

# **Corso di Astrofisica Stellare**

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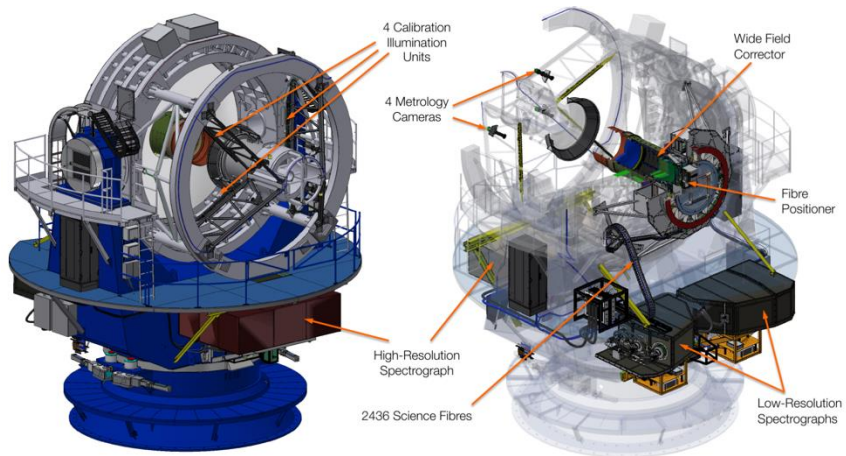
# Lezioni

- Lunedì 11 -13
- Giovedì 13:30-15

# Esami

3 quesiti che spaziano sul corso (alcuni conti bisogna saperli fare)  
contattarmi a [gabriele.cescutti@inaf.it](mailto:gabriele.cescutti@inaf.it)

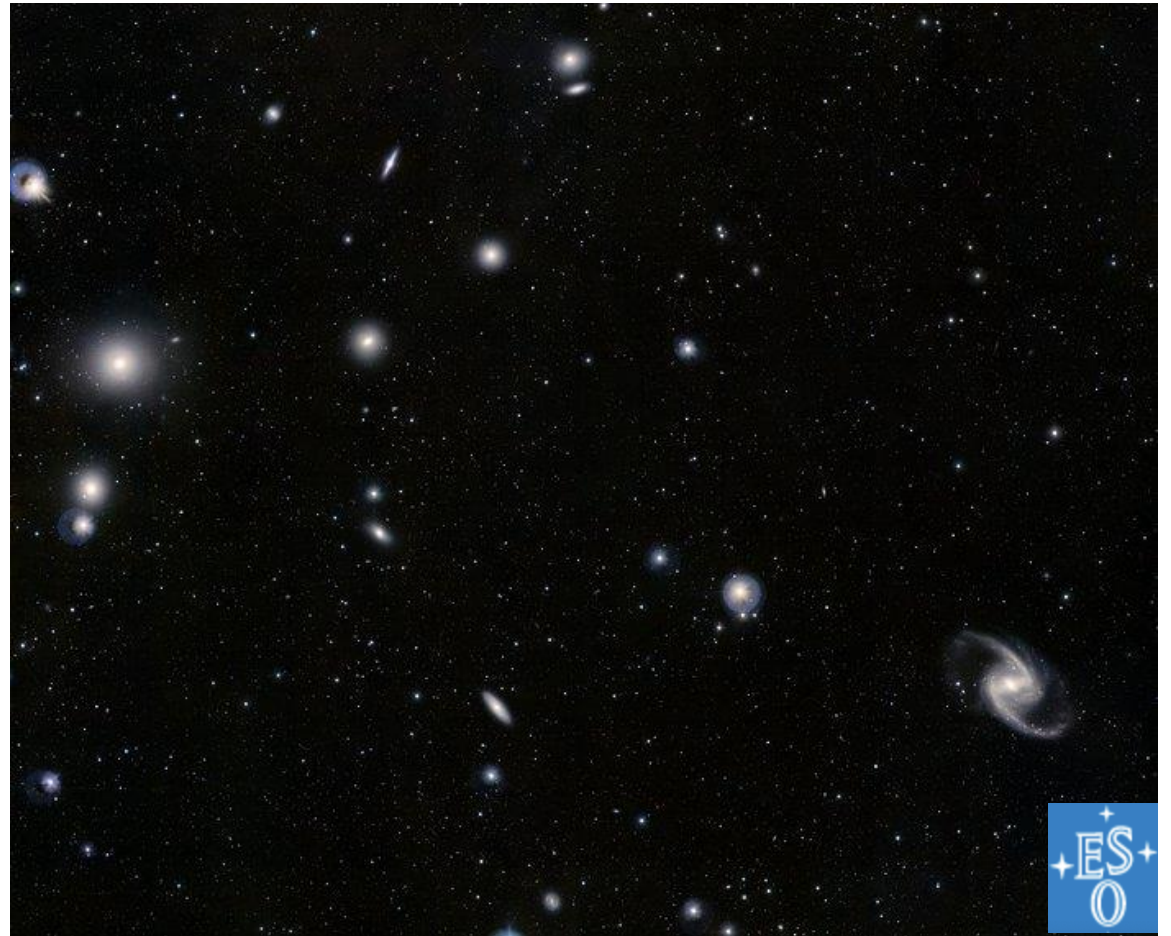
- Perché Astrofisica stellare è importante? (fondamentale!)



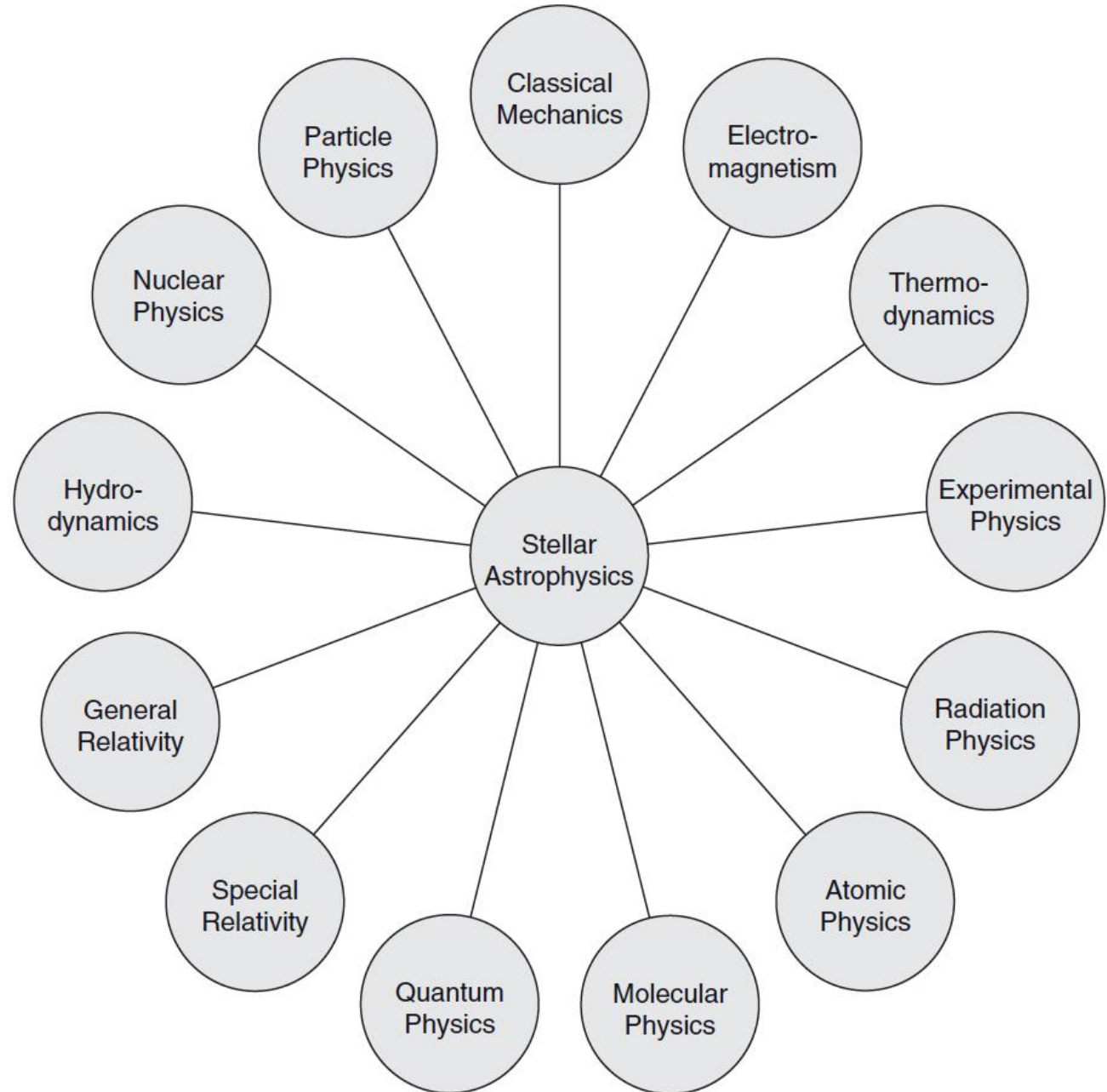
Galassie



Ammasso di Fornax



# EASY?



*An Introduction to  
Stellar Astrophysics*

*FRANCIS LEBLANC  
Université de Moncton, Canada*

# Programma ASTROFISICA STELLARE

Introduzione del corso,  
Il Sole come stella tipica proprietà delle stelle

Equazioni della struttura stellare

Risoluzione equazioni della struttura stellare

Coefficiente produzione energia nucleare,  
Reazioni nucleari di fusione

Elementi leggeri e nucleosintesi primordiale

Fasi di pre-sequenza stellare

Evoluzione di stelle di massa piccola ed intermedia

Evoluzione di stelle massicce

Supernovae da collasso del nucleo

Sistemi binari. Variabili cataclismatiche-Supernovae Ia

Mass loss, rotazione stellare e stelle pulsanti

Asterosismologia

Cenni di Atmosfere Stellari

Cenni di Struttura della Galassia

Evoluzione Chimica con soluzioni analitiche

Modelli di evoluzione chimica numerici

**Approfondimento su modelli di stellar evolution con rotazione**

**Approfondimento con modelli di evoluzione stellare 3D**

# Fundamental physical constants required in this course

$a$	radiation density constant	$7.55 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$
$c$	velocity of light	$3.00 \times 10^8 \text{ m s}^{-1}$
$G$	gravitational constant	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
$h$	Planck's constant	$6.62 \times 10^{-34} \text{ J s}$
$k$	Boltzmann's constant	$1.38 \times 10^{-23} \text{ J K}^{-1}$
$m_e$	mass of electron	$9.11 \times 10^{-31} \text{ kg}$
$m_H$	mass of hydrogen atom	$1.67 \times 10^{-27} \text{ kg}$
$N_A$	Avogadro's number	$6.02 \times 10^{23} \text{ mol}^{-1}$
$\sigma$	Stefan Boltzmann constant	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ ( $\sigma = ac/4$ )
$R$	gas constant ( $k/m_H$ )	$8.26 \times 10^3 \text{ J K}^{-1} \text{ kg}^{-1}$
$e$	charge of electron	$1.60 \times 10^{-19} \text{ C}$
$L_{\odot}$	luminosity of Sun	$3.86 \times 10^{26} \text{ W}$
$M_{\odot}$	mass of Sun	$1.99 \times 10^{30} \text{ kg}$
$T_{\text{eff}\odot}$	effective temperature of sun	5780 K
$R_{\odot}$	radius of Sun	$6.96 \times 10^8 \text{ m}$
Parsec ( <i>unit of distance</i> )		$3.09 \times 10^{16} \text{ m}$

# Lecture 1: The observed properties of stars

Learning outcomes: Students will

- Understand what parameters of stars we can measure
- Appreciate the use of stars and star clusters as laboratories for stellar astrophysics
- Begin to understand how we will constrain stellar models with hard observational evidence





# Observable properties of stars

Basic parameters to compare theory and observations:

- Mass ( $M$ )
- Luminosity ( $L$ )
  - The total energy radiated per second i.e. power (in W)

$$L = \int_0^{\infty} L_{\lambda} d\lambda$$

- Radius ( $R$ )
- Effective temperature ( $T_e$ )
  - The temperature of a black body of the same radius as the star that would radiate the same amount of energy. Thus

$$L = 4\pi R^2 \sigma T_e^4$$

where  $\sigma$  is the Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ )

**3 independent quantities**

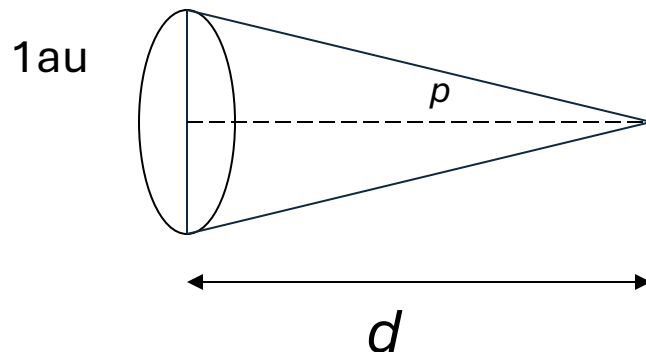
# Recap Level 2/3 - definitions

Measured energy flux depends on distance to star  
(inverse square law)

$$F = L / 4\pi d$$

Hence if  $d$  is known then  $L$  determined

Can determine distance if we measure parallax - apparent stellar motion to orbit of earth around Sun.



For small angles

$$p = 1 \text{ au} / d$$

$$d = 1/p \text{ parsecs}$$

If  $p$  is measured in arcsecs

SMALL DETOUR

# The cosmic distance ladder

How far from the Earth to the Moon?

How far from the Earth to the Sun?

How far from the Sun to nearby stars?

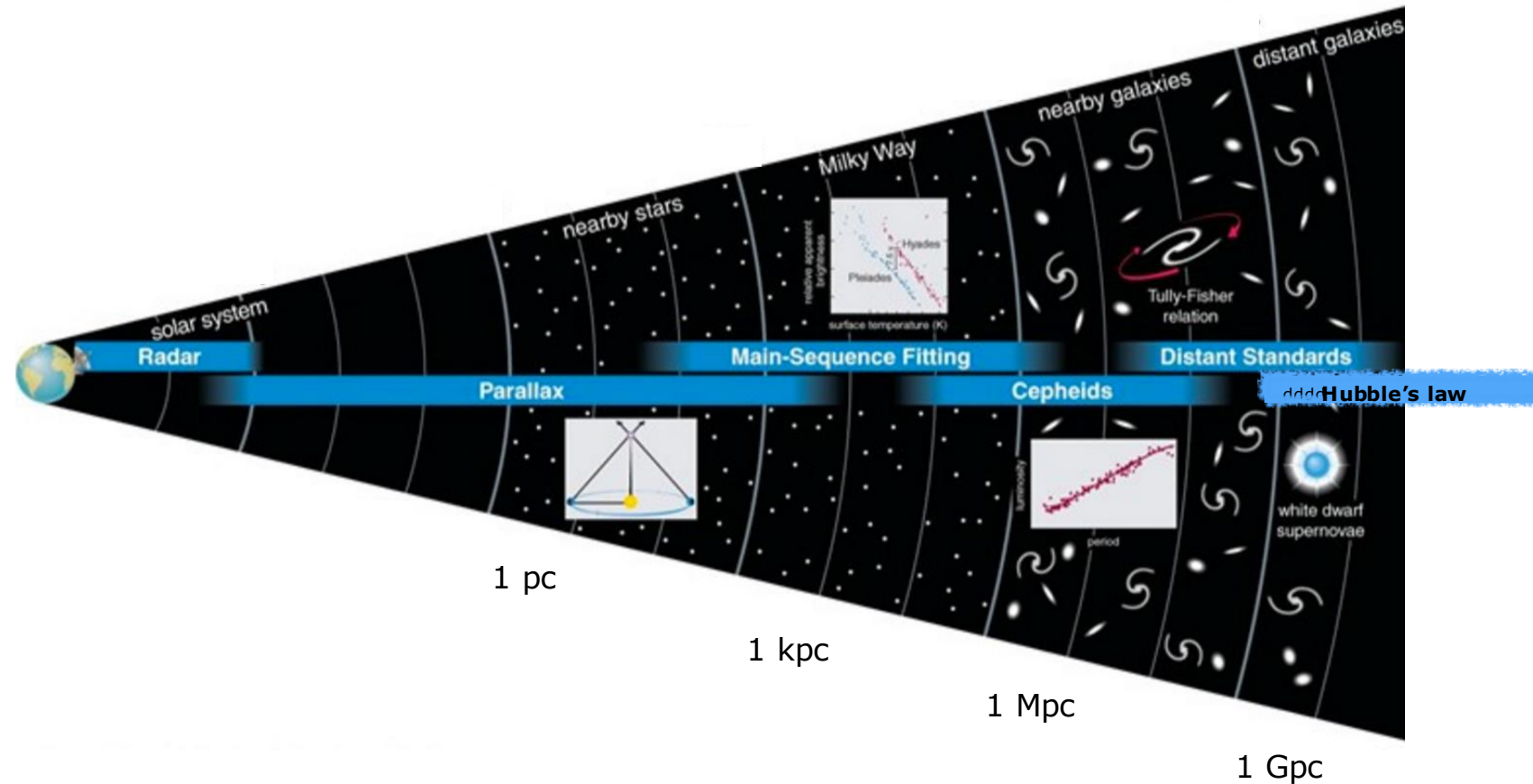
How far from the Sun to distant stars?

How far from the Sun to nearby galaxies?

How far from the Sun to distant galaxies?

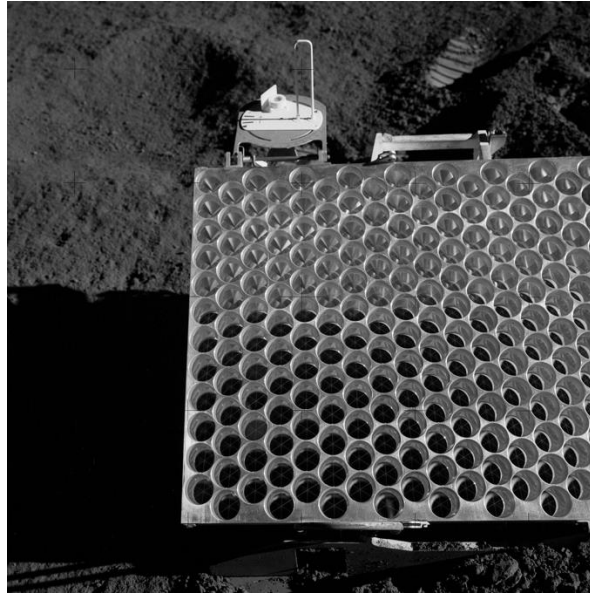
Not a single method,  
different methods should be applied!

In astronomy the distances are measured using a chain of interdependent techniques

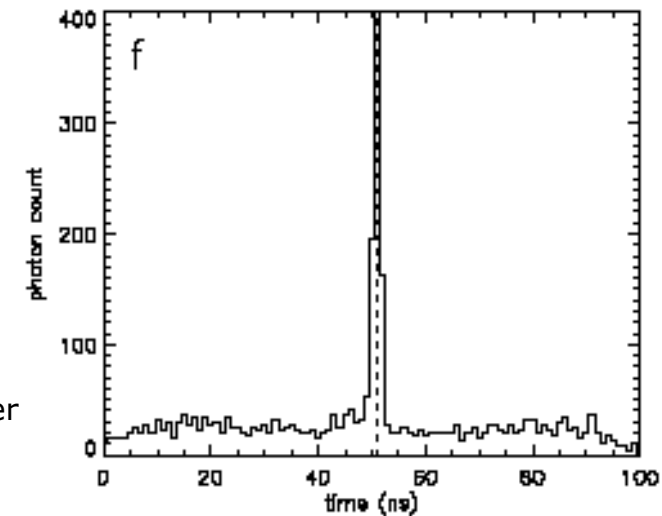


# Direct method: **Laser pulse**

Using telescopes on Earth, the reflectors on the Moon, and accurate timing of laser pulses scientists can measure the distance Earth - Moon to an accuracy of a few centimetres.



Apollo 15 Lunar Ranging Retro-Reflector .  
The small circles are corner cubes, which reflect light directly back in the direction from which it came.

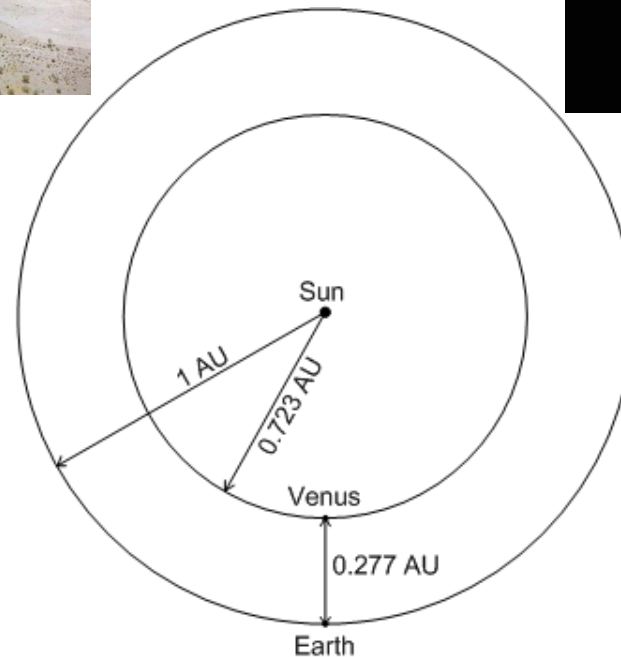
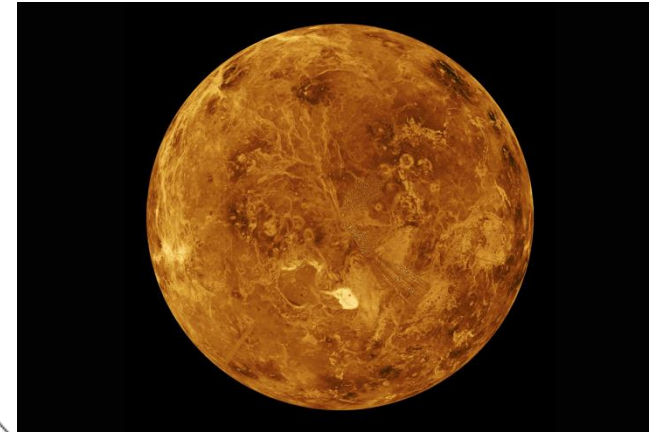


gate open after  
 $\sim 2.52$ s



# Direct method: **Radar distance**

Since 1961, the distance to Venus can be determined directly, by radar measurements, where a series of radio waves is transmitted from Earth and is received after it bounces off Venus and comes back to Earth. By measuring the time taken for the radar echo to come back, the distance can be calculated. Once this Earth-Venus distance is known, the distance between Earth and the Sun can be calculated.



Q The time for a radar signal to make the round trip between Earth and Venus is measured to be 276.2 seconds. Determine the astronomical unit.

# Direct method: **Stellar parallax**

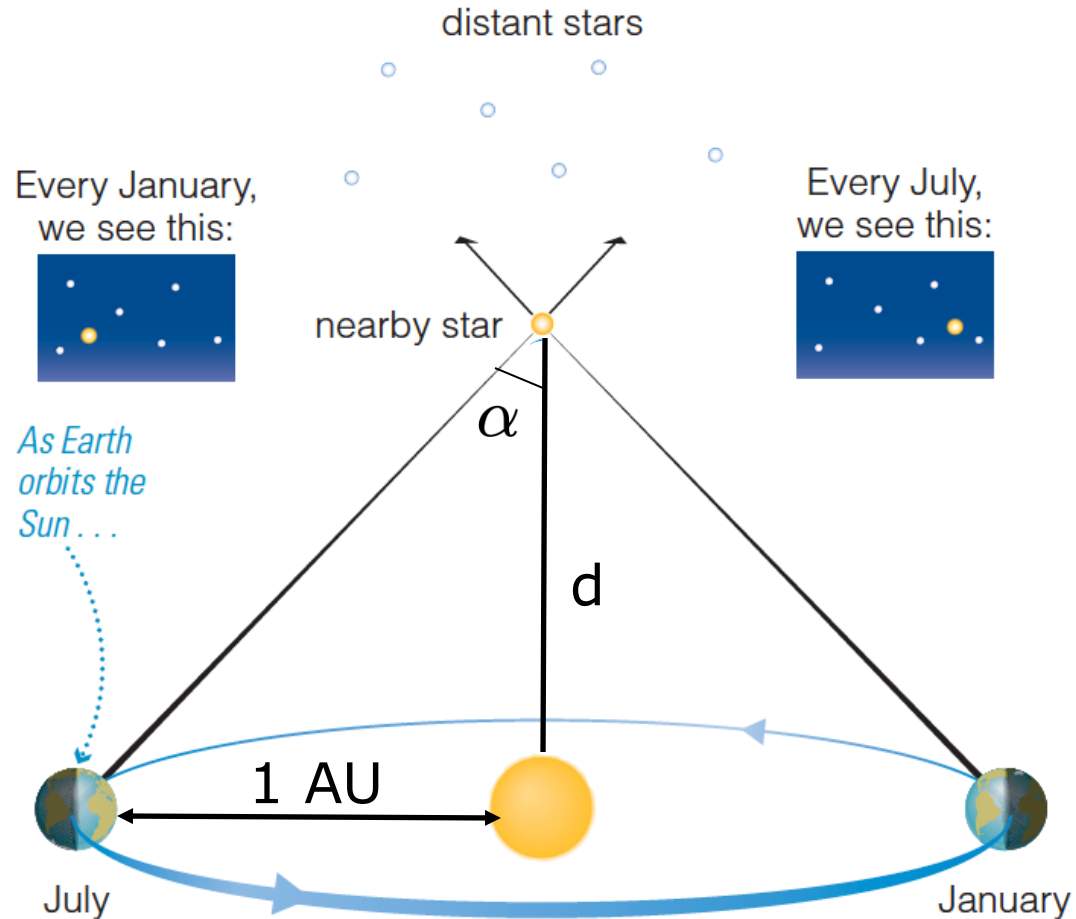
$$\tan(\alpha) = \frac{AU}{d}$$

$$\tan(\alpha) \sim \alpha \quad \text{if } \alpha \ll 1''$$

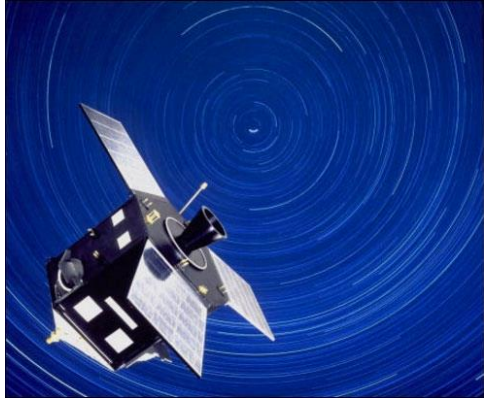
$$d(AU) = \frac{1}{\alpha(rad)}$$

Parsec (pc) is the distance at which one AU subtends an angle of one arc second. Therefore:

$$d(pc) = \frac{1}{\alpha(arcsec)}$$



Direct method: **Stellar parallax**



Hipparcos satellite 1989-1993  
120'000 stars with  
a precision of 2 - 4 mas

Q How far are the most distant stars measured by Hipparcos?

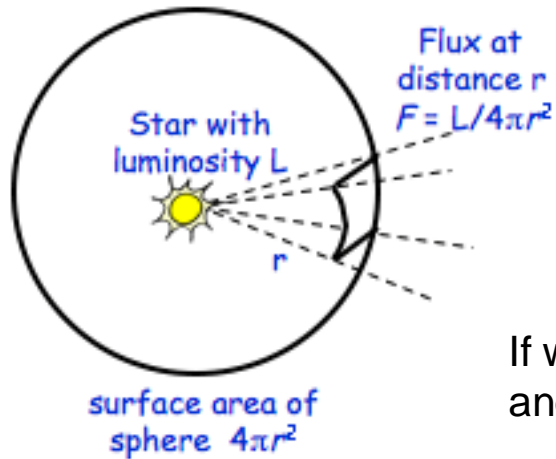


Gaia satellite 2013-2018  
**1 billion of stars(!)** with  
a precision up to 1  $\mu as$

Q Why always satellites?

## Indirect methods

# Standard candles



$$L = \text{const}$$

$$F(r) \sim 1/r^2$$

$F$  = observed flux

$d$  = distance

If we know the Luminosity of an object and we measure the flux from an unknown distance  $d$  then:

$$L = 4\pi F(d)d^2$$

therefore the unknown distance is:

$$d = \sqrt{\frac{L}{4\pi F}}$$

In astronomy it is common to use the apparent magnitude  $m$ :

$$m - m_0 = -2.5 \log_{10} \left( \frac{F}{F_0} \right)$$

where  $F_0$  and  $m_0$  are the flux and apparent magnitude for a reference object.

If we define the absolute magnitude  $M$  as the apparent magnitude of an object at 10 pc then:

$$m - M = 5 \log_{10}(d) - 5 \quad (\text{Q calculate this equation from the above definitions})$$

Q So what is the distance of Vega, if we know that it has an apparent magnitude of +0.03 and an absolute magnitude of 0.6?

In general the problem in astronomy is that we do not know “a priori” the absolute magnitude (or luminosity) of an astronomical object.

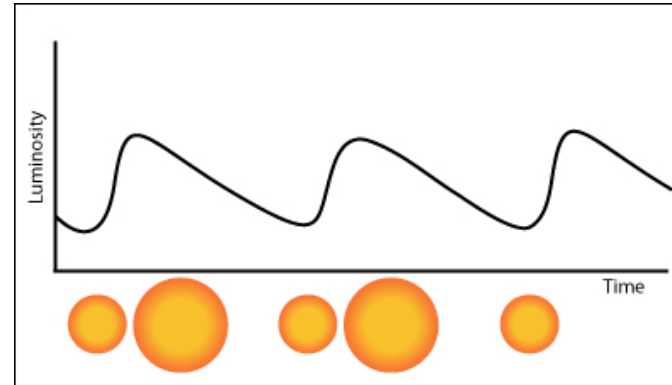
However, there are few astronomical objects that are reasonably close to have always the same Luminosity or a Luminosity that we can predict. They are called **standard candles**,

# Indirect methods

## Standard candles

# Cepheids

Cepheids are bright variable stars



Henrietta Swan Leavitt in 1912 discovered an important relation between the period and the apparent magnitude of the cepheids in the Large Magellanic cloud.

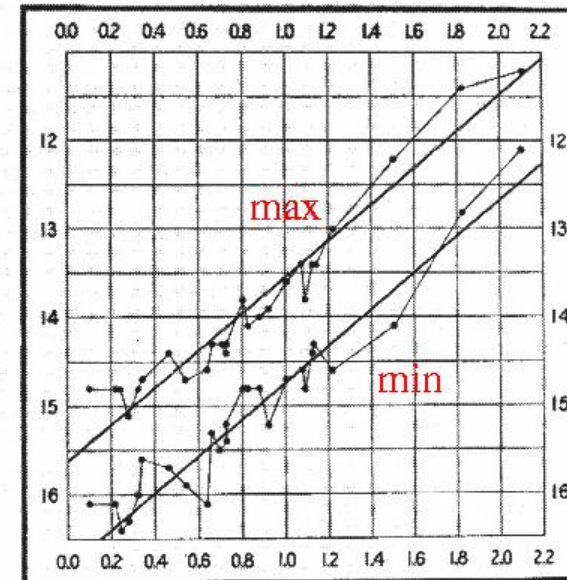


FIG. 2.

# Indirect methods

# Cepheids

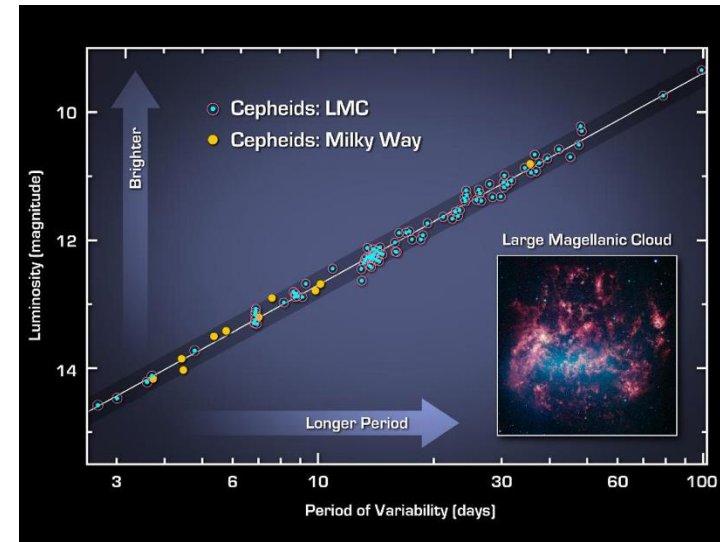
## Standard candles

The key point was that all these objects belong to the Large Magellanic cloud and therefore at (roughly) the same distance. Assuming a distance of 50 kpc for the Large Magellanic clouds, she obtained this relation:

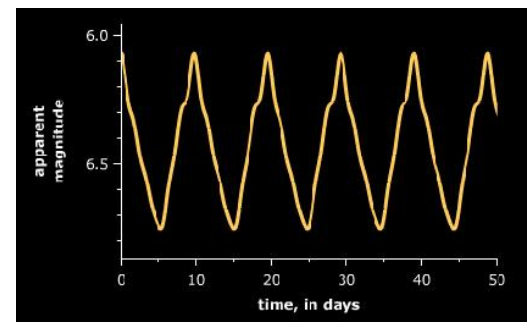
$$M = -2.87 \log_{10} P - 1.40$$

The cepheids are bright young objects and it is possible to measure them up to 10Mpc!

Thanks to this method it was possible for the first time to move from our Galaxy and explore the nearby Universe.



Q what is the distance of this cepheid if we know that it apparent magnitude is  $m=15$ ?



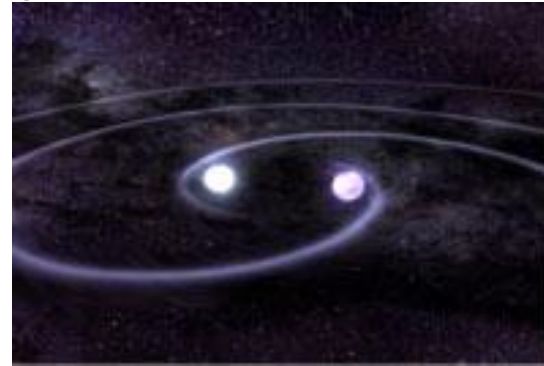
Indirect methods

Standard candles

# Supernovae Ia

When a white dwarf exceeds the 1.4 Msun (Chandrasekhar mass), it becomes unstable and explodes. Two scenarios:

Mass transfer from a binary companion or Merging with another white dwarf



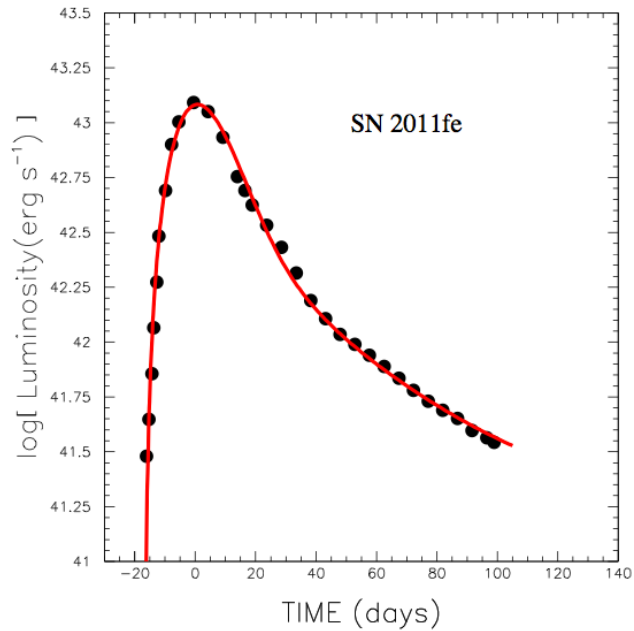
The key point is that the mechanism is always similar and it can produce in both case the same event: a Supernova Ia



Crab Nebula  
the remnant of a SNIa  
explosion in our Galaxies  
500y ago



## Indirect methods Standard candles



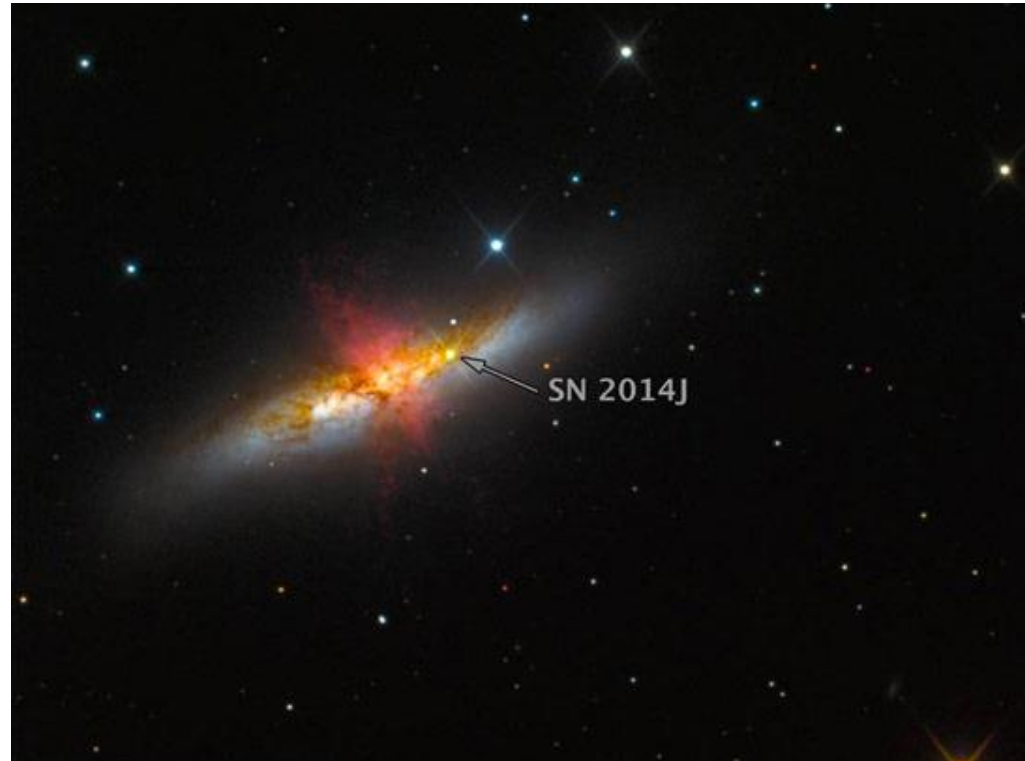
Contrary to Cepheids which are quite numerous, the SNIa are rare events.

The closest type Ia supernova recently discovered SN 2014J in the nearby galaxy M82.

# Supernovae Ia

SNe Ia are generated always by the same event and therefore the peak of the luminosity for these objects is constant as well as the shape of their luminosity vs time.

Therefore, we can recognise them and because of their brightness we are able to measure distance up to a few Gpc away.



Since nearest stars  $d > 1\text{pc}$  ; must measure  $p < 1\text{ arcsec}$   
e.g. and at  $d=100\text{ pc}$ ,  $p= 0.01\text{ arcsec}$

Telescopes on ground have resolution  $\sim 1''$  Hubble has  
resolution  $0.05''$  difficult !

Hipparcos satellite measured  $10^5$  bright stars with  
 $\delta p \sim 0.001'' \Rightarrow$  confident distances for stars with  $d < 100\text{ pc}$

Hence  $\sim 100'000$  stars with well measured parallax  
distances

now with GAIA  $\Rightarrow$  1billion!!!

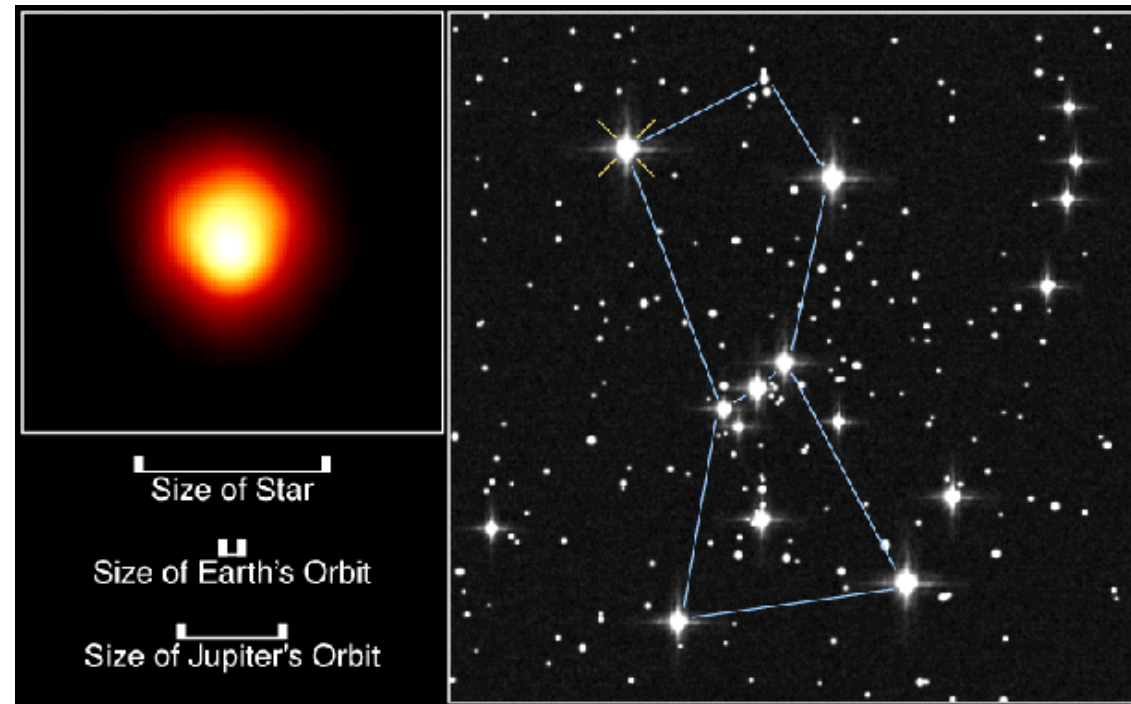
# Stellar radii

Angular diameter of sun at distance of 10pc:

$$\theta = 2R_{\odot}/10\text{pc} = 5 \cdot 10^{-9} \text{ radians} = 10^{-3} \text{ arcsec}$$

Compare with Hubble  
resol. of  $\sim 0.05$  arcsec

→ very difficult to  
measure  $R$  directly



Radii of  $\sim 600$  stars measured with techniques such as interferometry and eclipsing binaries.

# Observable properties of stars

Basic parameters to compare theory and observations:

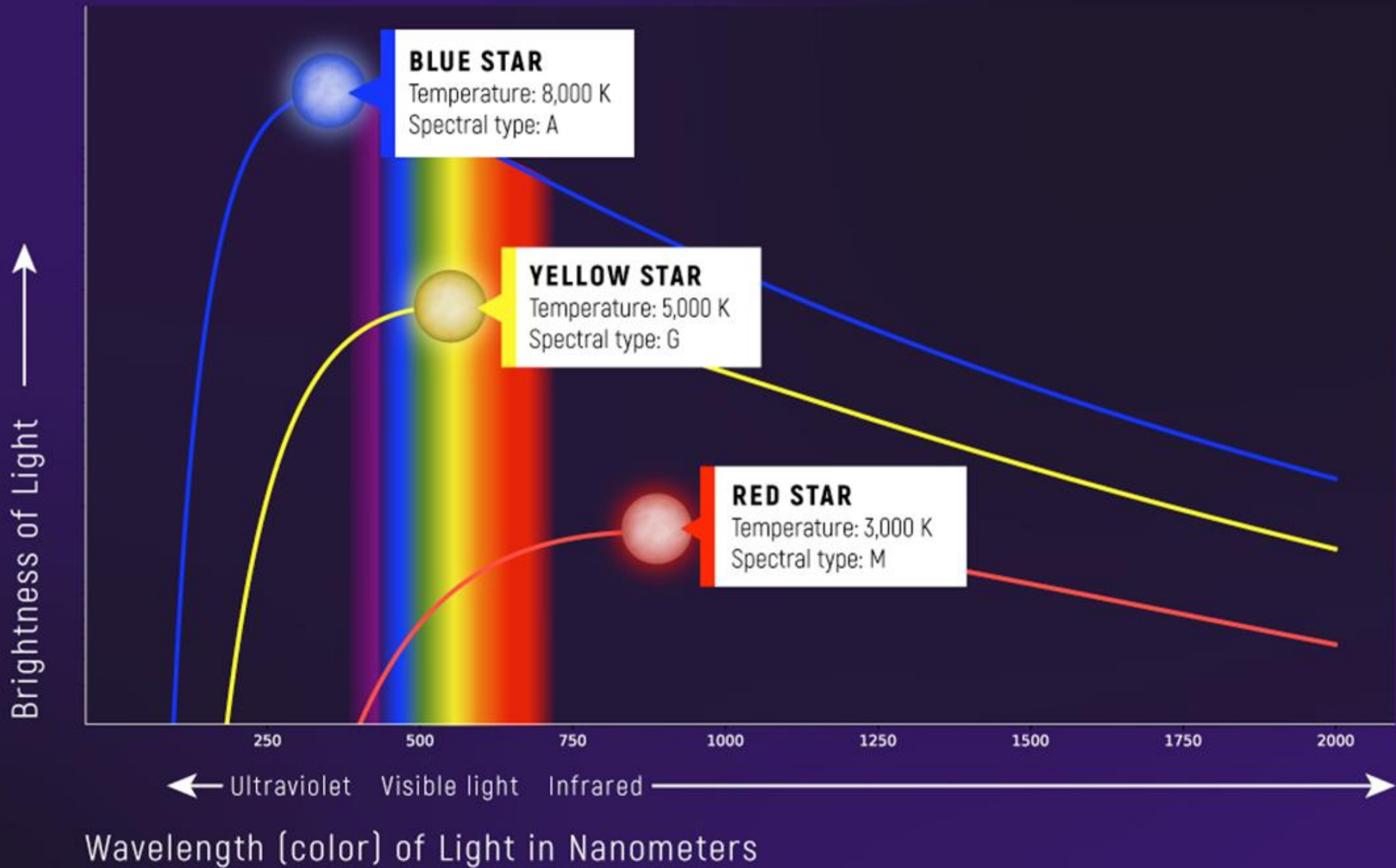
- Mass ( $M$ )
- Luminosity ( $L$ )
  - The total energy radiated per second i.e. power (in W)
- Radius ( $R$ )
- Effective temperature ( $T_e$ )
  - The temperature of a black body of the same radius as the star that would radiate the same amount of energy. Thus

$$L = \int_0^{\infty} L_{\lambda} d\lambda$$

$$L = 4\pi R^2 \sigma T_e^4$$

where  $\sigma$  is the Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ )

**3 independent quantities**

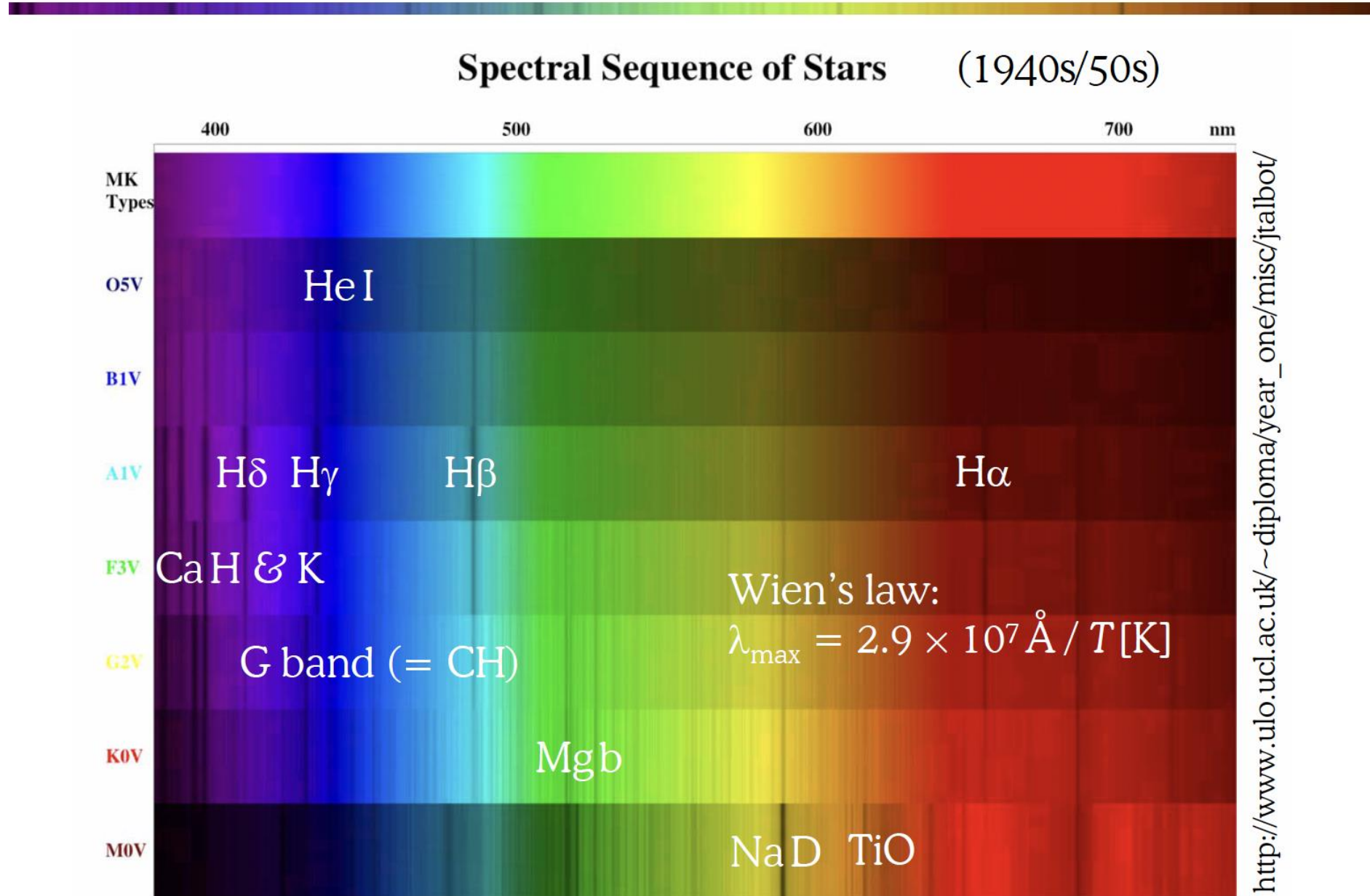


# Proprietà delle stelle

Vengono classificate sulla base delle righe spettrali che predominano nel loro spettro. La variabile fisica che determina il tipo spettrale è la **TEMPERATURA SUPERFICIALE** (più che la composizione chimica).

**O B A F G K M R N S**

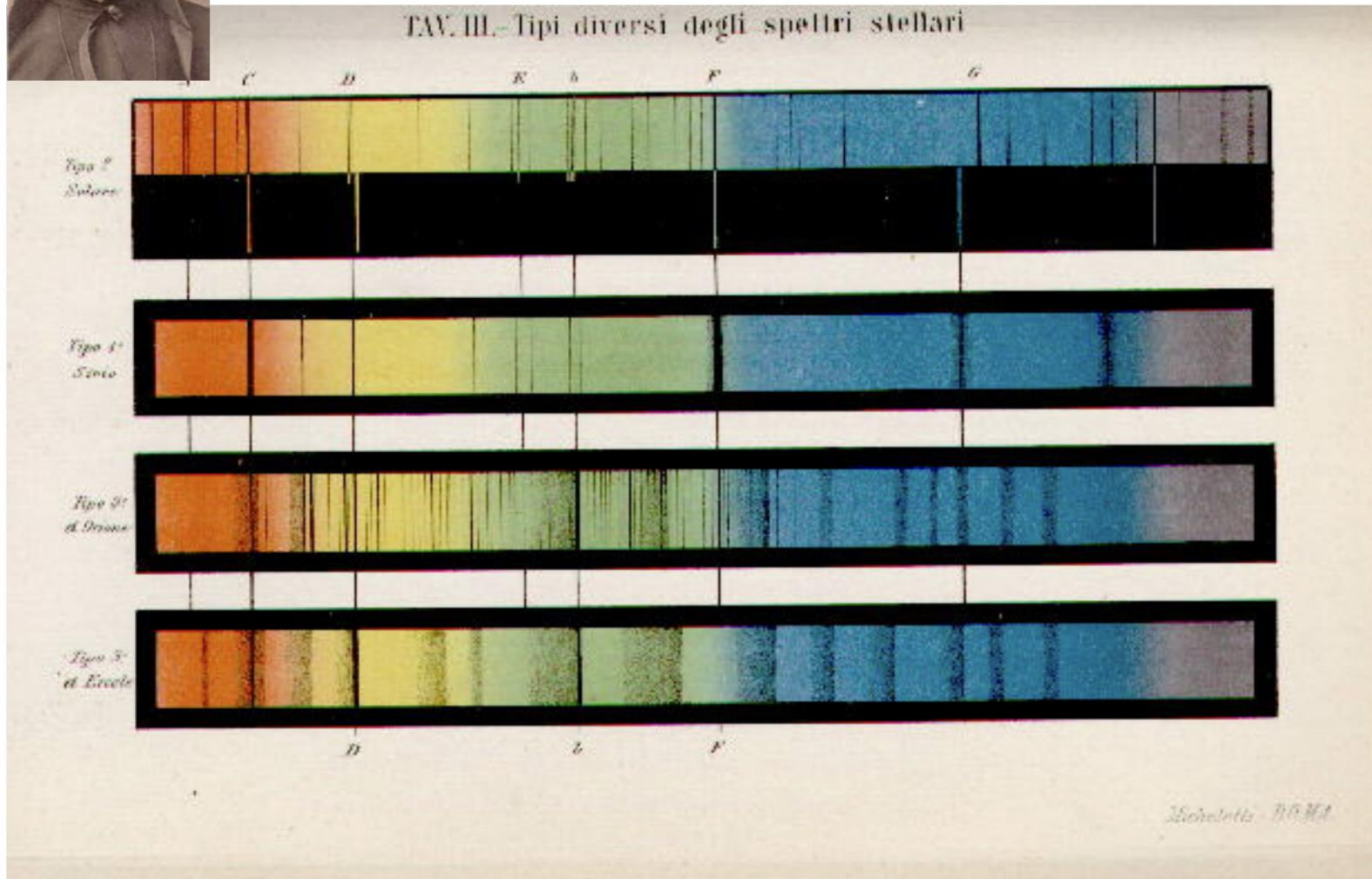
# Morgan-Keenan(-Kellman) classification





# Padre Angelo Secchi

(1864) four-category scheme





Ulteriore suddivisione all'interno del tipo  
spettrale va da **0 a 9**

Il sole per esempio è una stella di G2 di sequenza  
principale.

E' possibile distinguere stelle in diverse fasi evolutive dai  
loro spettri (o colori)?

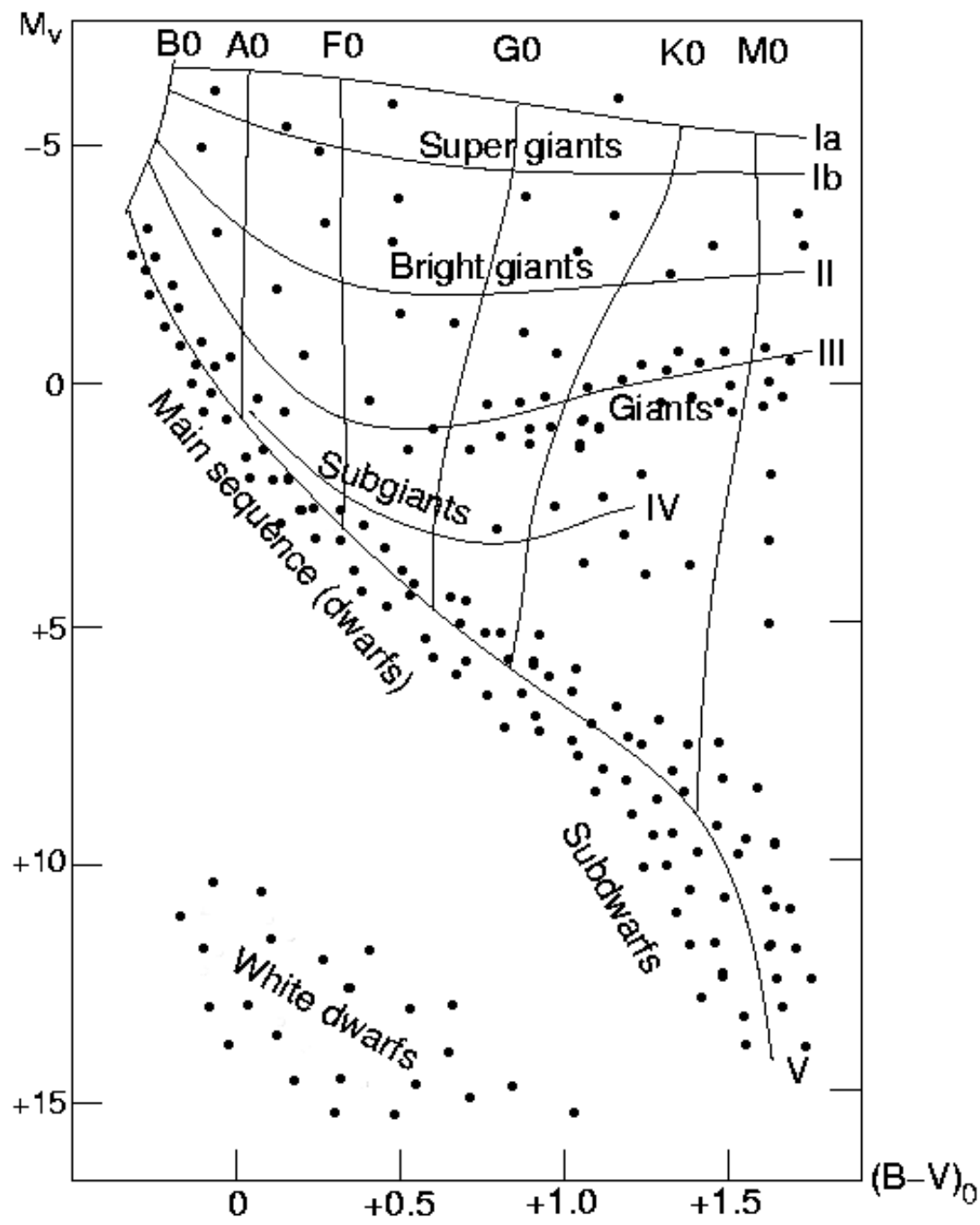
Le stelle irradiano come un corpo nero, quindi se due stelle  
hanno la stessa **temperatura superficiale ( $T_{\text{eff}}$ )**  
la più luminosa dev'essere quella di raggio maggiore

Stelle con raggi grandi vengono dette giganti, hanno densità minori delle stelle normali (dette nane).

Hanno anche un diverso grado di ionizzazione (influenzato dalla diversa gravità e densità superficiale) delle nane

Le righe spettrali sono diverse (vedremo perché) e quindi è possibile distinguerle anche se non sappiamo la luminosità assoluta di una stella.

In base a questo definiamo ulteriormente classi di Luminosità da **I (supergiganti) a V (nane)**



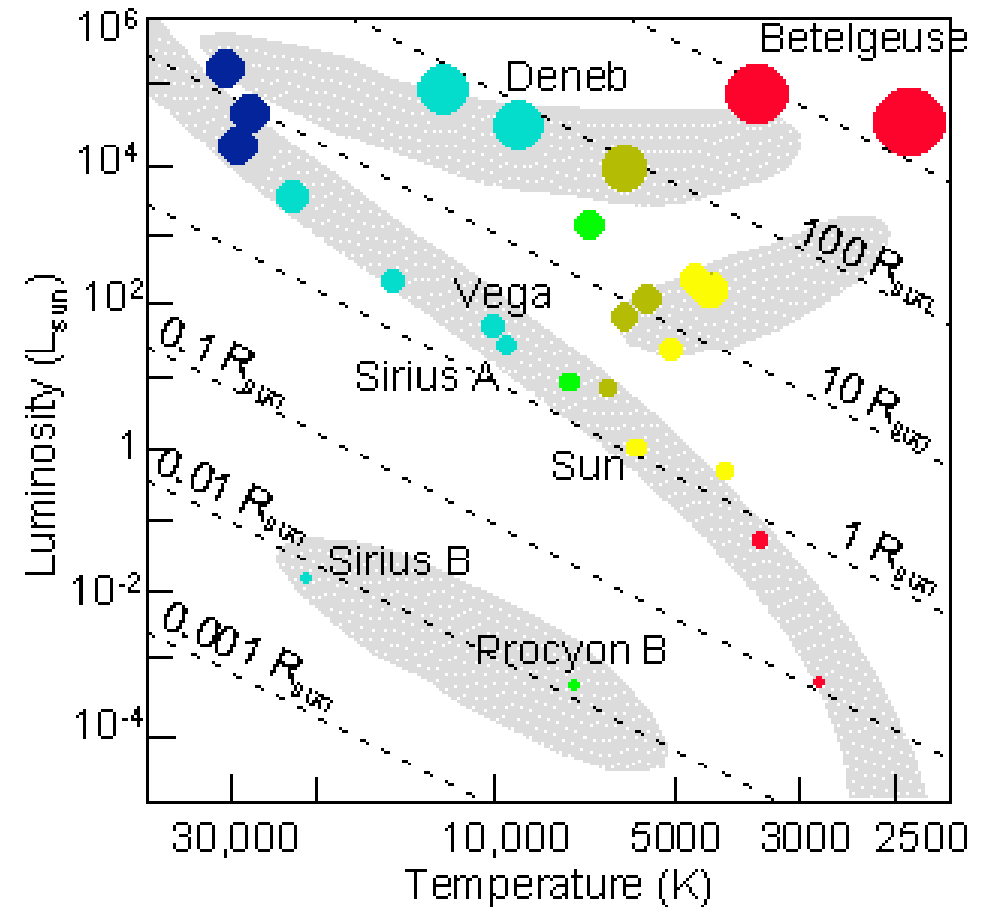
# The Hertzsprung-Russell diagram

$M$ ,  $R$ ,  $L$  and  $T_e$  do not vary independently.

Two major relationships

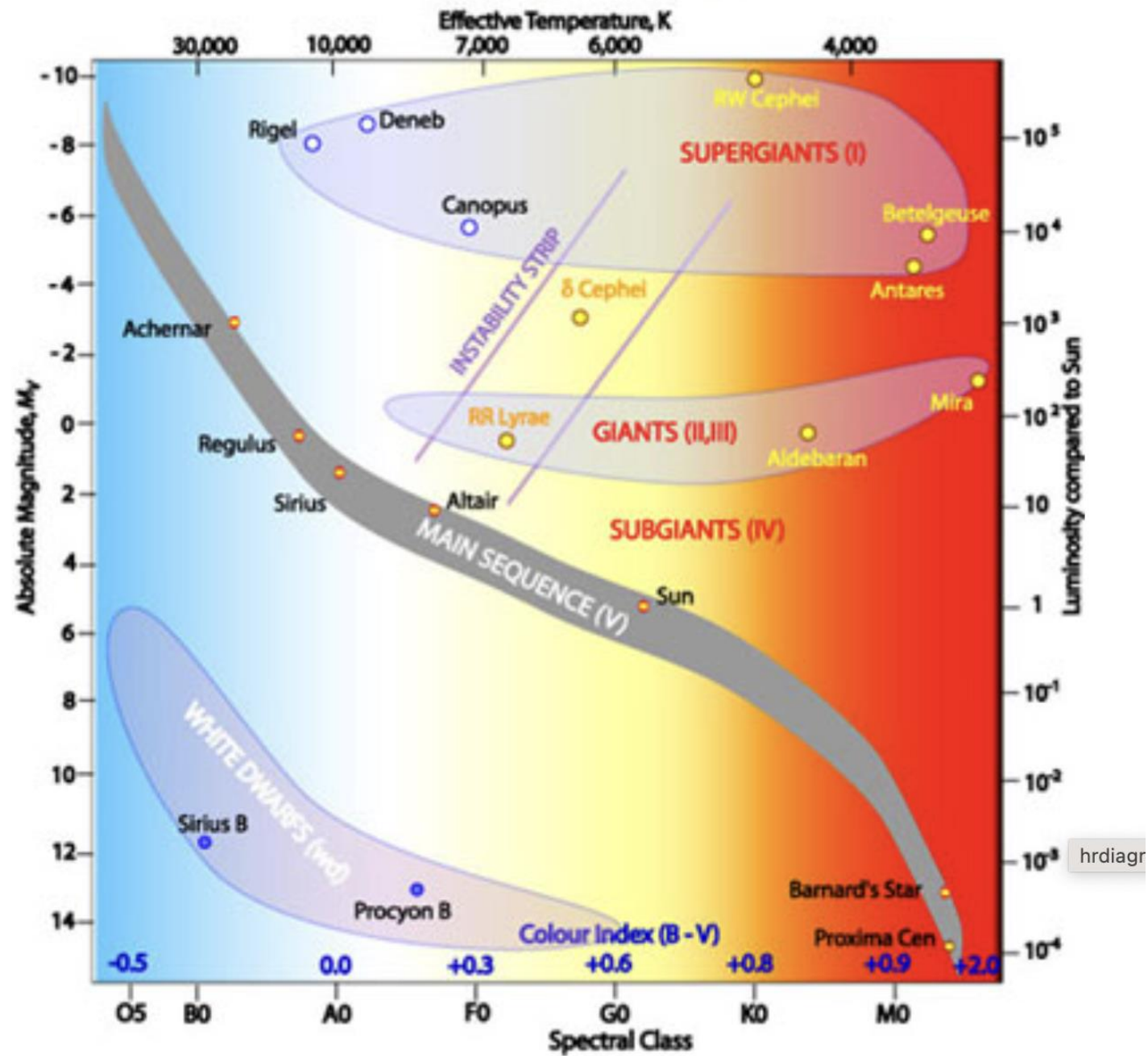
- $L$  with  $T$
- $L$  with  $M$

The first is known as the *Hertzsprung-Russell* (HR) diagram or the *colour-magnitude* diagram.

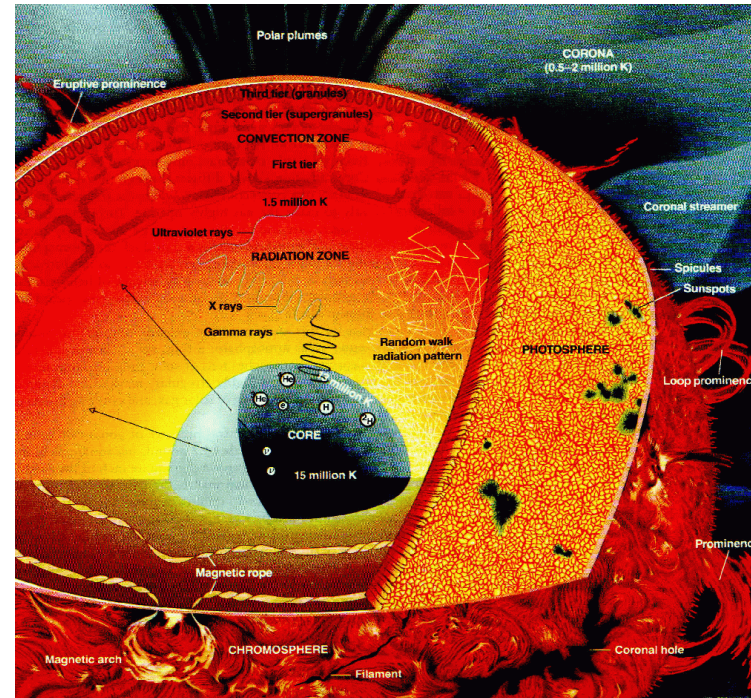
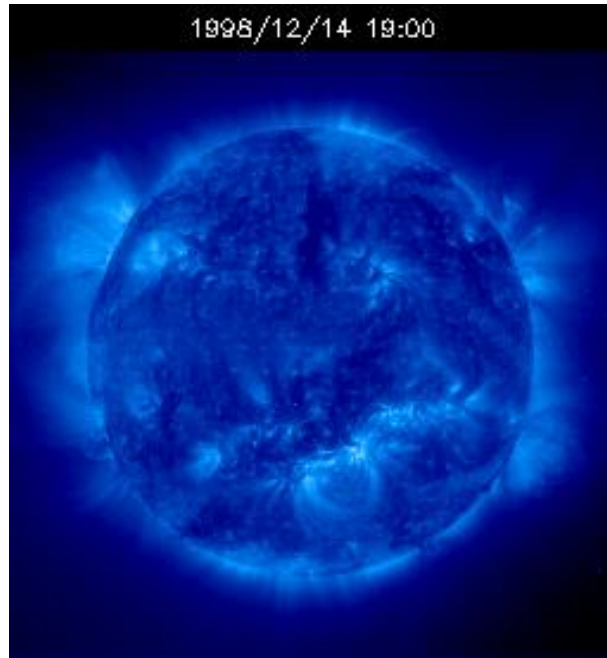


Colour Index (B-V)	-0.6	0	+0.6	+2.0
Spectral type	O B	A F	G K	M

# Hertzsprung-Russell Diagram



# The Sun – best studied example



Stellar interiors not directly observable. Solar neutrinos emitted at core and detectable. Helioseismology - vibrations of solar surface can be used to probe density structure

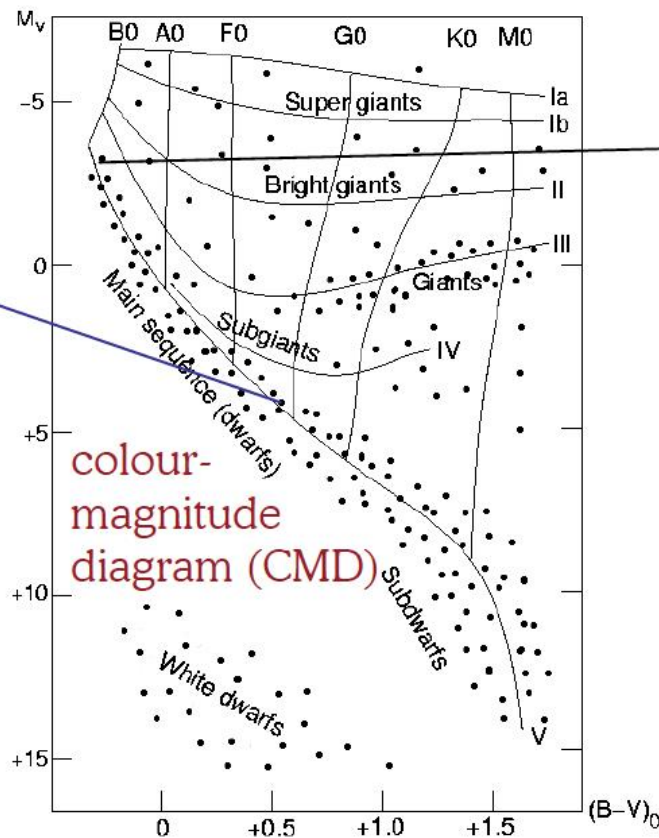
Must construct models of stellar interiors – predictions of these models are tested by comparison with observed properties of individual stars

# The Sun

$M = 2 \times 10^{33} \text{ g} = M_{\odot}$   
 $R = 7 \times 10^{10} \text{ cm} = R_{\odot}$   
 $L = 4 \times 10^{33} \text{ erg/s} = L_{\odot}$

photosphere:

$\Delta R \approx 200 \text{ km} < 10^{-3} R_{\odot}$   
 $n \approx 10^{15} \text{ cm}^{-3}$   
 $T \approx 6000 \text{ K}$

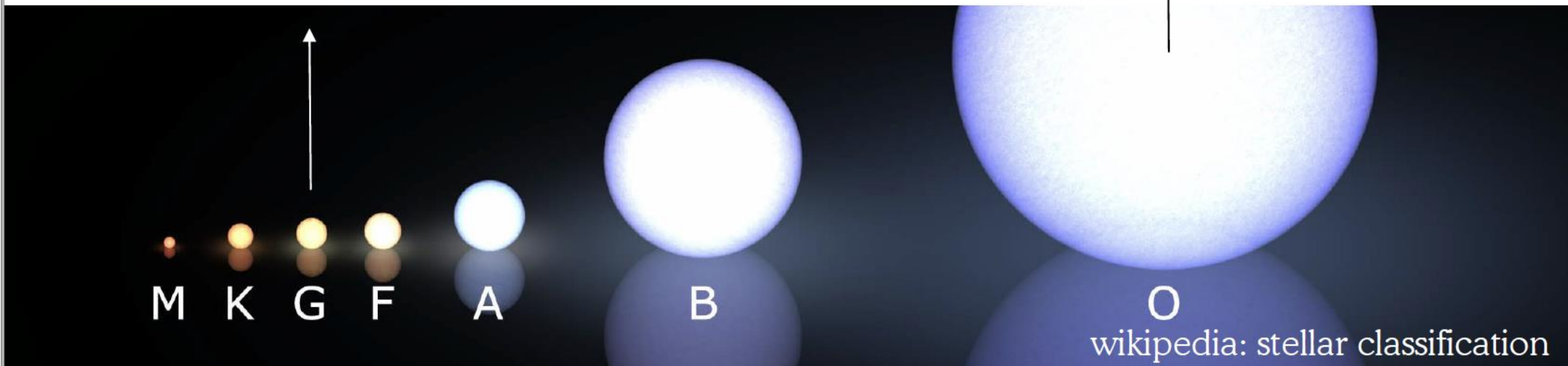


# an O star

$M \sim 50 M_{\odot}$   
 $R \sim 20 R_{\odot}$   
 $L \sim 10^6 L_{\odot} (\propto M^3)$

photosphere:

$\Delta R \approx 0.1 R_{\odot}$   
 $n \approx 10^{14} \text{ cm}^{-3}$   
 $T \approx 40000 \text{ K}$



# Colour-magnitude diagrams

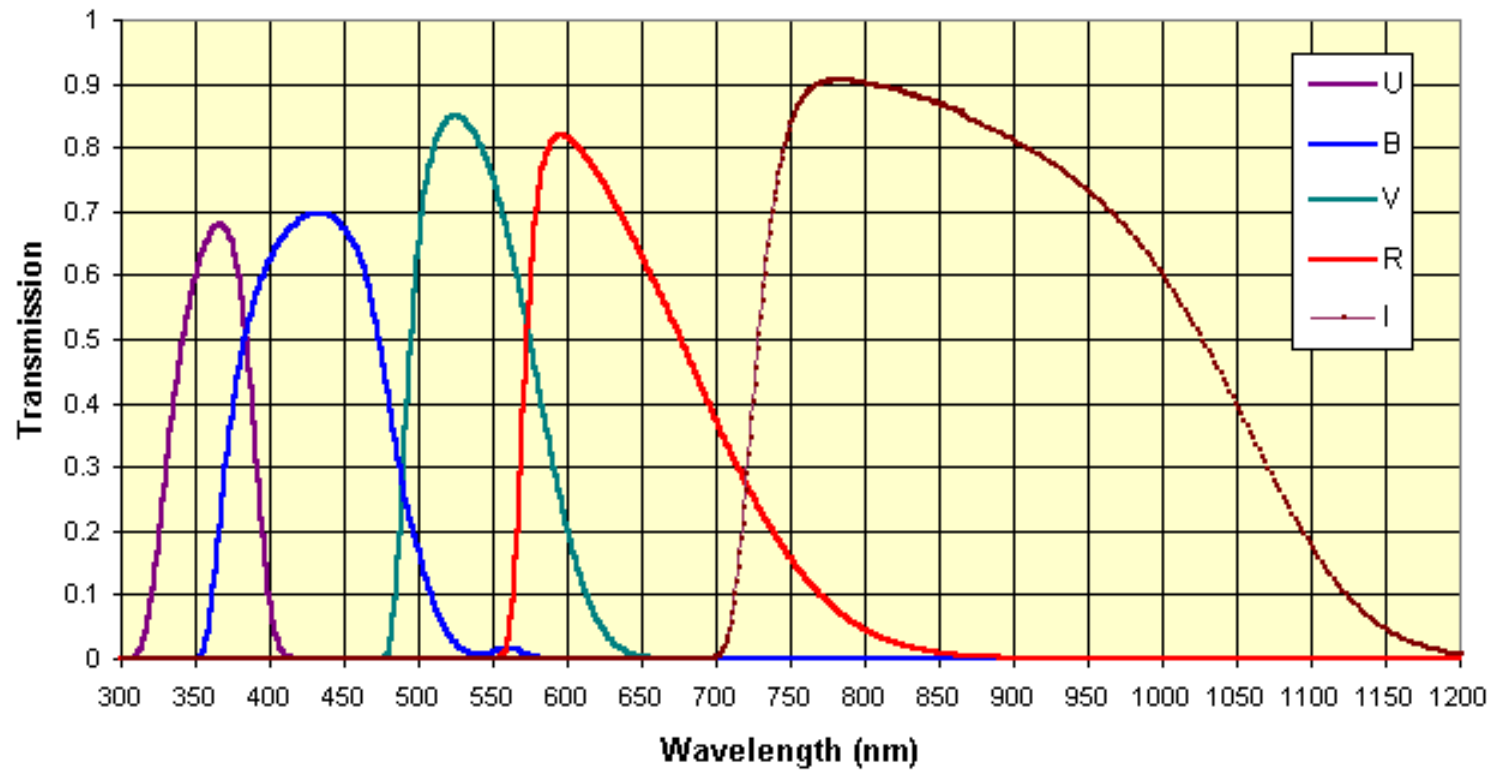
Measuring accurate  $T_e$  for  $\sim 10^2$  or  $10^3$  stars is intensive task – spectra needed and model atmospheres

Magnitudes of stars are measured at different wavelengths: standard system is *UBVRI*

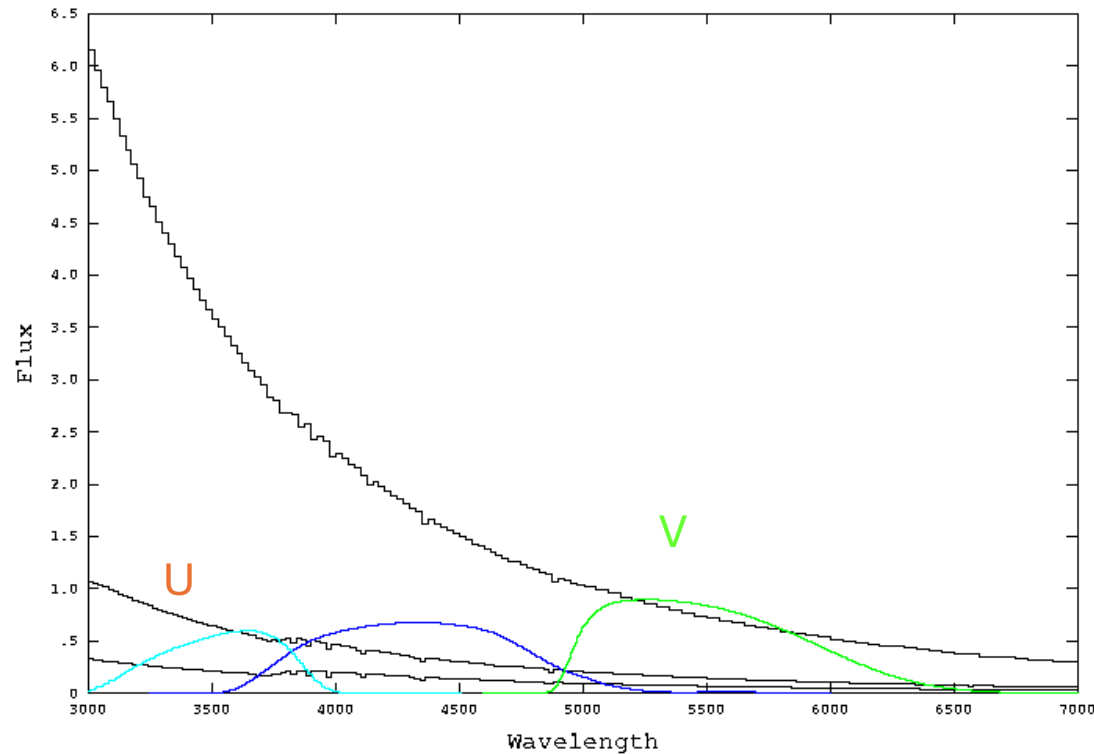
Band	<i>U</i>	<i>B</i>	<i>V</i>	<i>R</i>	<i>I</i>
$\lambda/\text{nm}$	365	445	551	658	806
$W/\text{nm}$	66	94	88	138	149



UBVRI Filter Characteristics



# Magnitudes and Colours (see also next slides)

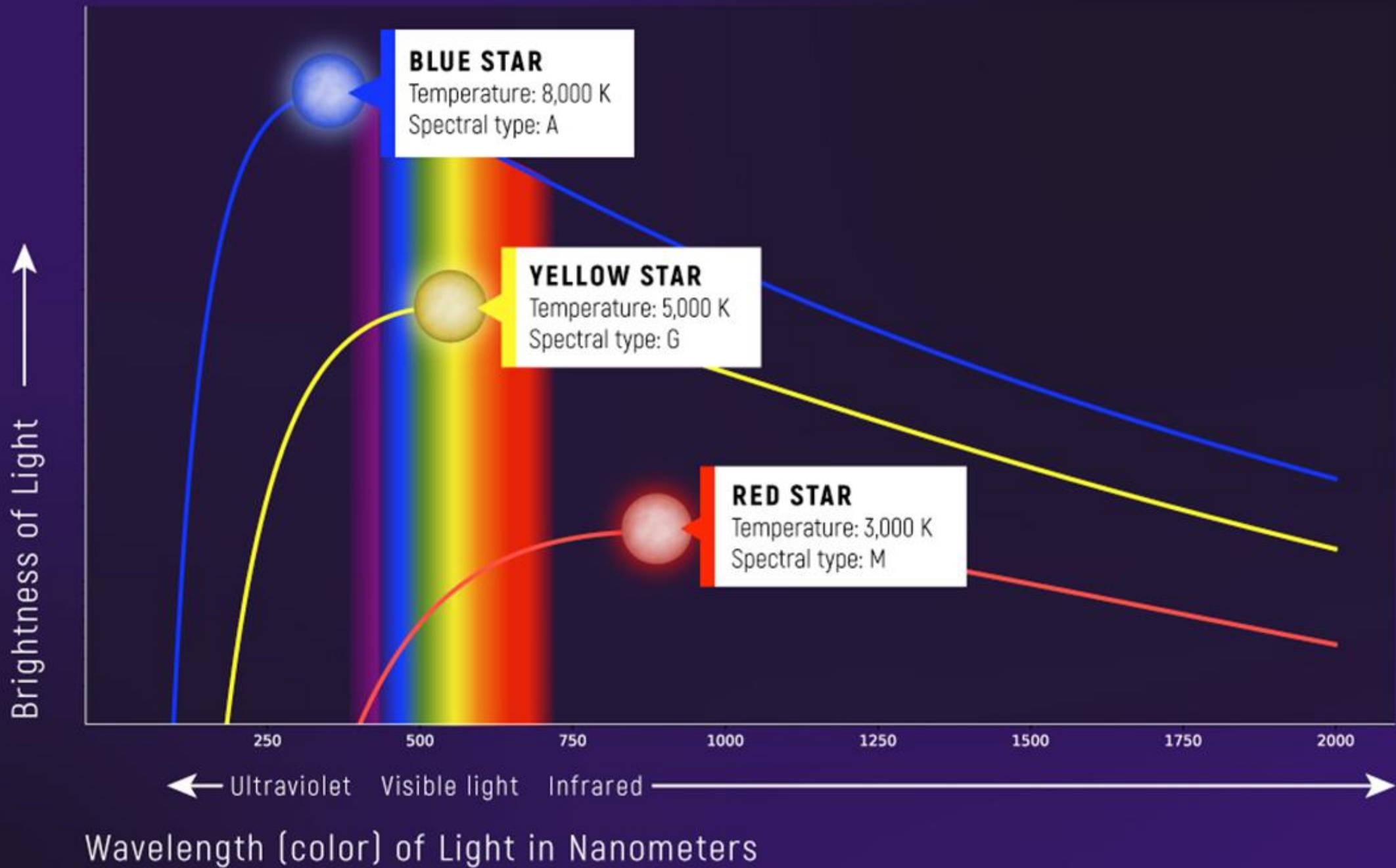


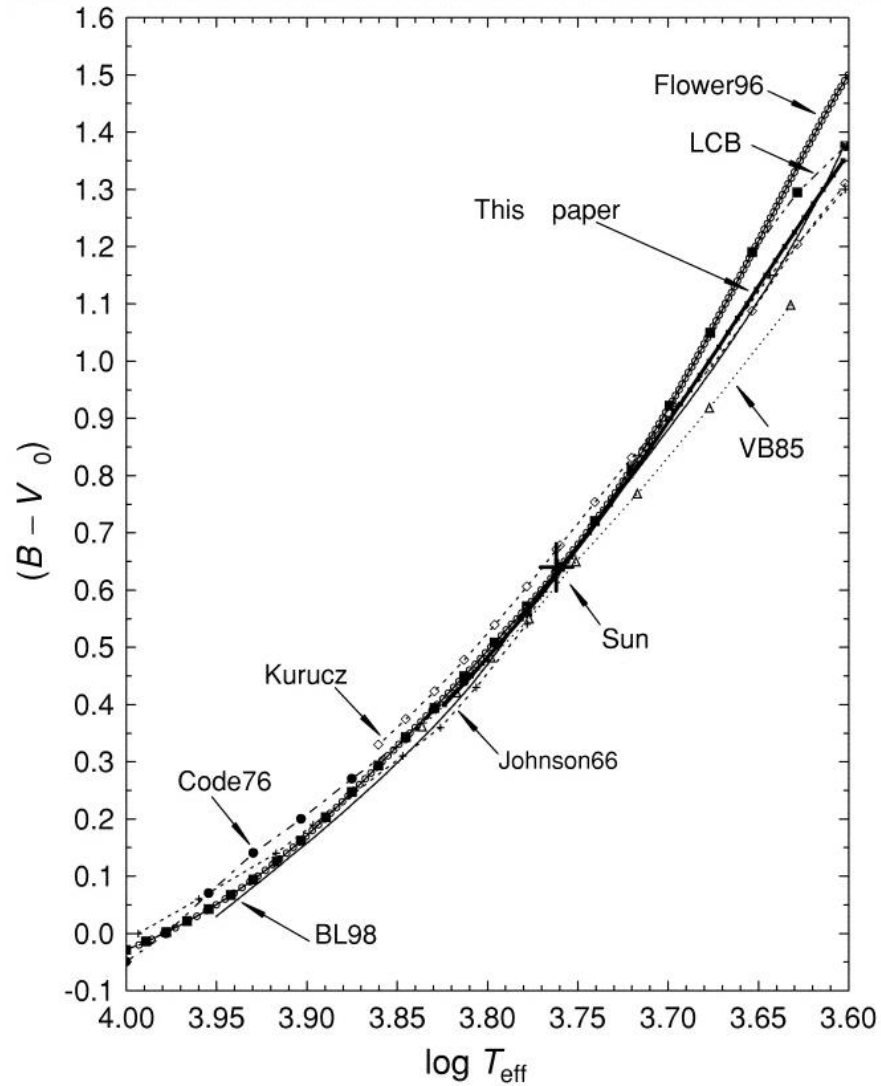
Model Stellar  
spectra  $T_e =$   
40,000, 30,000,  
20,000K

e.g.

$$B-V = f(T_e)$$

troms





Various calibrations can be used to provide the colour relation:

$$B-V = f(T_e)$$

Remember that observed  $(B-V)$  must be corrected for interstellar extinction to  $(B-V)_0$

# Absolute magnitude and bolometric magnitude

- **Absolute Magnitude**  $M$  defined as apparent magnitude of a star if it were placed at a distance of 10 pc

$$m - M = 5 \log(d/10) - 5$$

where  $d$  is in pc

- Magnitudes are measured in some wavelength band e.g. *UBV*. To compare with theory it is more useful to determine **bolometric magnitude** – defined as absolute magnitude that would be measured by a bolometer sensitive to all wavelengths. We define the bolometric correction to be

$$BC = M_{bol} - M_V$$

Bolometric luminosity is then

$$M_{bol} - M_{bol}^{\odot} = -2.5 \log L/L_{\odot}$$

# For Main-Sequence Stars

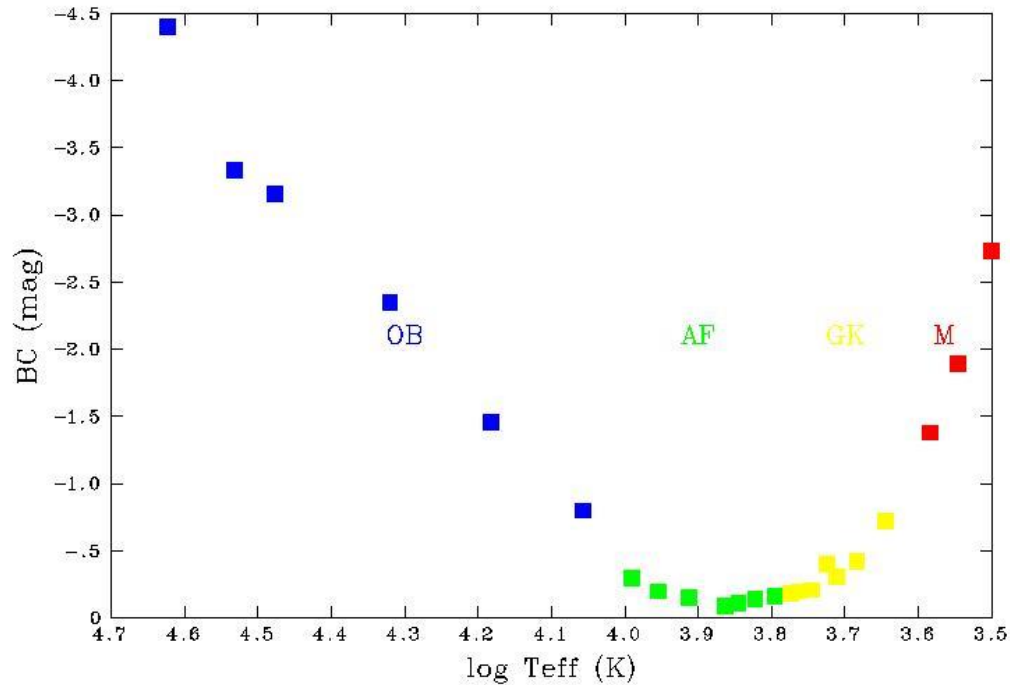


Table 15.7. Calibration of MK spectral types.

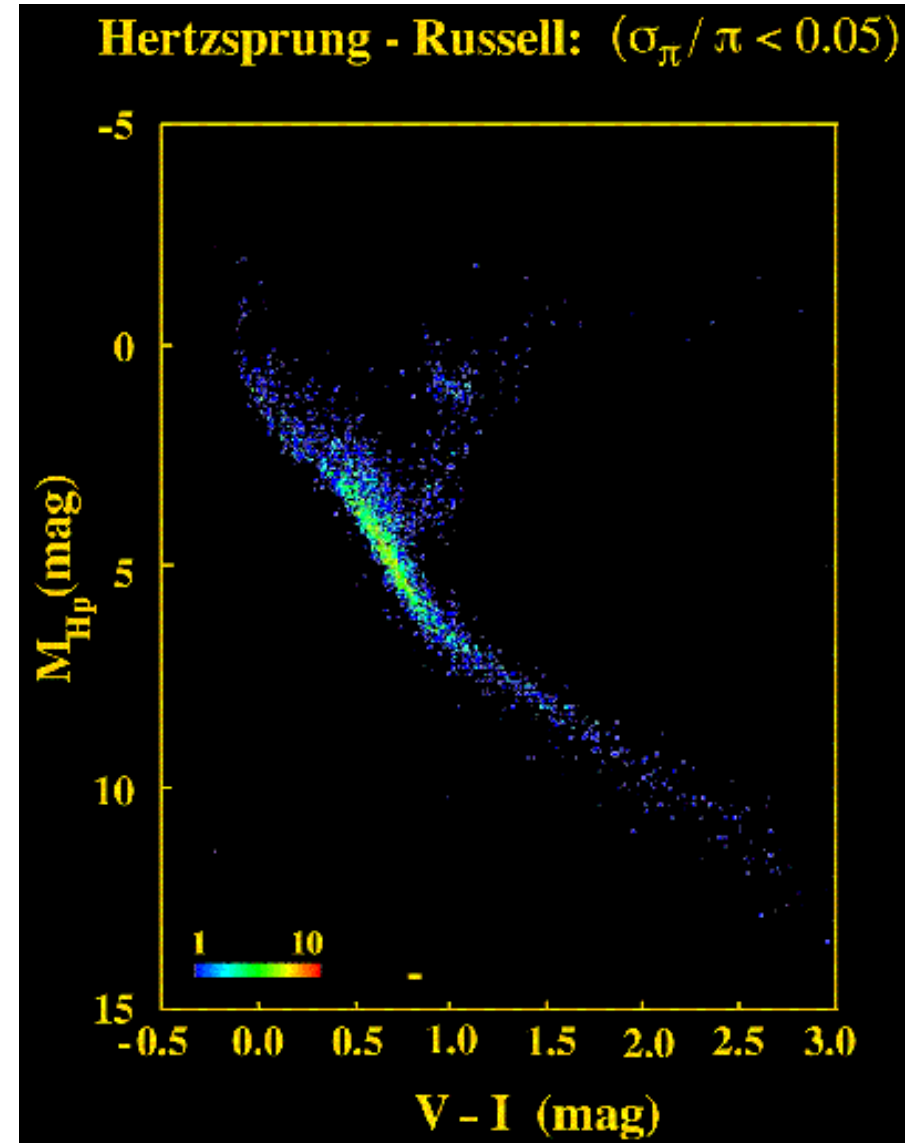
$Sp$	$M(V)$	$B - V$	$U - B$	$V - R$	$R - I$	$T_{\text{eff}}$	BC
MAIN SEQUENCE, V							
O5	-5.7	-0.33	-1.19	-0.15	-0.32	42 000	-4.40
O9	-4.5	-0.31	-1.12	-0.15	-0.32	34 000	-3.33
B0	-4.0	-0.30	-1.08	-0.13	-0.29	30 000	-3.16
B2	-2.45	-0.24	-0.84	-0.10	-0.22	20 900	-2.35
B5	-1.2	-0.17	-0.58	-0.06	-0.16	15 200	-1.46
B8	-0.25	-0.11	-0.34	-0.02	-0.10	11 400	-0.80
A0	+0.65	-0.02	-0.02	0.02	-0.02	9 790	-0.30
A2	+1.3	+0.05	+0.05	0.08	0.01	9 000	-0.20
A5	+1.95	+0.15	+0.10	0.16	0.06	8 180	-0.15
F0	+2.7	+0.30	+0.03	0.30	0.17	7 300	-0.09
F2	+3.6	+0.35	0.00	0.35	0.20	7 000	-0.11
F5	+3.5	+0.44	-0.02	0.40	0.24	6 650	-0.14
F8	+4.0	+0.52	+0.02	0.47	0.29	6 250	-0.16
G0	+4.4	+0.58	+0.06	0.50	0.31	5 940	-0.18
G2	+4.7	+0.63	+0.12	0.53	0.33	5 790	-0.20
G5	+5.1	+0.68	+0.20	0.54	0.35	5 560	-0.21
G8	+5.5	+0.74	+0.30	0.58	0.38	5 310	-0.40
K0	+5.9	+0.81	+0.45	0.64	0.42	5 150	-0.31
K2	+6.4	+0.91	+0.64	0.74	0.48	4 830	-0.42
K5	+7.35	+1.15	+1.08	0.99	0.63	4 410	-0.72
M0	+8.8	+1.40	+1.22	1.28	0.91	3 840	-1.38
M2	+9.9	+1.49	+1.18	1.50	1.19	3 520	-1.89
M5	+12.3	+1.64	+1.24	1.80	1.67	3 170	-2.73

From Allen's Astrophysical Quantities (4<sup>th</sup> edition)

# The HRD from Hipparcos

HRD from Hipparcos

HR diagram for 4477 single stars from the Hipparcos Catalogue with distance precision of better than 5%

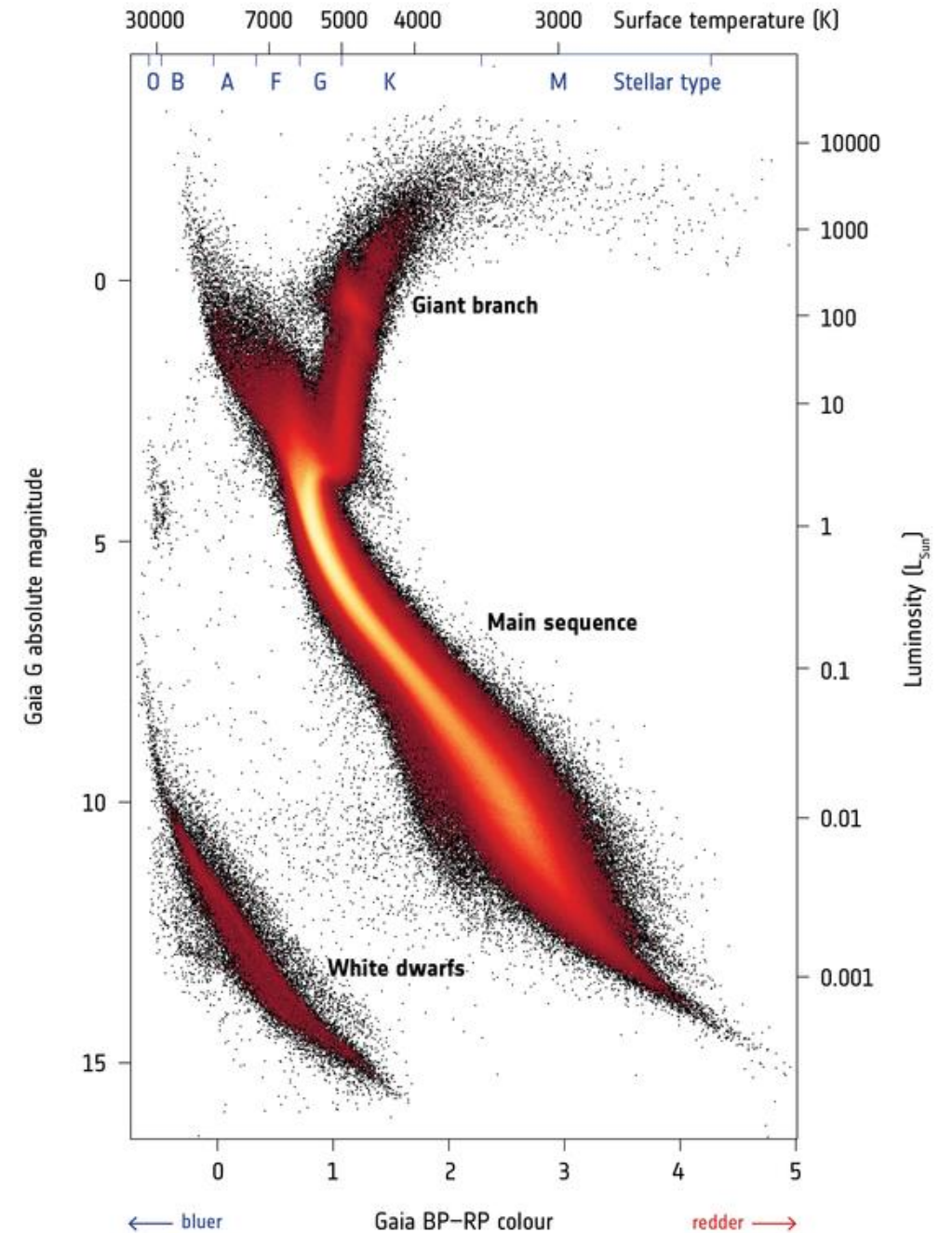


# GAIA

HRD from GAIA

HR diagram for 4 millions single stars from the GAIA DR3

These stars are within 1.5Kpc





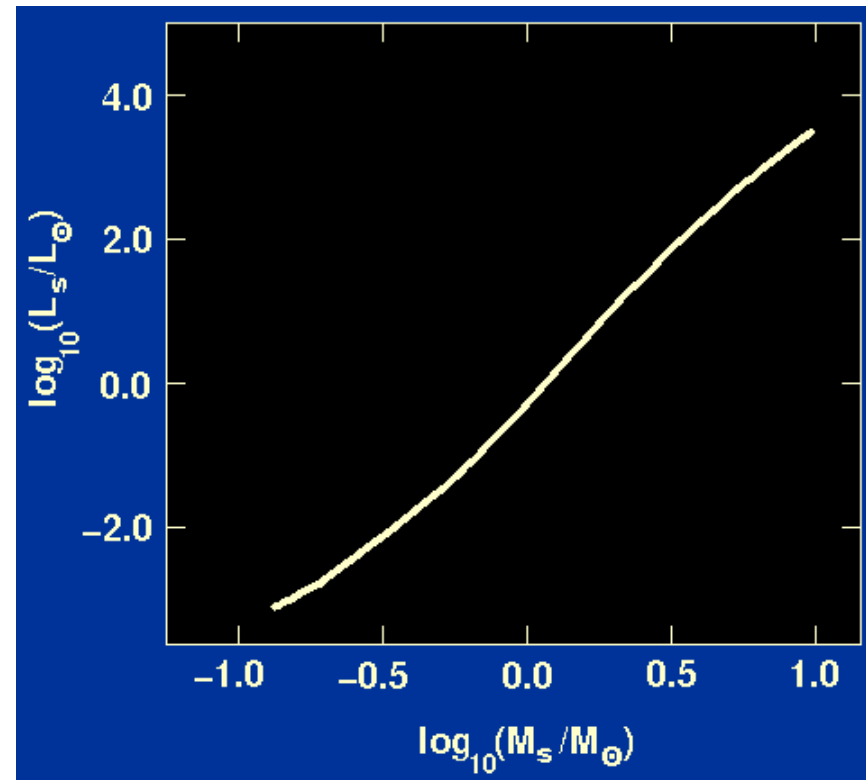
# Mass-luminosity relation

For the few main-sequence stars for which masses are known, there is a *Mass-luminosity relation*.

$$L \propto M^n$$

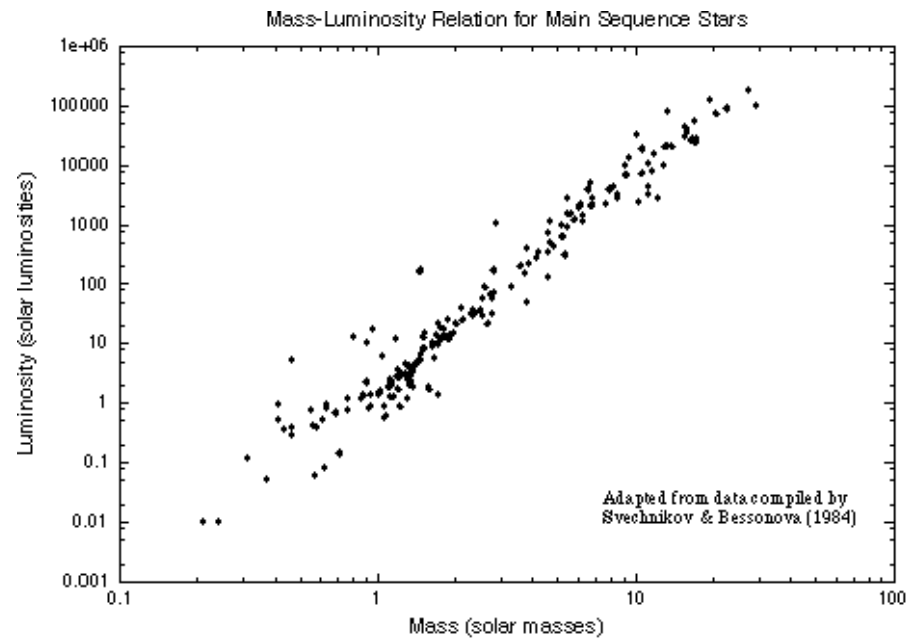
Where  $n=3-5$ . Slope changes at extremes, less steep for low and high mass stars.

This implies that the main-sequence (MS) on the HRD is a function of mass i.e. from bottom to top of main-sequence, stars increase in mass

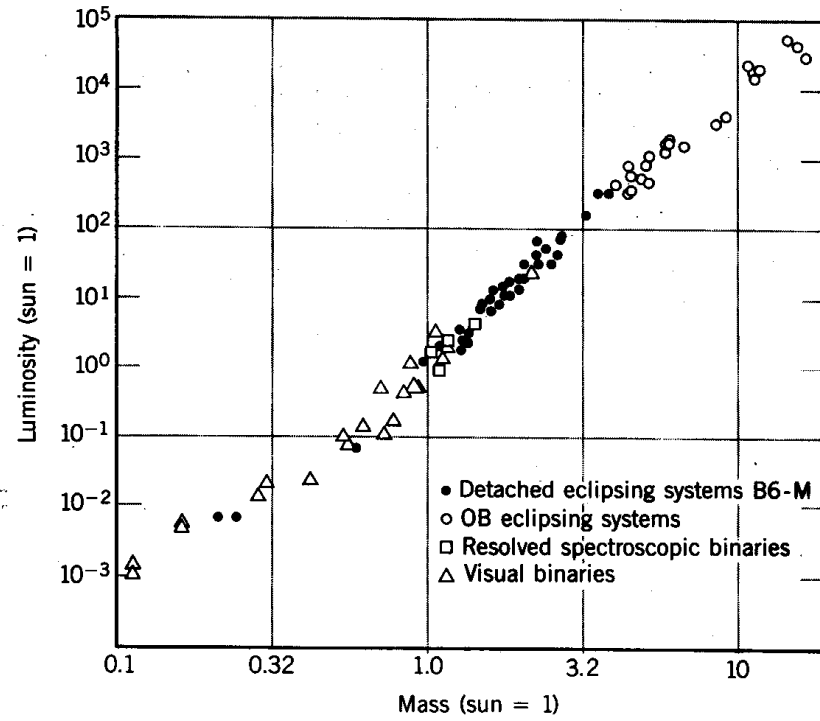


We must understand the  $M-L$  relation and  $L-T_e$  relation theoretically.

Models must reproduce observations



The mass - luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.



# Age and metallicity

There are two other fundamental properties of stars that we can measure – age ( $t$ ) and chemical composition

Composition parameterised with

$X, Y, Z \equiv$  mass fraction of H, He and all other elements

e.g.  $X_{\odot} = 0.747$  ;  $Y_{\odot} = 0.236$  ;  $Z_{\odot} = 0.017$

Note –  $Z$  often referred to as *metallicity*

Would like to studies stars of same age and chemical composition – to keep these parameters constant and determine how models reproduce the other observables

# Star clusters

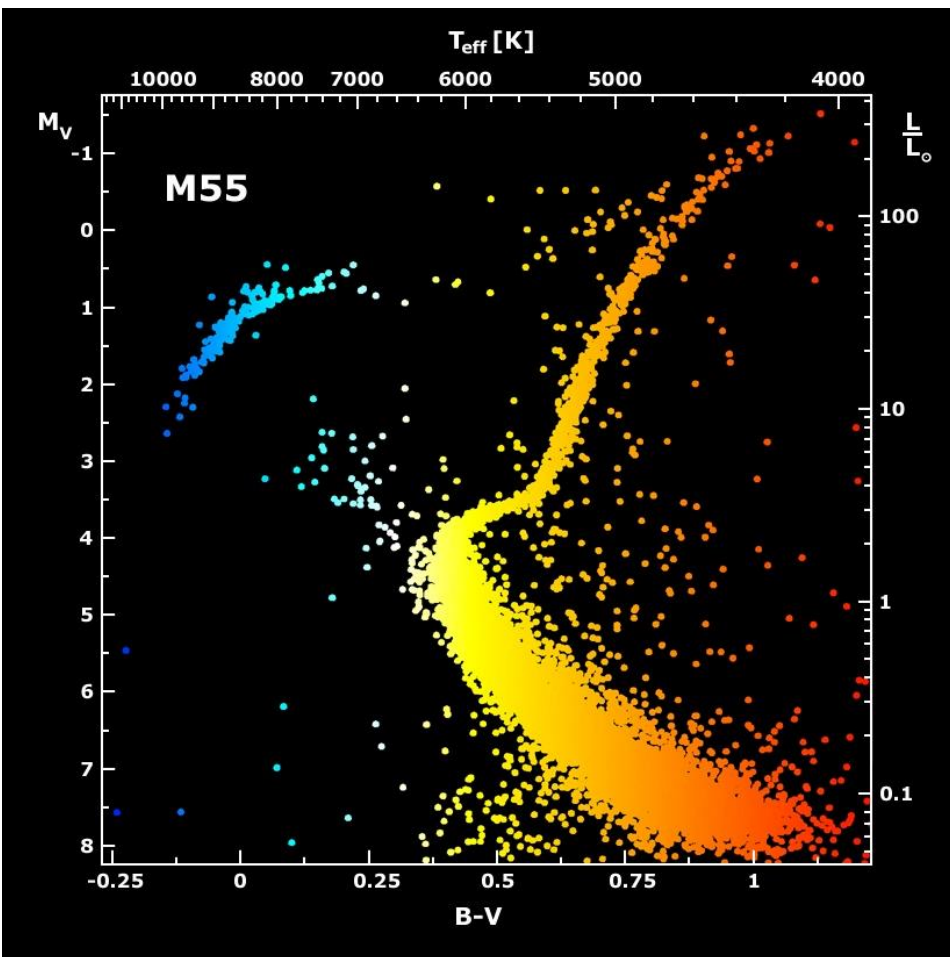


NGC3293 - Open cluster



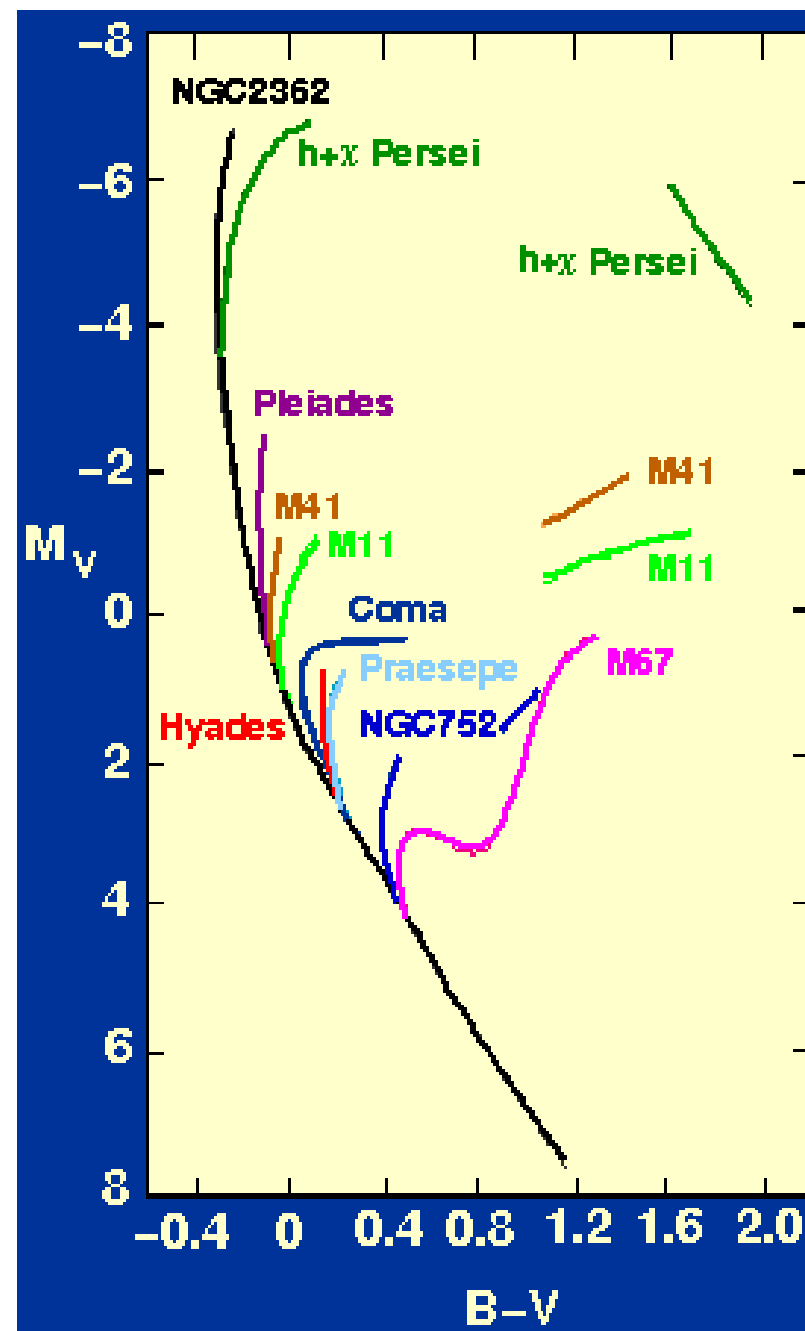
47 Tuc – Globular cluster

## Globular cluster M3 example



- In clusters,  $t$  and  $Z$  must be same for all stars
- Hence differences must be due to  $M$
- Stellar evolution assumes that the differences in cluster stars are due only (or mainly) to initial  $M$
- Cluster HR (or colour-magnitude) diagrams are quite similar – age determines overall appearance

## Selection of Open clusters



# Globular vs. Open clusters

Globular	Open
<ul style="list-style-type: none"><li>• MS turn-off points in similar position. Giant branch joining MS</li><li>• Horizontal branch from giant branch to above the MS turn-off point</li><li>• Horizontal branch often populated only with variable RR Lyrae stars</li></ul>	<ul style="list-style-type: none"><li>• MS turn off point varies massively, faintest is consistent with globulars</li><li>• Maximum luminosity of stars can get to <math>M_V \approx -10</math></li><li>• Very massive stars found in these clusters</li></ul>

The differences are interpreted due to age – open clusters lie in the disk of the Milky Way and have large range of ages. The Globulars are all ancient, with the oldest tracing the earliest stages of the formation of Milky Way ( $\sim 12 \cdot 10^9$  yrs)

# Summary

- Four fundamental observables used to parameterise stars and compare with models  $M$ ,  $R$ ,  $L$ ,  $T_e$
- $M$  and  $R$  can be measured directly in small numbers of stars (will cover more of this later)
- Age and chemical composition also dictate the position of stars in the HR diagram
- Stellar clusters very useful laboratories – all stars at same distance, same  $t$ , and initial  $Z$
- We will develop models to attempt to reproduce the  $M$ ,  $R$ ,  $L$ ,  $T_e$  relationships and understand HR diagrams