

# Neurofunctional Techniques

**Lessons 3 & 4**

7, 9 October 2024

- 1)  $\text{Ca}^{2+}$  indicators**
- 2)  $\text{Ca}^{2+}$  binding**
- 3) Diffusion**

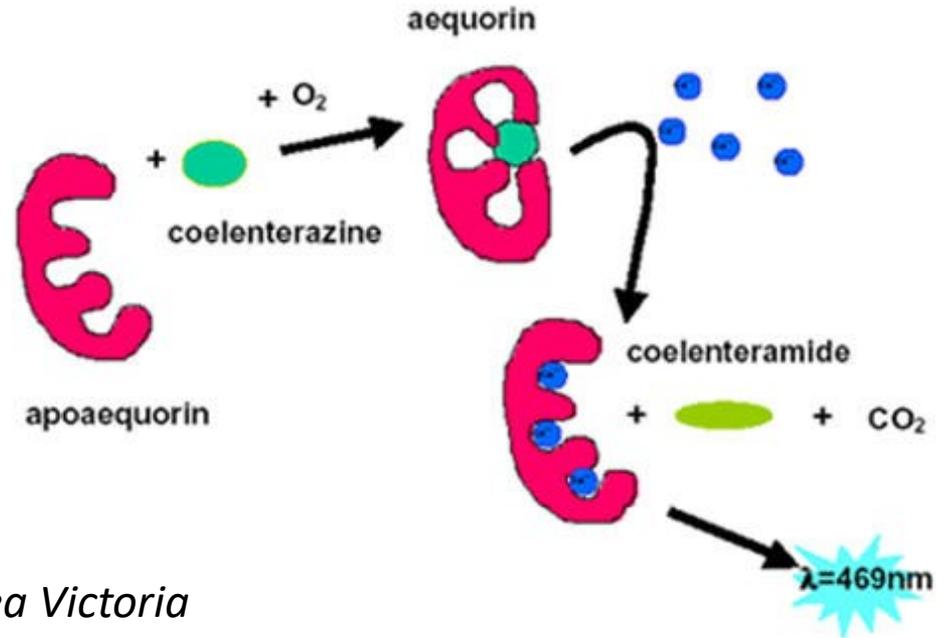
# Calendar

- **M 30 Sept:** Course introduction
- **W 2 Oct:** Functional imaging
- **F 4 Oct:** Statistics (Cesca)
- **M 7 Oct:** Functional imaging
- **W 9 Oct:** Biophysics of diffusion
- **F 11 Oct:** Statistics (Cesca)
- **M 14 Oct:** Functional imaging
- **W 16 Oct:** General introduction to the papers for the presentations
- **F 18 Oct:** Statistics (Cesca)
- **M 21 Oct:** Modeling in neuroscience
- **W 23 Oct:** Practical exercises on the first part of the course
- **F 25 Oct:** Statistics (Cesca)
- **F 25 Oct:** Laboratory (14:00- 18:00) to be confirmed!
- **M 28 Oct:** Molecular approaches in modern neuroscience
- **W 30 Oct:** Genome editing in neuroscience (Dr. Jaudon)
- **M 4 Nov:** Optogenetics
- **W 6 Nov:** Papers assignment to the groups; introductions to the specific papers
- **T 12 Nov:** X-genetics + Practical exercises on the second part of the course
- **W 13 Nov:** Introductions to the specific papers
- **M 18 Nov:** Introductions to the specific papers
  
- **9, 10, 11 Dic (15:00-19:00):** Paper presentation (all 3 days!)
- **Week 16 Dic:** Test (to be confirmed!)

# Ca<sup>2+</sup> imaging

- Review Ca<sup>2+</sup> signaling
- **Ca<sup>2+</sup> indicators**
  - Small molecule indicators (SMIs)
  - Genetically-encoded Ca<sup>2+</sup> indicators (GECIs)
- **Ca<sup>2+</sup> binding**
- **Ca<sup>2+</sup> diffusion**
- **Ca<sup>2+</sup>-dependent fluorescence properties**
- **Simplified models of Ca<sup>2+</sup> dynamics**
- **Imaging devices**
- **What can we do with it**

# A bit of history: Aequorin bioluminescence



**Aequorin** derived from the jellyfish *Aequorea Victoria*

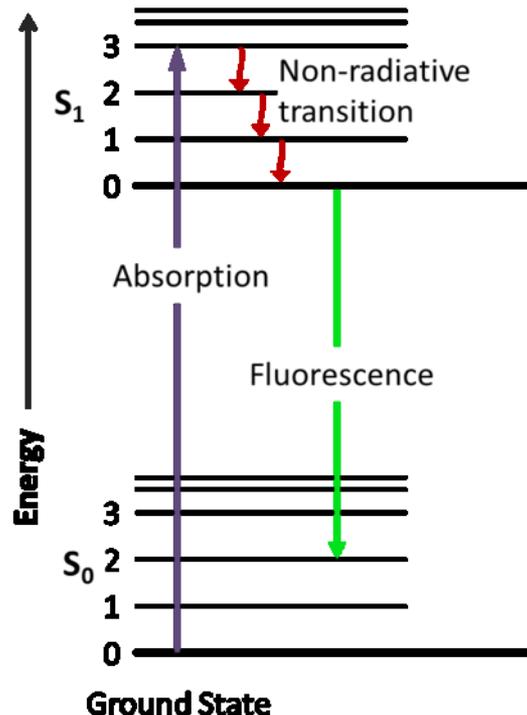
**Bioluminescence is the production of light by a living organism, for example using a chemical reaction.** Bioluminescence is different from fluorescence as it does not require external illumination.

**Advantages:** there are no problems of phototoxicity, photobleaching, autofluorescence.

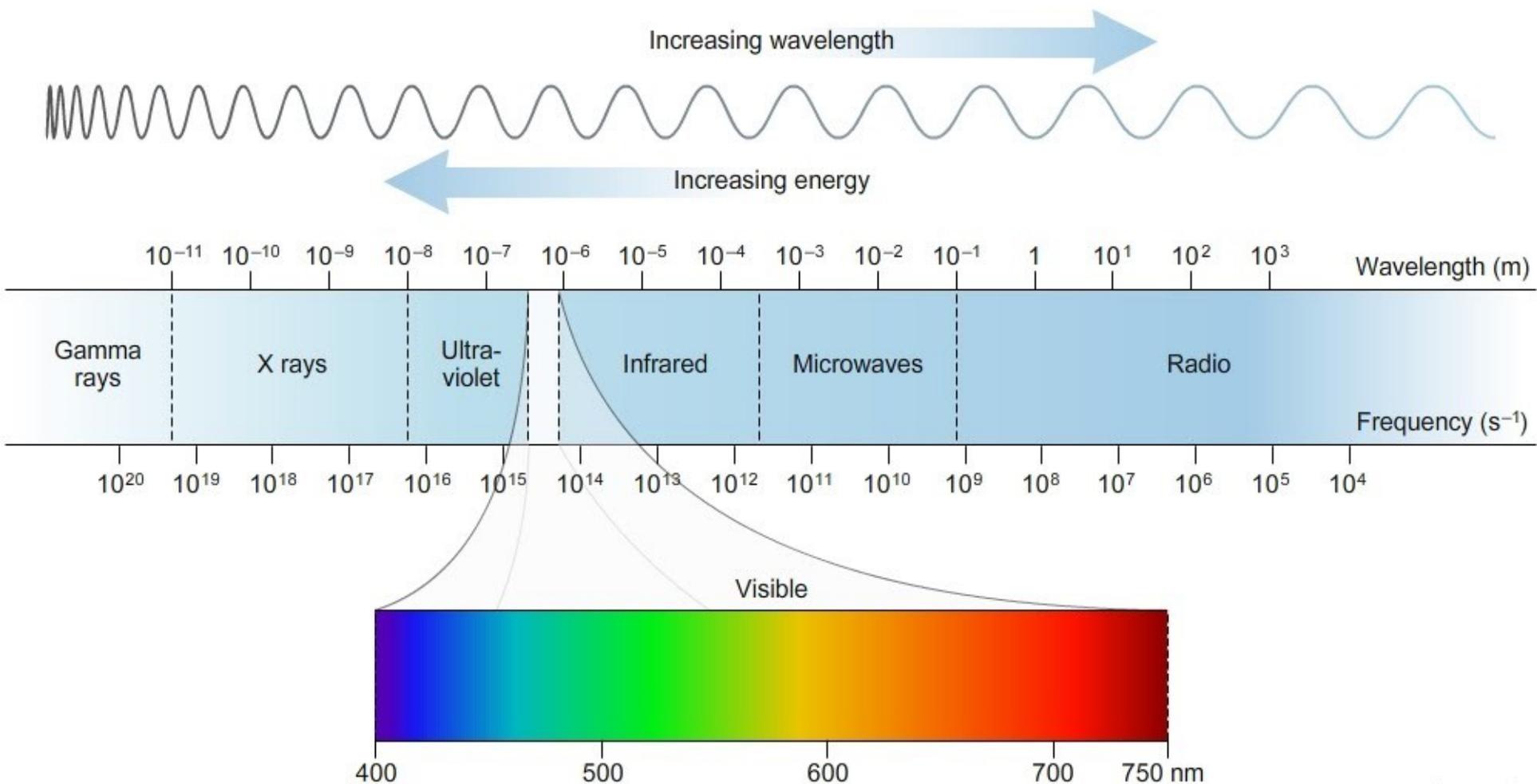
**Disadvantages:** each molecule performs only one emission cycle and recharging the coelenterazine is a **slow process**.

# Fluorescence

**Fluorescence** is not produced by the organism itself; fluorescent molecules absorb photons of light (preferentially of a specific wavelength), which temporarily excite electrons to a higher energy level, giving light off as a waste product. **The emitted light has a longer wavelength**, and therefore lower energy, than the absorbed light.



# Electromagnetic spectrum



# An example: GFP (green fluorescent protein)



Getting Started | Save | Visualizza informazioni sul sito | Getting Started | Come iniziare

## Spectra Viewer – EGFP

Legend Fluorochromes Sets Parts

Fluorochromes

EGFP [Hide Individual Filters](#) x

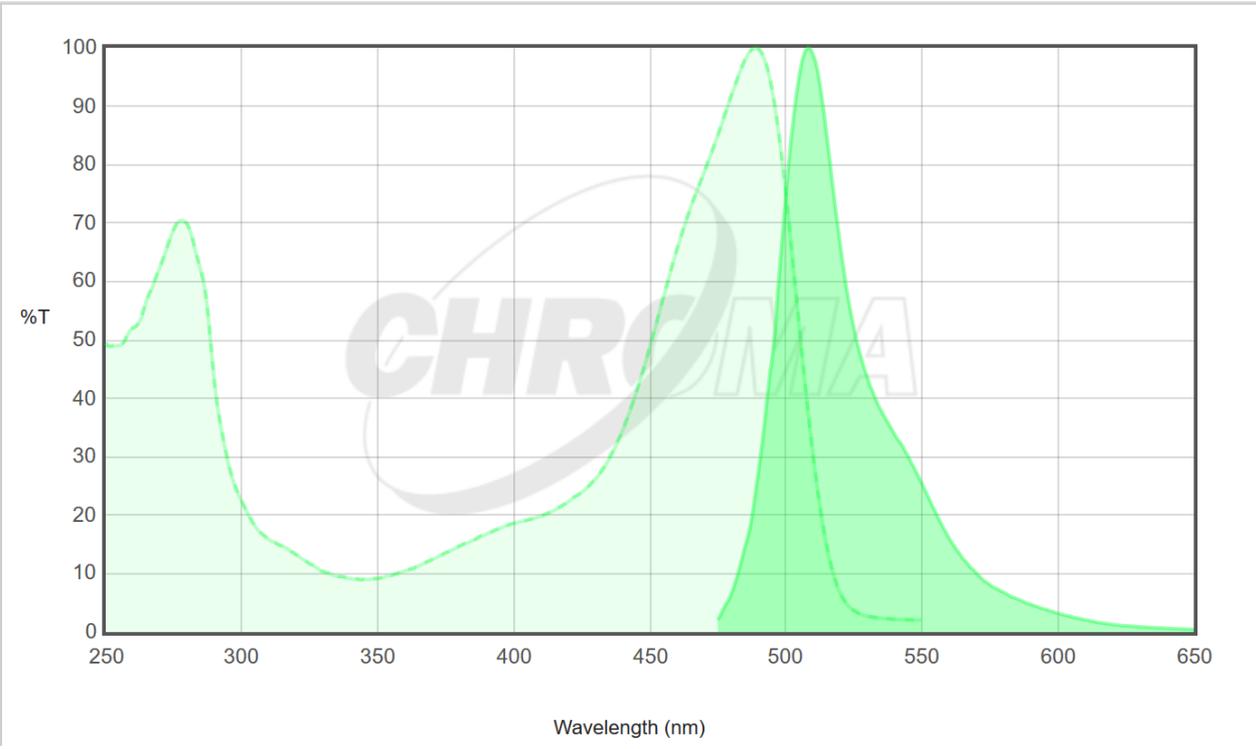
PLOT	FILTERS	TYPE	%T-OD	AOI	DATA
<input checked="" type="checkbox"/>	EGFP Excitation Spectra	EX	49.06 T at 255nm	0°	ASCII
<input checked="" type="checkbox"/>	EGFP Emission Spectra	EM	0.00 T at 255nm	0°	ASCII

Reload Plot

### Attributes

Ex $\lambda$	Em $\lambda$
488	507

View By:  %T  OD Wavelength 250 to 650 [update](#) [reset](#) [export](#)



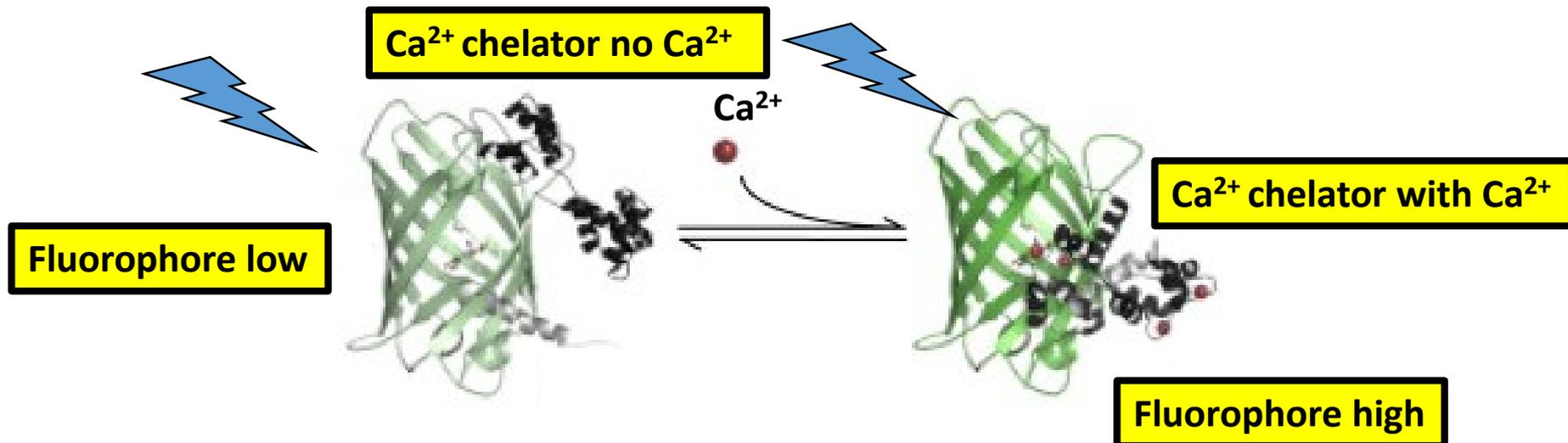
# Fluorescent Ca<sup>2+</sup> indicator

Fluorescent indicators are advantageous because even at **low concentration** they allow for high-contrast labeling

## How do you make a fluorescent Ca<sup>2+</sup> indicator?

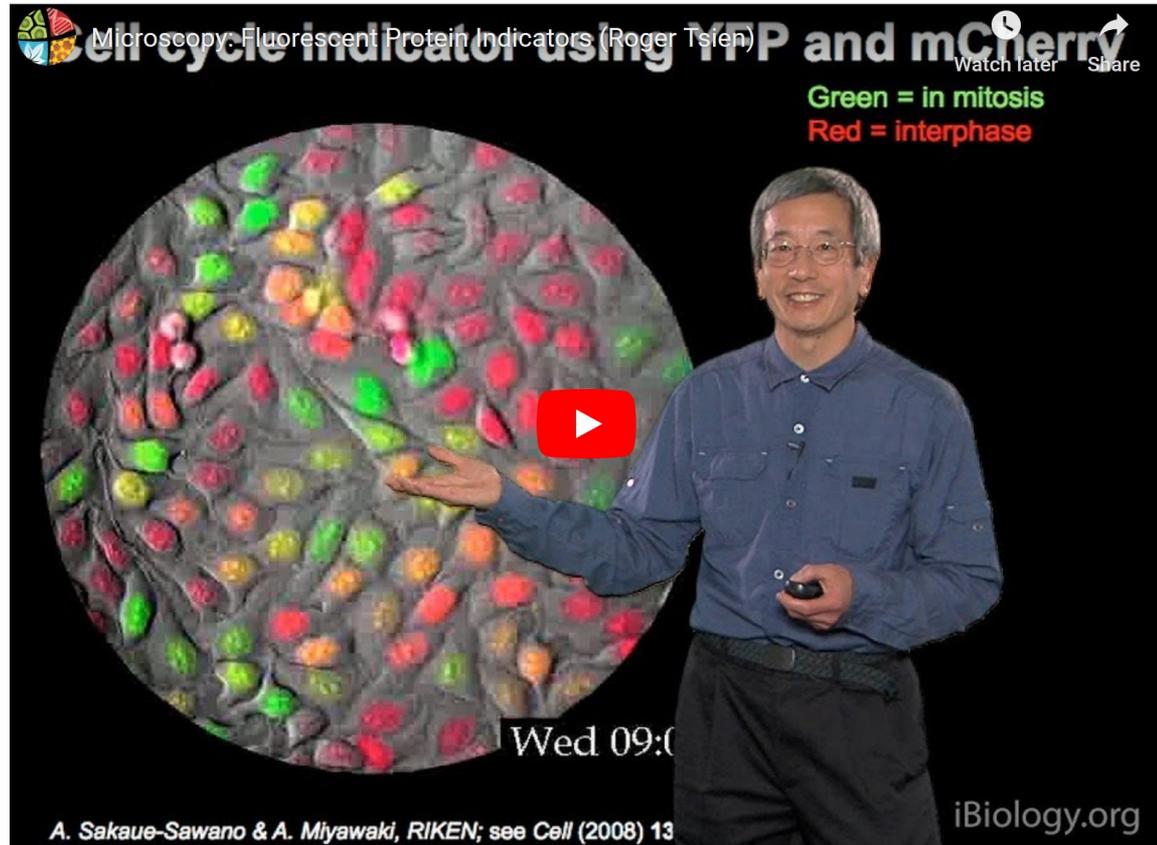
It should have two moieties:

- 1 acting as Ca<sup>2+</sup> buffer/chelator
- 1 acting as fluorophore
- Ca<sup>2+</sup> binding to the chelator moiety must affect some property of the fluorophore

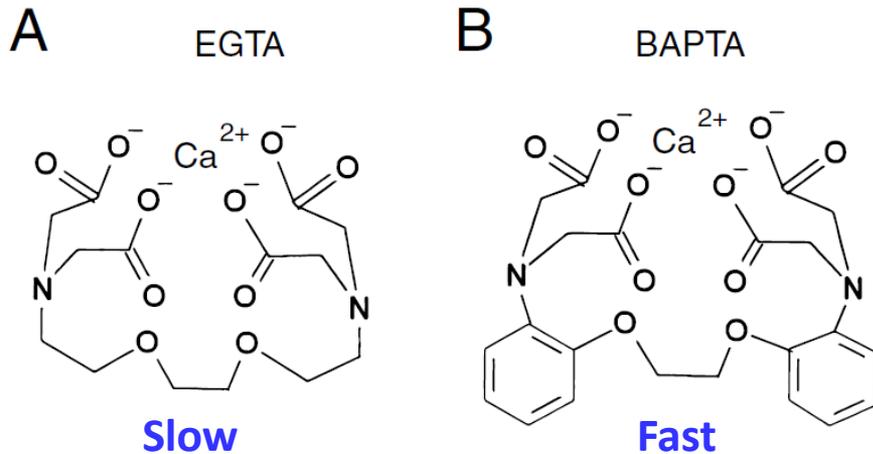


# There are 2 major types of fluorescent $\text{Ca}^{2+}$ indicators:

- **Small molecule indicators (SMIs)**, synthetic organic dyes developed since the beginning of the 1980s, mainly by Roger Tsien
- **Genetically-encoded  $\text{Ca}^{2+}$  indicators (GECIs)**



# Small molecule indicators (SMIs) are based on:



EGTA and BAPTA, the 2 major exogenous  $\text{Ca}^{2+}$  buffers

1. **They bind 1  $\text{Ca}^{2+}$**  → important for quantitative estimation of  $[\text{Ca}^{2+}]$
2. **with affinities comparable to endogenous  $\text{Ca}^{2+}$  binding proteins** → they can 'read' physiological levels of  $[\text{Ca}^{2+}]$

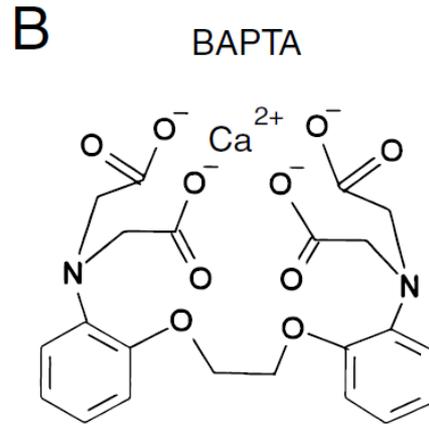
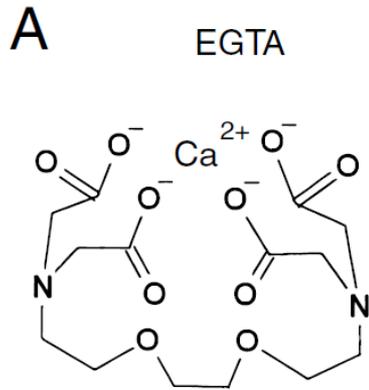
Table 1 | **Physicochemical properties of exogenous and endogenous  $\text{Ca}^{2+}$  buffers**

Chelator/ $\text{Ca}^{2+}$ -binding protein	$\text{Ca}^{2+}$ -binding rate ( $k_{\text{on}}$ )	$\text{Ca}^{2+}$ -unbinding rate ( $k_{\text{off}}$ )	Affinity ( $K_{\text{D}}$ )	Refs
BAPTA*	$4 \times 10^8 \text{ M}^{-1} \text{ s}^{-1}$	$88 \text{ s}^{-1}\ddagger$	220 nM	22,33,139
EGTA*	$1 \times 10^7 \text{ M}^{-1} \text{ s}^{-1}$	$0.7 \text{ s}^{-1}\ddagger$	70 nM	22,77
Calbindin	$7.5 \times 10^7 \text{ M}^{-1} \text{ s}^{-1}$	$29.5 \text{ s}^{-1}$	$293 \text{ nM}^\ddagger$	77,79
Calretinin <sup>§</sup>	$1.8 \times 10^6 \text{ M}^{-1} \text{ s}^{-1}$ (T)	$1.29 \text{ s}^{-1}$ (T)	$717 \text{ nM}^\ddagger$	78
	$3.1 \times 10^8 \text{ M}^{-1} \text{ s}^{-1}$ (R)	$1.73 \text{ s}^{-1}$ (R)	$5.6 \text{ nM}^\ddagger$	
Calmodulin N-lobe <sup>§</sup>	$7.7 \times 10^8 \text{ M}^{-1} \text{ s}^{-1}$ (T)	$1.6 \times 10^5 \text{ s}^{-1}$ (T)	$208 \mu\text{M}^\ddagger$	79
	$3.2 \times 10^{10} \text{ M}^{-1} \text{ s}^{-1}$ (R)	$2.2 \times 10^4 \text{ s}^{-1}$ (R)	$688 \text{ nM}^\ddagger$	
Calmodulin C-lobe <sup>§</sup>	$8.4 \times 10^7 \text{ M}^{-1} \text{ s}^{-1}$ (T)	$2.6 \times 10^3 \text{ s}^{-1}$ (T)	$31 \mu\text{M}^\ddagger$	79
	$2.5 \times 10^7 \text{ M}^{-1} \text{ s}^{-1}$ (R)	$6.5 \text{ s}^{-1}$ (R)	$260 \text{ nM}^\ddagger$	

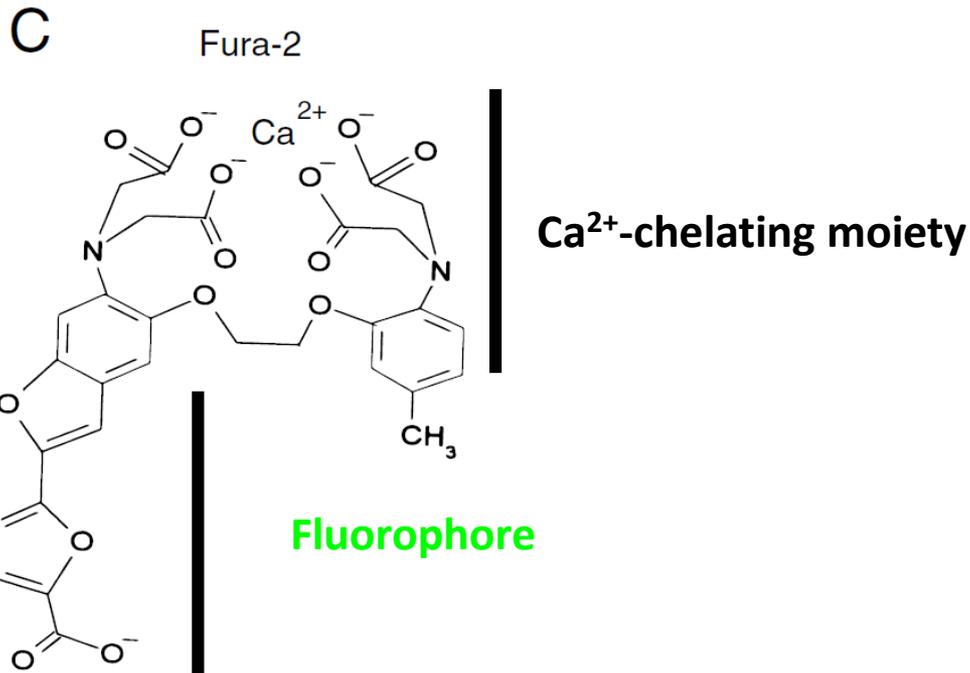
\*For the exogenous chelators, the  $\text{Ca}^{2+}$ -binding rate (on rate) is **~40 times** higher for BAPTA than for EGTA. By contrast, the affinity values are comparable; in fact the affinity is threefold lower for BAPTA than for EGTA. <sup>‡</sup> This value was calculated using  $K_{\text{D}} = k_{\text{off}} / k_{\text{on}}$ .

<sup>§</sup>For the  $\text{Ca}^{2+}$ -binding proteins calretinin and calmodulin,  $\text{Ca}^{2+}$  binding is highly cooperative. Therefore, rates are given separately for tense (T) and relaxed (R) conformations of the protein.

# Small molecule indicators (SMIs) are based on:



EGTA and BAPTA are the 2 major exogenous  $\text{Ca}^{2+}$  buffers



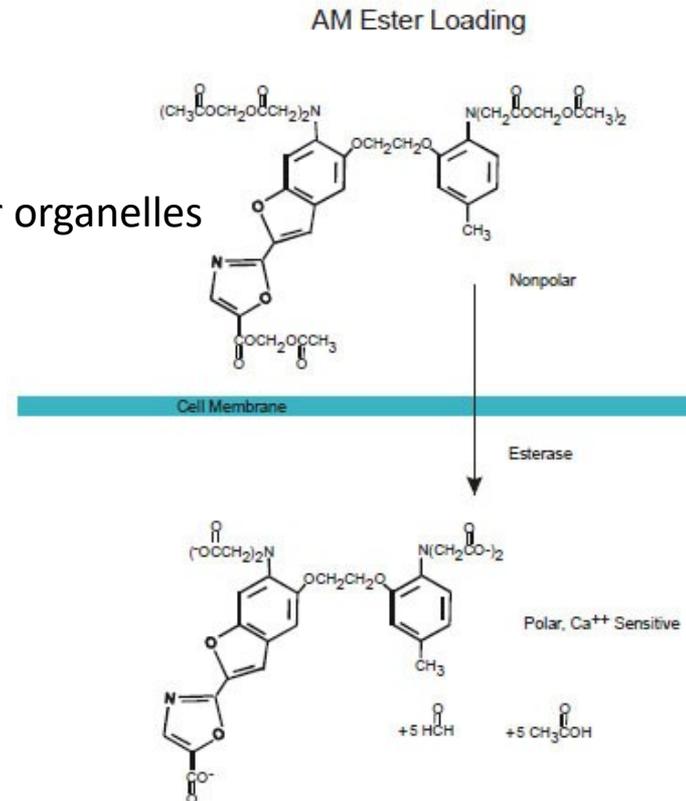
**Fura-2**: an example of SMI based on BAPTA

# Small molecule indicators (SMIs)

- **SMIs are water soluble and diffuse readily through the cytosol**
  - ❑ Pros: you can know the concentration → important for quantitative estimation of  $[Ca^{2+}]$
  - ❑ Cons: difficult to load, one cell at the time
- **Membrane permeable variants (acetoxymethyl ester (AM)-conjugated) have been developed**

- ❑ Pros: easy to load
- ❑ Cons: unknown concentration

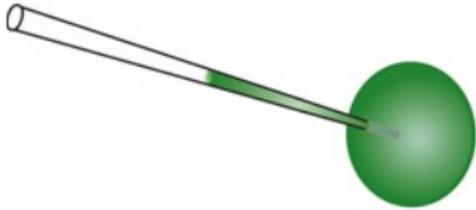
They tend to accumulate in intracellular organelles



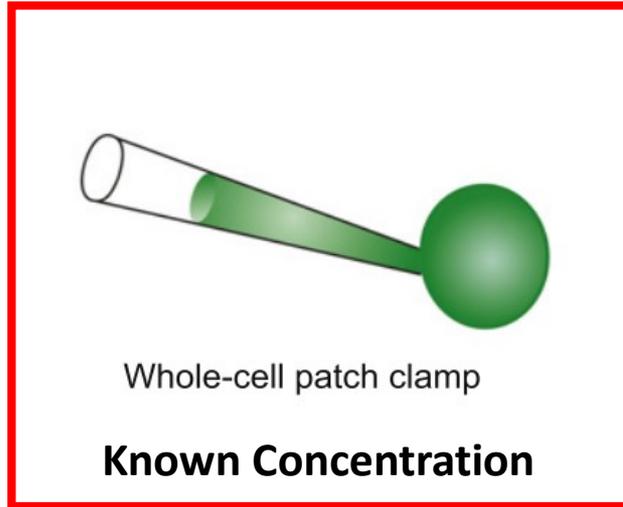
# SMI loading

**A**

Single cell loading

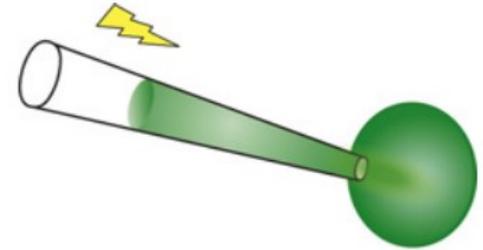


Sharp electrode



Whole-cell patch clamp

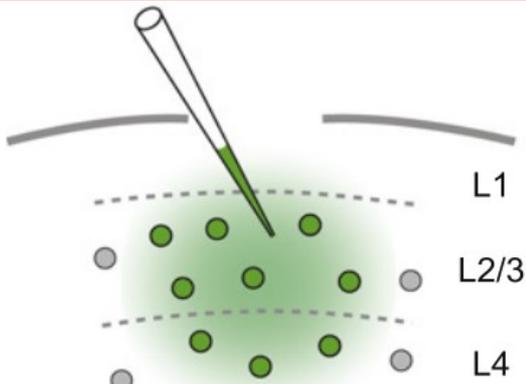
**Known Concentration**



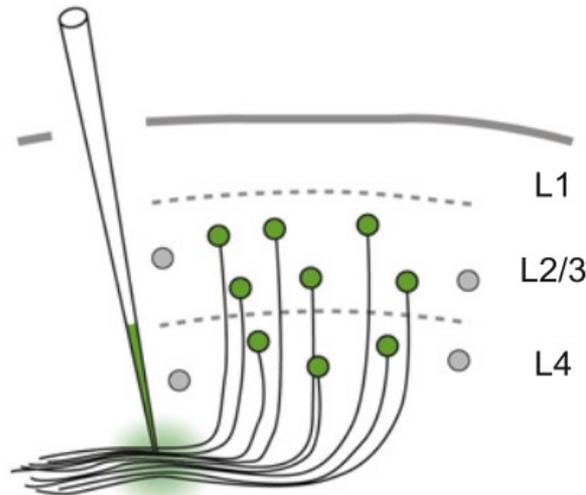
Single cell electroporation

**B**

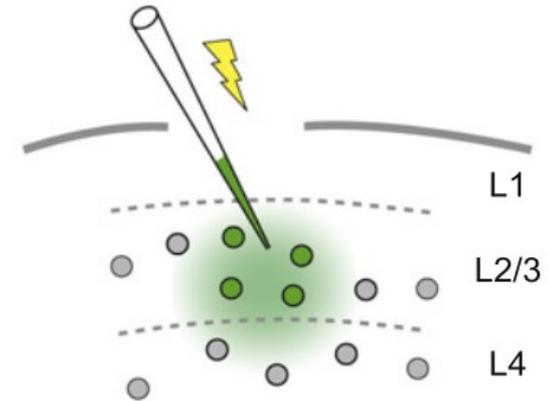
'Acute' network loading



AM loading

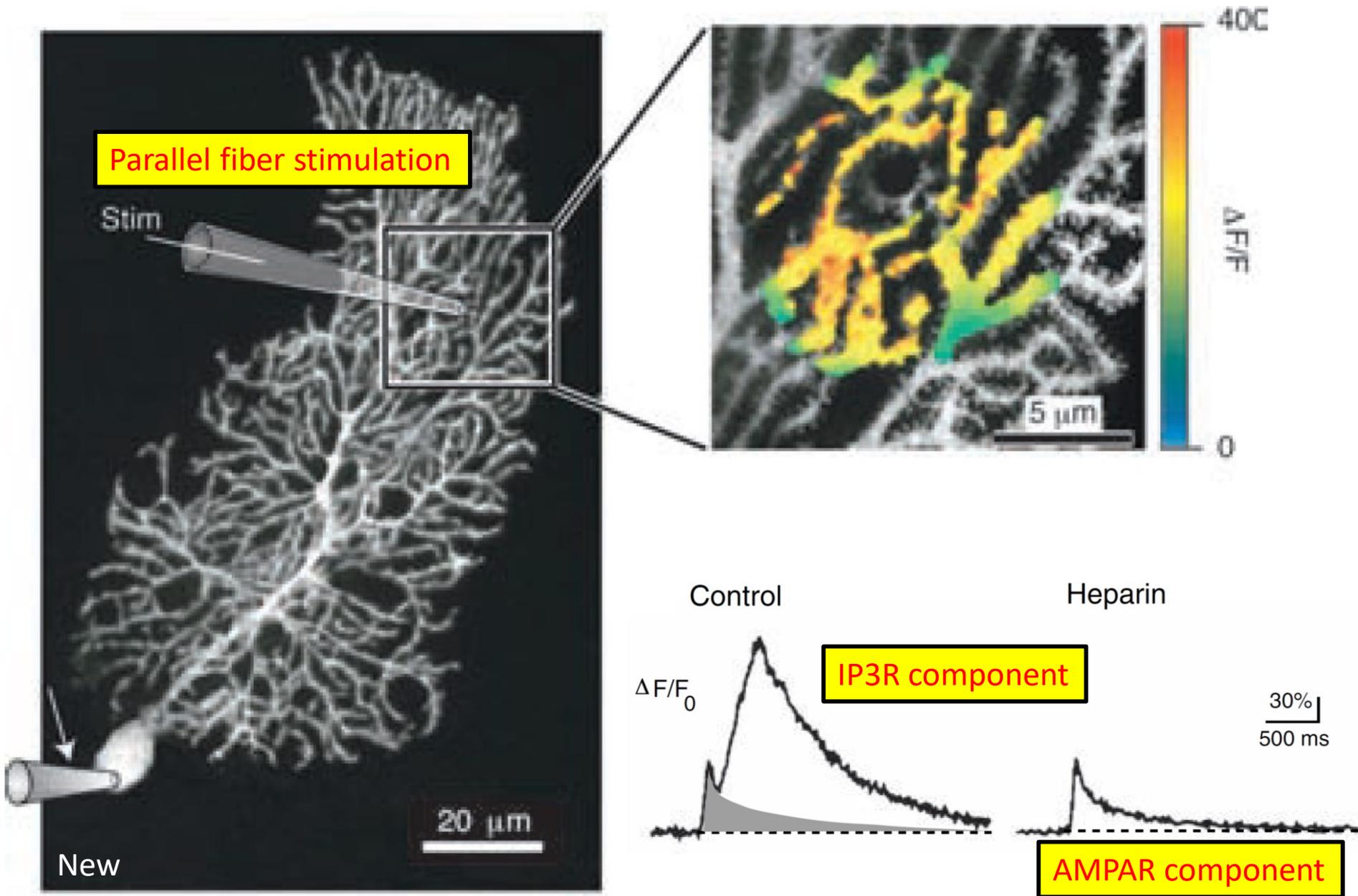


Dextran-conjugate loading



Bulk electroporation

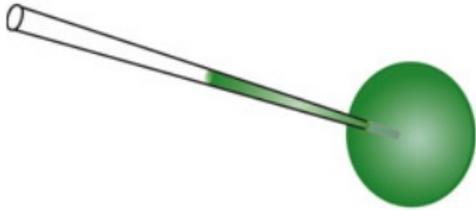
# mGluRs



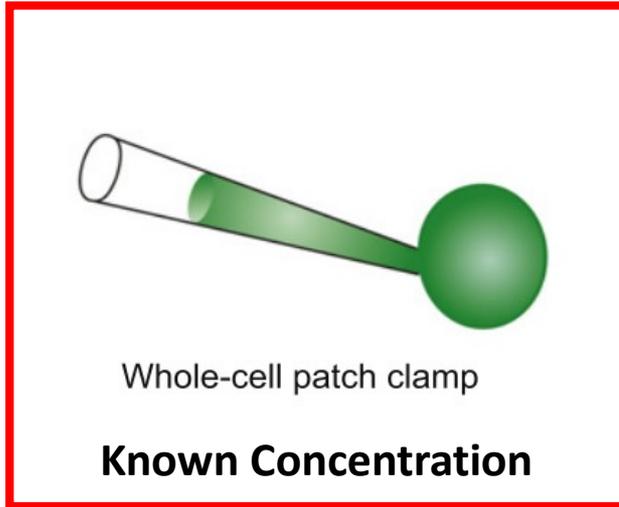
# SMI loading

**A**

Single cell loading

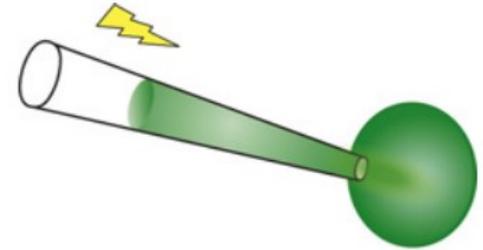


Sharp electrode



Whole-cell patch clamp

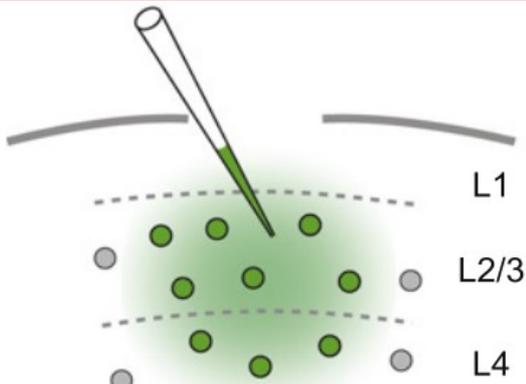
**Known Concentration**



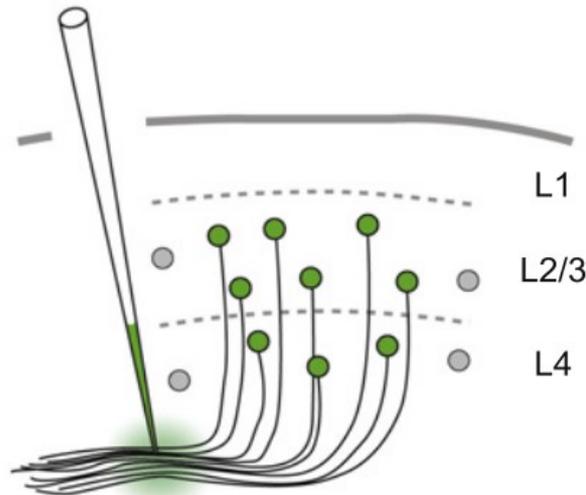
Single cell electroporation

**B**

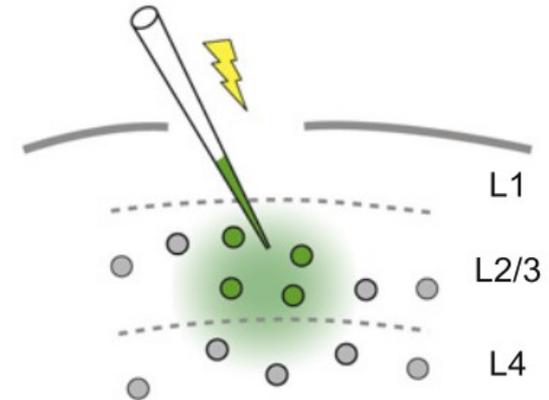
'Acute' network loading



AM loading



Dextran-conjugate loading



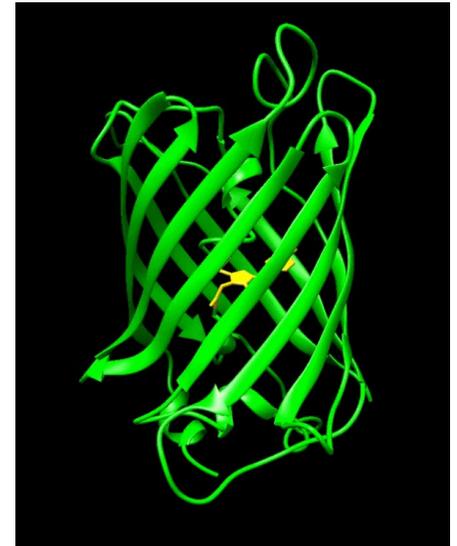
Bulk electroporation

# Genetically-encoded Ca<sup>2+</sup>-indicators

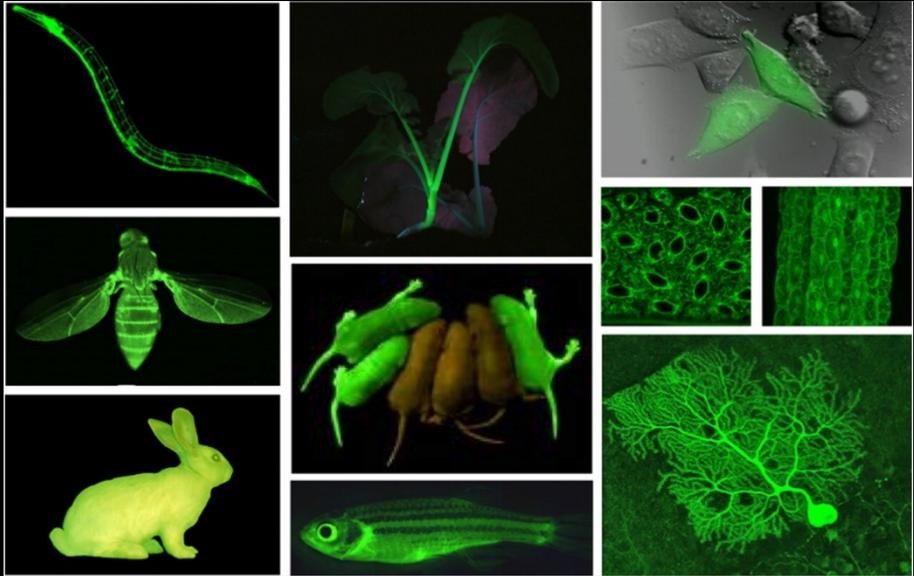
Most of them are based on **EGFP**

We can distinguish 2 major classes:

- Fluorescence resonance energy transfer (FRET)-based GECIs (e.g. **Cameleon** family)
- Single-protein indicators (e.g. **GCaMP** family)



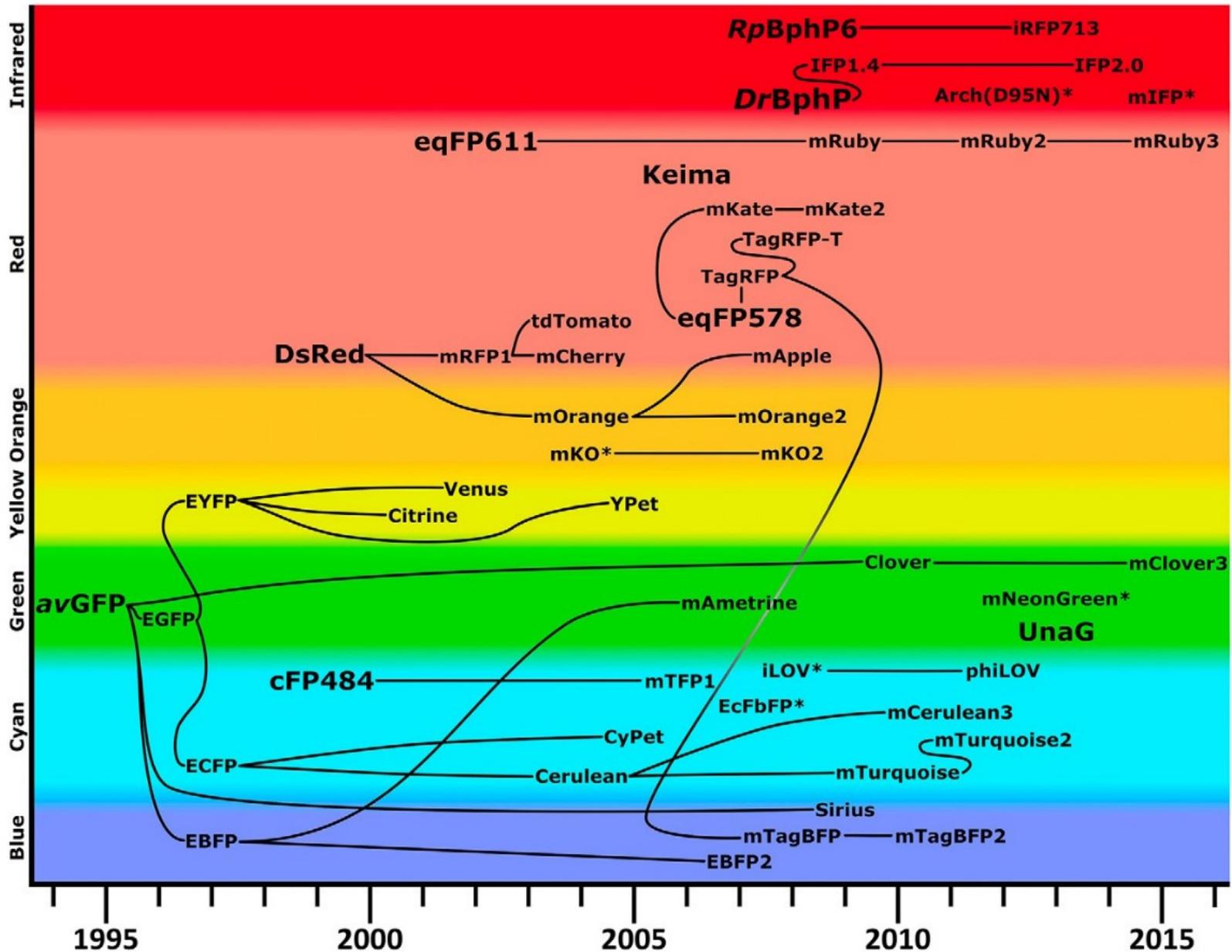
# An example: **GFP** (green fluorescent protein)



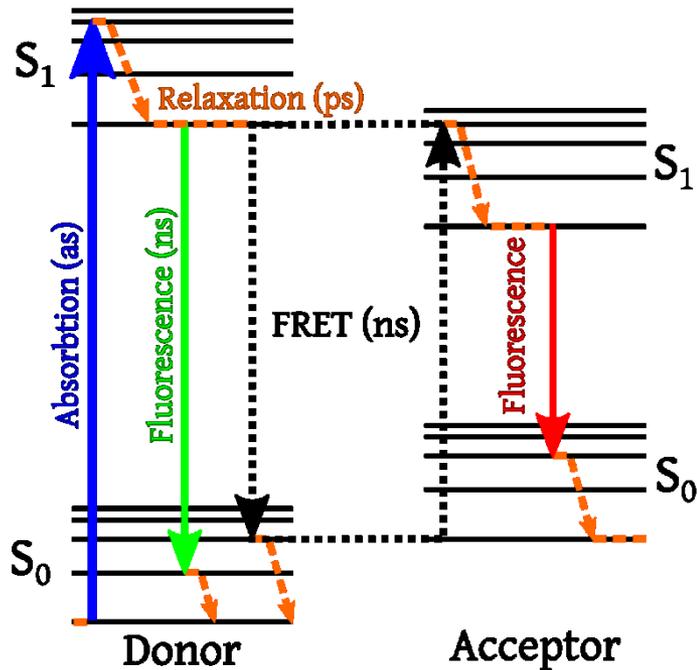
**Aequorea victoria**

Nobel price 2008 for chemistry: Martin Chalfie ,  
Osamiu Shimomura and Roger Tsien,

# Fluorescent proteins

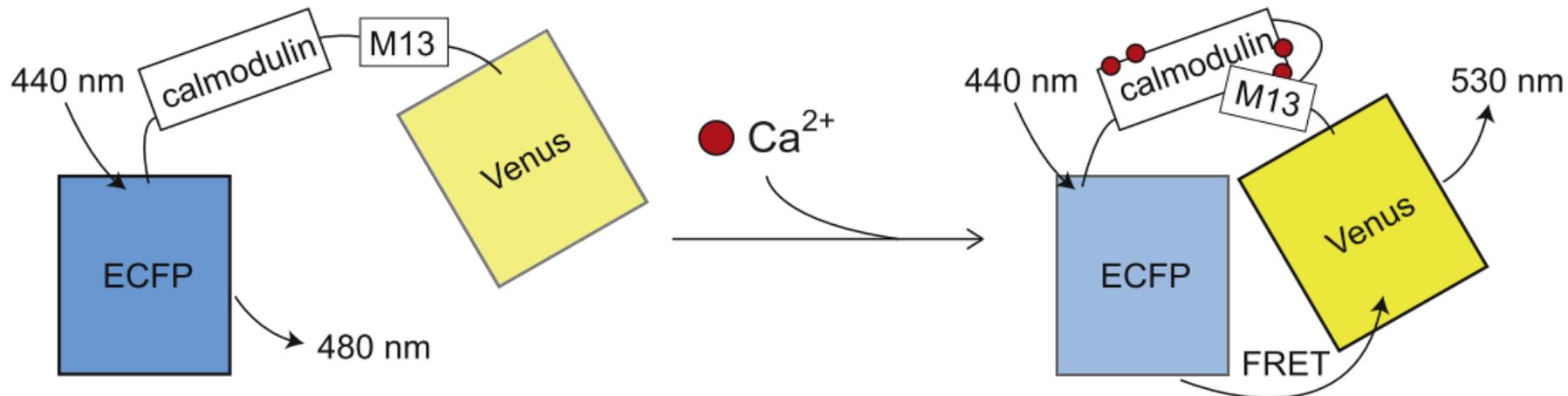


# Fluorescence resonance energy transfer (FRET)-based GECIs



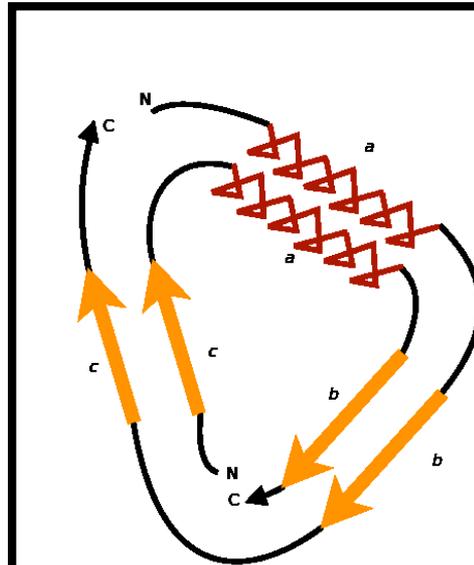
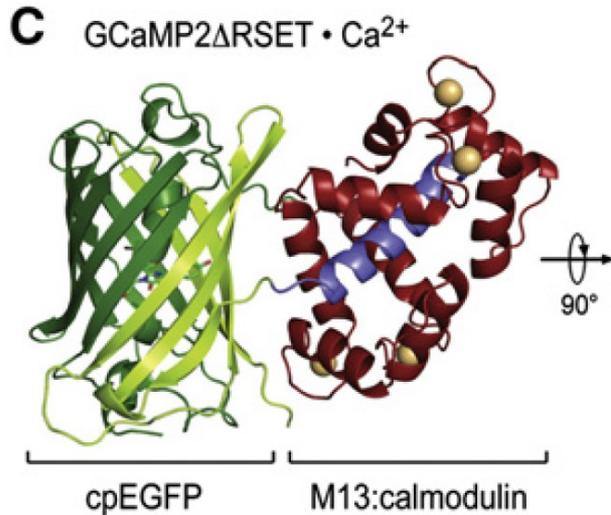
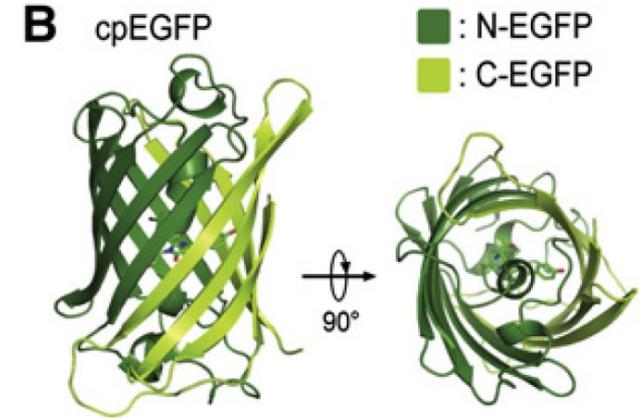
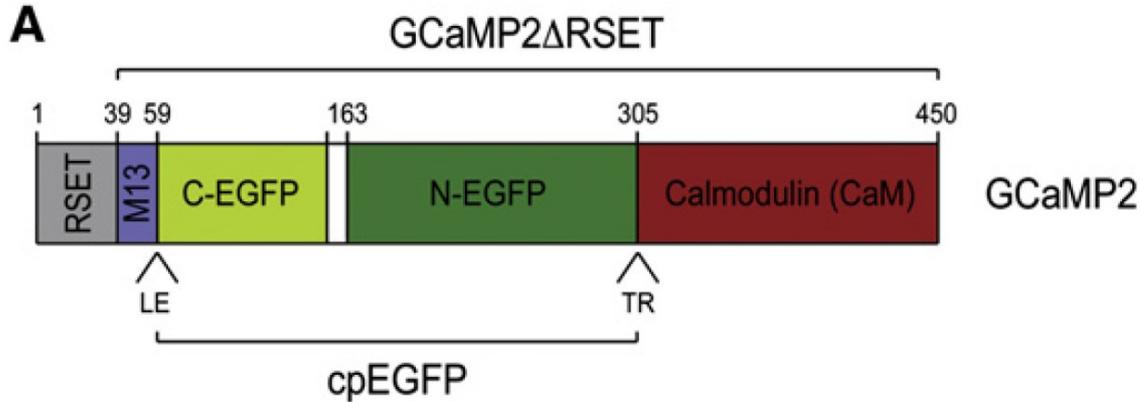
C FRET-based GECI

- The efficiency of this energy transfer is inversely proportional to the **sixth power of the distance** between donor and acceptor, making FRET extremely sensitive to small changes in distance
- Pros: **ratiometric** (we will come back to this)
- FRET has broad applications beyond  $\text{Ca}^{2+}$  imaging for example to detect **protein-protein interactions, enzymatic reaction, protein cleavage, mechanical forces...**



# GCaMP family

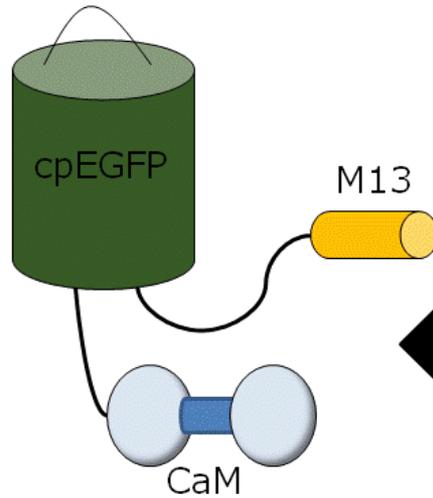
## Circularly permuted EGFP



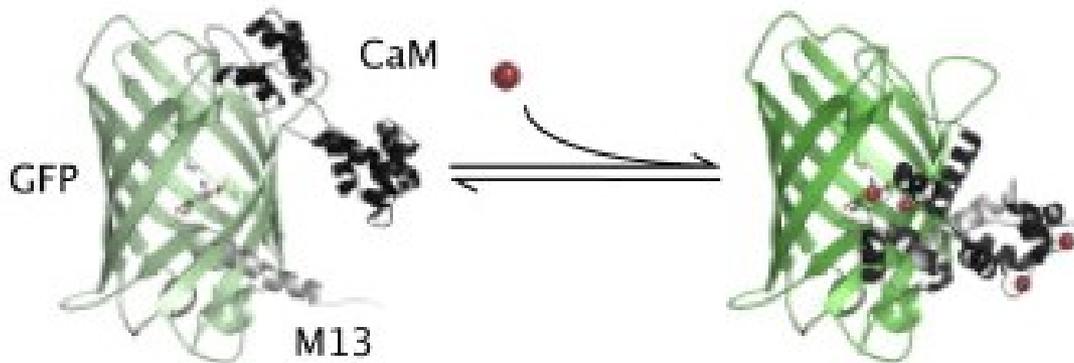
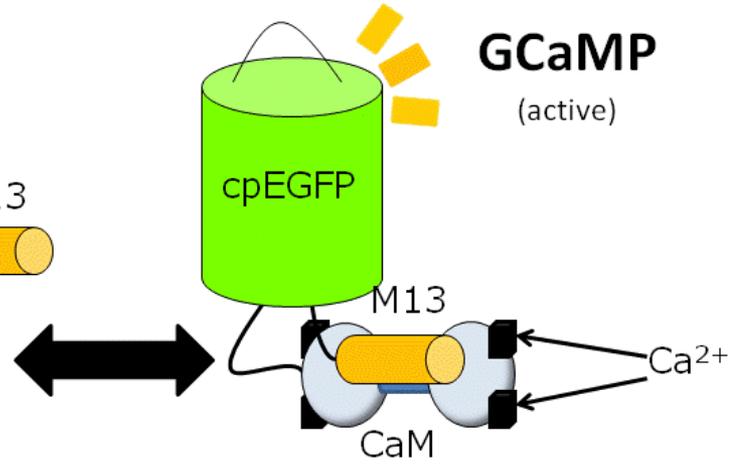
A **circular permutation** is a relationship between proteins whereby the proteins have a changed order of amino acids in their peptide sequence. The result is a protein structure with different connectivity, but overall similar 3D shape.

# GCaMP family

**GCaMP**  
(inactive)

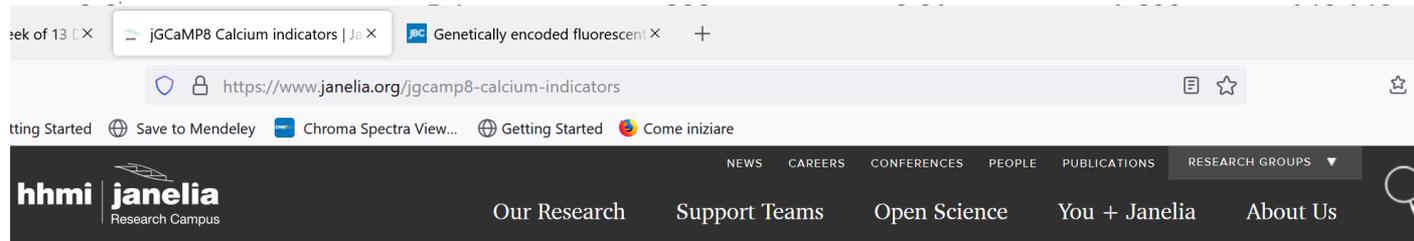


**GCaMP**  
(active)



# Genetically-encoded Ca<sup>2+</sup>-indicators

GECI	Maximum $\Delta F/F$ <i>in vitro</i> <sup>a</sup>	Ca <sup>2+</sup> -free brightness (mM <sup>-1</sup> cm <sup>-1</sup> ) <sup>b</sup>	Ca <sup>2+</sup> -saturated brightness (mM <sup>-1</sup> cm <sup>-1</sup> ) <sup>b</sup>	$K_d$ <i>in vitro</i> (nM) <sup>c</sup>	$\Delta F/F$ per AP in tissue <sup>d</sup>	Half-decay rate in tissue (ms) <sup>e</sup>	Refs.
YC3.60	-0.66 (ECFP) +0.77 (cpVenus)	8.8 <sup>f</sup> 2.4 <sup>f</sup>	3.1 11	780	-0.01 +0.02	410	137,138
YC3.60 3GS	-0.66 (ECFP) +0.77 (cpVenus)	8.8 <sup>g</sup> 2.4 <sup>g</sup>	3.1 11	140	-0.01 +0.01	470	139,140
D3cpV	-0.46 (ECFP) +1.1 (cpVenus)	7.3 <sup>h</sup> 4.8 <sup>h</sup>	3.6 10	530	-0.03 +0.02	9,500	141,142
TN-XXL	-0.5 (ECFP) +1.0 (cpCitrine)						
Twitch-2B	-0.77 (mCerulean3) +0.87 (cpVenus)						
GCaMP3	+12						
GCaMP5k	+9.4						
GCaMP6f	+52						
GCaMP6s	+63						
R-CaMP2	+4.8						
jRGECO1a	+11						
jRCaMP1b	+6.2						



## jGCaMP8 Calcium indicators

Ultra-sensitive protein calcium sensors with fast kinetics

The [Looger Lab](#) and the [GENIE Project Team](#) at HHMI Janelia have developed a new suite of jGCaMP8 calcium indicators, built on the GCaMP scaffold. The jGCaMP8 sensors have fast kinetics without compromising sensitivity, setting a new standard for *in vivo* imaging. Sensors that have been extensively tested in mammalian neurons *in vivo* and *in vitro* are:

- **jGCaMP8f** (fast): 4x faster rise time, 2.5x faster decay time than jGCaMP7f
- **jGCaMP8m** (medium): almost 4x faster rise time and 3.5x more sensitive than jGCaMP7f
- **jGCaMP8s** (sensitive): 2x more sensitive than jGCaMP7s, >2x faster than jGCaMP7f (at 1 AP)

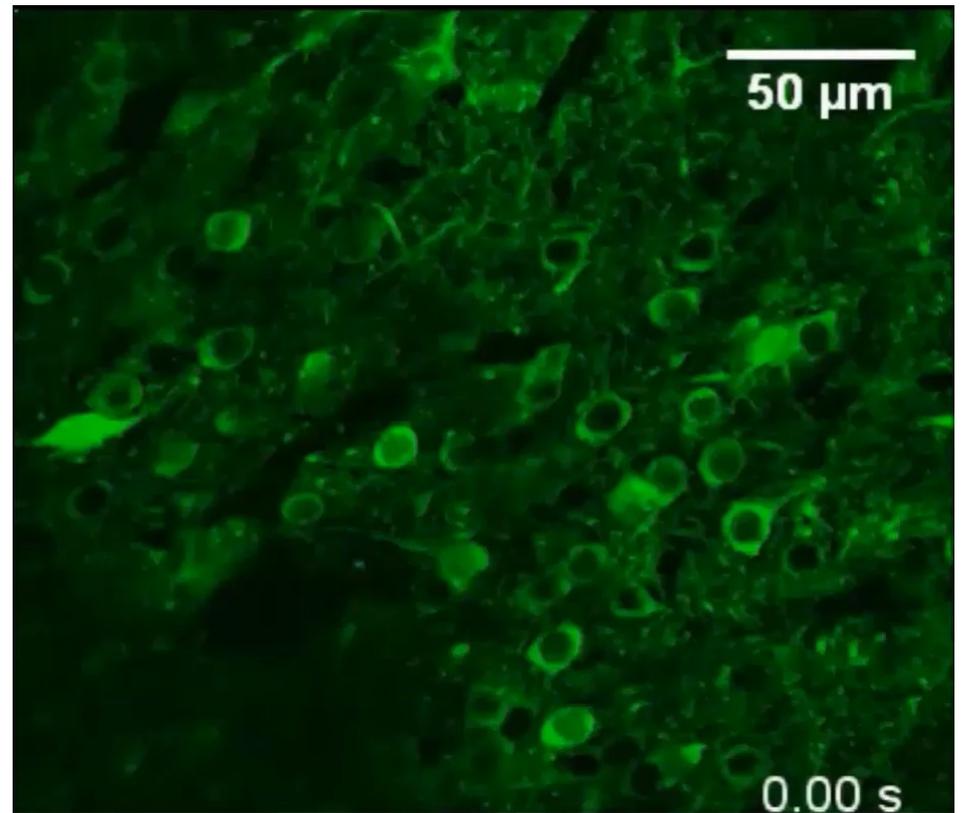
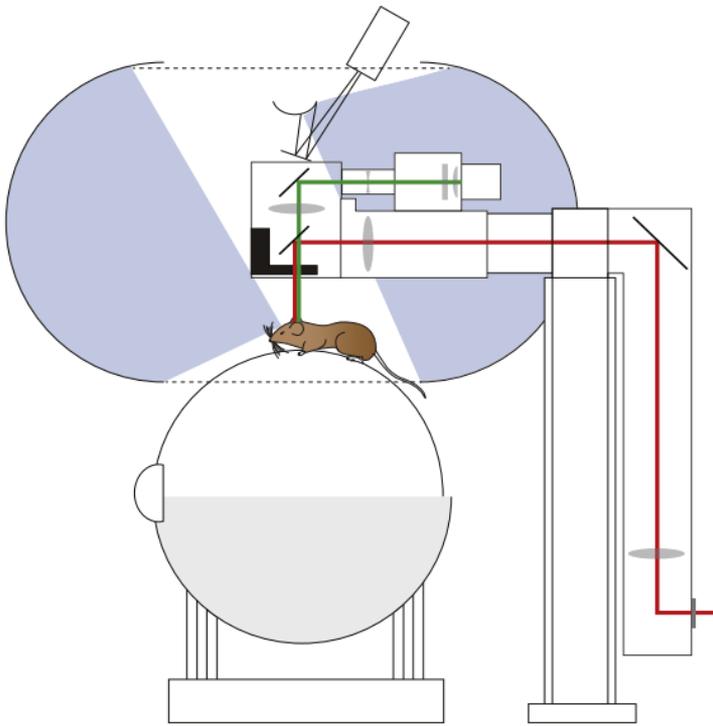
[The plasmids are available on Addgene!](#)

**Update (6/23/2021):**

jGCaMP8-expressing flies are now available from [The Bloomington Drosophila Stock Center](#). Search for "jGCaMP8" to see all available strains. See results of testing in *Drosophila* [below](#).

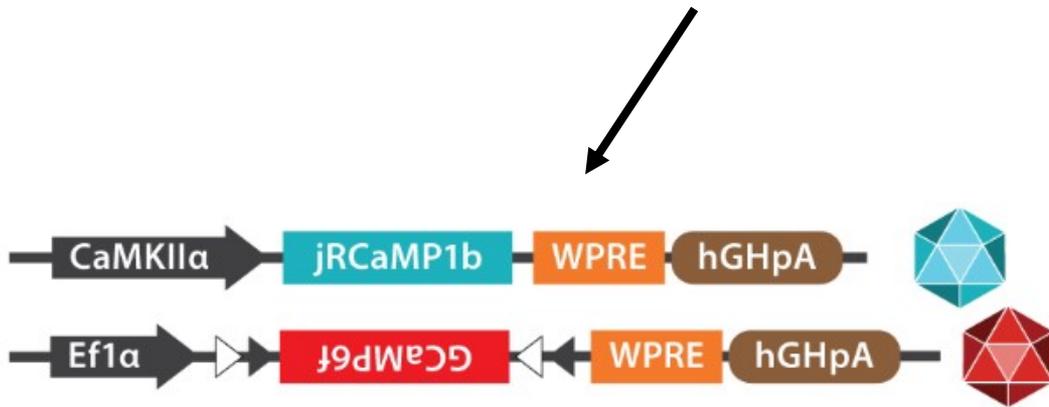
# Pros and Cons of GECIs

- 1) Long-term (days, weeks, months) expression and imaging *in vivo*



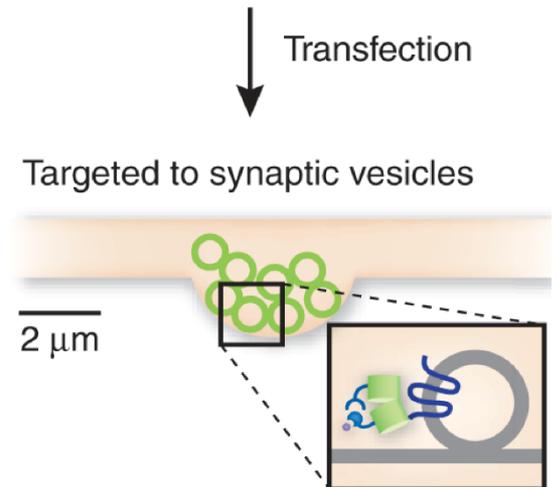
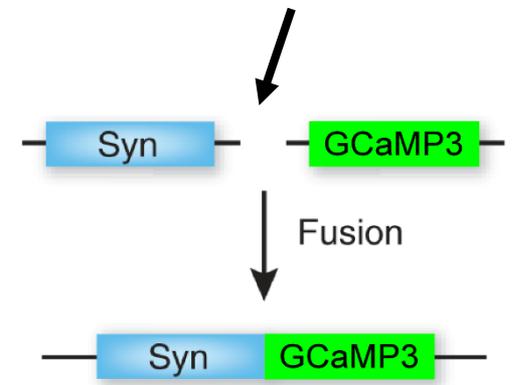
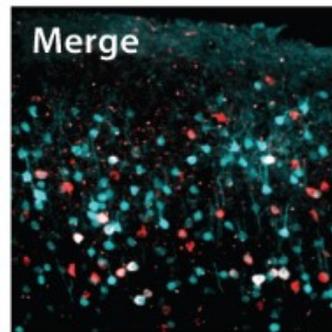
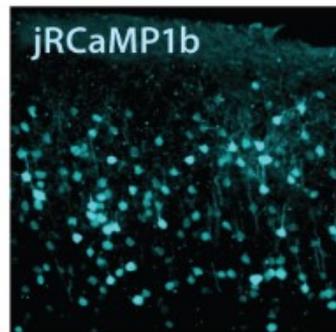
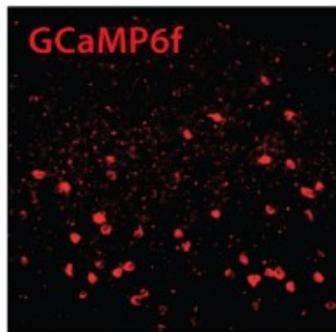
# Pros and Cons of GECIs

2) Targeting to (i) specific subtypes of neurons or (ii) subcellular locations



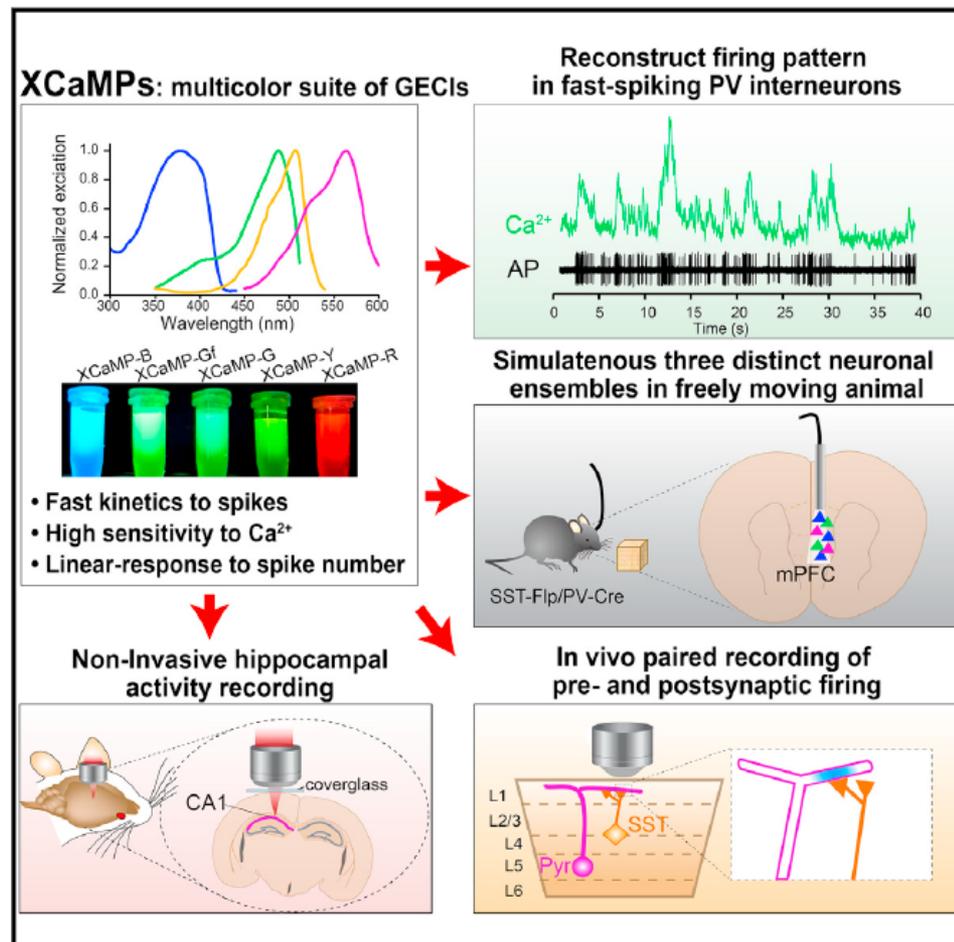
**Inhibitory**  
(Gad2<sup>+</sup>)

**Excitatory**  
(CaMKII $\alpha$ )



# Rational Engineering of XCaMPs, a Multicolor GEI Suite for *In Vivo* Imaging of Complex Brain Circuit Dynamics

## Graphical Abstract



## Authors

Masatoshi Inoue, Atsuya Takeuchi, Satoshi Manita, ..., Karl Deisseroth, Kazuo Kitamura, Haruhiko Bito

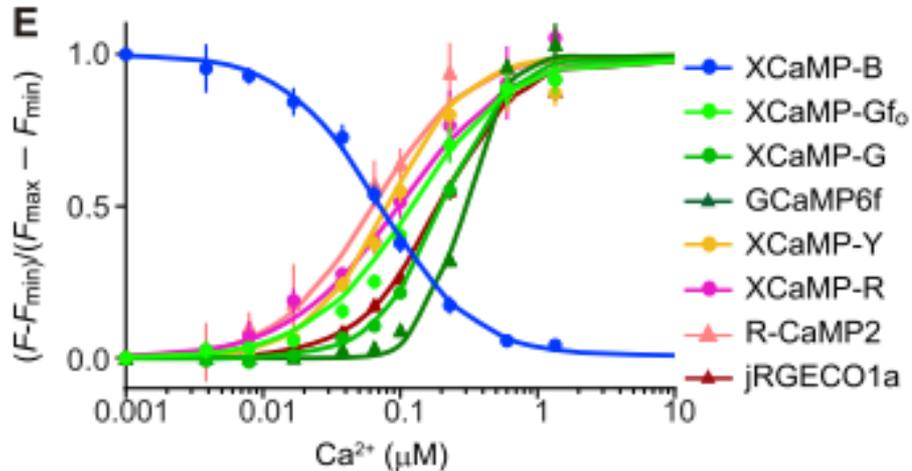
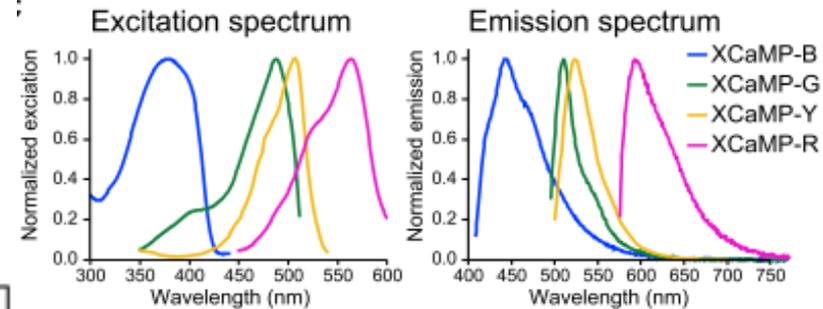
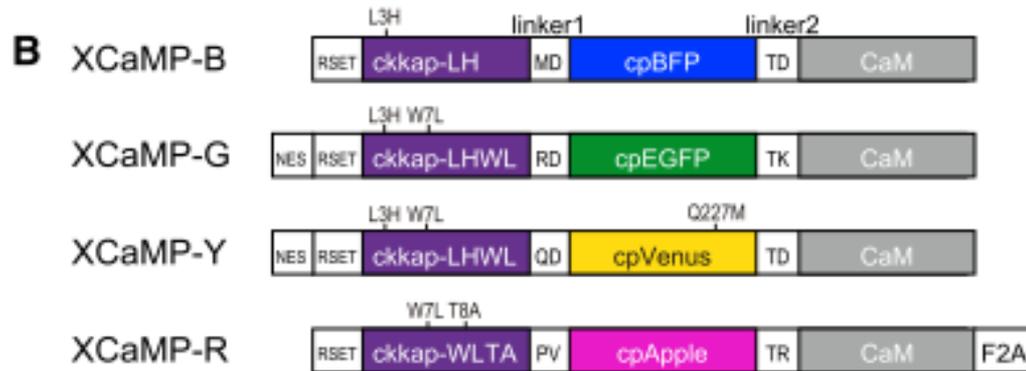
## Correspondence

hbito@m.u-tokyo.ac.jp

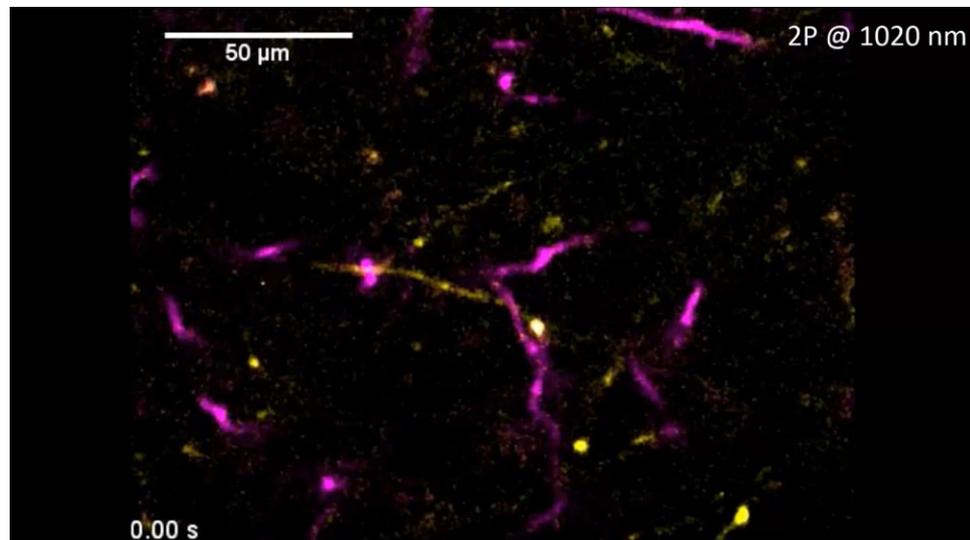
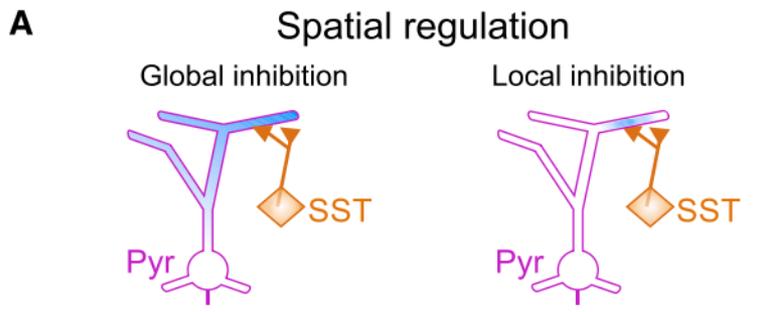
## In Brief

Quadricolor suite of genetically encoded calcium indicators for multiplex recording in the brain.

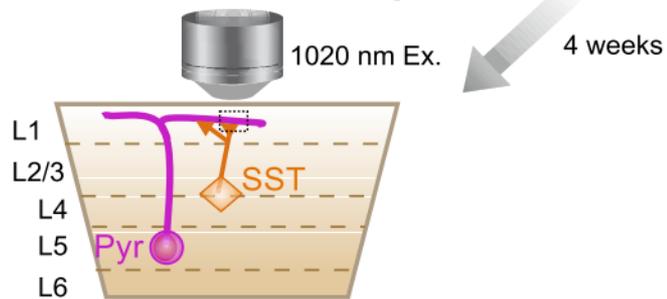
# Multicolor XCaMPs



# Dual color simultaneous imaging of axons and dendrites in the barrel cortex

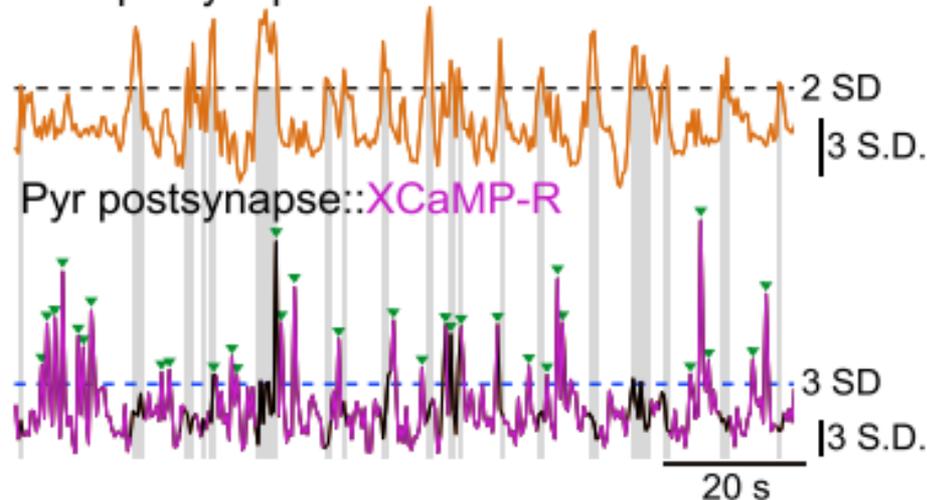


*In vivo* dual color simultaneous image

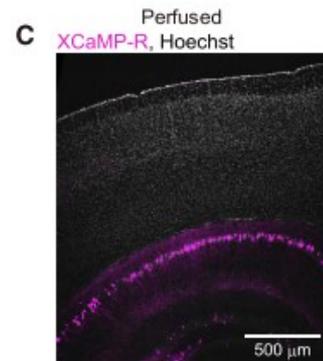
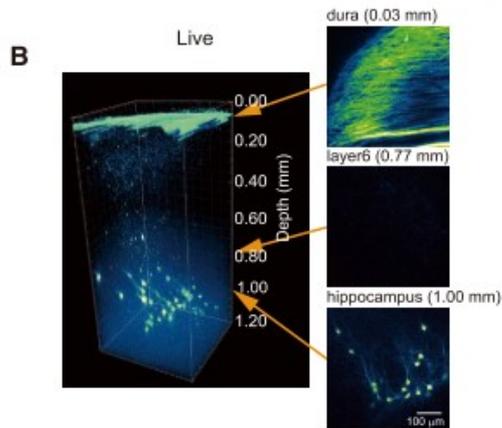
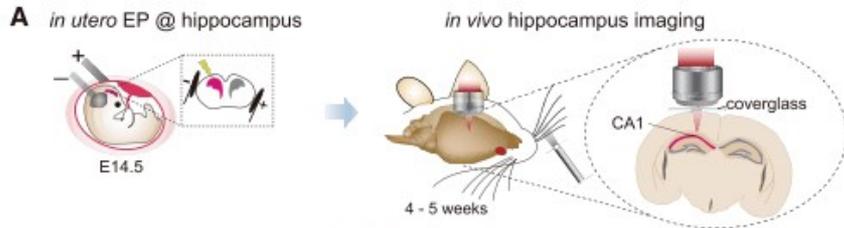


**E** SST presynapse::XCaMP-Y

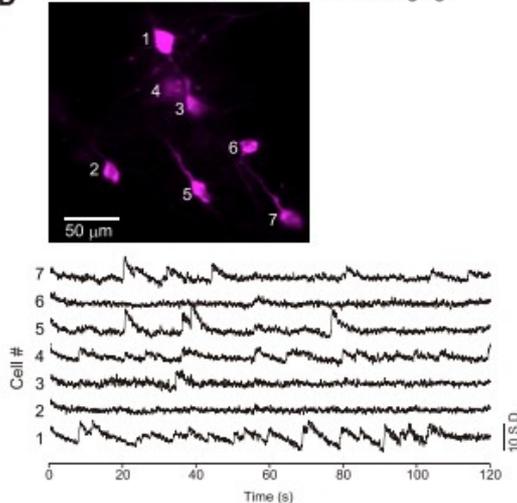
**F** SST presynapse::XCaMP-Y



# Can we image deep brain structures non-invasively?



**D**  $\lambda=1064$  nm GSLD, Resonant 30 Hz imaging



Fluorescence imaging with longer wavelength has the advantage of minimizing both the scattering of excitation light and the absorption of emission when compared to green probes

# Pros and Cons of GECIs

- 1) Indicator concentration not known
- 2) Because (often) based on calmodulin (4 cooperative  $\text{Ca}^{2+}$  binding sites) →  $\text{Ca}^{2+}$  binding is cooperative



**Difficult to relate  $\Delta F$  to  $\Delta[\text{Ca}^{2+}]$**

# Conclusions

1.  $\text{Ca}^{2+}$  indicators are **fluorescent**  $\text{Ca}^{2+}$  indicators

2. Small molecule indicators (SMIs)

a. Based on EGTA/BAPTA

b. Soluble or membrane permeable

c. Pros: They bind 1  $\text{Ca}^{2+}$

For the soluble ones, we know the concentration

} Quantitative

3. Genetically-encoded  $\text{Ca}^{2+}$  indicators (GECIs)

a. Based on EGFP or other fluorescent proteins

b. The most commonly used are those based on circularly permuted fluorescent proteins (e.g. GCaMPs)

c. Pros: Long-term *in vivo* experiments

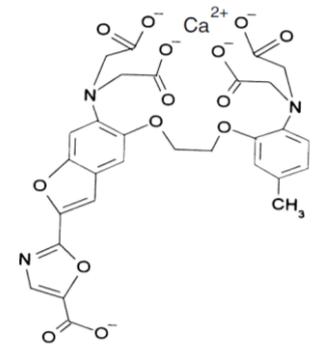
Cell-type specific expression (e.g. excitatory vs. inhibitory neurons)

Specific subcellular locations (e.g. presynaptic bouton)

# Ca<sup>2+</sup> imaging

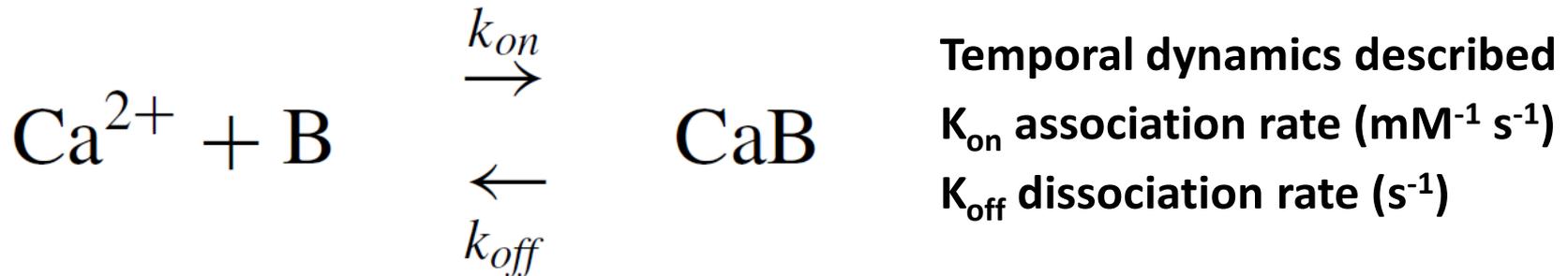
- Review Ca<sup>2+</sup> signaling
- Ca<sup>2+</sup> indicators
- **Ca<sup>2+</sup> binding**
  - Dissociation constant
  - Calcium binding ratio
  - Cooperative binding
- **Ca<sup>2+</sup> diffusion**
- **Ca<sup>2+</sup>-dependent fluorescence properties**
- **Simplified models of Ca<sup>2+</sup> dynamics**
- **Imaging devices**
- **What can we do with it**

# Independent $\text{Ca}^{2+}$ binding



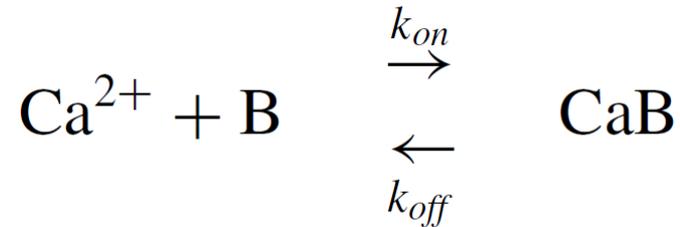
The indicator binds 1  $\text{Ca}^{2+}$

(Alternatively it binds multiple  $\text{Ca}^{2+}$  but it does so independently, non-cooperatively)



$$\begin{aligned}\frac{\partial [\text{Ca}]}{\partial t} &= -k_{on} [\text{Ca}] [\text{B}] + k_{off} [\text{CaB}] \\ \frac{\partial [\text{B}]}{\partial t} &= -k_{on} [\text{Ca}] [\text{B}] + k_{off} [\text{CaB}] \\ \frac{\partial [\text{CaB}]}{\partial t} &= -\frac{\partial [\text{B}]}{\partial t} = k_{on} [\text{Ca}] [\text{B}] - k_{off} [\text{CaB}]\end{aligned}$$

# Independent $\text{Ca}^{2+}$ binding



At steady state, once equilibrium has been reached:

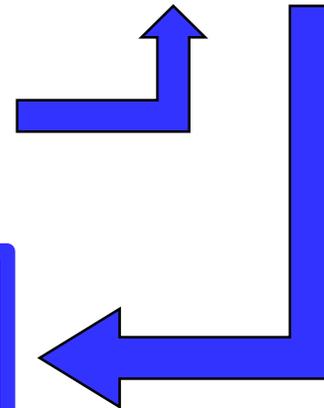
$$0 = -k_{\text{on}} [\text{Ca}] [\text{B}] + k_{\text{off}} [\text{CaB}]$$

Law of  
mass action

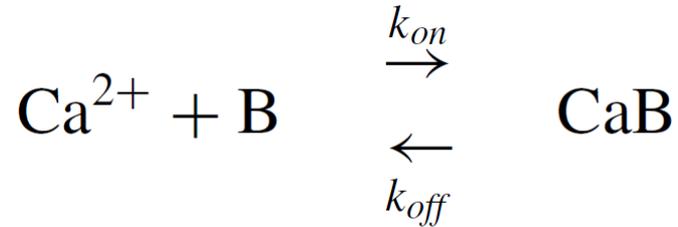
$$K_d = \frac{k_{\text{off}}}{k_{\text{on}}} = \frac{[\text{Ca}] [\text{B}]}{[\text{CaB}]} = \frac{[\text{Ca}] ([\text{B}]_T - [\text{CaB}])}{[\text{CaB}]}$$

$$[\text{B}]_T = [\text{B}] + [\text{CaB}]$$

$$S = \frac{[\text{CaB}]}{[\text{B}]_T} = \frac{[\text{Ca}]}{[\text{Ca}] + K_d}$$



# Independent $\text{Ca}^{2+}$ binding



$$K_d = \frac{k_{\text{off}}}{k_{\text{on}}} = \frac{[\text{Ca}][\text{B}]}{[\text{CaB}]}$$

$$S = \frac{[\text{CaB}]}{[\text{B}]_T} = \frac{[\text{Ca}]}{[\text{Ca}] + K_d}$$

$K_d$  = Dissociation constant. When  $[\text{Ca}^{2+}] = K_d$  50% of the buffer is saturated.

Attention: Low  $K_d$  = high affinity

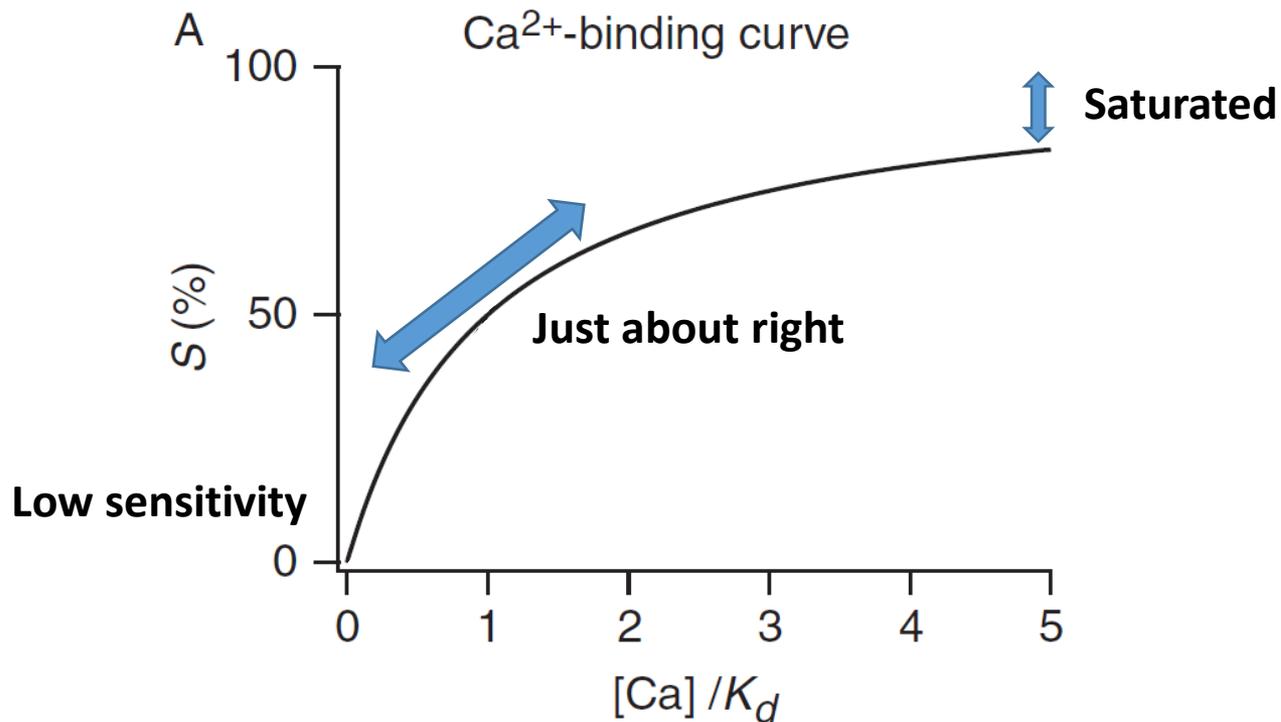
High  $K_d$  = low affinity

The affinity of the indicator should match the expected range of  $[\text{Ca}^{2+}]$  'seen' by the indicator

Affinity too low ( $K_d$  high)  $\rightarrow$  not enough sensitivity    Affinity too high ( $K_d$  low)  $\rightarrow$  saturation

# Independent $\text{Ca}^{2+}$ binding

$$S = \frac{[\text{CaB}]}{[\text{B}]_T} = \frac{[\text{Ca}]}{[\text{Ca}] + K_d}$$



# An example: **Fluo** family of $\text{Ca}^{2+}$ indicators



**ThermoFisher**  
SCIENTIFIC

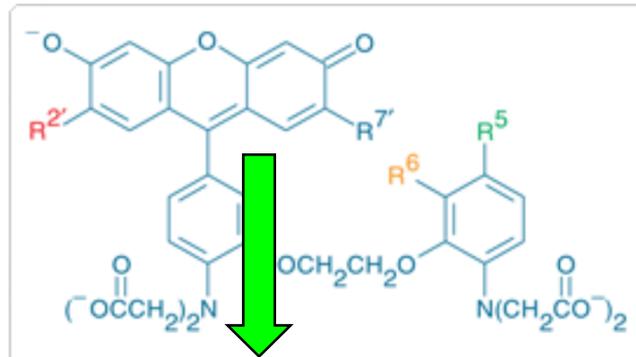
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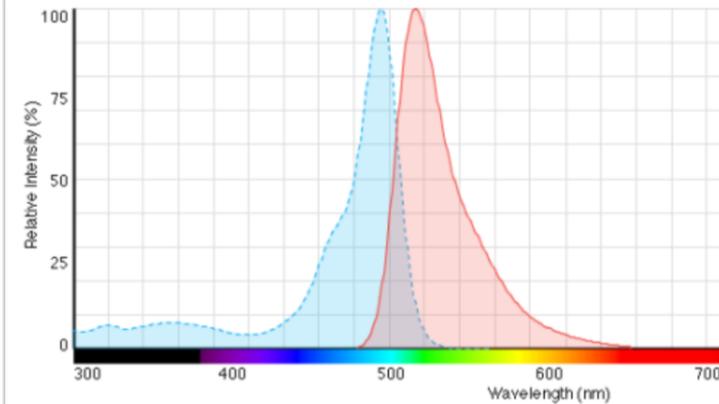
List of Tables

Technical Notes and Product Highlights



Indicator	$K_d(\text{Ca}^{2+})$	R <sup>2'</sup>	R <sup>7'</sup>	R <sup>5</sup>	R <sup>6</sup>
Fluo-3	0.39 $\mu\text{M}$	Cl	Cl	CH <sub>3</sub>	H
Fluo-4	0.35 $\mu\text{M}$	F	F	CH <sub>3</sub>	H
Fluo-5F	2.3 $\mu\text{M}$	F	F	F	H
Fluo-5N	90 $\mu\text{M}$	F	F	NO <sub>2</sub>	H
Fluo-4FF	9.7 $\mu\text{M}$	F	F	F	F

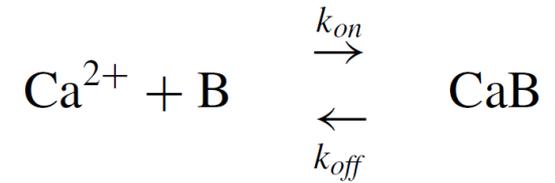
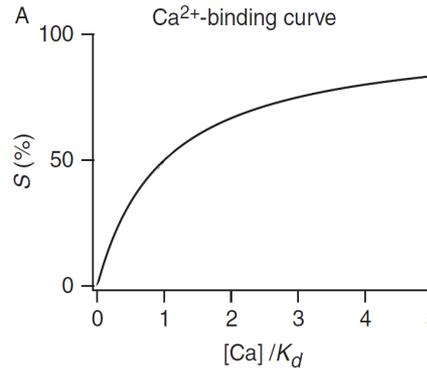
Figure 19.3.1 Fluo indicators.



# Ca<sup>2+</sup>-binding ratio

Buffering capacity = buffering strength = Ca<sup>2+</sup>-binding efficiency = Ca<sup>2+</sup>-binding ratio =  $\kappa_B$

While  $K_d$  is an intrinsic property of the buffer,  $\kappa_B$  depends on intrinsic properties of B ( $K_d$ ), on [B] and [Ca<sup>2+</sup>]



$$S = \frac{[\text{CaB}]}{[\text{B}]_T} = \frac{[\text{Ca}]}{[\text{Ca}] + K_d}$$

↓

$$[\text{CaB}] = \frac{[\text{B}]_T [\text{Ca}]}{[\text{Ca}] + K_d} \xrightarrow{\text{Differentiate relative to } [\text{Ca}^{2+}]} \kappa_B = \frac{\partial [\text{CaB}]}{\partial [\text{Ca}]} = \frac{[\text{B}]_T K_d}{([\text{Ca}] + K_d)^2}$$

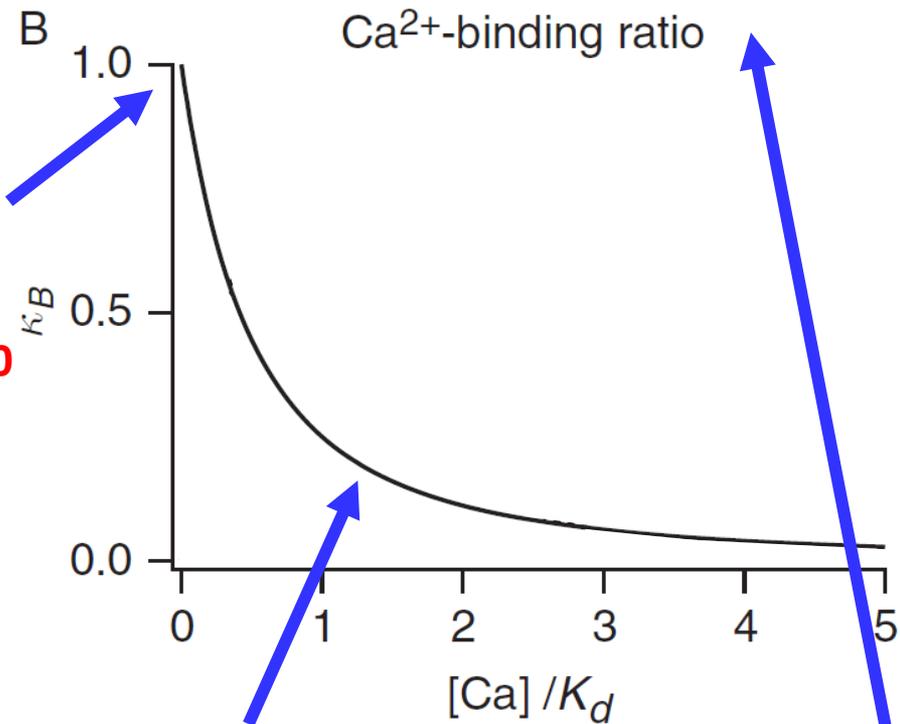
$\kappa_B$  = how fast [CaB] changes relative to  $\Delta[\text{Ca}^{2+}]$   
 = capacity of absorbing extra Ca<sup>2+</sup>  
 = Ca<sup>2+</sup> buffering capacity

# Ca<sup>2+</sup>-binding ratio

$$\kappa_B = \frac{\partial[\text{CaB}]}{\partial[\text{Ca}]} = \frac{[\text{B}]_T K_d}{([\text{Ca}] + K_d)^2}$$

$$[\text{Ca}^{2+}] = 0 \rightarrow \kappa_B = \frac{[\text{B}]_T}{K_d}$$

Buffering capacity is maximal at  $[\text{Ca}^{2+}] = 0$   
(it makes sense!)



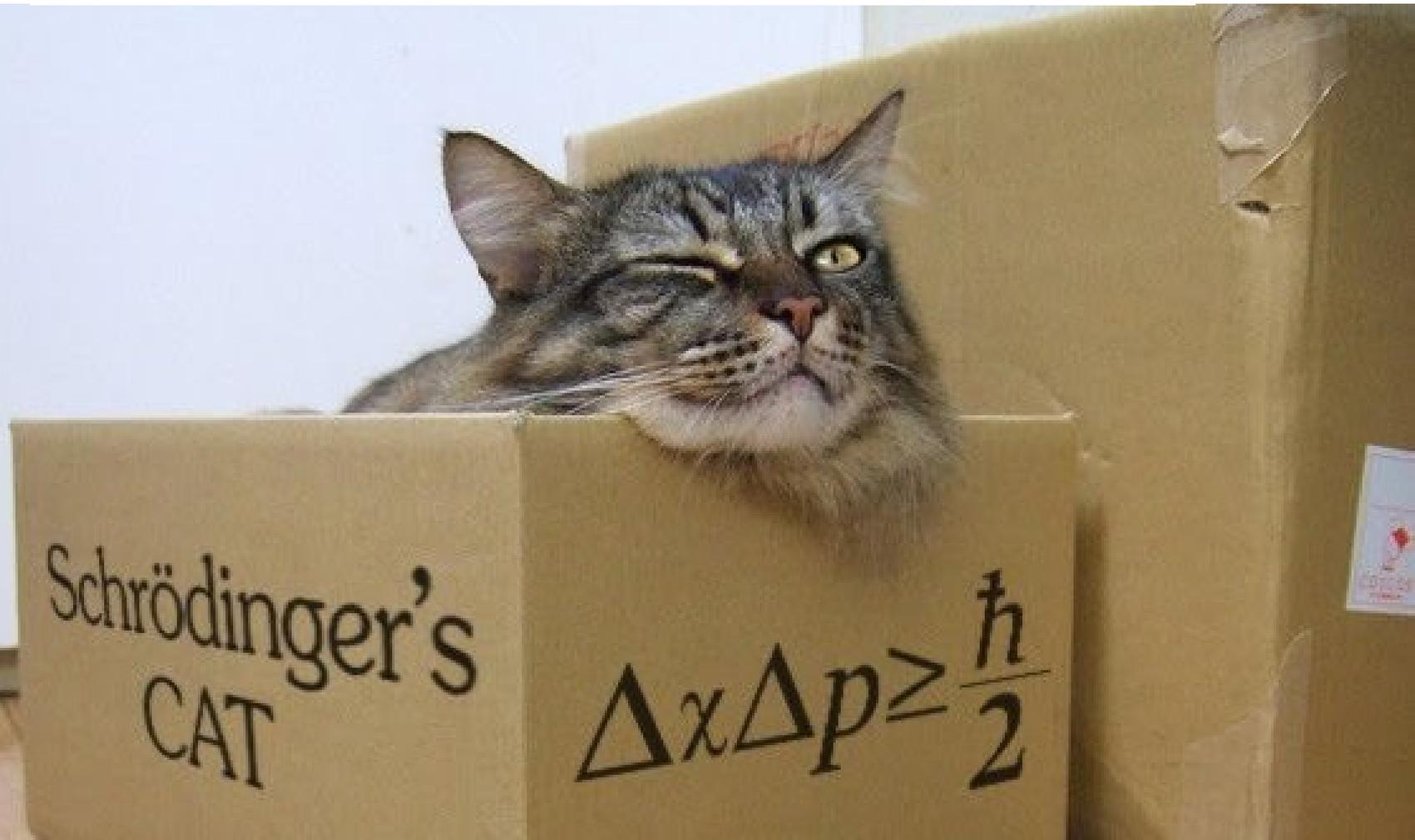
$\kappa_B$  decreases when  $[\text{Ca}^{2+}]$  increases according to the 2 power of  $[\text{Ca}^{2+}]$

Buffering capacity decreases quickly the more  $\text{Ca}^{2+}$  you give to B

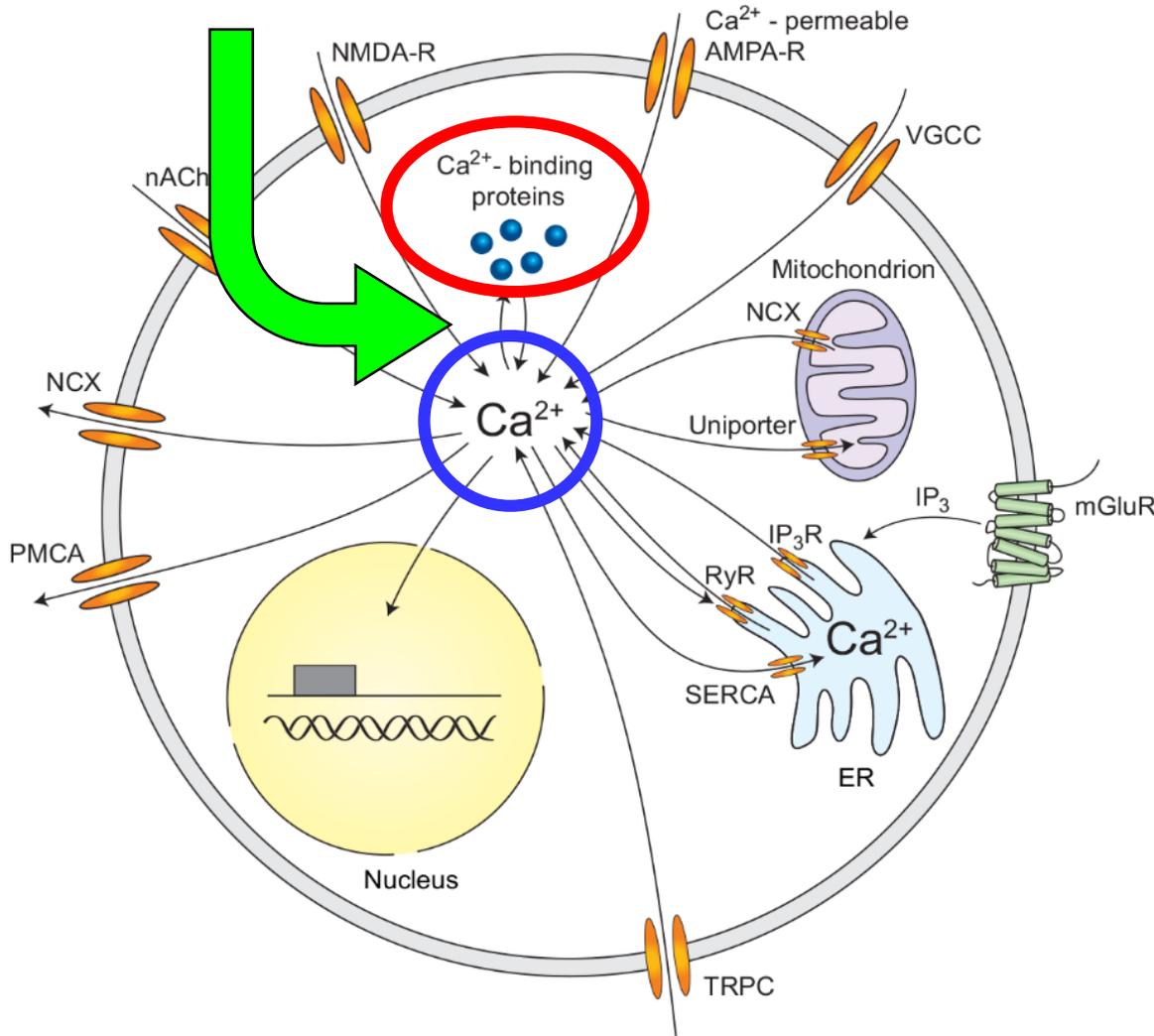
(This also makes sense!)

## Opinion

## An Inconvenient Truth: Calcium Sensors Are Calcium Buffers

Shane M. McMahon<sup>1</sup> and Meyer B. Jackson<sup>1,\*</sup>

# Exogenous vs. endogenous $\text{Ca}^{2+}$ -binding ratio

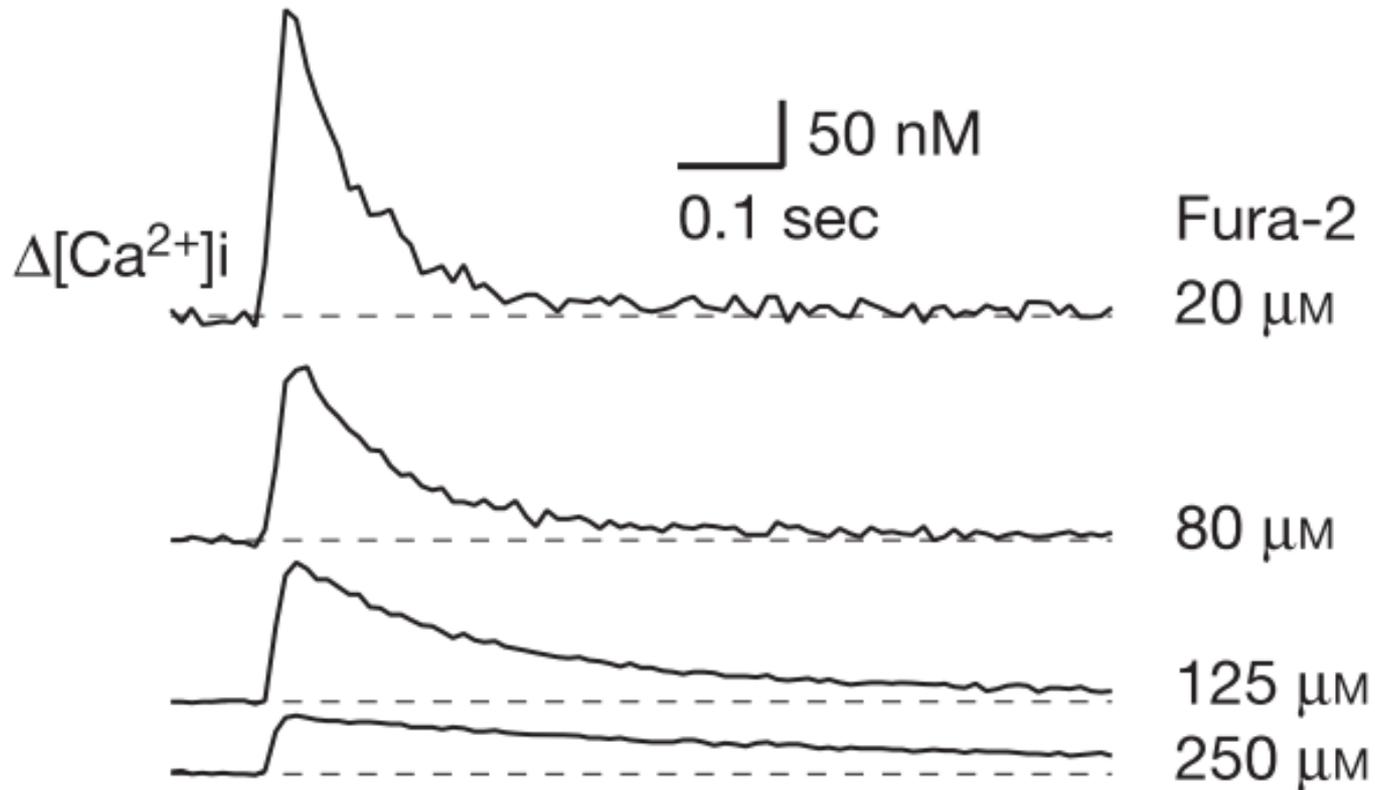


At any given moment, **free**  $[\text{Ca}^{2+}]_i$  is determined by the balance between

- $\text{Ca}^{2+}$  influx
- $\text{Ca}^{2+}$  efflux
- $\text{Ca}^{2+}$  exchange with internal stores
- $\text{Ca}^{2+}$  sequestration by endogenous buffers ( $\text{Ca}^{2+}$ -binding proteins)

It is important to have a good estimation of  $\kappa_b$  of the added fluorescent indicator relative to the  $\kappa_b$  of the endogenous buffers, in order not to alter (too much) the free  $\text{Ca}^{2+}$  signal.

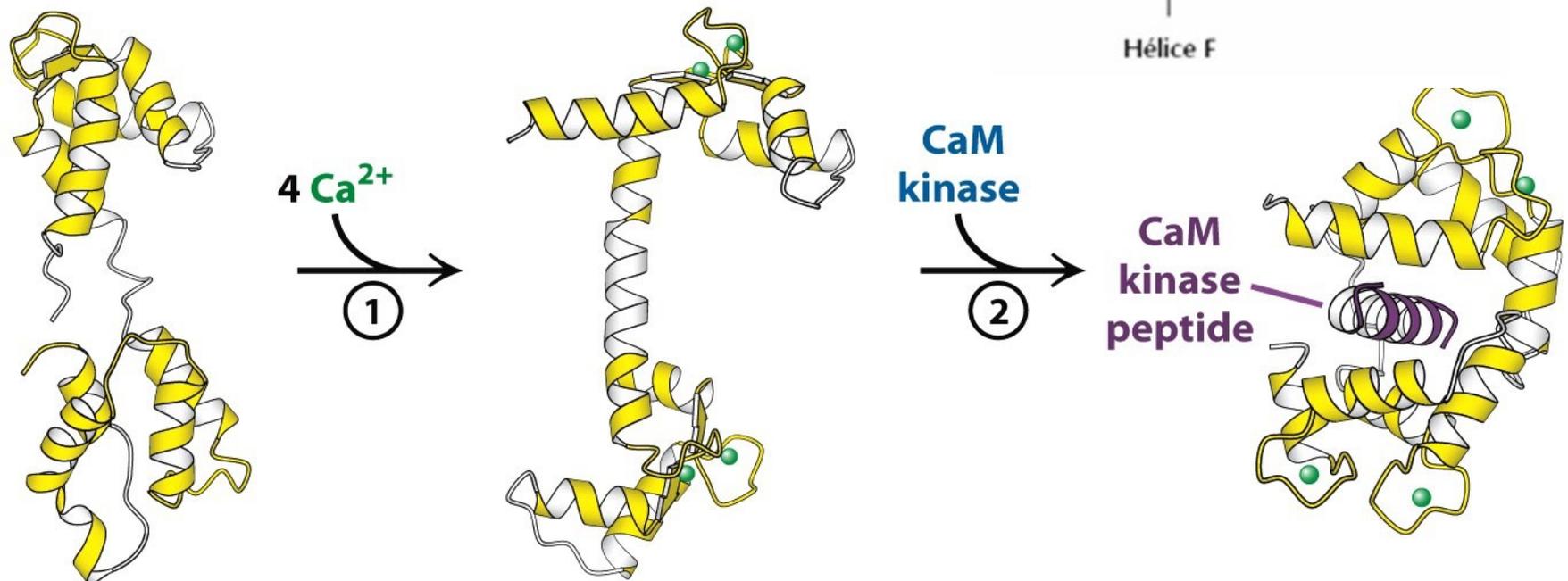
An added exogenous  $\kappa_B$  larger than the endogenous  $\kappa_B$  of the neuron deforms the very same signal we wish to measure



It is important to have a good estimation of  $\kappa_B$  of the added fluorescent indicator relative to the  $\kappa_B$  of the endogenous buffers, in order not to alter (too much) the free  $Ca^{2+}$  signal.

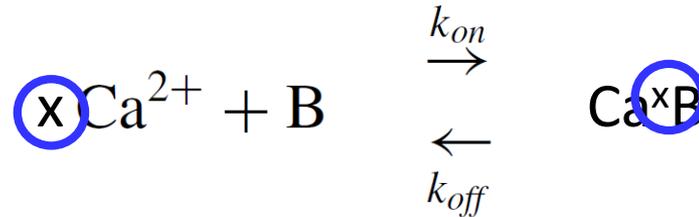
# Many GECs exhibit however cooperative $\text{Ca}^{2+}$ binding

because many of them are based on Calmodulin



# Cooperative Ca<sup>2+</sup> binding

The buffer binds  $\underline{x}$  Ca<sup>2+</sup> non-independently, cooperatively (for calmodulin  $x = 4$ )



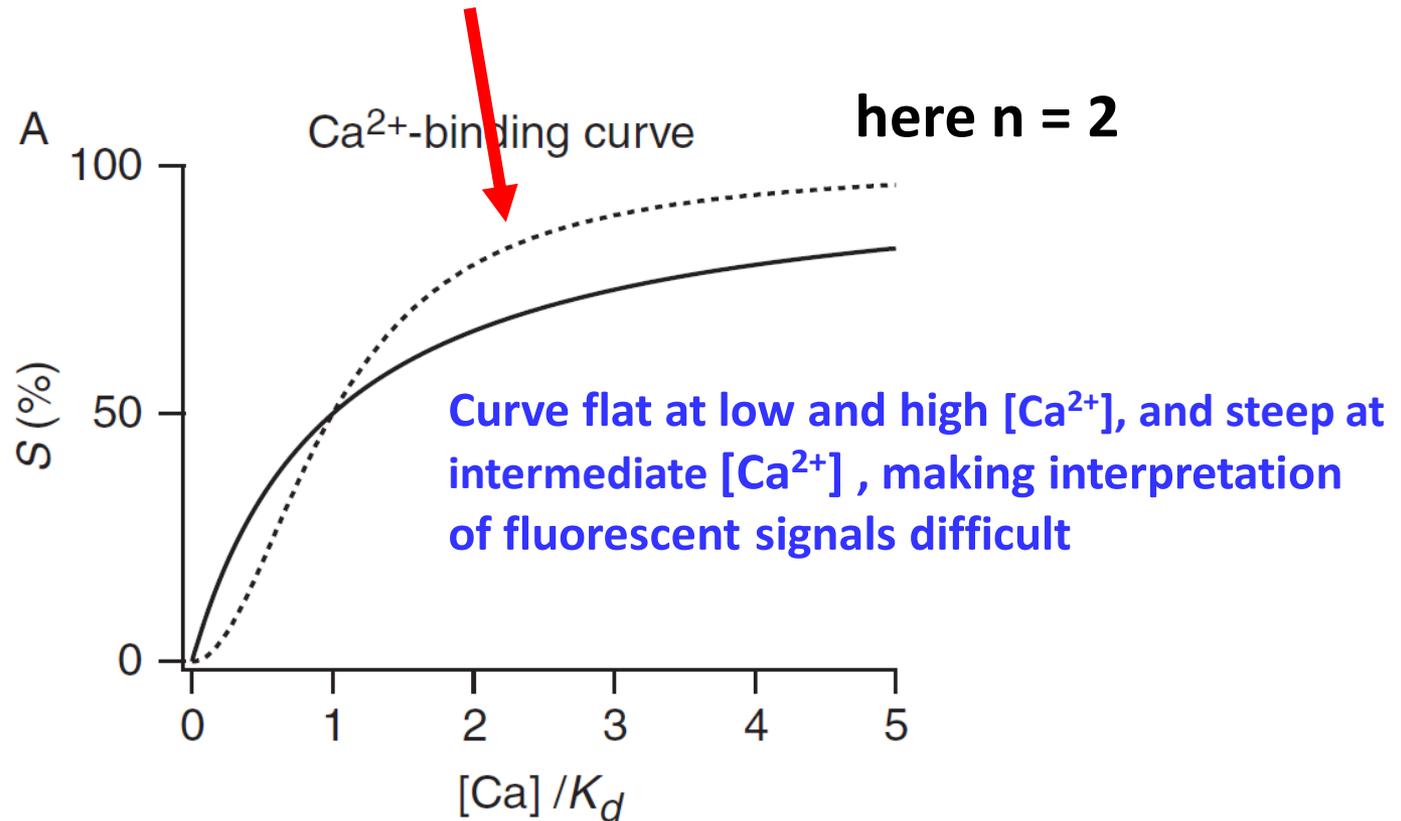
$$K_A = \frac{[\text{Ca}]^n [\text{B}]}{[\text{Ca}^n \text{B}]}$$

$n$  = Hill coefficient (it is not the number of Ca<sup>2+</sup> binding sites, but an empirical parameter describing cooperativity; generally  $\leq$  binding sites)

$K_A$  = apparent dissociation constant =  $K_d^n$

# Cooperative Ca<sup>2+</sup> binding

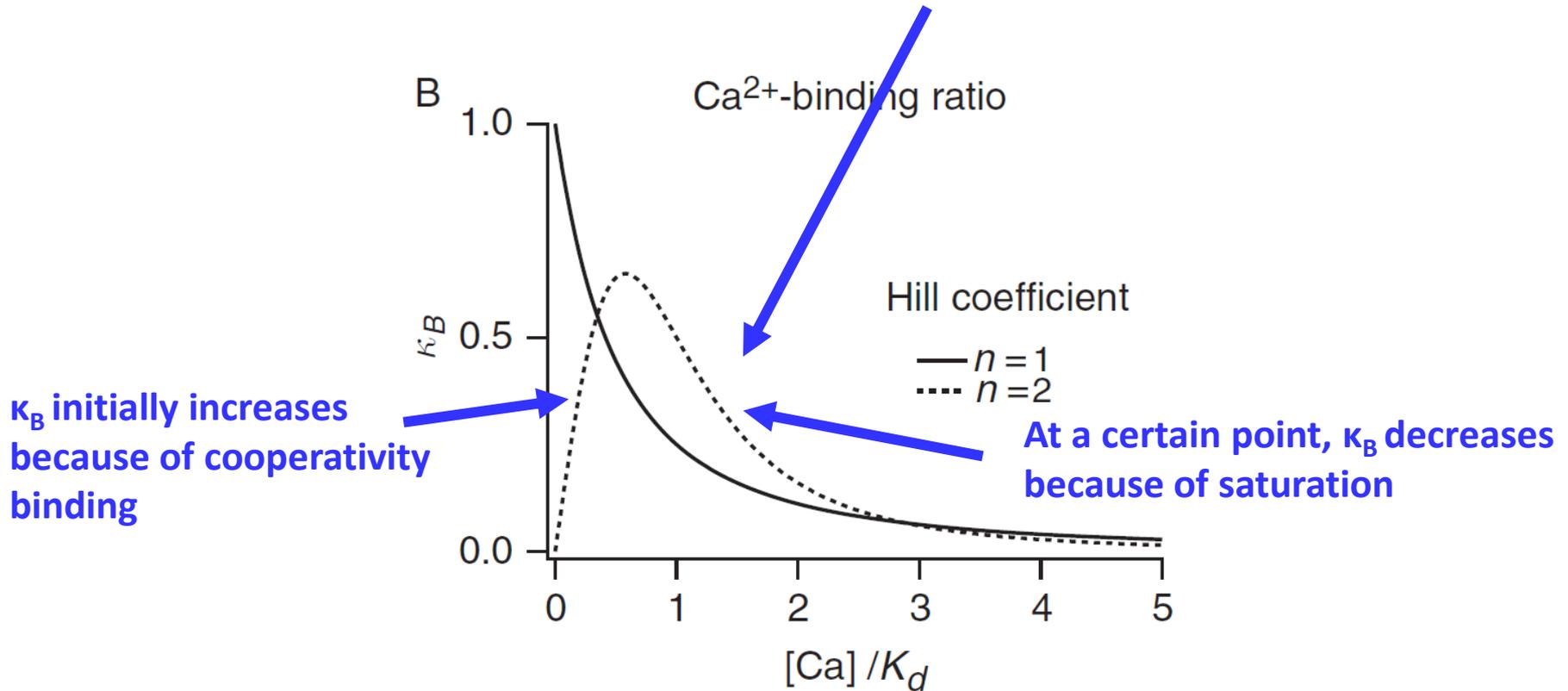
$$S = \frac{[\text{Ca}^n \text{B}]}{[\text{B}]_T} = \frac{[\text{Ca}]^n}{[\text{Ca}]^n + K_A} = \frac{[\text{Ca}]^n}{[\text{Ca}]^n + K_d^n}$$



(the [Ca<sup>2+</sup>] at half-maximal occupancy is  $K_d$ , not  $K_A = K_d^n$ )

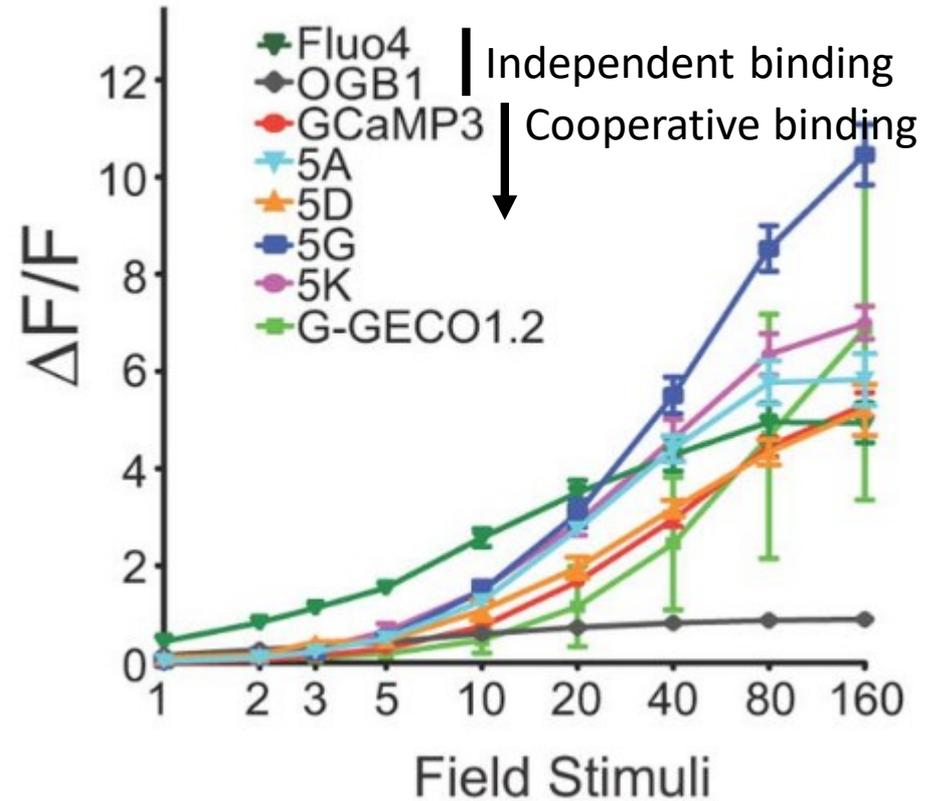
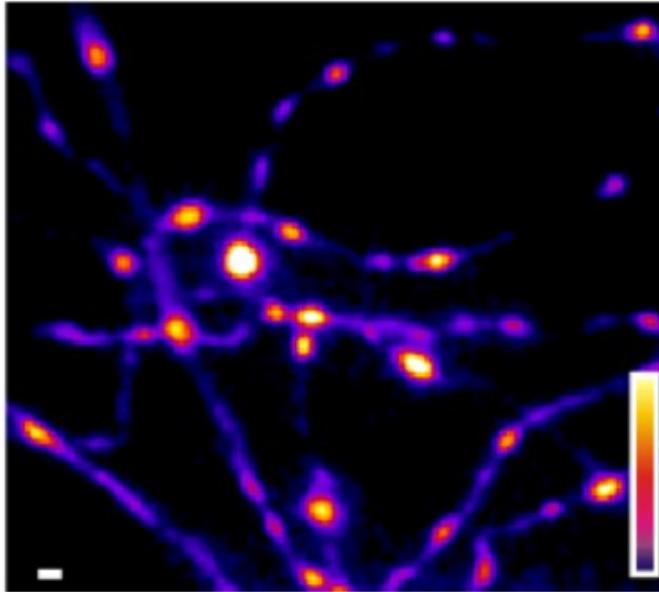
# Cooperative Ca<sup>2+</sup> binding

$$\kappa_B = \frac{\partial [\text{Ca}^n \text{B}]}{\partial [\text{Ca}]} = [\text{B}]_T \frac{n [\text{Ca}]^{n-1} K_d^n}{([\text{Ca}]^n + K_d^n)^2}$$



Relationship non-monotonic, making interpretation of fluorescent signals extremely difficult

# Cooperative Ca<sup>2+</sup> binding



Cooperative binding makes it difficult to correlate fluorescence to [Ca<sup>2+</sup>]

# Conclusion

## 1. $K_d$ = Dissociation constant

- a. It depends on the intrinsic properties of B
- b. It needs to match the  $[Ca^{2+}]$  we want to detect

## 2. $\kappa_B$ = $Ca^{2+}$ -binding ratio

- a. It depends on
  - [B]
  - intrinsic properties of B ( $K_d$ )
  - $[Ca^{2+}]$
- b.  $\kappa_B$  of exogenous indicators needs not to overwhelm  $\kappa_B$  of endogenous buffers

## 3. Cooperativity binding makes quantitative $Ca^{2+}$ imaging arduous

# Further readings



Neuron  
Primer

## Imaging Calcium in Neurons

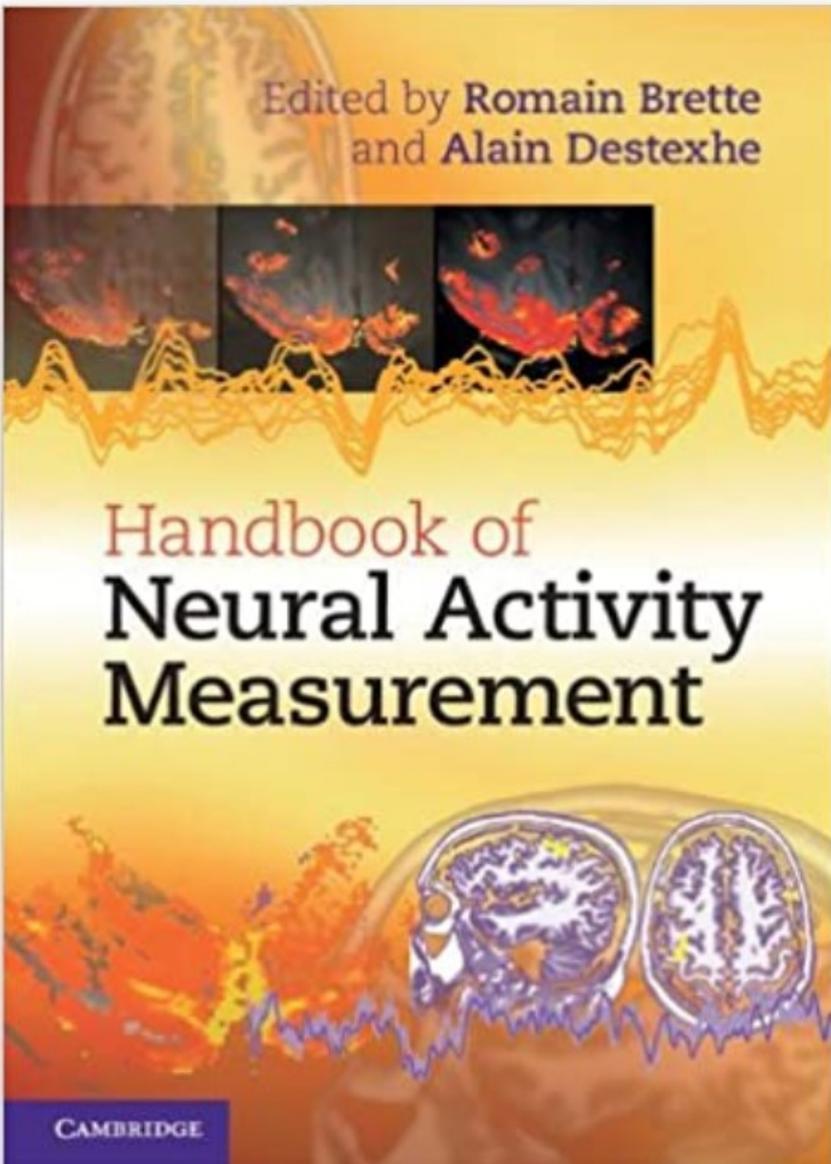
Christine Grienberger<sup>1</sup> and Arthur Konnerth<sup>1,\*</sup>

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\*Correspondence: arthur.konnerth@lrz.tum.de

DOI 10.1016/j.neuron.2012.02.011

Calcium ions generate versatile intracellular signals that control key functions in all types of neurons. Imaging calcium in neurons is particularly important because calcium signals exert their highly specific functions in well-defined cellular subcompartments. In this Primer, we briefly review the general mechanisms of neuronal calcium signaling. We then introduce the calcium imaging devices, including confocal and two-photon microscopy as well as miniaturized devices that are used in freely moving animals. We provide an overview of the classical chemical fluorescent calcium indicators and of the protein-based genetically encoded calcium indicators. Using application examples, we introduce new developments in the field, such as calcium imaging in awake, behaving animals and the use of calcium imaging for mapping single spine sensory inputs in cortical neurons in vivo. We conclude by providing an outlook on the prospects of calcium imaging for the analysis of neuronal signaling and plasticity in various animal models.



## Handbook of Neural Activity Measurement

CAMBRIDGE

# Diffusion

- Review Ca<sup>2+</sup> signaling
- Ca<sup>2+</sup> indicators
- Ca<sup>2+</sup> binding
- **Ca<sup>2+</sup> diffusion**
- **Ca<sup>2+</sup>-dependent fluorescence properties**
- **Simplified models of Ca<sup>2+</sup> dynamics**
- **Imaging devices**
- **Applications**

# Objective

To understand how the timescale of diffusion  
relates to length scales

# Thermal energy

**For every degree of freedom** (= for every way that a particle can move, either forward and backward, left and right, up and down, or rotations), **a particle has a kinetic energy proportional to the temperature.**

The proportionality constant is the **Boltzmann constant** ( $k$ ) =  $1.38 \times 10^{-23}$  J/K

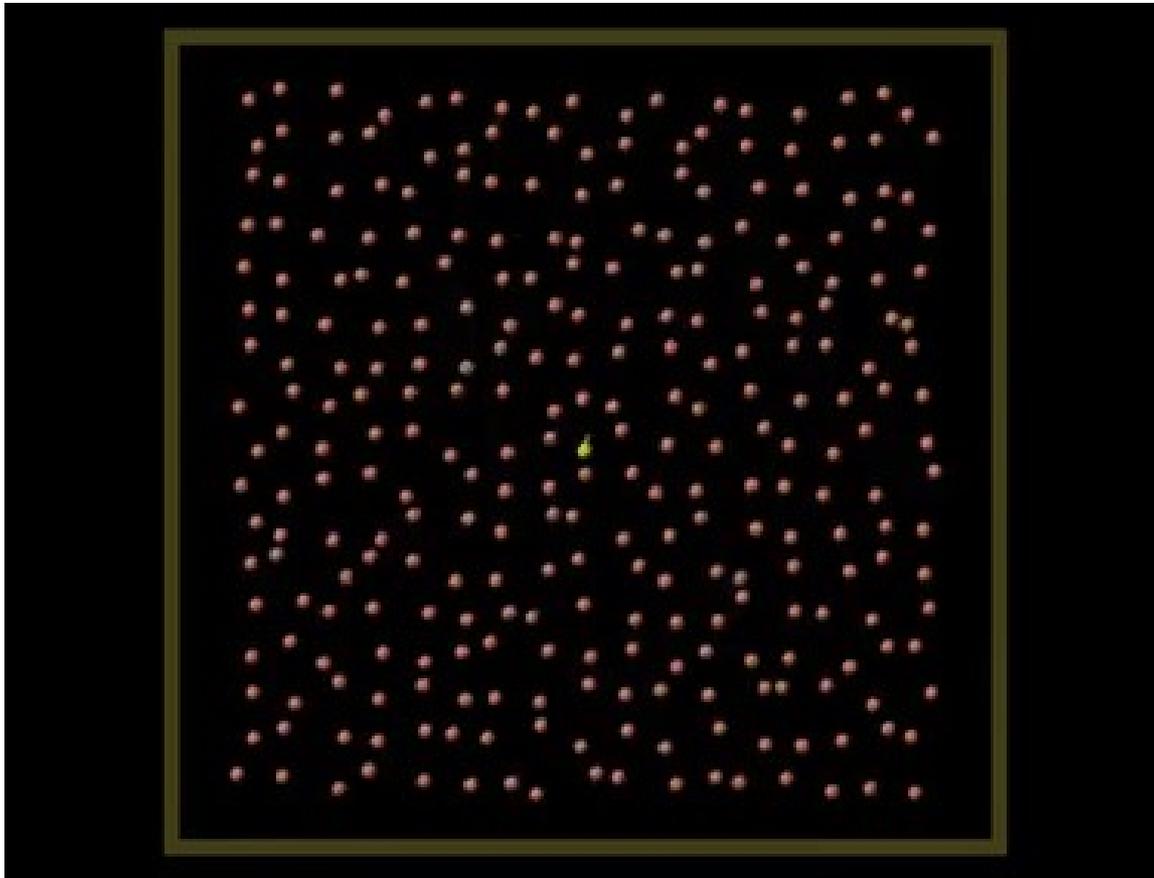
Kinetic energy:  $\left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{1}{2} kT$

The mass of a calcium ion is  $6.66 \times 10^{-26}$  Kg; Room temperature = 300 K

$$\left\langle v_x^2 \right\rangle = 6 \times 10^4 \text{ m}^2/\text{s}^2 \quad \rightarrow \quad \overline{v_x} = 250 \text{ m/s (900 km/hr)}$$

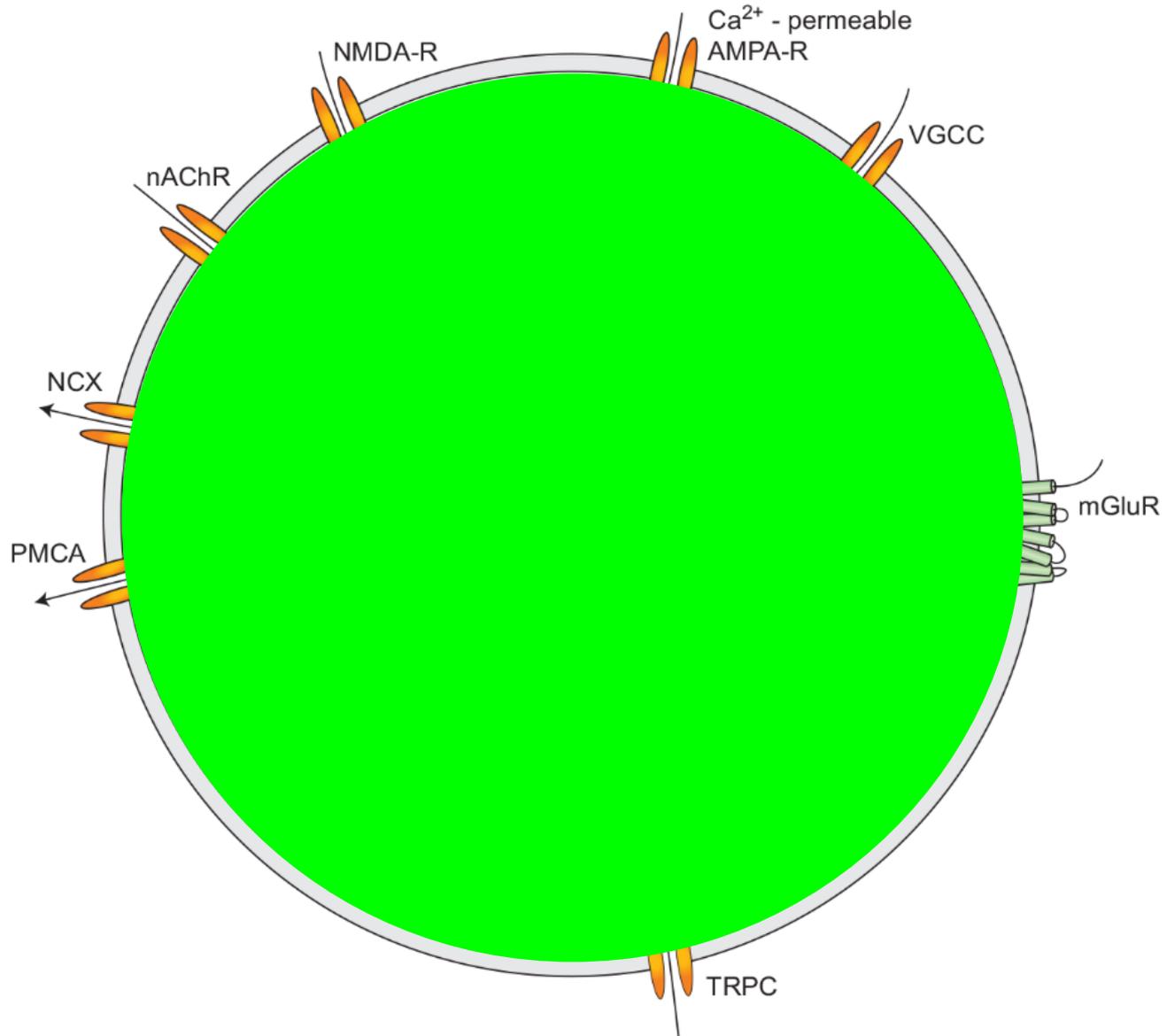
# Collisions produce a **random walk**

A particle in solution constantly ( $10^{13}$  times per second) bumps into  $\text{H}_2\text{O}$  and other molecules, which constantly changes its direction.

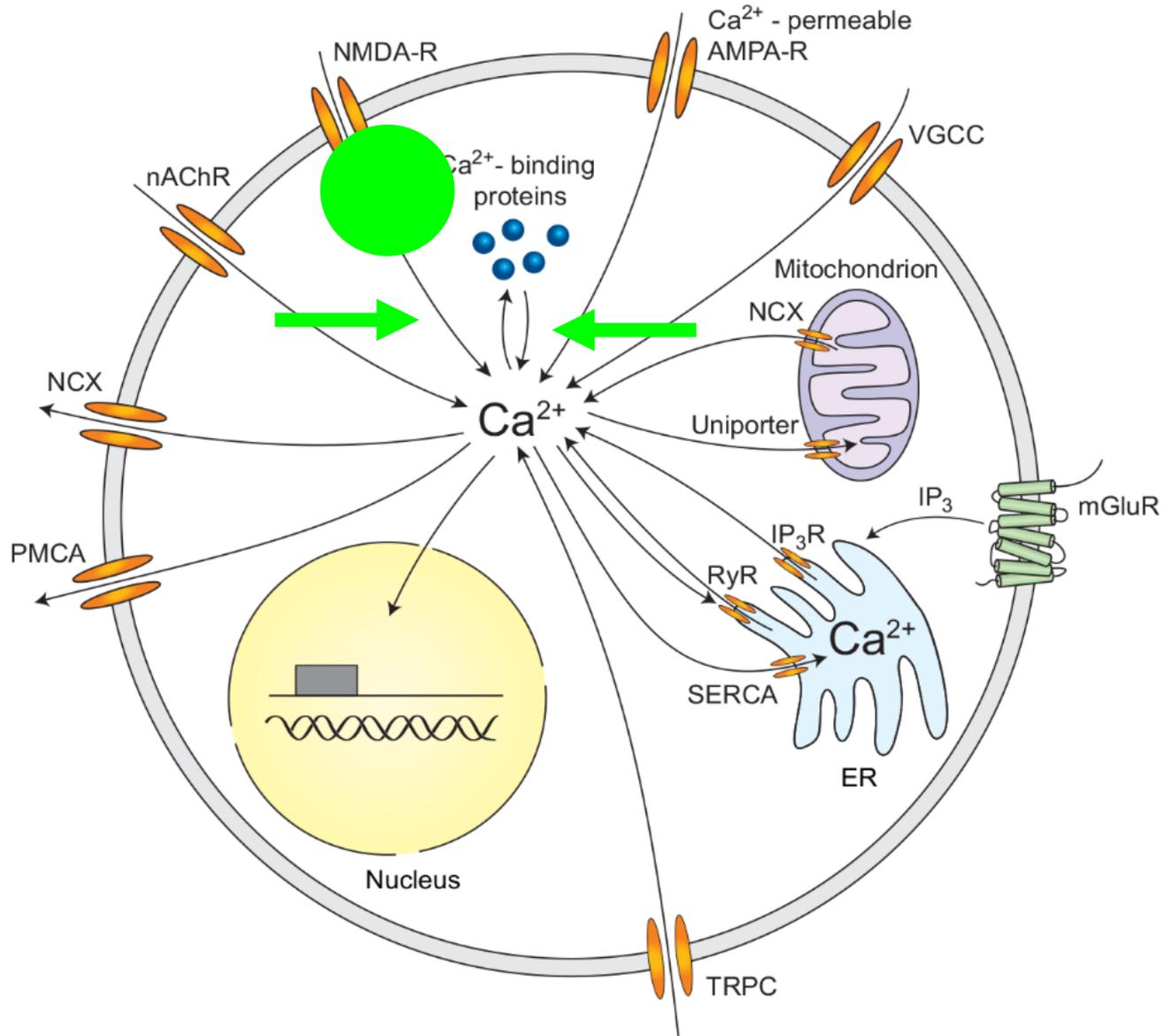


The so called **brownian motion**  
(Robert Brown, botanist)

# Ca<sup>2+</sup> diffusion



# Ca<sup>2+</sup> diffusion



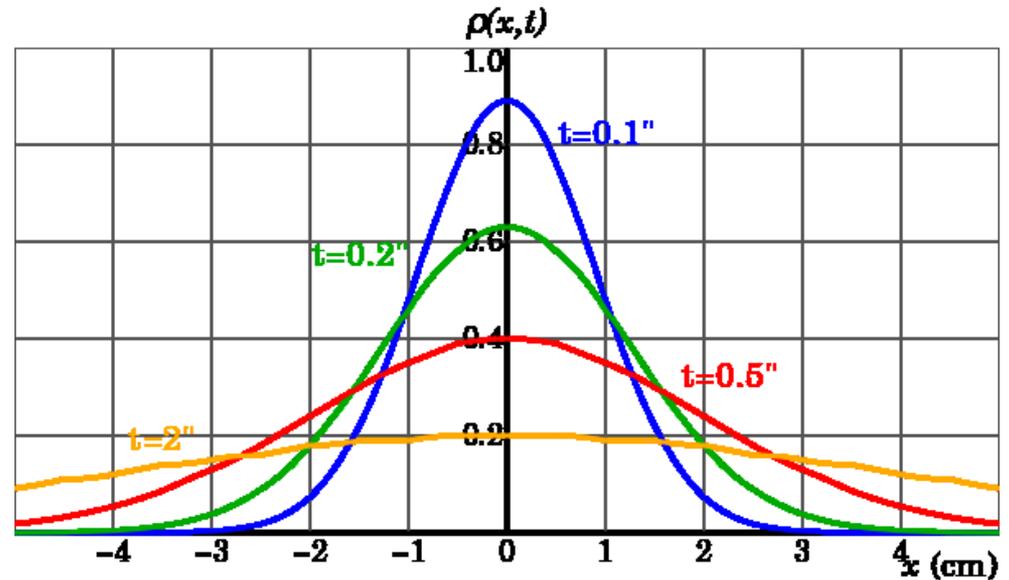
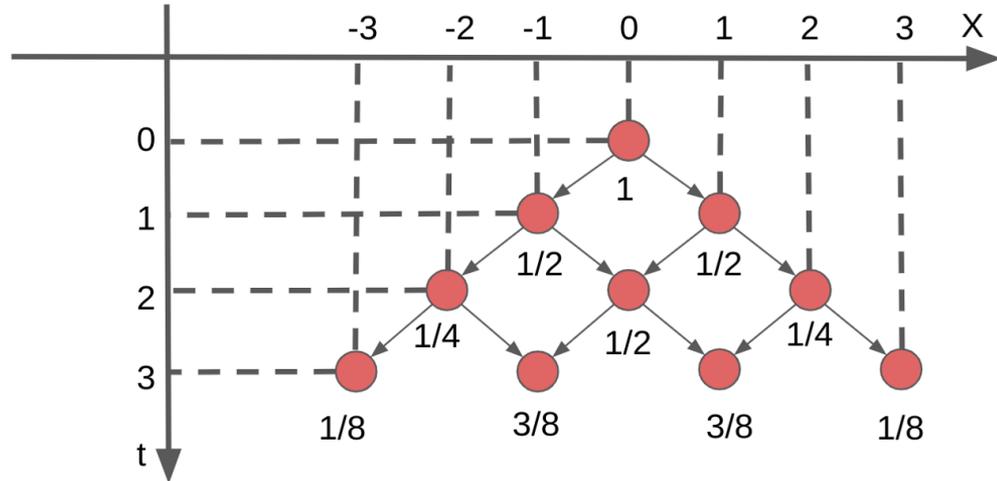
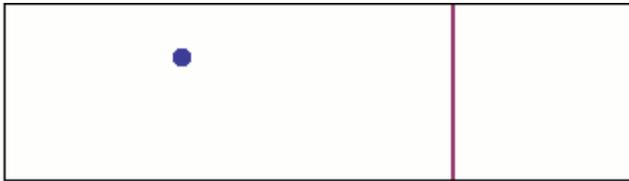
# Relationship between spatial and temporal scales

Diffusion is fast at short length scales and becomes slower and slower at long length scales

- To diffuse across a **synapse (1  $\mu\text{m}$ )**, it takes  $\text{Ca}^{2+}$  **8 ms (not 4 ns)**
- To diffuse across a **soma (10  $\mu\text{m}$ )**, it takes  $\text{Ca}^{2+}$  **800 ms (not 40 ns)**
- To diffuse down a **dendrite (1 mm)**, it takes  $\text{Ca}^{2+}$  **???**
- To diffuse down an **axon (1 m)**, it takes  $\text{Ca}^{2+}$  **????**

# Diffusion in 1-D

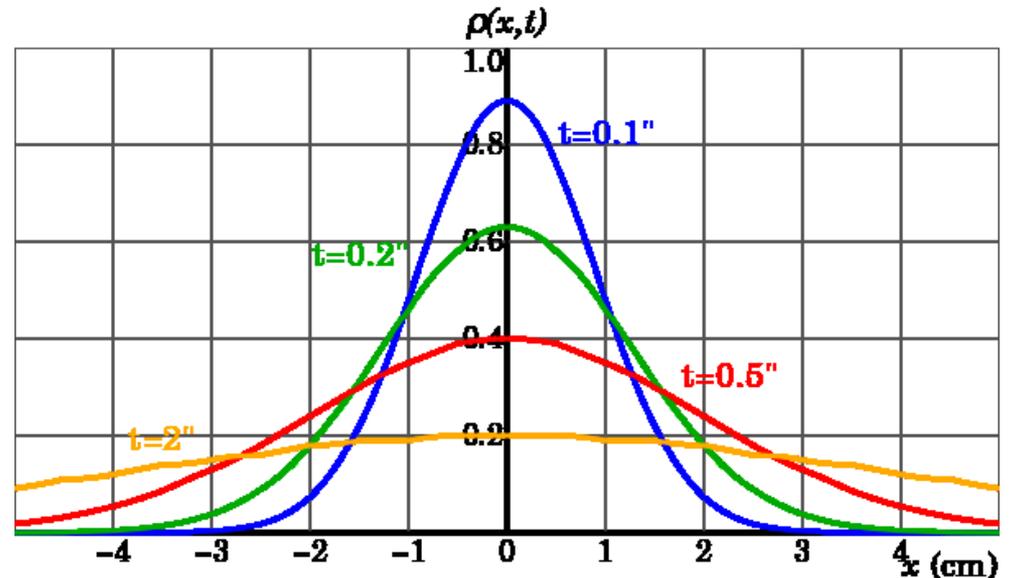
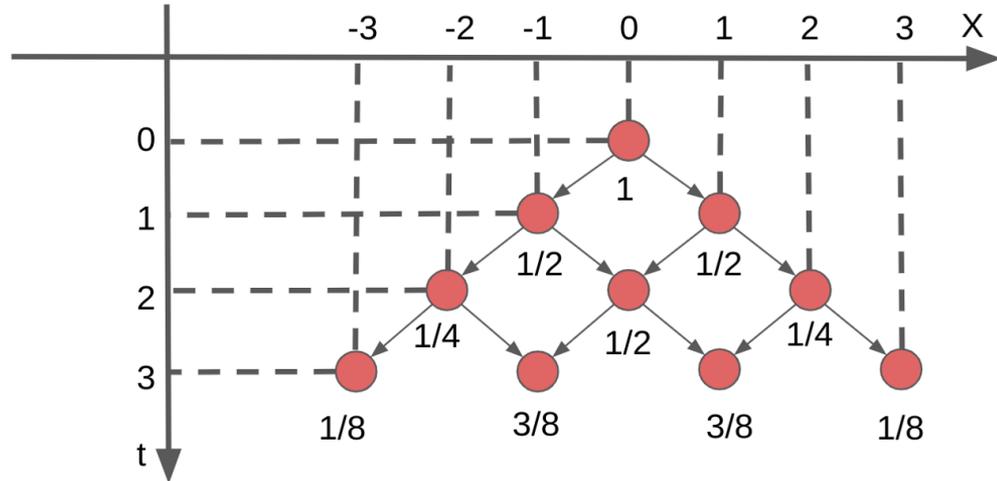
You can get an intuition why this is the case by analyzing how ions diffuse in 1-D, for example along an axon



# Diffusion in 1-D

What is the average position of all the particles?

- Each particle is moving left ( $p=0.5$ ) or right ( $p=0.5$ ) by a distance  $\delta$  at a velocity  $\overline{v_x}$  for some time  $\tau$  before a collision.
- After each collision  $\overline{v_x}$  is randomly reset to the left or to the right.
- Each particle is independent
- On average, particles stay clustered around the initial position = the peak of the Gaussian stays fixed at the center ( $\mu = 0$ )



# What is diffusion?

On average, how far do particles travel/get from where they started?

This is simply how wide the Gaussian distribution is

$$\rho(x, t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}}$$

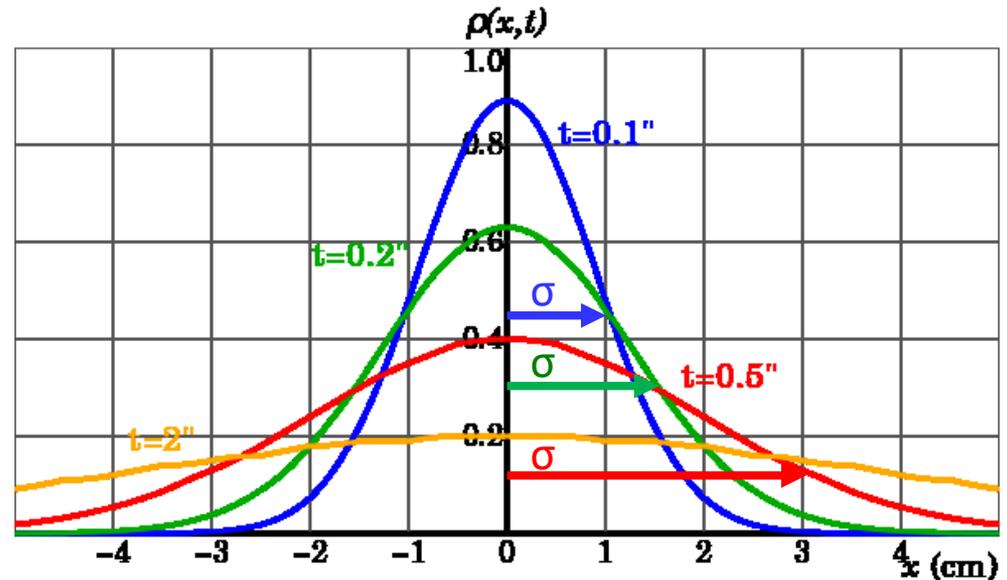
With  $\mu = 0$

$$2\sigma^2 = 4Dt$$

$$\sigma^2 = 2Dt$$

$$\sigma = \sqrt{2Dt}$$

(D = diffusion coefficient)



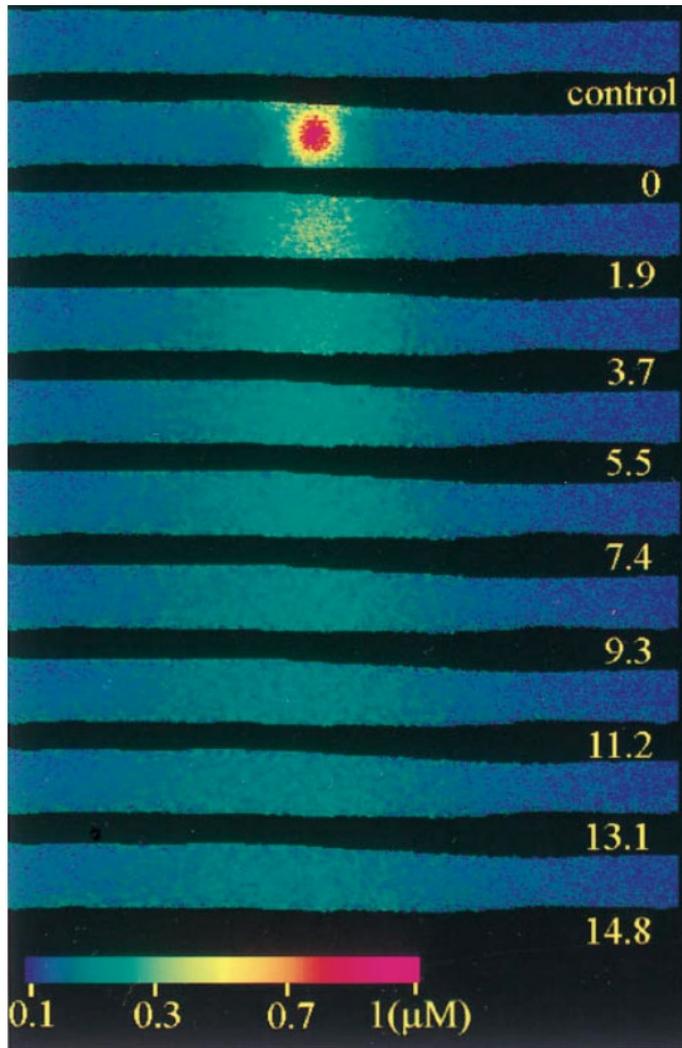
Mean displacement =  $r$  = standard deviation =  $\sigma = \sqrt{Var} = \sqrt{2Dt}$

with  $D = \frac{\delta^2}{2\tau}$  (D is the diffusion coefficient)

(I have posted an easy to understand derivation of this important result)

$$f(x | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

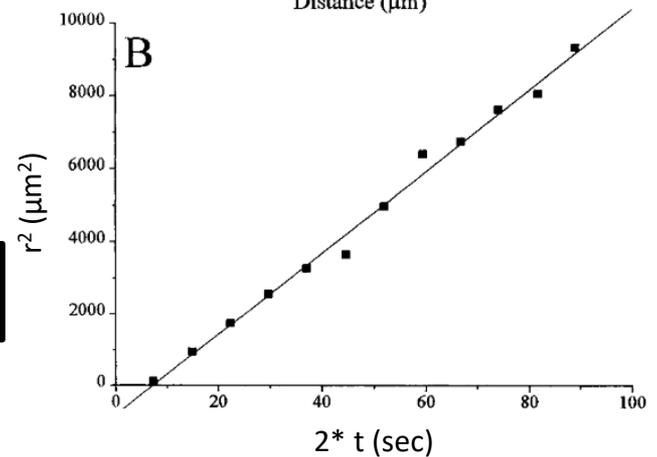
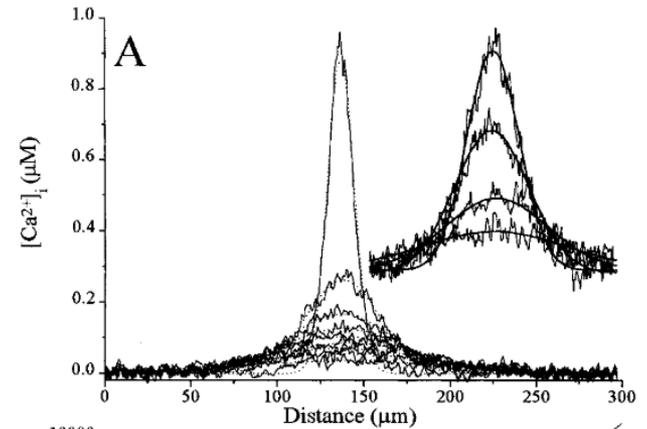
# Low Mobility of the Ca<sup>2+</sup> Buffers in Axons of Cultured Aplysia Neurons



- Neuron loaded with the indicator Fura-2
- Ca<sup>2+</sup> (100 μM) pressure injected in the center of the axon at time 0

$$r^2 = \sigma^2 = D_{Ca} 2t$$

$$D_{Ca} \approx 20 \mu\text{m}^2\text{s}^{-1}$$

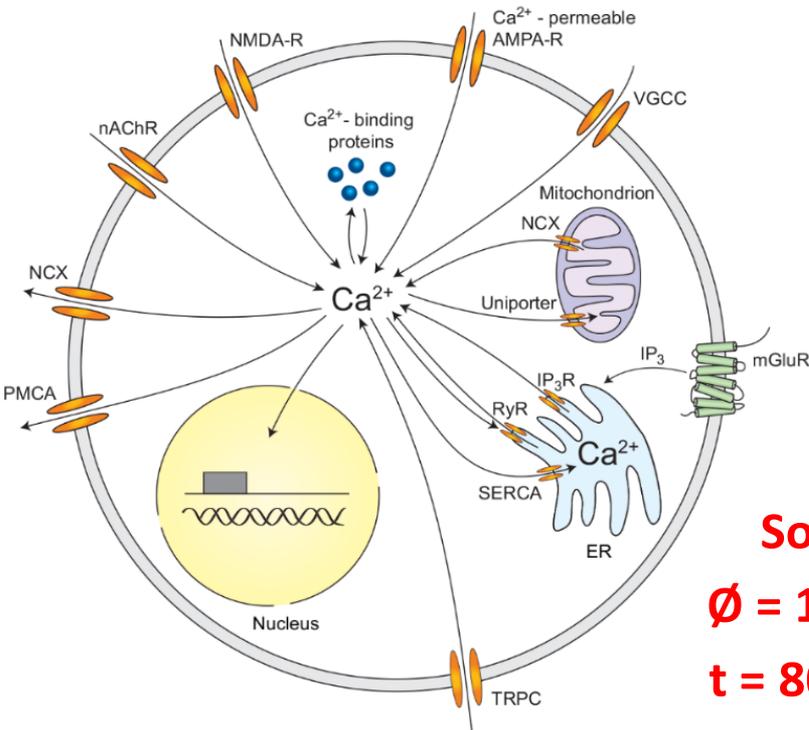


# How long will it take for $\text{Ca}^{2+}$ to be uniformly distributed across a subcellular compartment?

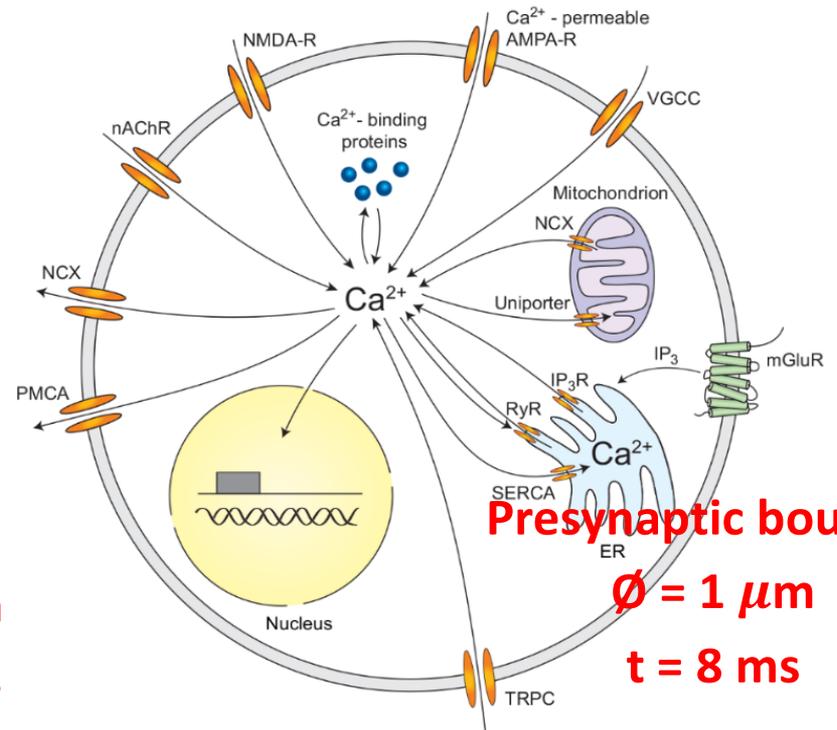
In one dimension: mean displacement =  $r = \sigma = \sqrt{2Dt} \rightarrow t = \frac{r^2}{2D}$

In two dimension: mean displacement =  $r = \sigma = \sqrt{4Dt} \rightarrow t = \frac{r^2}{4D}$

In three dimension: mean displacement =  $r = \sigma = \sqrt{6Dt} \rightarrow t = \frac{r^2}{6D}$



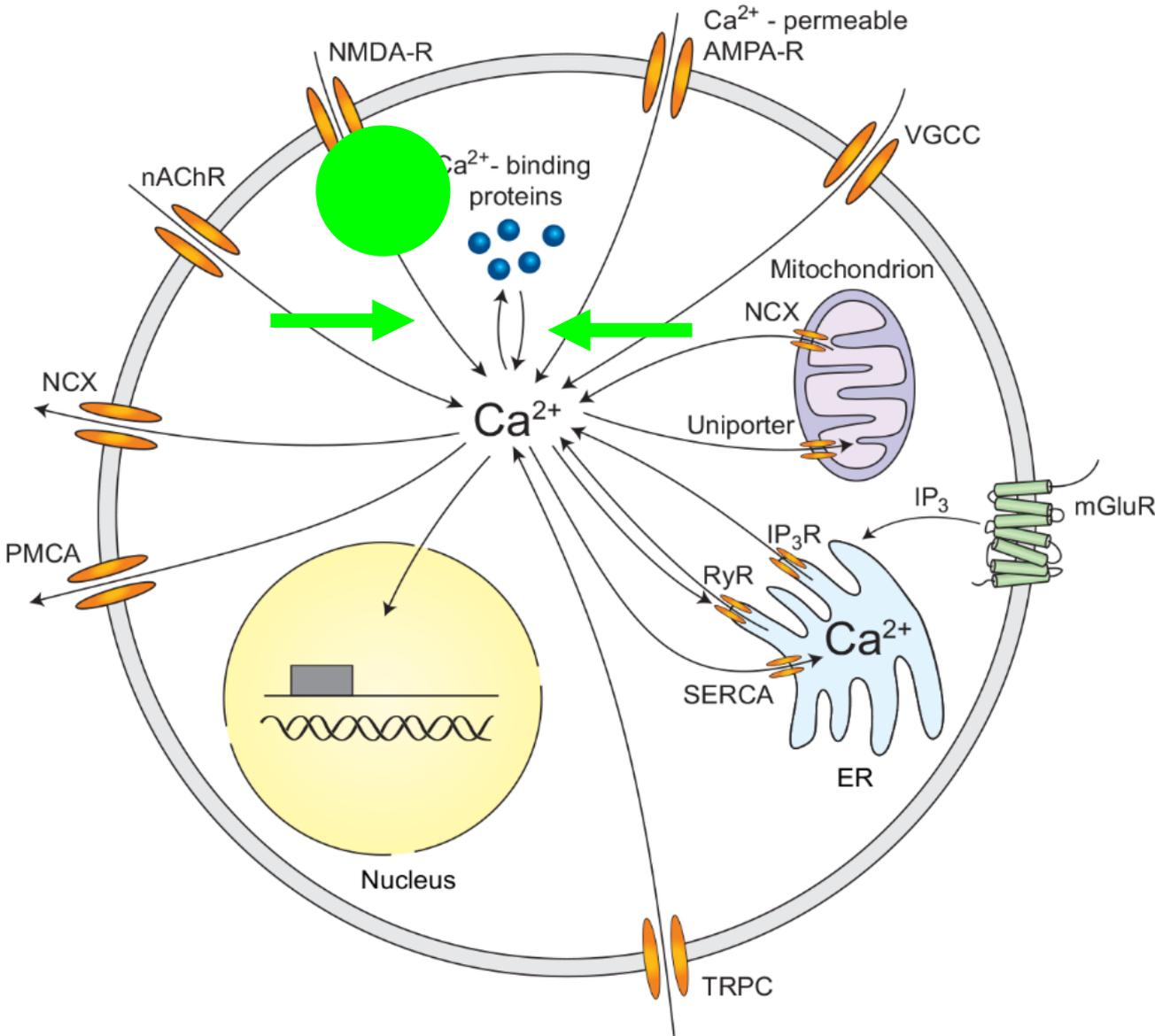
**Soma**  
 $\varnothing = 10 \mu\text{m}$   
 $t = 800 \text{ ms}$



**Presynaptic boutons**  
 $\varnothing = 1 \mu\text{m}$   
 $t = 8 \text{ ms}$

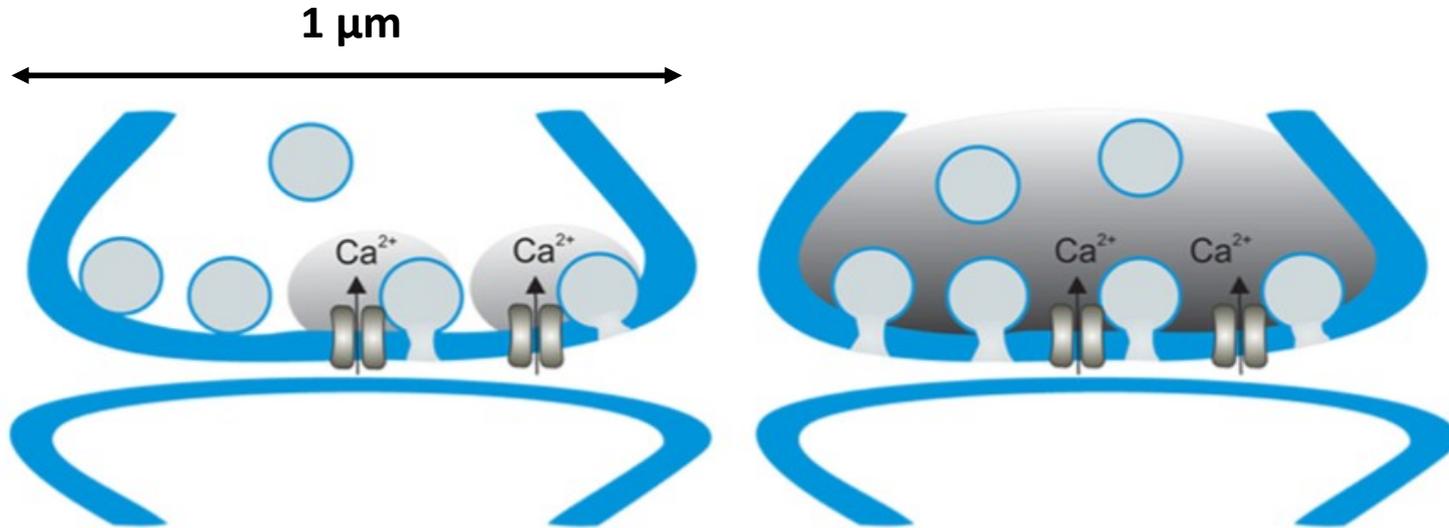
- 5) Some axons in your brain can be 1 m long. Assuming a diffusional constant for  $\text{Ca}^{2+}$  ( $D_{\text{Ca}}$ ) of  $20 \mu\text{m}^2\text{s}^{-1}$ , how long would it take for  $\text{Ca}^{2+}$  to diffuse from the soma to the terminal bouton if there were only free diffusion in 1 D?
- a) 80 min
  - b) 80 hours
  - c) 80 days
  - d) 8 months
  - e) 8 years
  - f) 80 years
  - g) 800 years

# Lower limit of (Ca<sup>2+</sup>) imaging (1)



1) We can only image the **average 'residual' [Ca<sup>2+</sup>]** that remains after initial concentration gradients have equilibrated by diffusion

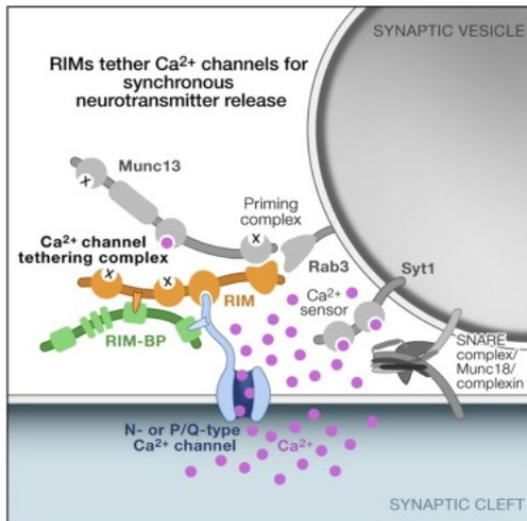
# Lower limit of (Ca<sup>2+</sup>) imaging (1)



Ca<sup>2+</sup> important for synaptic transmission

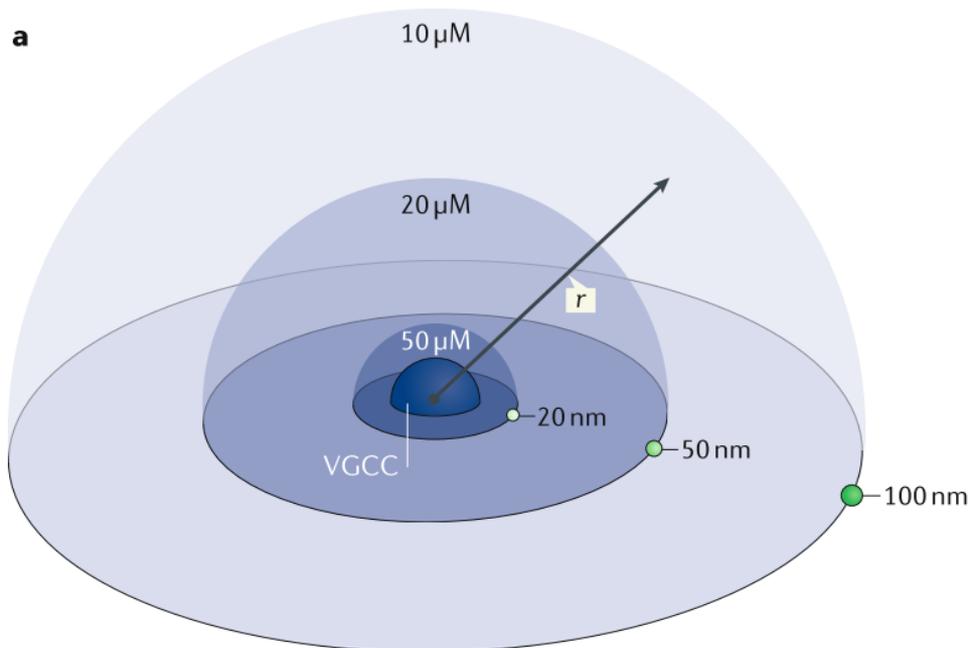
Average residual Ca<sup>2+</sup> we can 'see' with

Ca<sup>2+</sup> imaging after diffusion and equilibration with endogenous and exogenous (= Ca<sup>2+</sup> indicators) buffers



# Lower limit of ( $\text{Ca}^{2+}$ ) imaging (1)

Many  $\text{Ca}^{2+}$ -dependent events (e.g. synaptic transmission) depends on local **nanodomains (few hundreds nanometers)** of high  $\text{Ca}^{2+}$  ( **$100 \mu\text{M}$** ) that develop and collapse within **tens to hundreds of microseconds** following the opening of  $\text{Ca}^{2+}$  permeable channels



**10 - 100  $\mu\text{M}$   $\text{Ca}^{2+}$**  are required for synaptic transmission

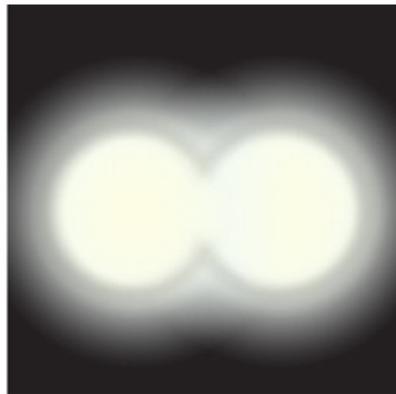
Residual  $\text{Ca}^{2+}$  in the presynaptic bouton, as measured with  $\text{Ca}^{2+}$  imaging, is  **$\approx 1 \mu\text{M}$**

# Lower limit of ( $\text{Ca}^{2+}$ ) imaging (2)

Due to the diffraction limit ( $\approx 200 \text{ nm}$ )



(a)

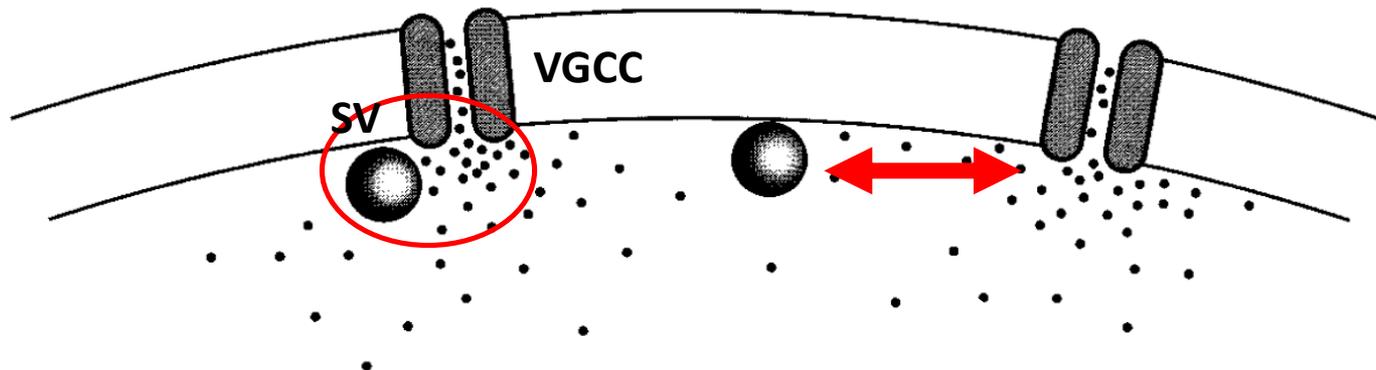


(b)



(c)

# Comparing slow (EGTA) and fast (BAPTA) $\text{Ca}^{2+}$ buffers to estimate the distance between intracellular targets and $\text{Ca}^{2+}$ channels



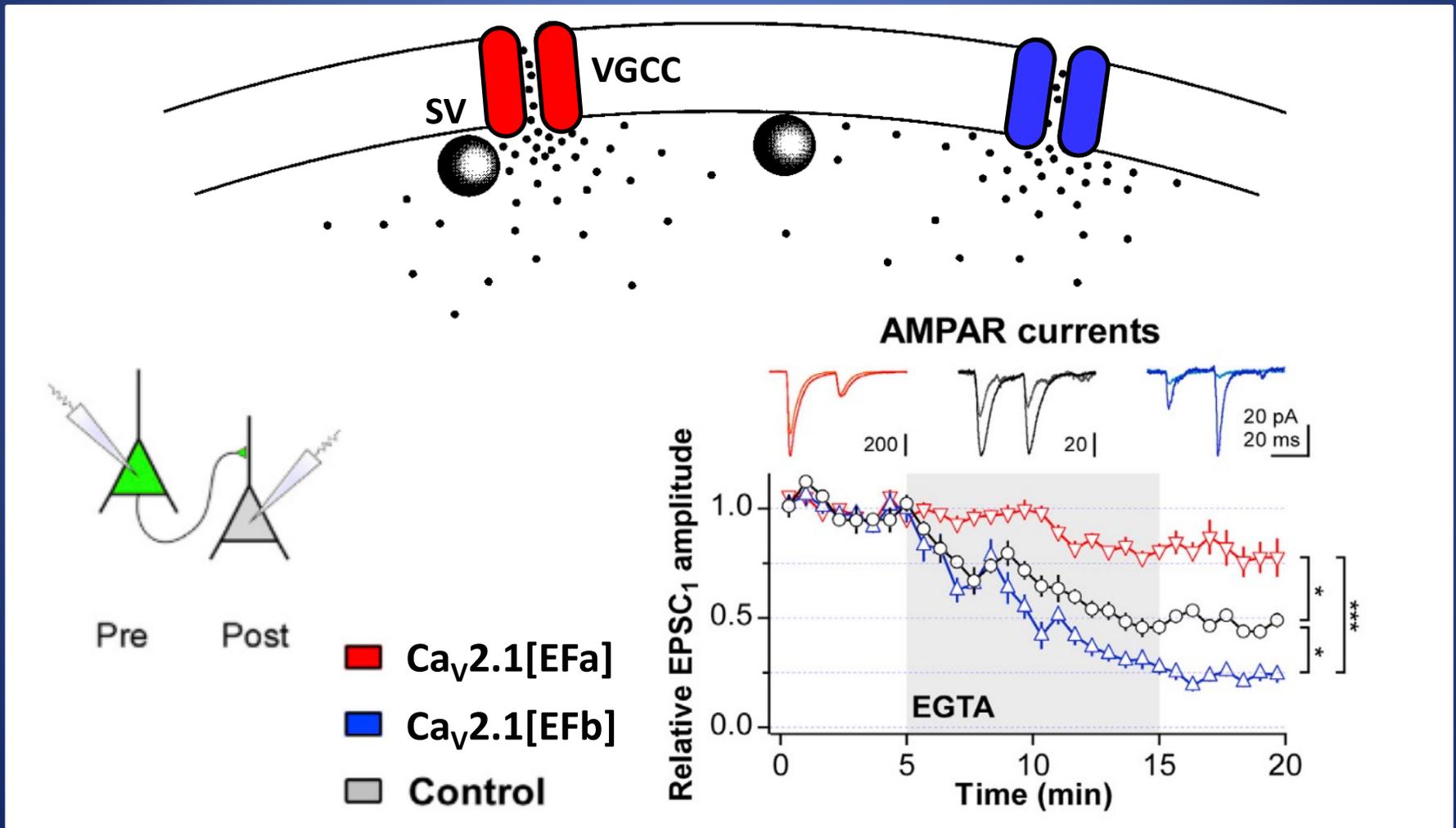
## At 10-20 nm distance:

- high  $[\text{Ca}^{2+}]$ , high  $P_r$
- **Sensitive to BAPTA**
- **Insensitive to EGTA**

## At 100-200 nm distance:

- low  $[\text{Ca}^{2+}]$ , low  $P_r$
- **Sensitive to BAPTA**
- **Sensitive to EGTA**

# Example: sensitivity of different $Ca_v2.1$ splice variants to EGTA-AM

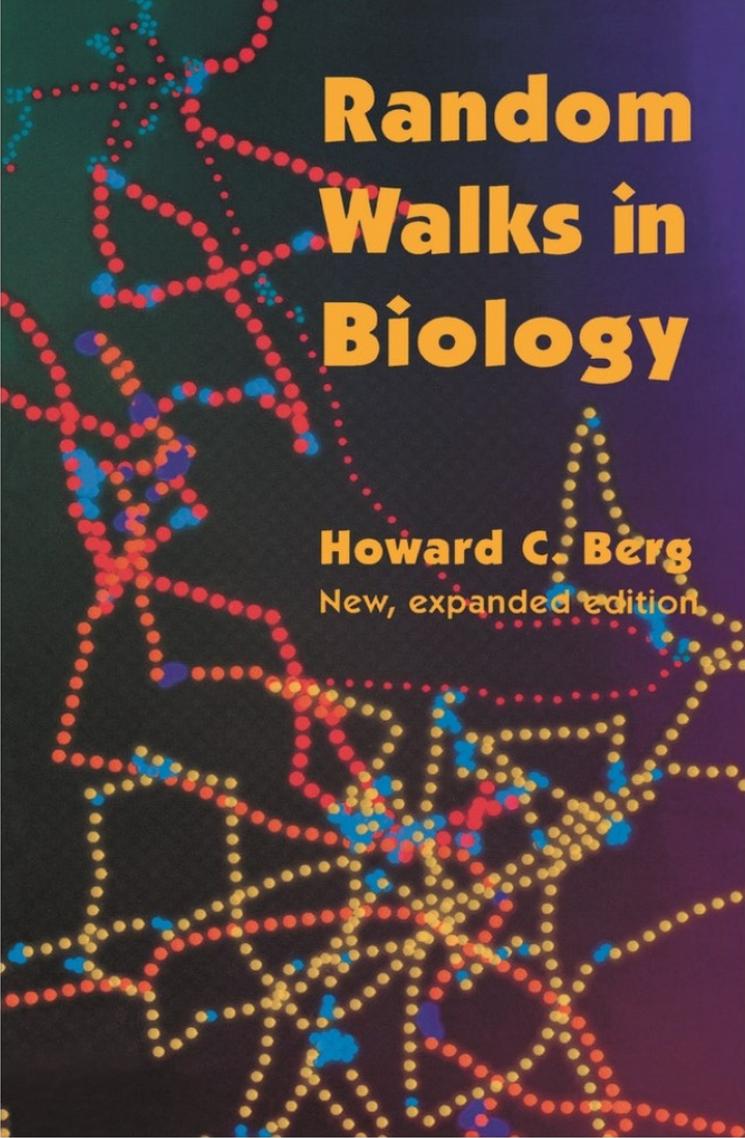


# Conclusion

1. A particle has a kinetic energy proportional to the temperature ( $\overline{v_x} = 250$  m/s at RT for  $\text{Ca}^{2+}$ )
2. A particle in solution constantly ( $10^{13}$  times per second) bumps into other particles, which changes its direction
3. As a result, diffusion is fast at short length scales and slower at long length scales or
4.  $t_{\text{diffusion}} = \frac{r^2}{2D}$  (in one dimension)
5. We can only image the average 'residual'  $[\text{Ca}^{2+}]$  that remains after initial concentration gradients have equilibrated by diffusion

# Further readings

## REVIEWS



### Random Walks in Biology

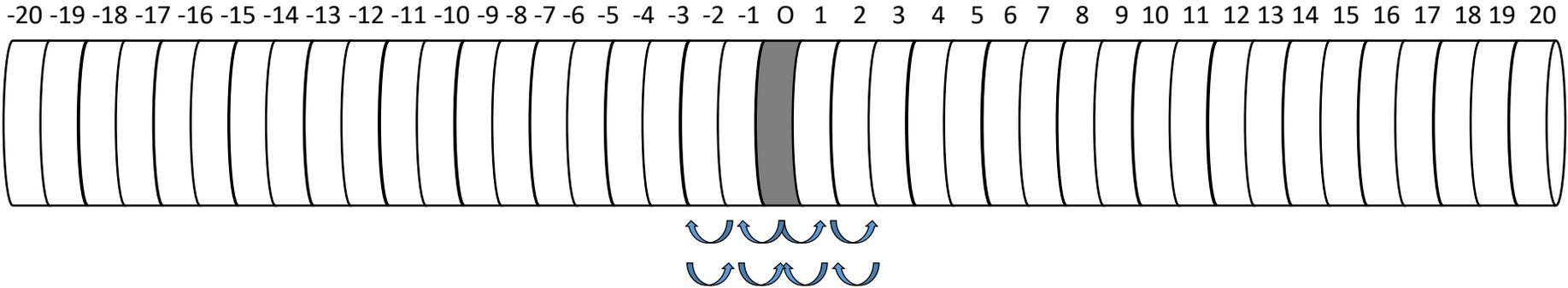
**Howard C. Berg**  
New, expanded edition

## The control of release probability at nerve terminals

*Jeremy S. Dittman\** and *Timothy A. Ryan* \*

Abstract | Exocytosis is a fundamental membrane fusion process by which the soluble or membrane-associated cargoes of a secretory vesicle are delivered to the extracellular milieu or the cell surface. While essential for all organs, the brain relies on a specialized form of exocytosis to mediate information flow throughout its vast circuitry. Neurotransmitter-laden synaptic vesicles fuse with the plasma membrane on cue with astonishing speed in a probabilistic process that is both tightly regulated and capable of a fascinating array of plasticities. Here, we examine progress in the molecular understanding of synaptic vesicle fusion and its control.

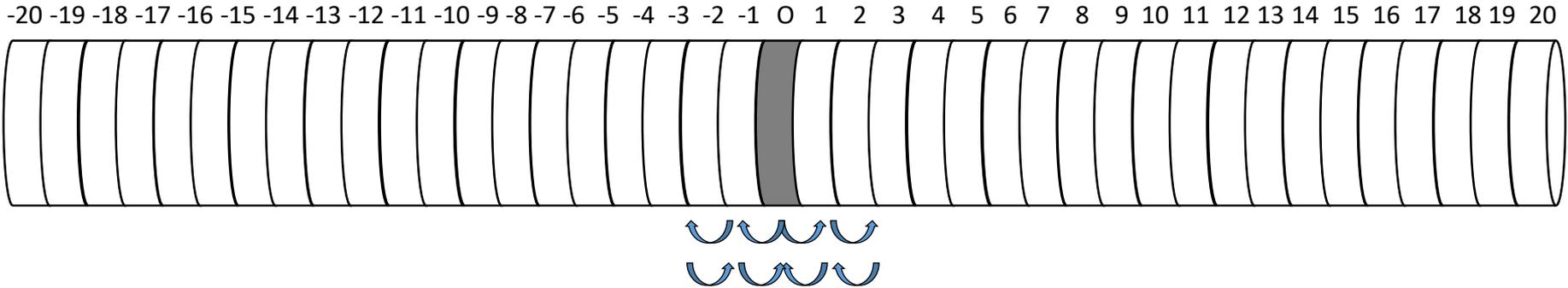
# Supplementary material



- We have  $N$  particles at position  $x = 0$  at time  $t = 0$
- At every time step  $\tau$ , each particle moves independently from the others:
  - Half of the particles step right by a distance  $\delta = +\overline{v_x} \tau$
  - Half of the particles step left by a distance  $-\delta = -\overline{v_x} \tau$
- $x_i(n)$  = position of the  $i^{\text{th}}$  particle on time-step  $n$ , with  $n = \frac{t}{\tau}$
- Thus, we can write the position of each particle at time-step  $n$  as a function of the position at previous time-step:

$$x_i(n) = x_i(n - 1) \pm \delta$$

# Supplementary material



What is the average position of all the particles?

$$\langle x_i(n) \rangle = \frac{1}{N} \sum_i x_i(n)$$

$$\langle x_i(n) \rangle = \frac{1}{N} \sum_i [x_i(n-1) \pm \delta]$$

$$\langle x_i(n) \rangle = \frac{1}{N} \sum_i [x_i(n-1)] + \frac{1}{N} \sum_i (\pm \delta)$$

0

$$\langle x_i(n) \rangle = \langle x_i(n-1) \rangle$$

$$\langle x_i(n) \rangle = \mu = 0$$

# Supplementary material

On average, how far do particles travel in absolute terms?

$\langle |x_i(n)| \rangle \rightarrow$

$$x_i(n) = x_i(n-1) \pm \delta$$

$$x_i^2(n) = (x_i(n-1) \pm \delta)^2$$

$$x_i^2(n) = x_i^2(n-1) \pm 2 \delta x_i(n-1) + \delta^2$$

$\cap$



# Supplementary material

At each step the variance grows linearly by  $\delta^2$

$$\sigma^2 = \langle x_i^2(n) \rangle = \langle x_i^2(n-1) \rangle + \delta^2$$

$$\langle x_i^2(0) \rangle = 0, \quad \langle x_i^2(1) \rangle = \delta^2, \quad \langle x_i^2(2) \rangle = 2\delta^2, \quad \langle x_i^2(n) \rangle = n\delta^2$$

We can change time steps between collisions ( $n$ ) to continuous time, with  $n = \frac{t}{\tau}$

$$\sigma^2 = \langle x_i^2(t) \rangle = \frac{\delta^2}{\tau} t$$

$$\sigma^2 = \langle x_i^2(t) \rangle = 2Dt, \quad \text{with } 2D = \frac{\delta^2}{\tau} \quad (\text{Diffusion coefficient; length}^2/\text{time})$$

$$\sigma = \sqrt{\langle x_i^2(n) \rangle} = \sqrt{2Dt}$$