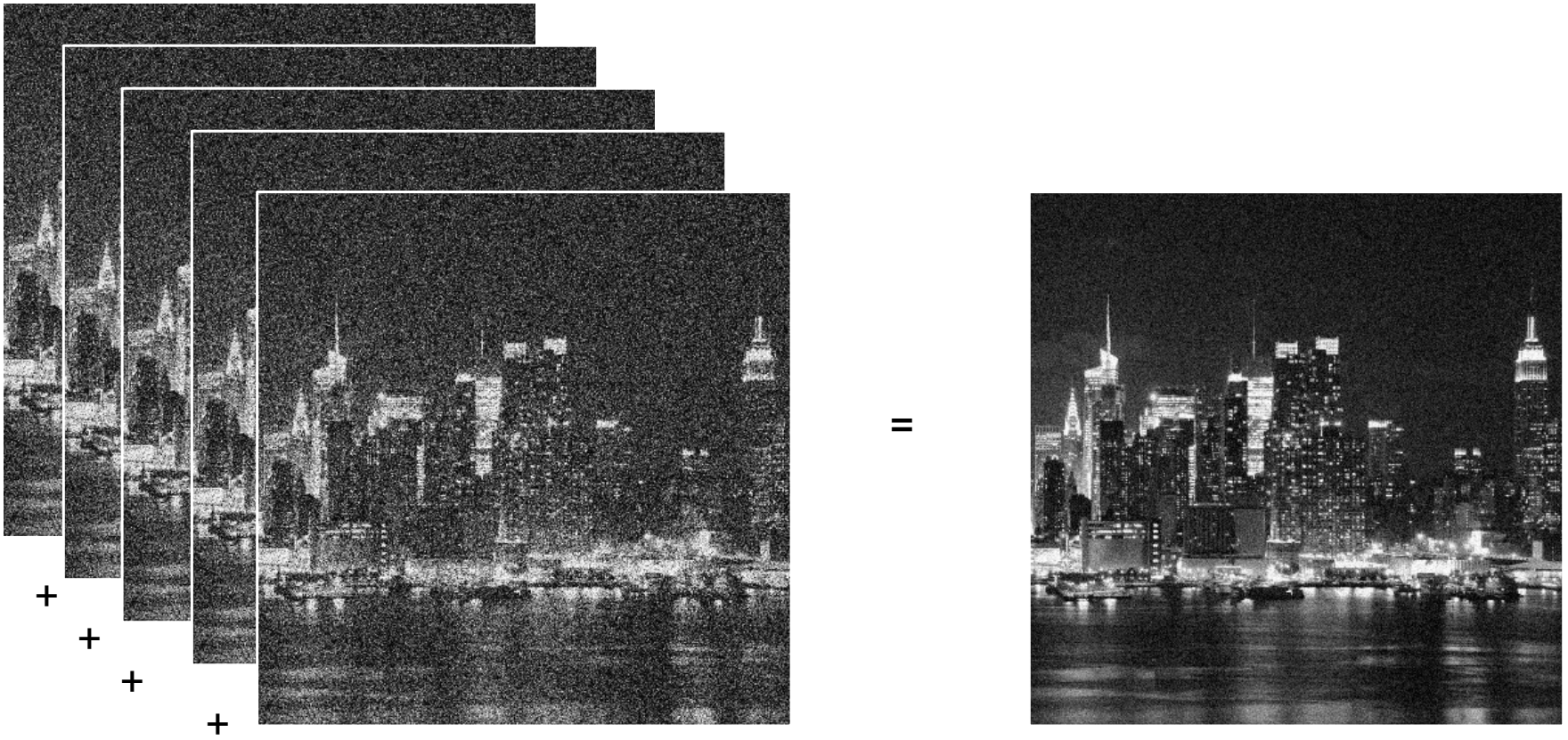


Image Multiplication Example

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

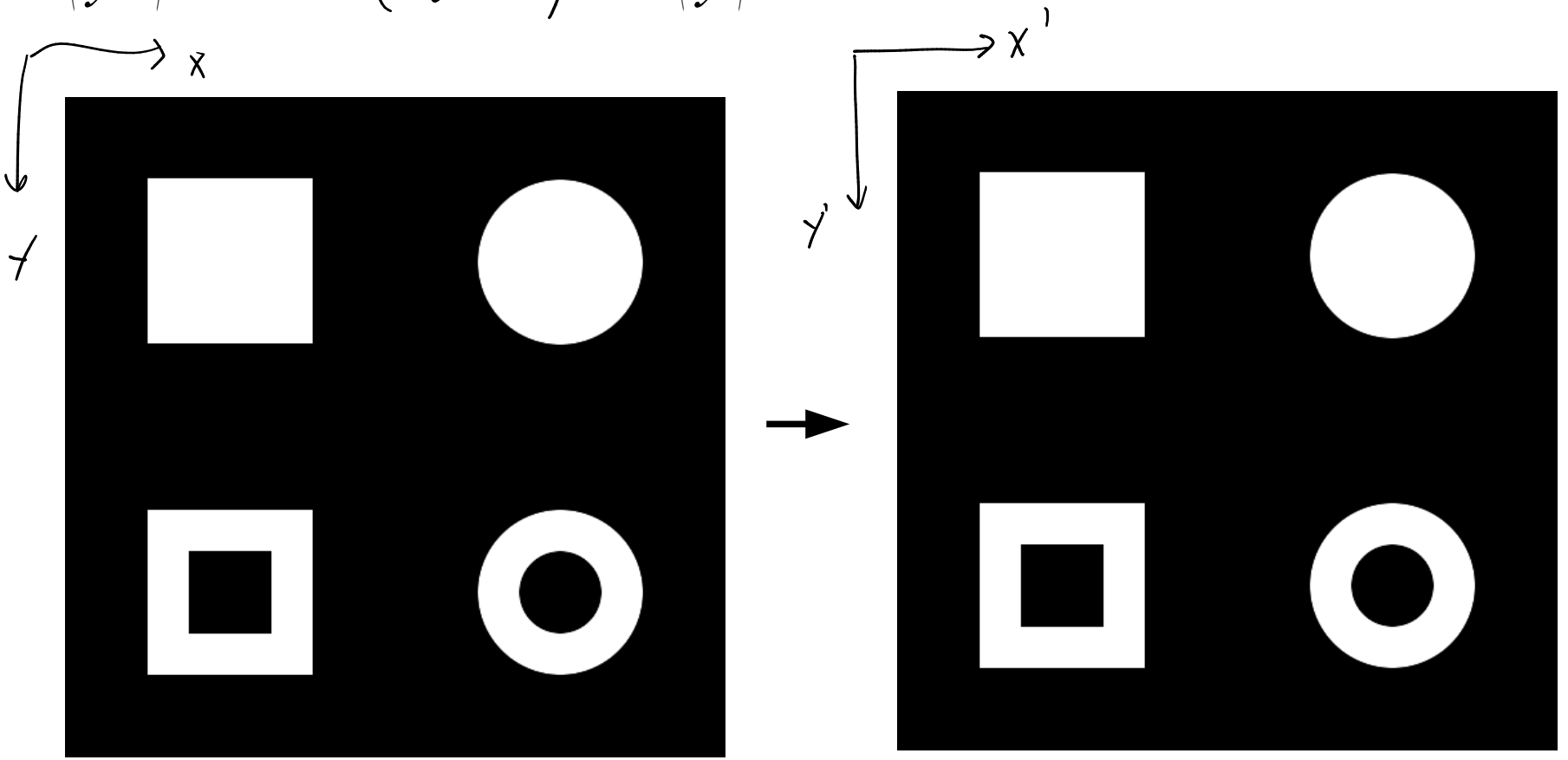
Image division - example: flat field correction

Source: Gonzales, Digital Image Processing

Affine transformations

- identity

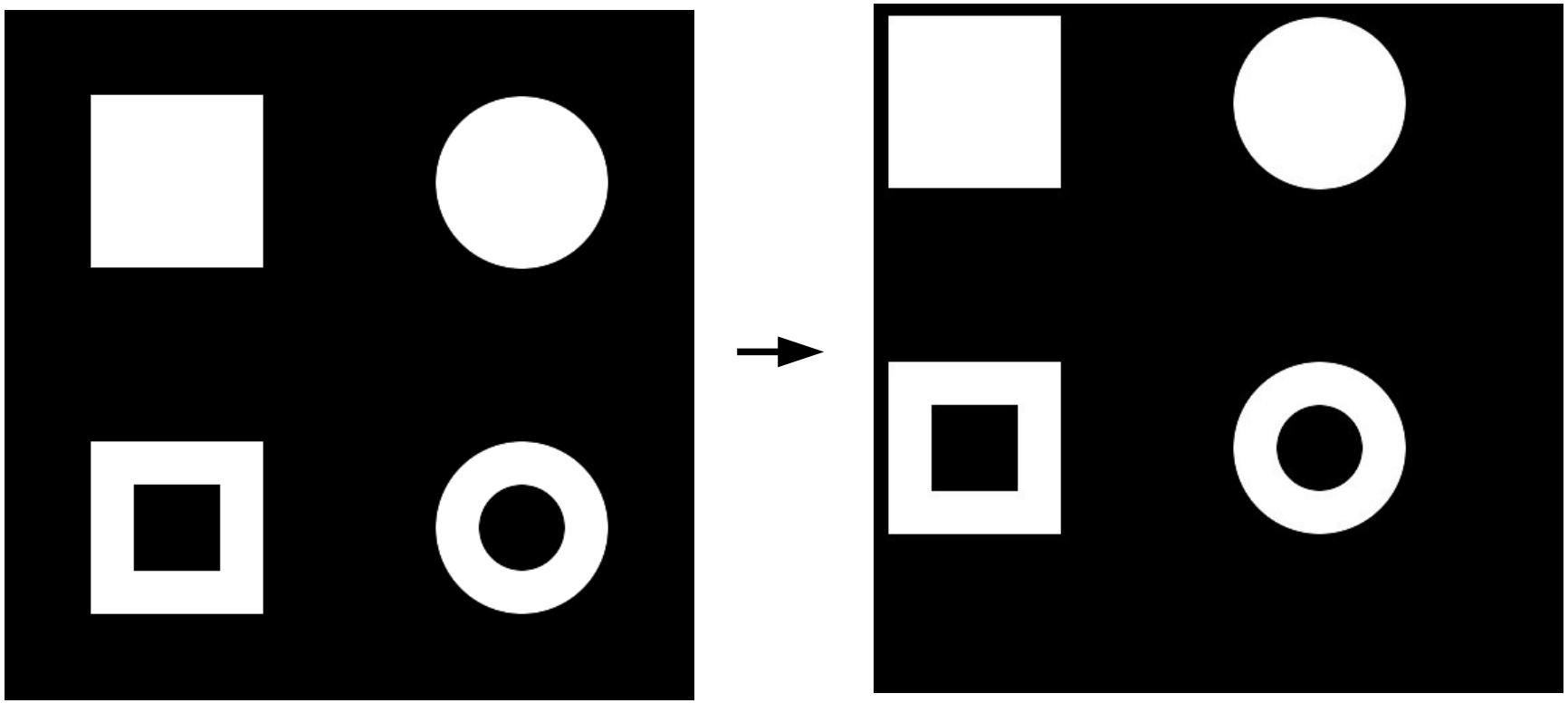
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

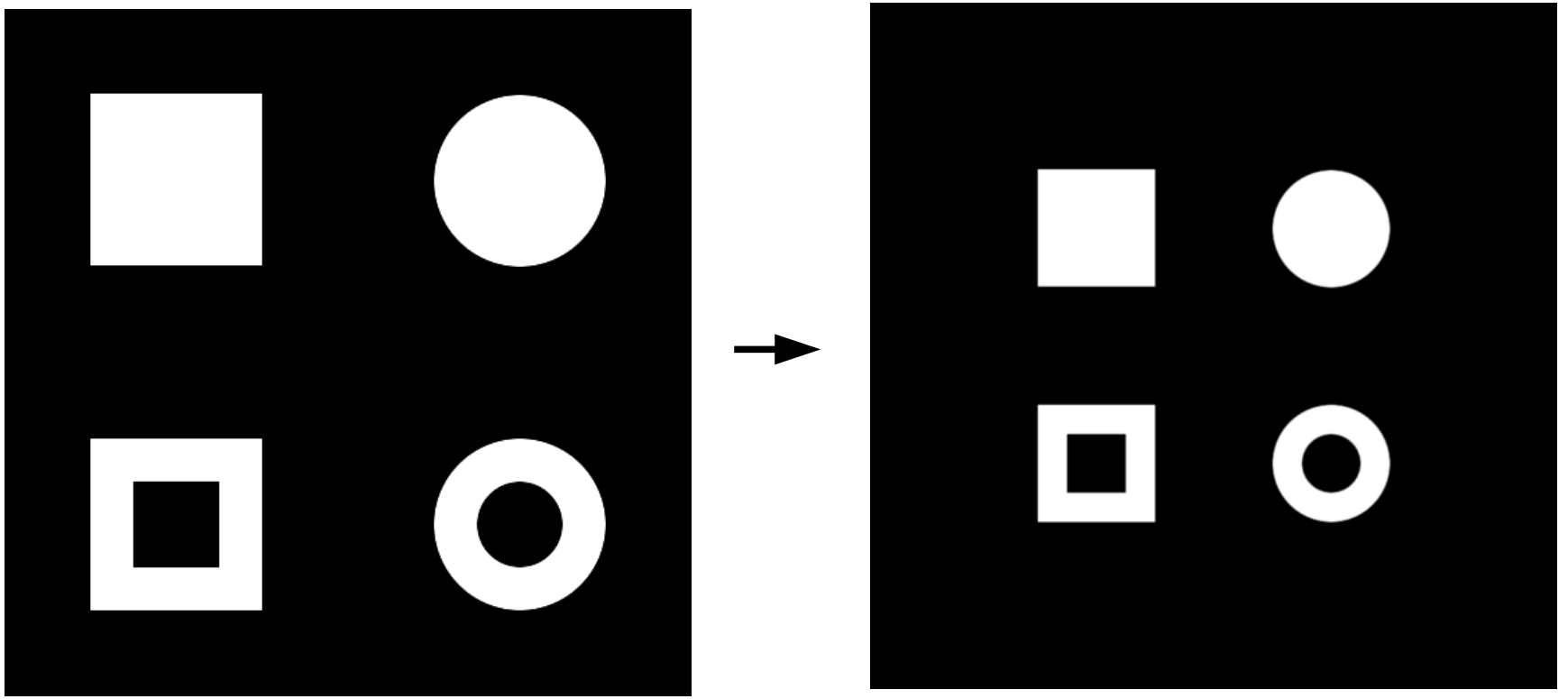


Affine transformations

- scaling

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

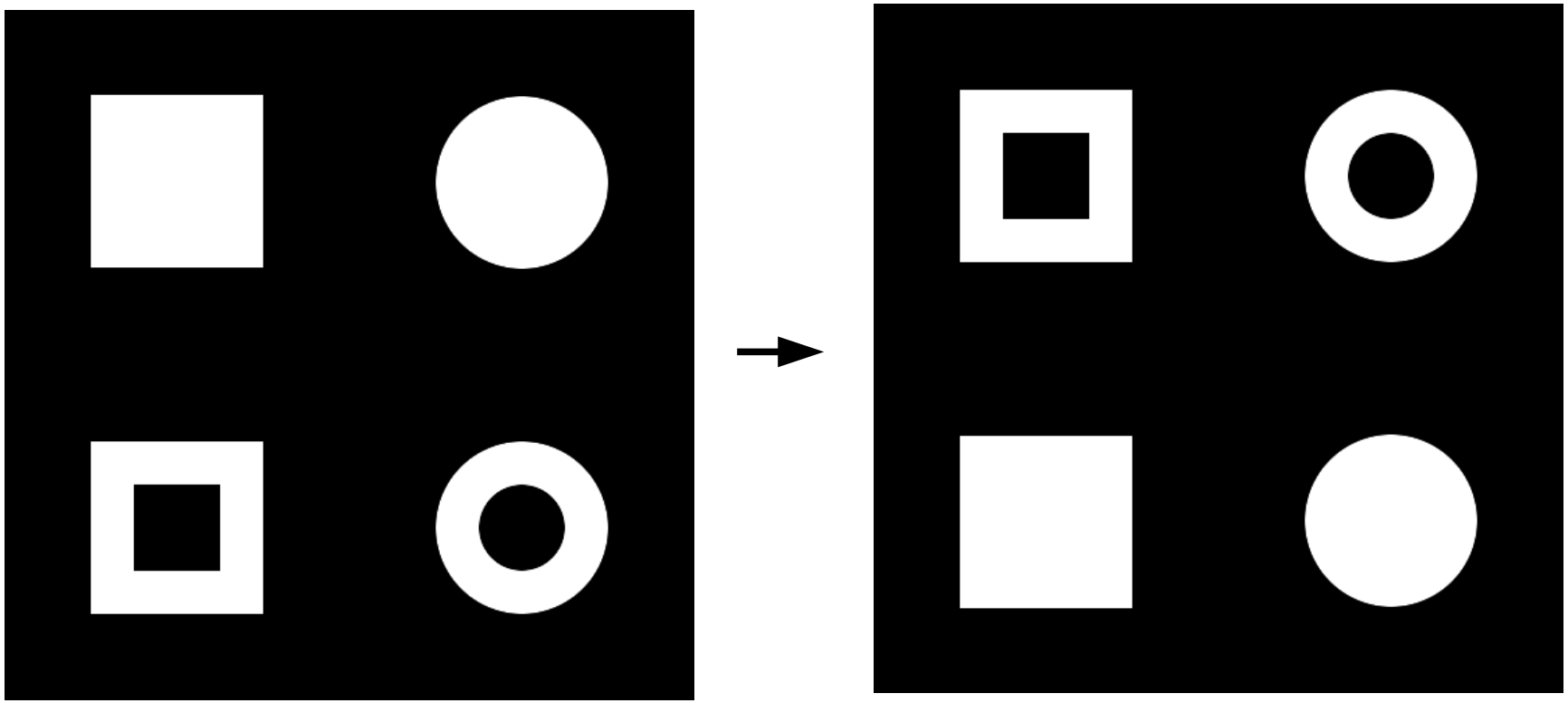
$a = b = \frac{1}{2}$



Affine transformations

- reflections

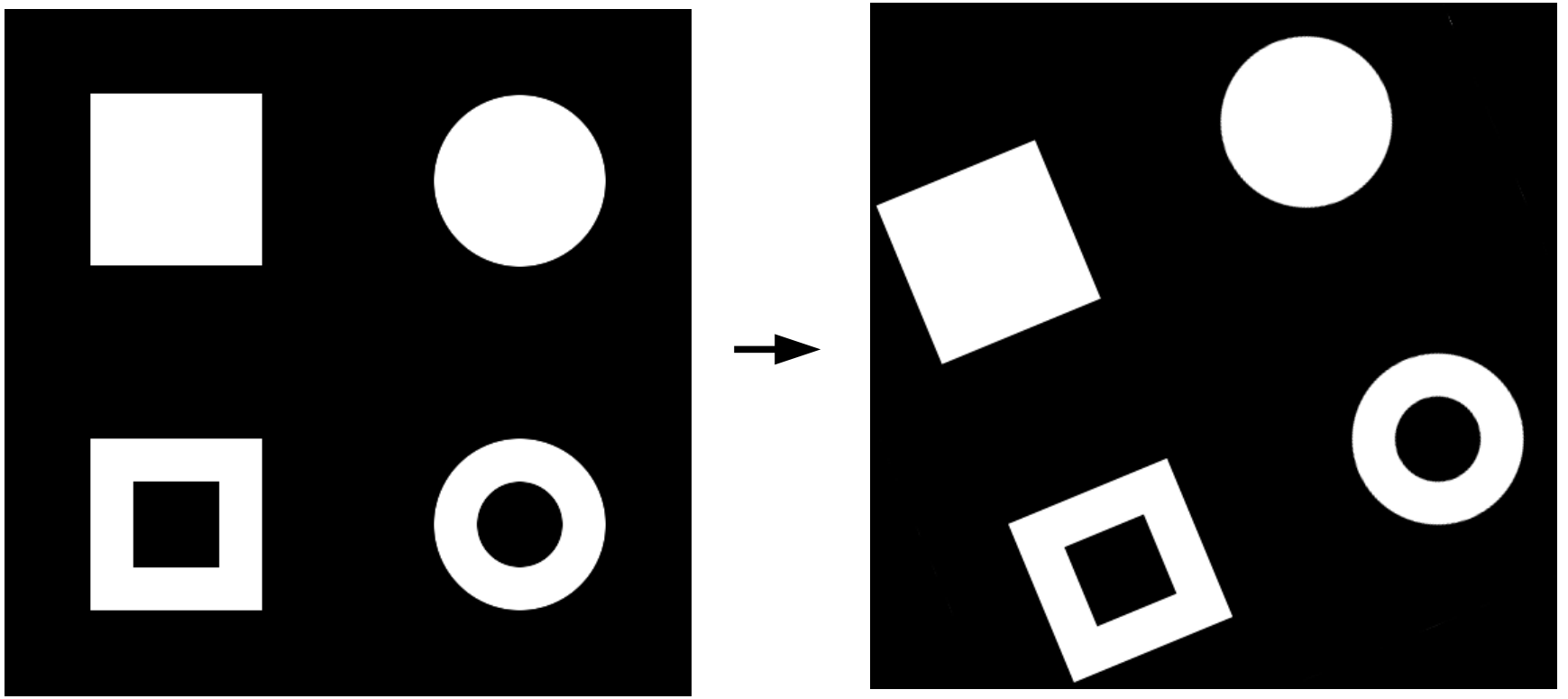
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- rotation

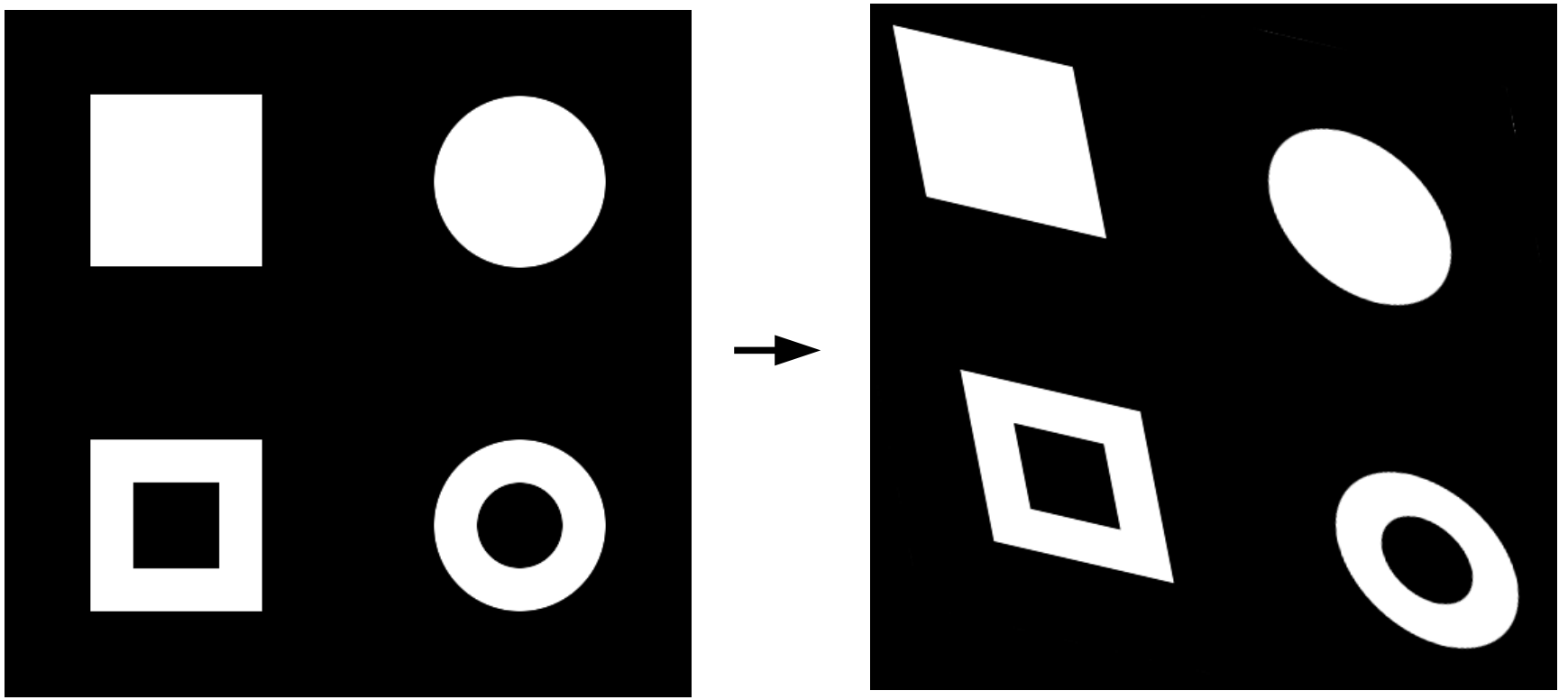
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

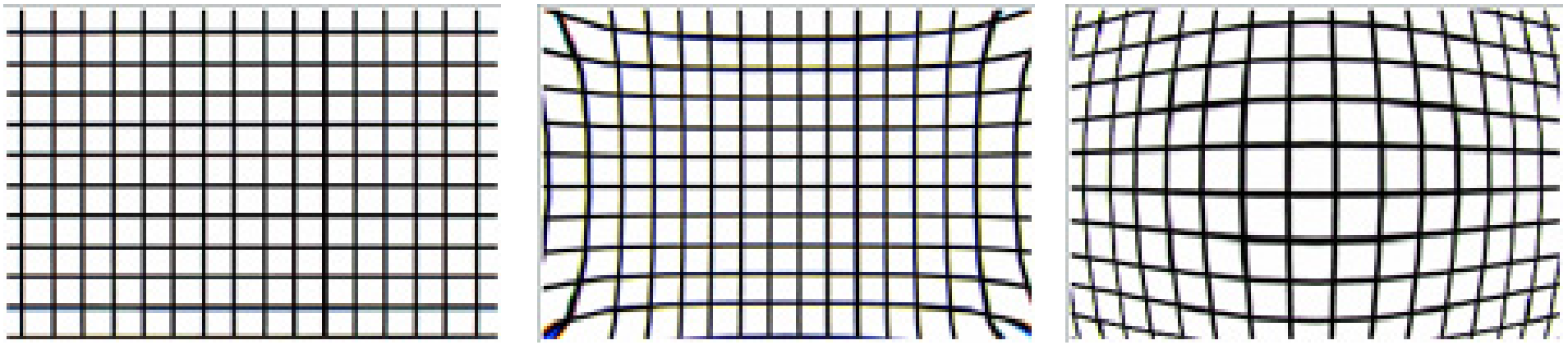
- shear

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Nonlinear coordinate transformation

- pincushion and barrel distortion

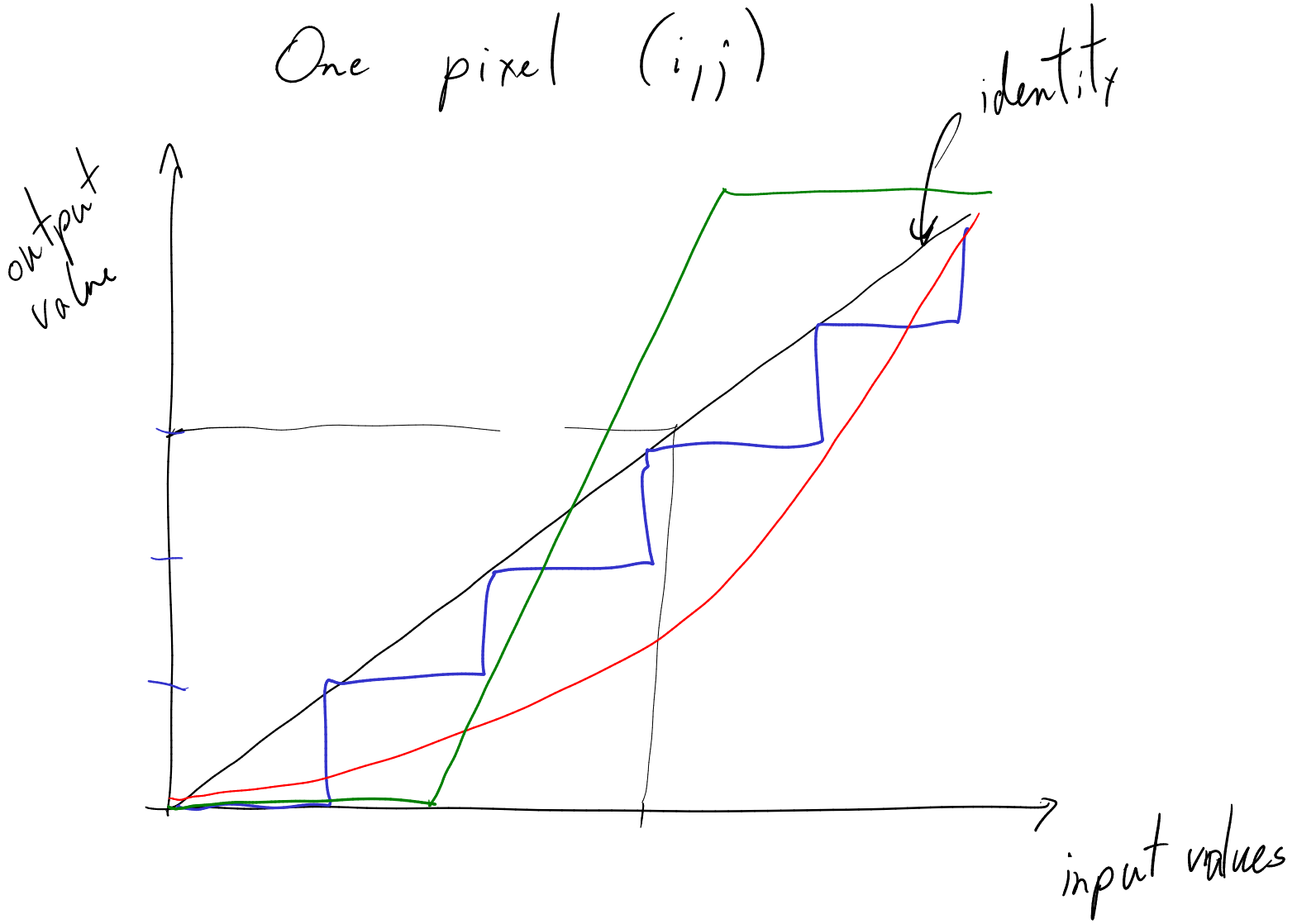


- mapping depends on radial distance from centre

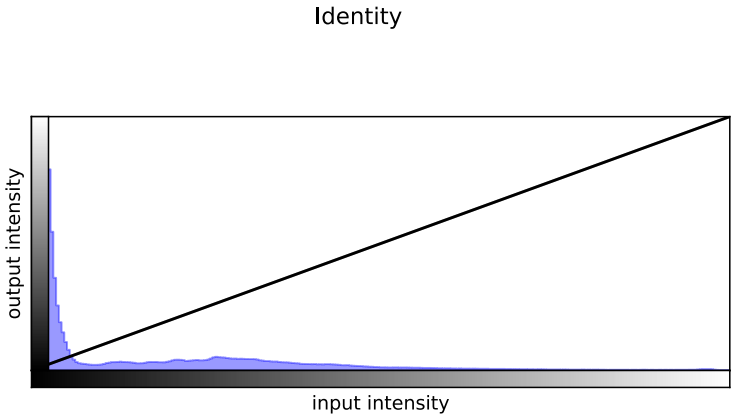
$$x' = x_0 + ax + by + ex^2 + fxy + gy^2$$

$$y' = y_0 + cx + dy + \ldots$$

Intensity mapping



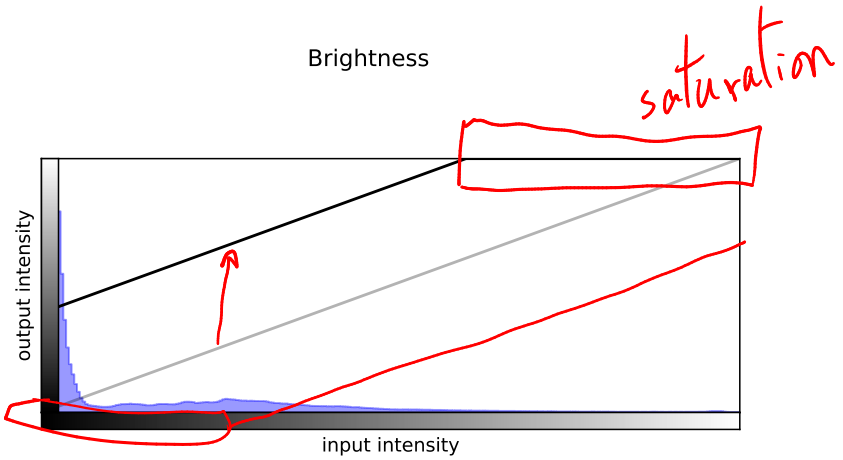
Intensity mappings



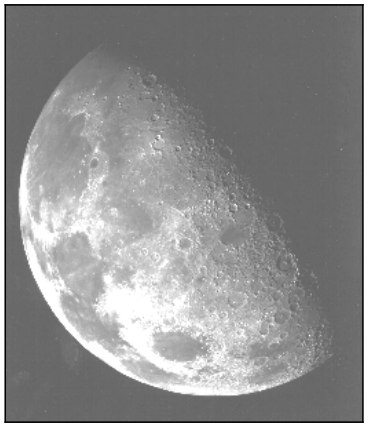

original


remapped

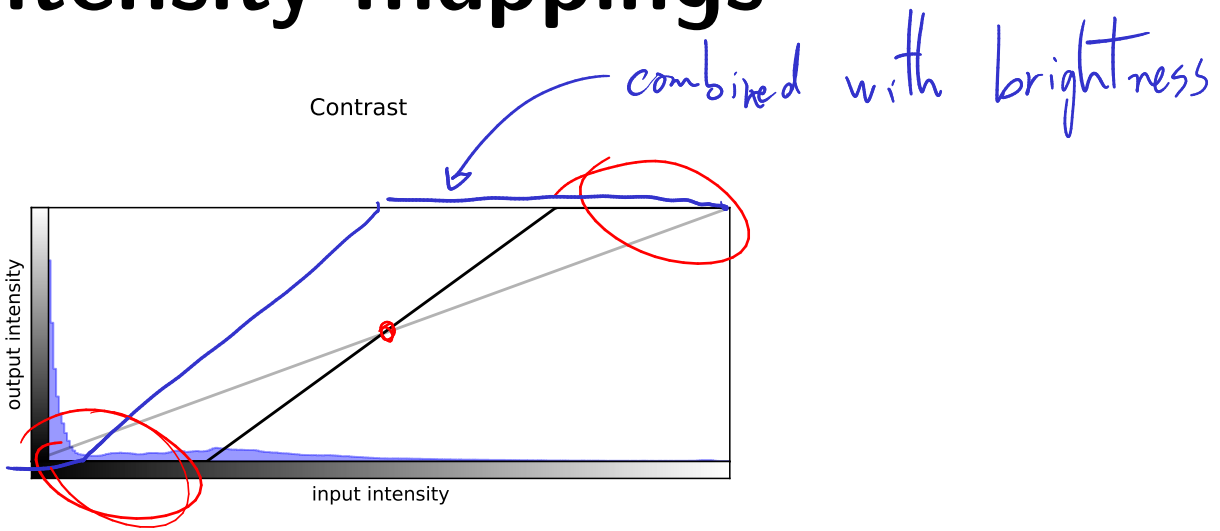
Intensity mappings

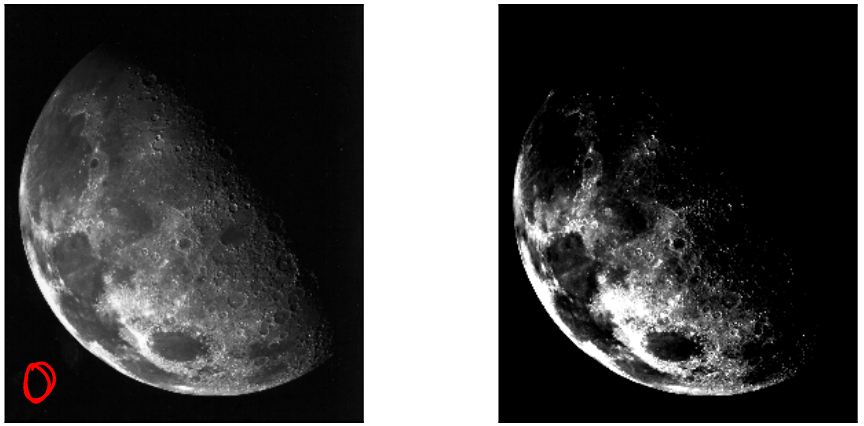



original


remapped

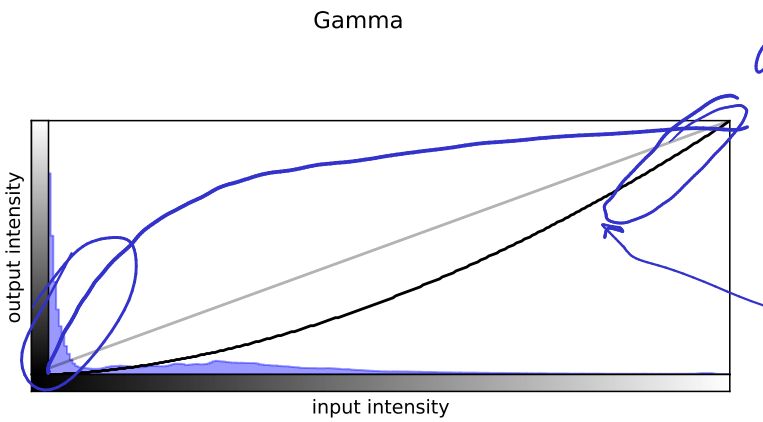
Intensity mappings




original

remapped

Intensity mappings



$output = \left(\frac{input}{max}\right)^{\gamma}$

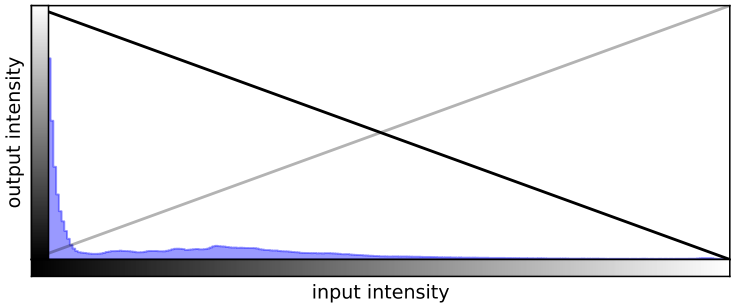
$\gamma > 1$


original

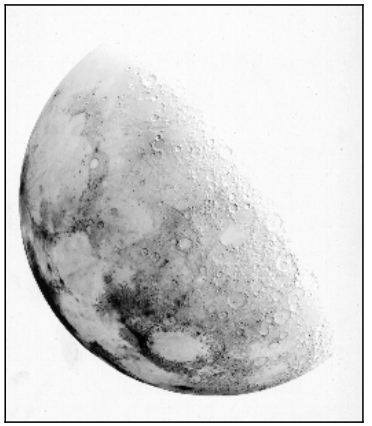

remapped

Intensity mappings

Inversion

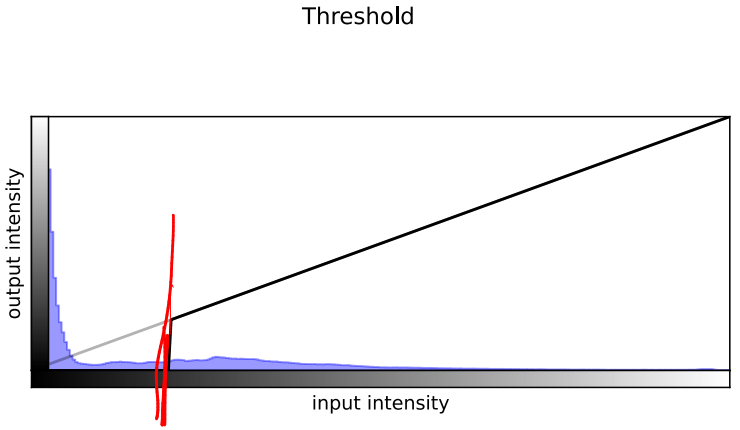


original



remapped

Intensity mappings

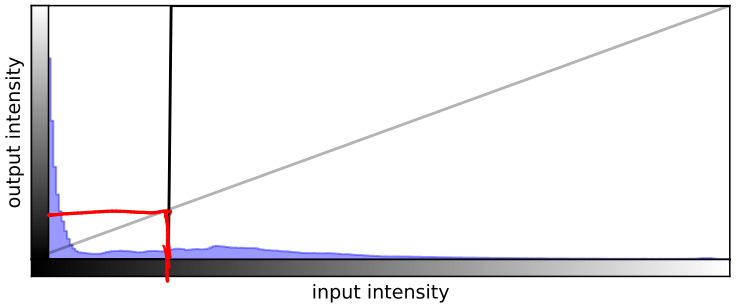



original


remapped

Intensity mappings

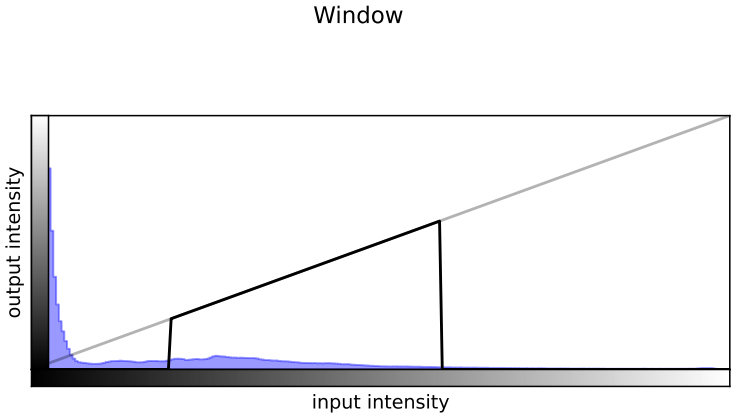
Binary threshold



original

remapped

Intensity mappings

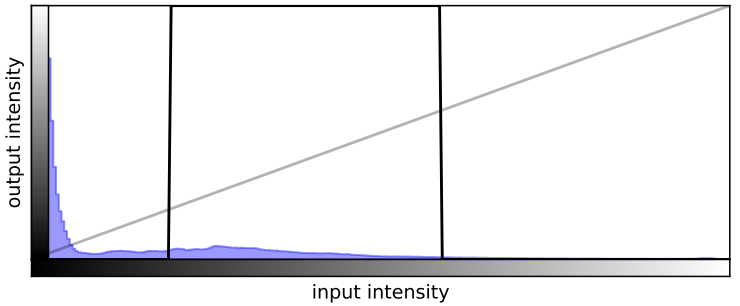



original

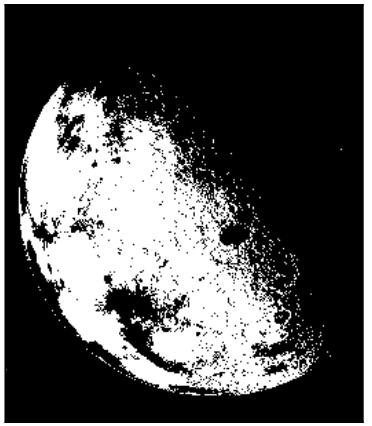

remapped

Intensity mappings

Binary window



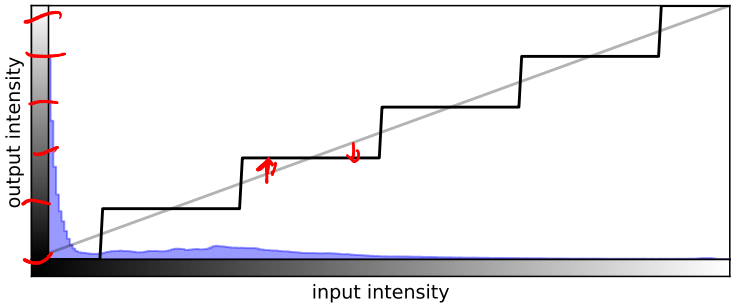
original



remapped

Intensity mappings

Posterization



original

remapped

Possible additional material:

#2 → 1. Machine Learning (neural networks, convolutional networks, diffusion, autoencoders, ...)

2. Non-linear filters (median filter, bilateral filter, histogram equalization, ...)

#1 → 3. Advanced optimization methods (stochastic descent, regularization, preconditioning, ...)

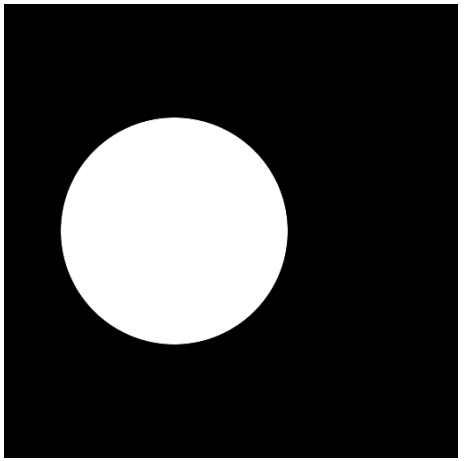
4. Time-resolved imaging (optical flow, temporal filtering, motion tracking, ...)

Morphological operations

- analyze morphology of image structures
  - based on set theory and topology
- extract image information
  - shape
  - size
  - connectivity
  - number
  - boundary
- mostly on binary images

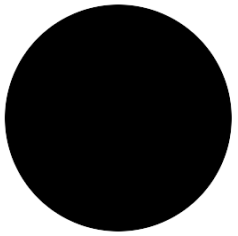
Set operations

binary mask
A

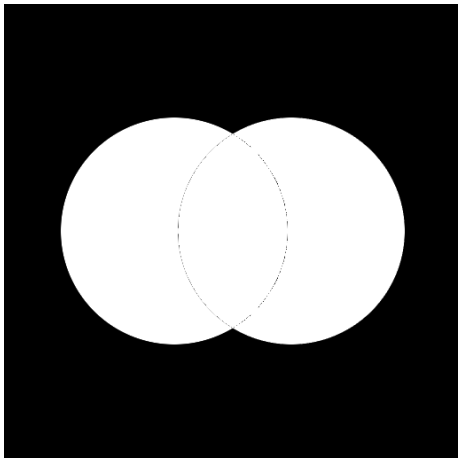


complement (background)
A^c

not



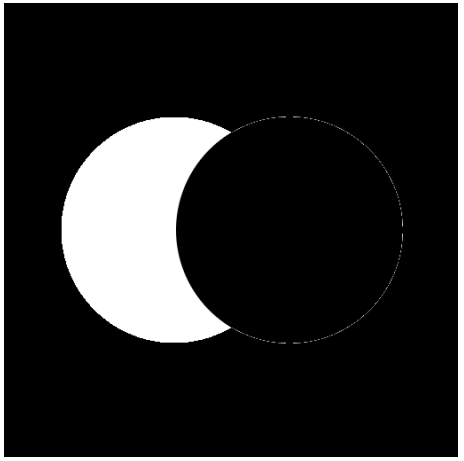
union
$A \cup B$



or

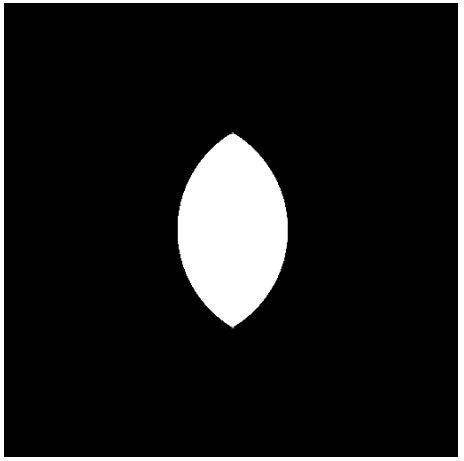
difference
$A \setminus B$

and not



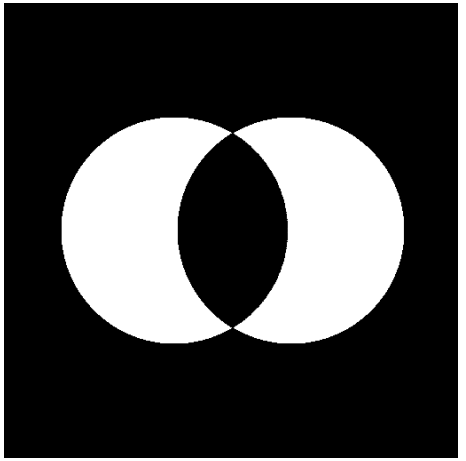
intersection
$A \cap B$

and



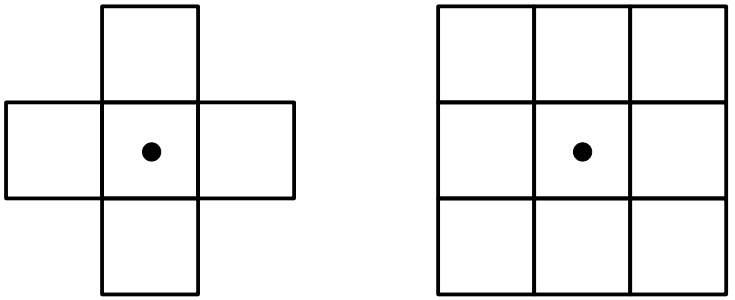
symmetric difference
$A \Delta B$

xor

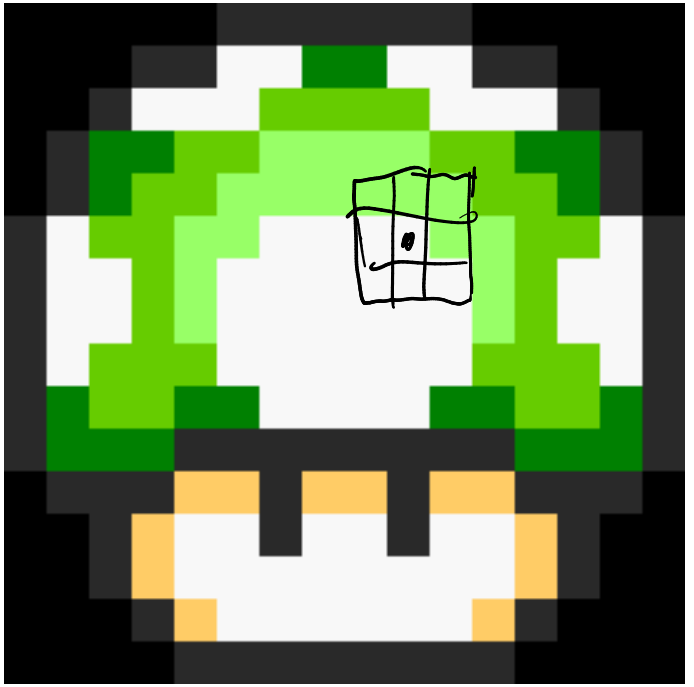


Structuring elements

- small bit mask to probe the image
- scan origin of SE over image
- check overlap between SE and image
- set pixel(s) to zero (or one)

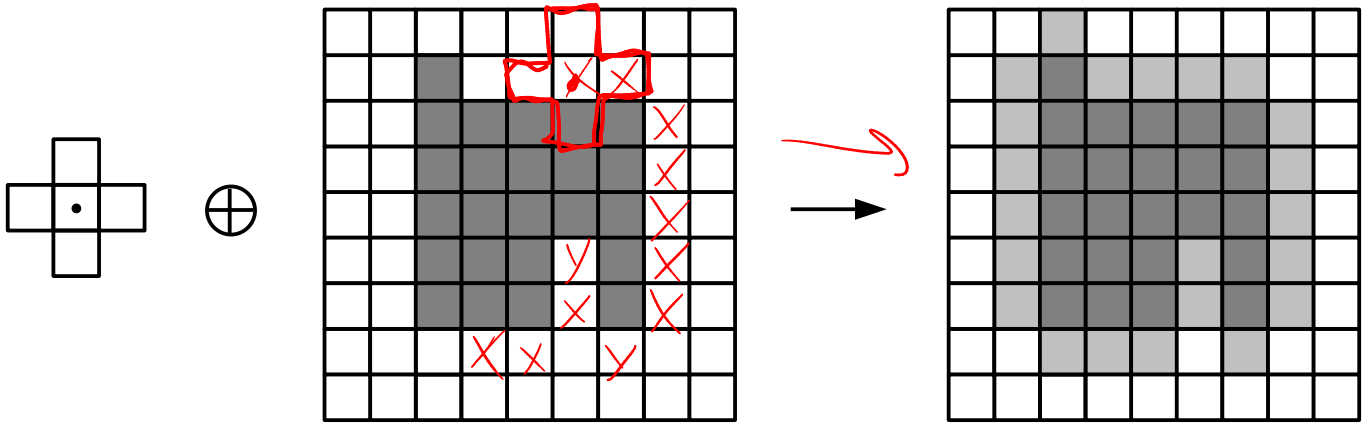


"footprint"

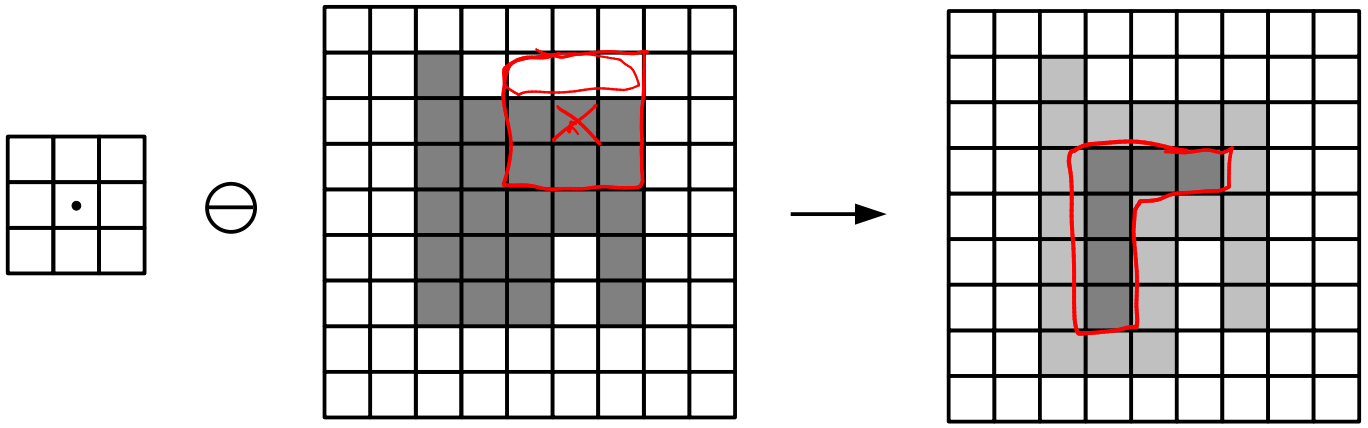


Basic operations

- Dilation: expand region

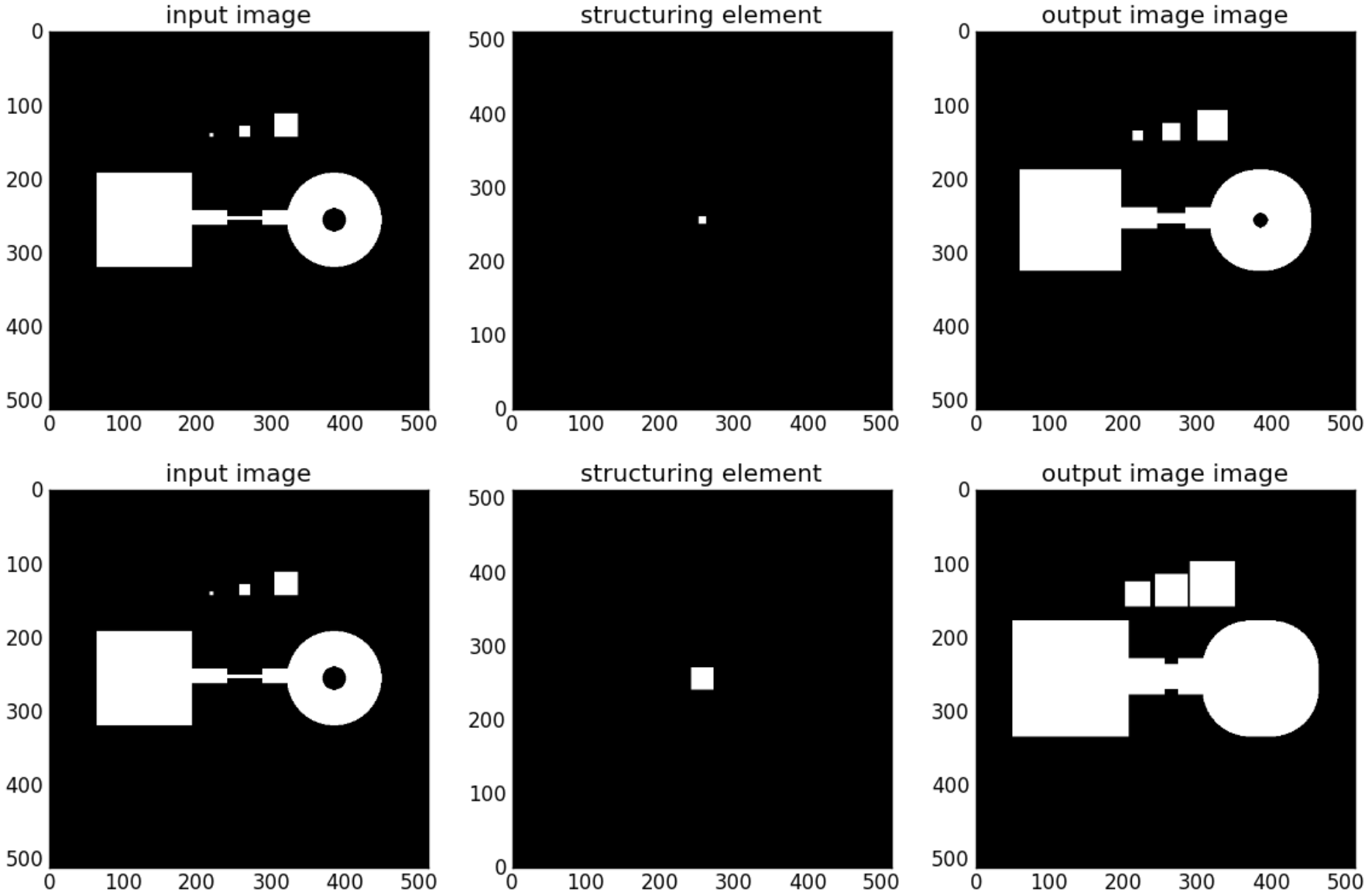


- Erosion: shrink region



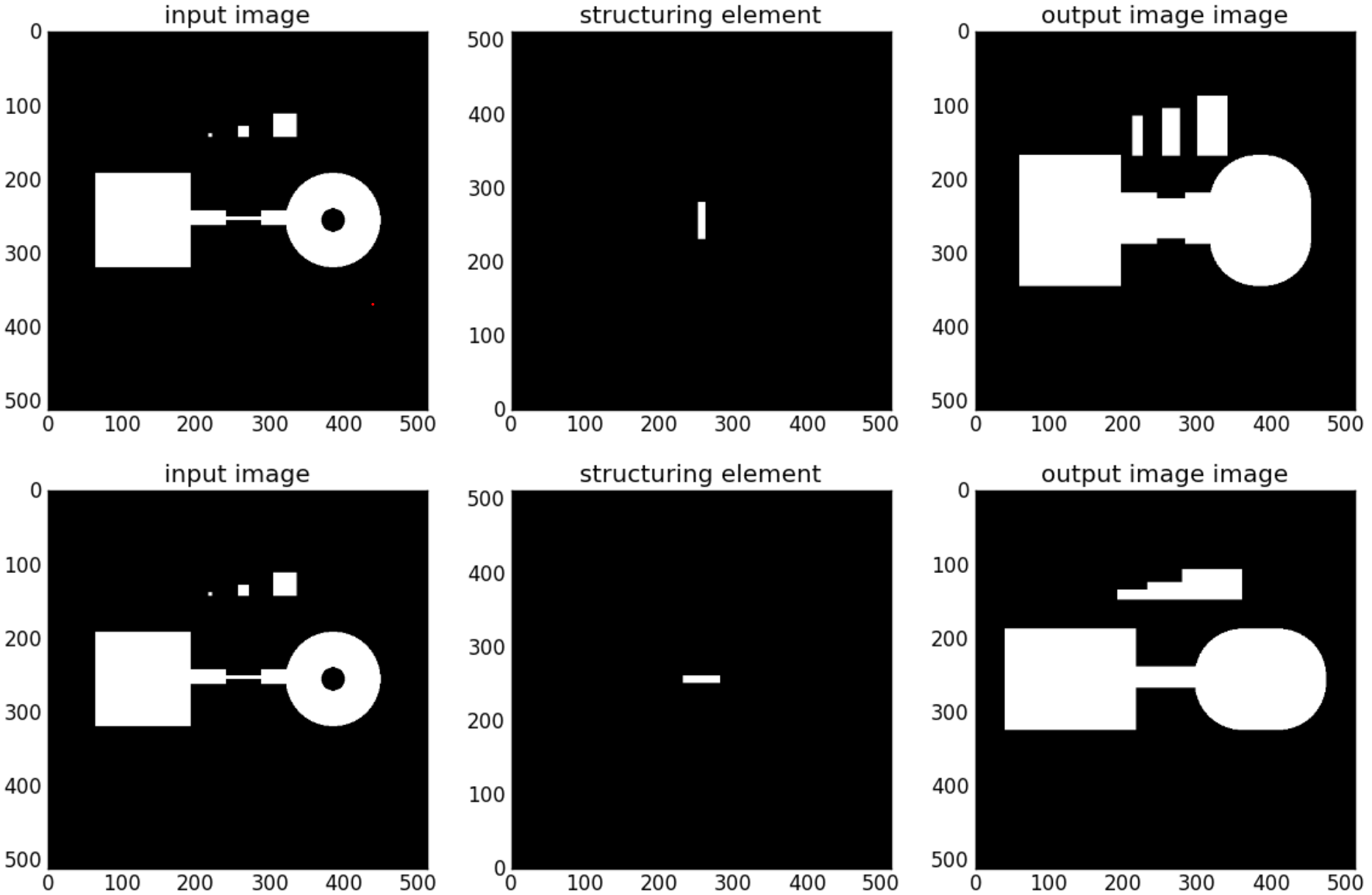
Morphological operations

- dilation



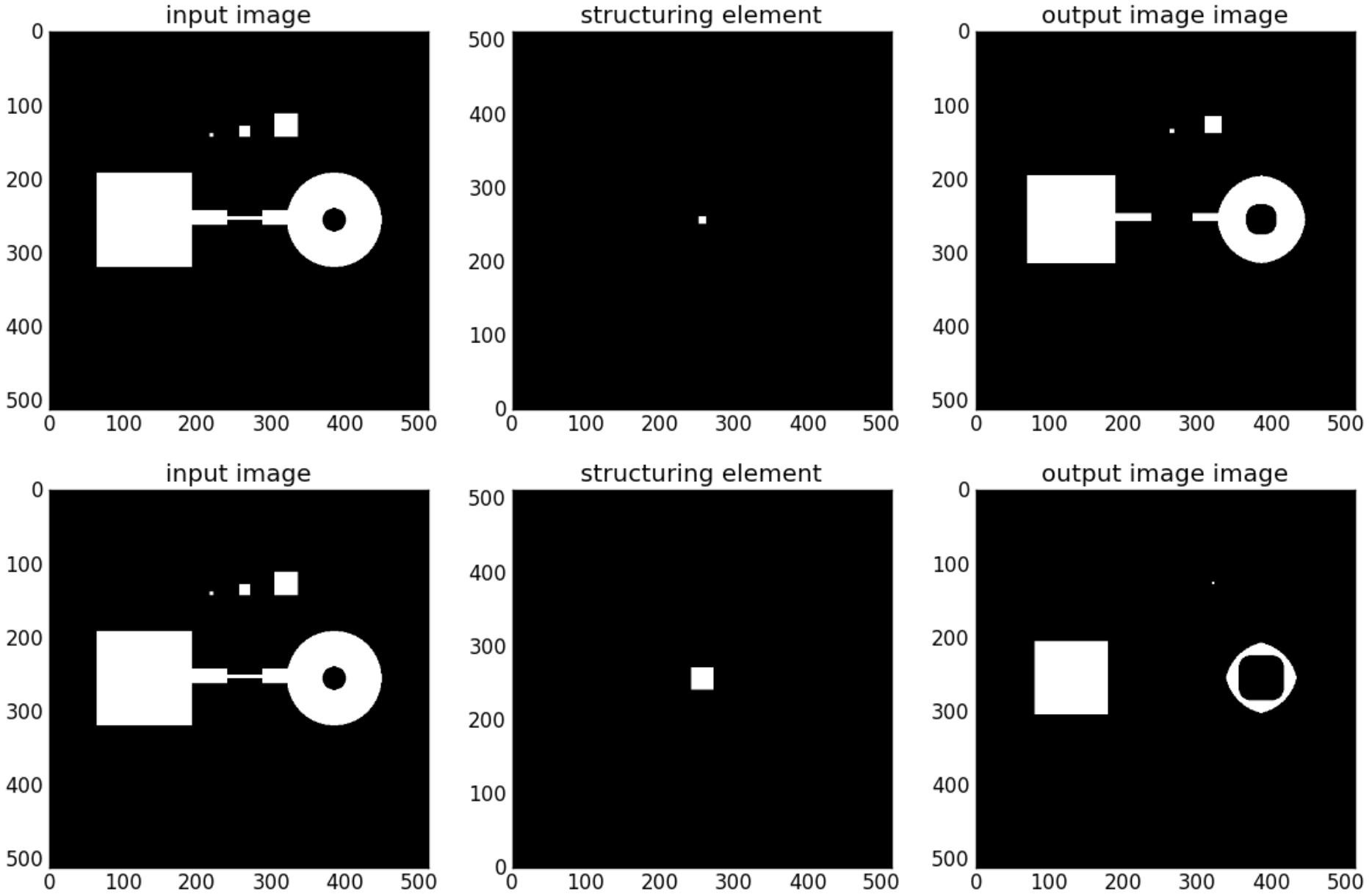
Morphological operations

- dilation



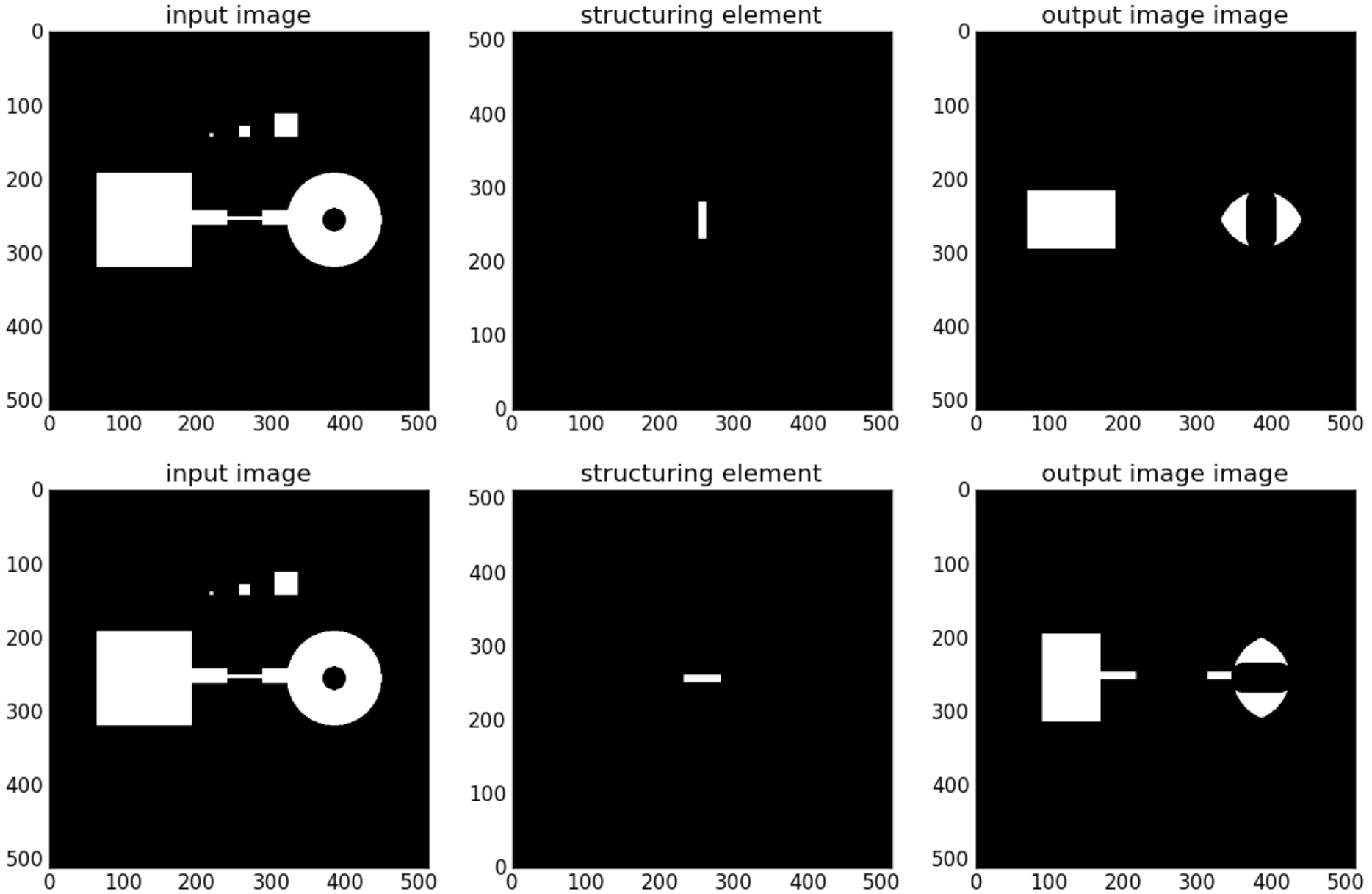
Morphological operations

- erosion



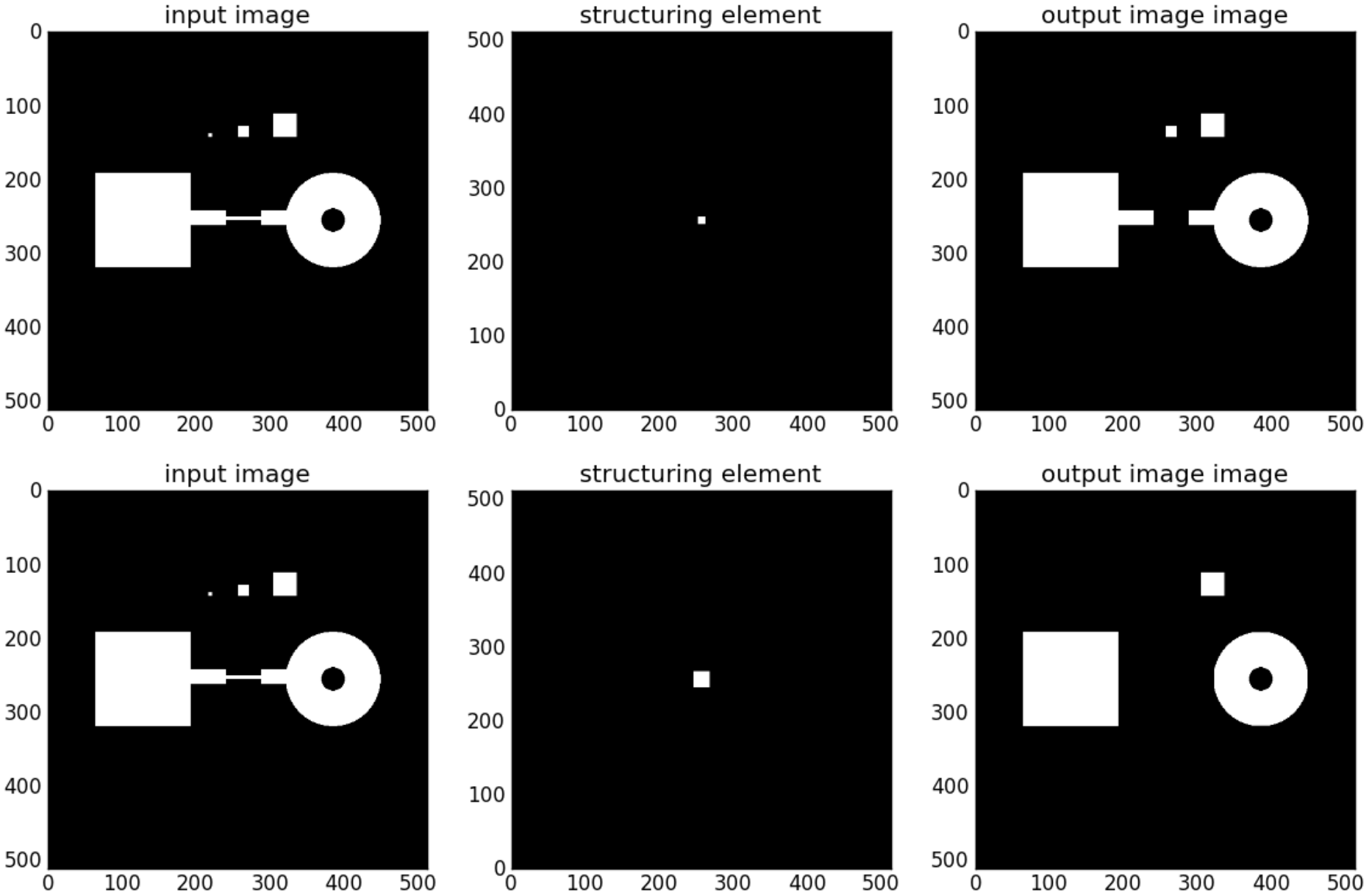
Morphological operations

- erosion



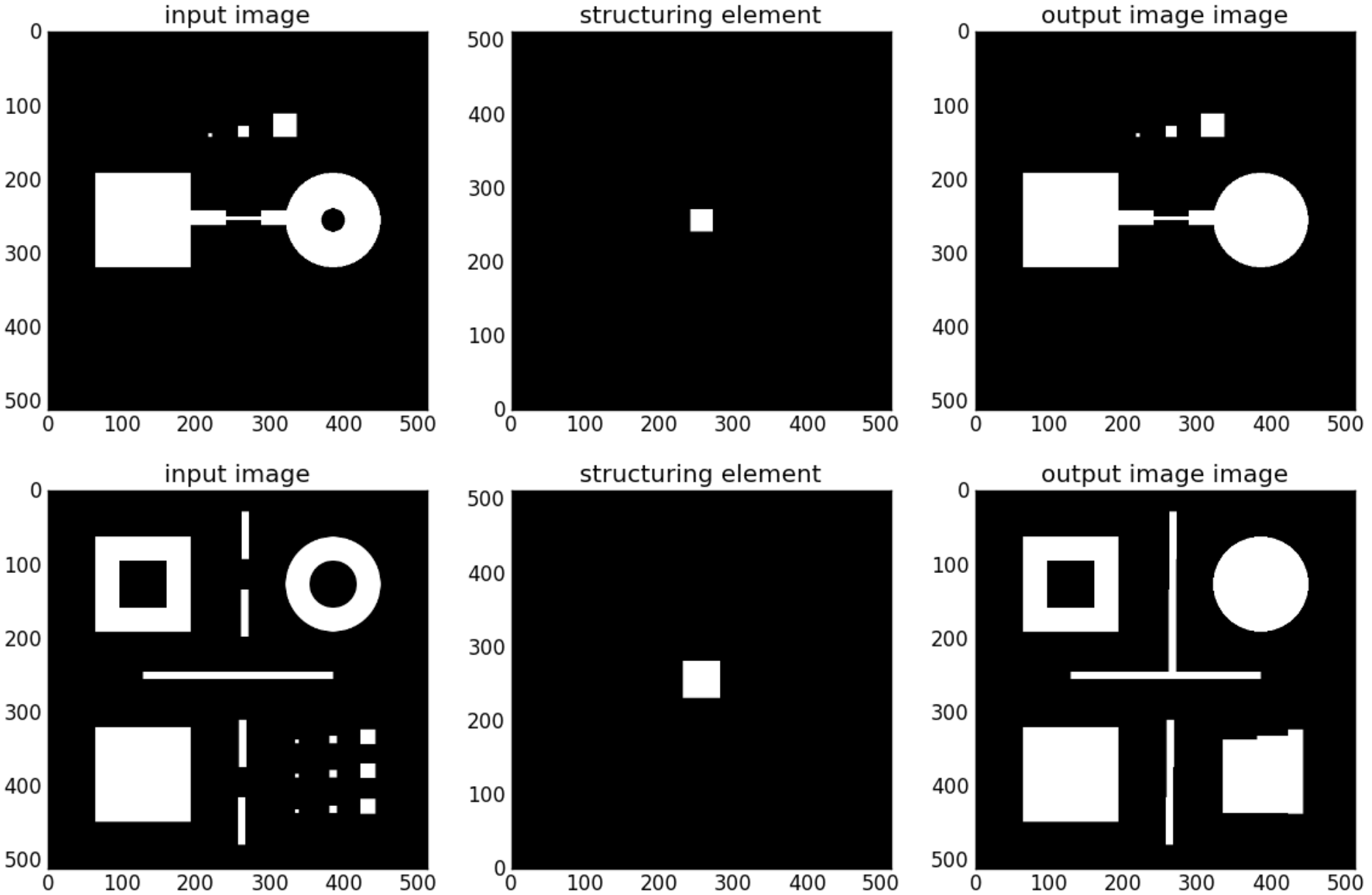
Morphological operations

- opening: first erosion, then dilation



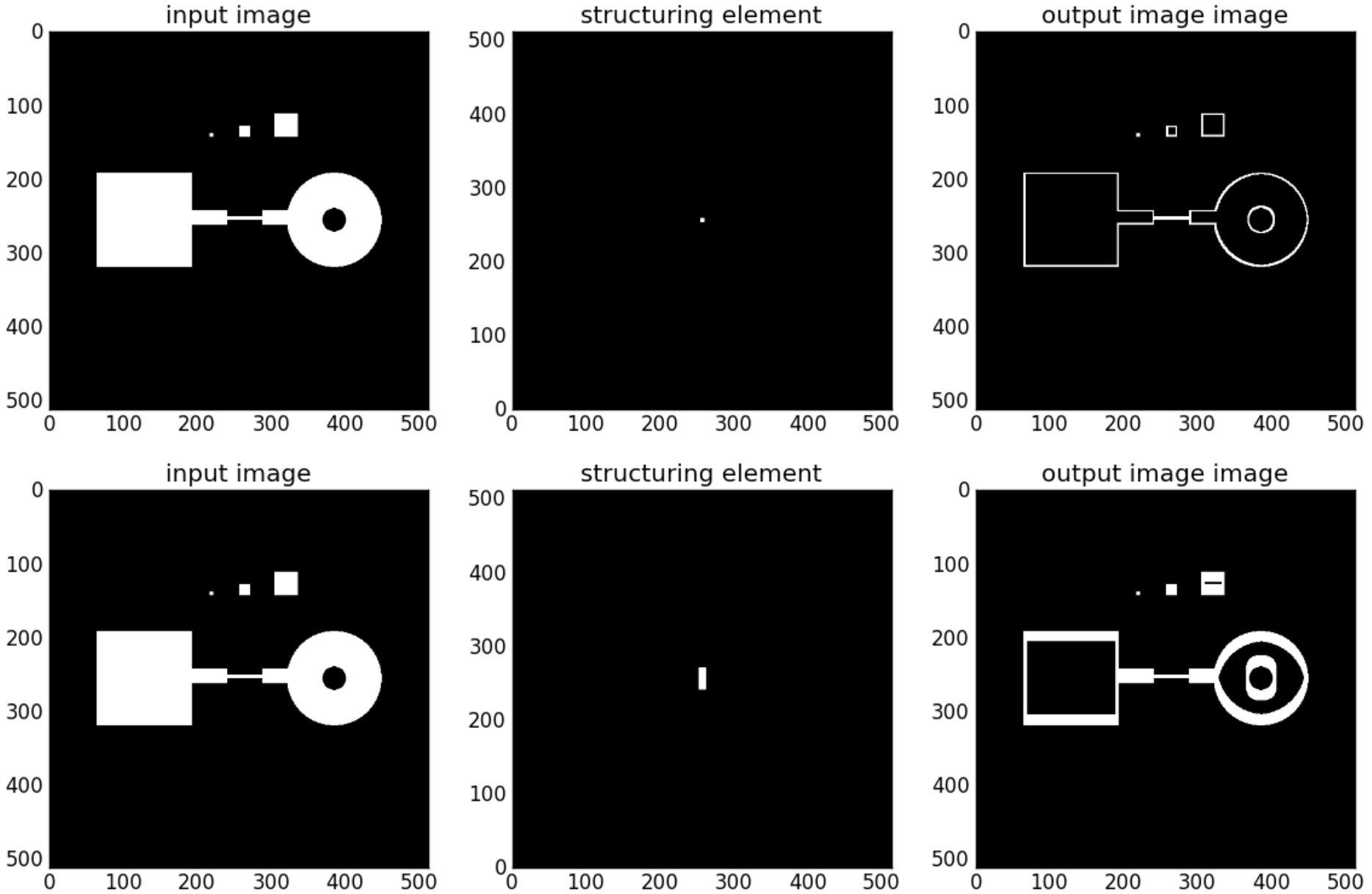
Morphological operations

- closing: first dilation, then erosion



Morphological operations

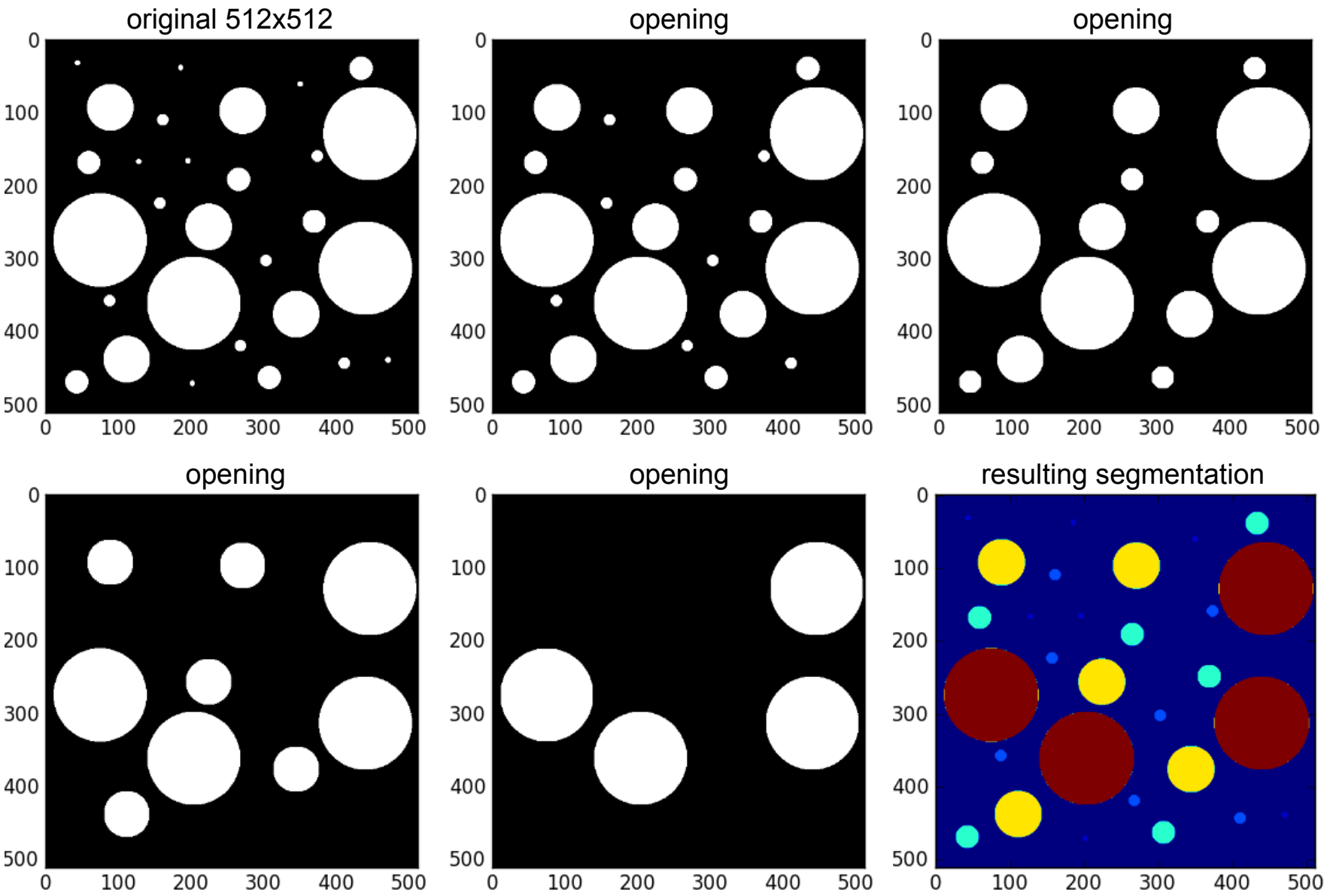
- boundary: original - erosion



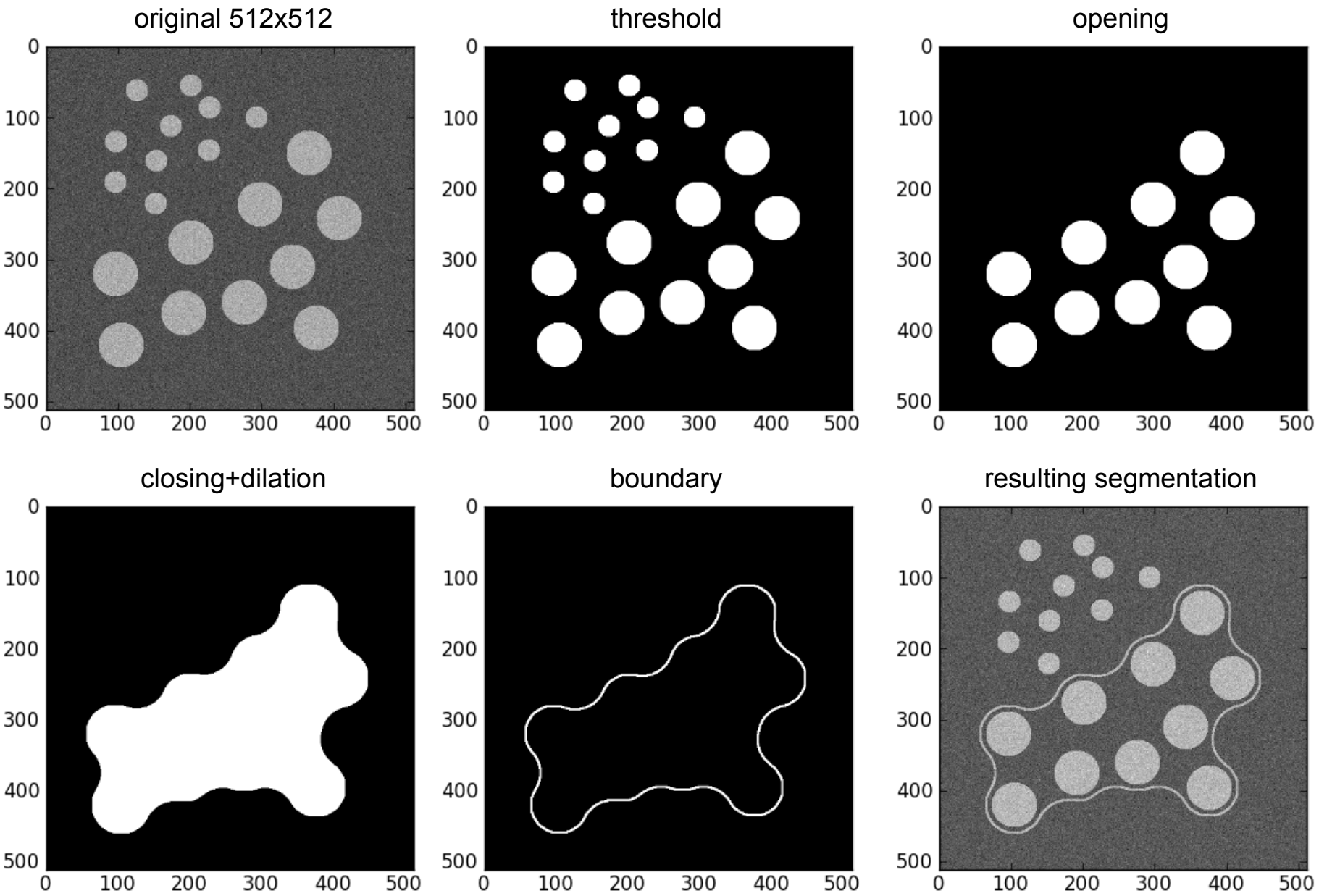
Segmentation: Motivation

- Partitioning of image by regions-of-interest
- various methods available
  - by morphology
  - by intensity
  - by region
  - by boundary
  - ...

Segmentation by morphology



Segmentation by morphology

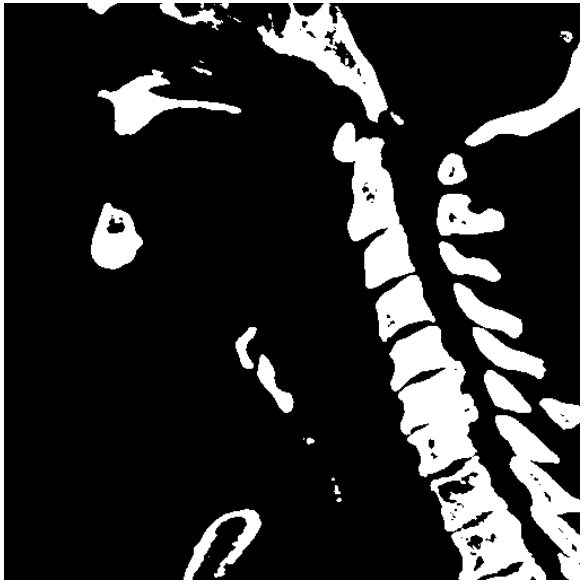


Segmentation by intensity

- easy
- widely used

original

high window



mid window

low window

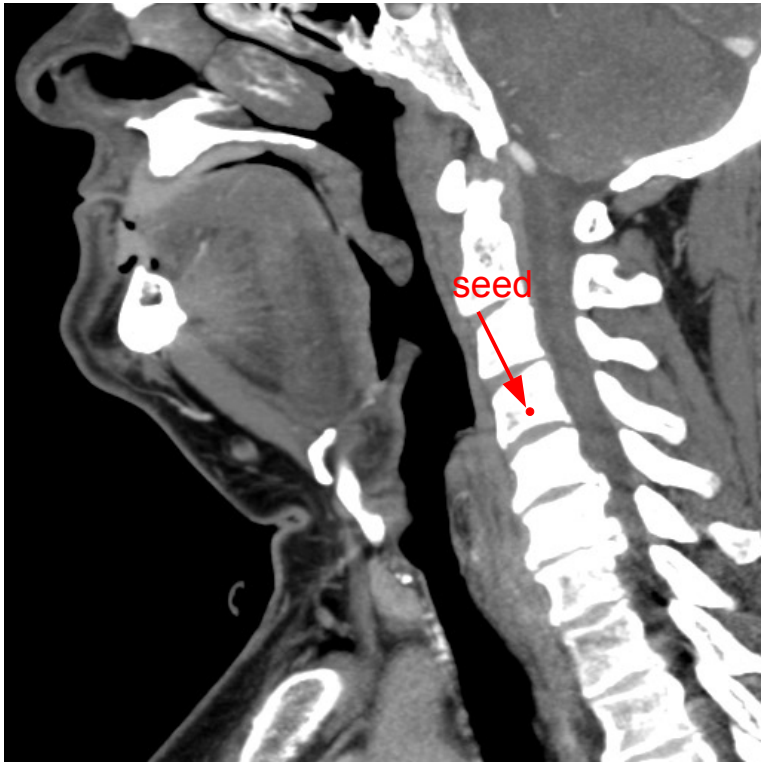
segmented

- noise prone
- no connectivity

Segmentation by region growth

- start with seed
- check intensity in neighborhood
- if intensity within window, set to 1
- iterate until no change

original



Segmentation by boundaries

- look for sharp changes in intensity
- more ~~next week~~... on Wednesday

original

laplace