Lezione 8 ottobre 2024

martedì 8 ottobre 2024 14:06

VENERÌ : 11/10/24 NO LEZIONE

RECUPERIAMO MARTEDÌ 15/10/24
(14:00 - 17:00)

Aula 3B H2 bis Lunedì 16-18

venerdì 11/10 no ricevimento

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Visto: a^x , $a>0$, $x \in \mathbb{R}$

$0<a<1$: a^x è strett. decr.

$a>1$: a^x è strett. cresc.

$$a^x : \mathbb{R} \longrightarrow (0,+\infty) = \{r \in \mathbb{R} : r>0\}$$

$a \neq 1$ è biiettiva

$\Longrightarrow$ ammette funzione inversa

$$a^x = y$$

l'inversa si chiama LOGARITMO in base a. $(a>0)$

$\log_a$

$$\log_a : (0,+\infty) \longrightarrow \mathbb{R}$$

$$\log_a(y) = \underset{\substack{\downarrow \\ \text{esponente}}}{\boxed{x}} \quad : \quad a^x = y$$

Es: $\log_2 8 = 3$ perché $2^3 = 8$

OSS.: $$\boxed{a^{\overset{= x \ : \ a^x = y}{\log_a y}} = y}$$

infatti: $f(x) = a^x$

$$f^{-1}(y) = \log_a y$$

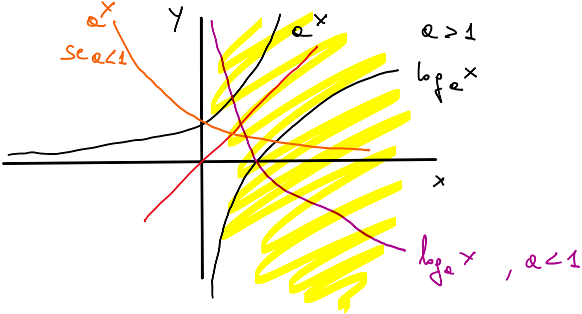
$$y = (f \circ f^{-1})(y) = f(f^{-1}(y)) = f(\log_a y)$$

$$= a^{\log_a y} = y$$

$$x = (f^{-1} \circ f)(x) = f^{-1}(f(x))$$

$$= f^{-1}(a^x)$$

$$\boxed{= \log_a(a^x) = x}$$



PROPRIETÀ DI $\log_a$:

- $\log_a 1 = 0$ $\qquad a^0 = 1$
- $\log_a a = 1$ $\qquad a^1 = a$
- $$\boxed{\underbrace{\log_a(b\cdot c)}_{=y} = \underbrace{\log_a b}_{y_1} + \underbrace{\log_a c}_{y_2}}$$

$$a^y = b\cdot c \qquad a^{y_1} = b \qquad a^{y_2} = c$$

$$\left.\begin{aligned} a^y &= a^{y_1}\cdot a^{y_2} \\ &= a^{y_1+y_2} \end{aligned}\right\} \Longrightarrow y = y_1 + y_2$$

a^x è iniettiva

$a \neq 1$

- $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$
- $\log_a(b^\alpha) = \alpha\cdot\log_a b$

FORMULA DI CAMBIO DI BASE

$$\log_q x = \frac{\log_p x}{\log_p q}$$

$p, q > 0$

$p \neq 1$

$q \neq 1$

Verifica:

$\log_q x = a \Rightarrow q^a = x \Rightarrow (p^c)^a = x$

$\log_p x = b \Rightarrow p^b = x = p^{c \cdot a} = x$

$\log_p q = c \Rightarrow p^c = q$

$$\Rightarrow p^b = p^{c \cdot a} \Rightarrow b = c \cdot a \; ; \; \boxed{a = \frac{b}{c}}$$

SCALA SEMILOGARITMICA:

Introduc. un nuovo sistema di coordinate nel piano: $\mathbb{R}^2$

$X = x$

$Y = \ln(y) = \log_e y$

$e = \#$ di Nepero $= \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$= 2,71 \ldots$

In questo sist. di coordinate

se cons. $f(x) = c \cdot a^x$

$Y = \ln(f(x))$

Grafico : $(x, Y) : Y = \ln(f(x))$

Grafico $\Gamma_f : \{(x, f(x)) : x \in D\}$

Grafico semilogaritmico : $\{(x, \ln(f(x))) : x \in D\}$

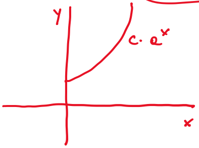
se $f(x) = c \cdot a^x$, $c > 0$, $a > 0$

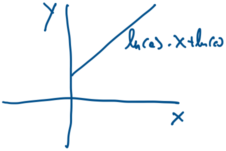
$$\ln(f(x)) = \ln(c\cdot e^{x}) = \ln(c) + \ln(e^{x})$$

$$\overset{!}{=} \ln(c) + x\cdot \ln(e)$$

$$= \ln(e)\cdot x + \ln(c)$$

$$\boxed{Y \overset{!}{=} \ln(e)\cdot X + \ln(c)}$$





SCALA LOGARITMICA

Nuovo sistema di coordinate:

$$\begin{cases} X = \ln(x) \\ Y = \ln(y) \end{cases}$$

Se cons. $f(x) = a\cdot x^{p}$

$a>0,\quad x>0$

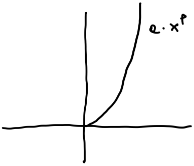
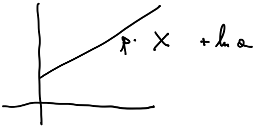
Il grafico LOGARITMICO

$$Y = \ln f(x)$$

$$\overset{!}{=} \ln(a\cdot x^{p}) = \ln(a) + \ln(x^{p})$$

$$= \ln a + p\cdot \underbrace{\ln(x)}_{= X}$$

$$= p\cdot X + \ln a$$

FUNZIONE lineare in X