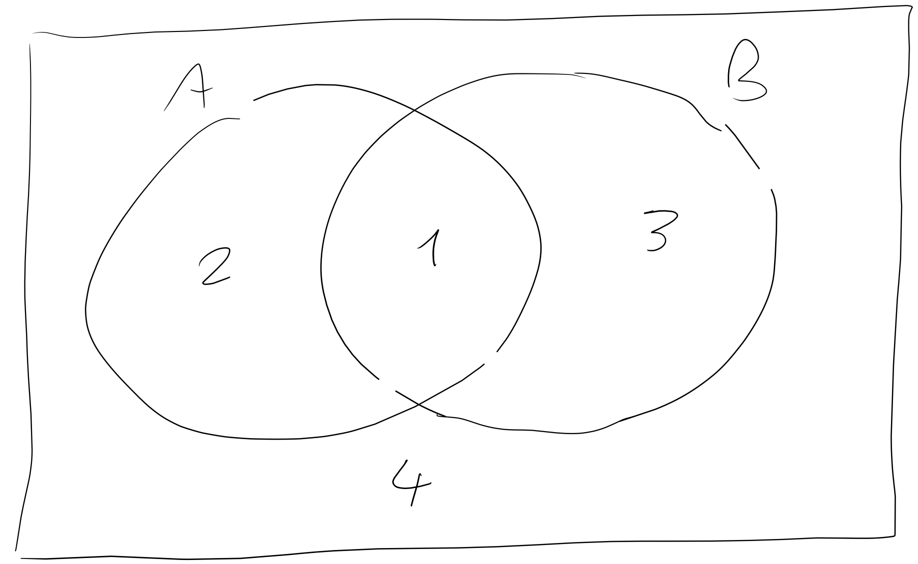
$\mathcal{C} = \{A, B\}$ $\quad \sigma(\mathcal{C}) = \sigma(\mathbb{P}_\sigma(\mathcal{C}))$

$A' \cap B'$




$$\mathbb{P}_\sigma(\mathcal{C}) = \{\overset{1}{A \cap B}, \overset{2}{A \cap \bar{B}}, \overset{3}{\bar{A} \cap B}, \underset{4}{\bar{A} \cap \bar{B}}\}$$

$$\sigma(\mathcal{C}) = \sigma(\mathbb{P}_\sigma(\mathcal{C})) = \{\emptyset, A \cap B, A \cap \bar{B}, \bar{A} \cap B, \bar{A} \cap \bar{B}, \overset{1 \cup 2}{A}, \overset{1 \cup 3}{B}, \underset{1 \cup 4}{(A \cap B) \cup (\bar{A} \cap \bar{B})}, \underset{2 \cup 3}{A \Delta B}, \underset{2 \cup 4}{\bar{B}}, \underset{3 \cup 4}{\bar{A}}, \underset{1 \cup 2 \cup 3}{A \cup B}, \underset{1 \cup 2 \cup 4}{A \cup \bar{B}}, \underset{1 \cup 3 \cup 4}{\bar{A} \cup B}, \underset{2 \cup 3 \cup 4}{\overline{A \cup B}}, \underset{1 \cup 2 \cup 3 \cup 4}{\Omega}\}$$

$\mathbb{P} = (A_n)_{n \geq 1}$

$\sigma(\mathbb{P}) = \left\{ \bigcup_{n \in I} A_n \mid I \subset \mathbb{N} - \{0\} \right\}$

$\mathbb{P} \subset \sigma(\mathbb{P})$ OVVIO

$\sigma(\mathbb{P})$ È UNA $\sigma$- ALGEBRA

$\Omega \in \sigma(\mathbb{P})$ $\quad \Omega = \bigcup_{n \geq 1} A_n$

$A \in \sigma(\mathbb{P})$ $\quad A = \bigcup_{n \in I} A_n$ $\quad \overline{A} = \bigcup_{n \in \overline{I}} A_n \in \sigma(\mathbb{P})$

$$A = \bigcup_{n \in I} A_n \qquad B = \bigcup_{n \in J} A_n \qquad I, J \subset \mathbb{N}_+$$

$$A \cup B = \bigcup_{n \in I \cup J} A_n$$

$\sigma(\mathbb{P})$ È LA PIÙ PICCOLA $\sigma$-ALGEBRA CHE CONTIENE $\mathbb{P}$ (COSTRUITA CON UNIONI DI EVENTI DI $\mathbb{P}$)

$$\sigma(\mathcal{C}) = \sigma(\mathbb{P}_G(\mathcal{C})) \qquad \mathcal{C} = \{A_1 \ldots A_n\}$$

$$\mathcal{C} \subset \sigma(\underbrace{\mathbb{P}_G(\mathcal{C})}_{A'_1 \cap \ldots \cap A'_n})$$

$$A_1 = \underbrace{(A_1 \cap A_2 \cap \ldots A_n) \cup (A_1 \cap \overline{A_2} \cap \ldots A_n) \cup \ldots \cup (A_1 \cap \overline{A_2} \cap \ldots \overline{A_n})}_{\in \sigma(\mathbb{P}_G(\mathcal{C}))}$$

QUINDI $\sigma(\mathcal{C}) \subset \sigma(\mathbb{P}_G(\mathcal{C}))$ (MINIMALITÀ)

ORA, $\underbrace{\mathbb{P}_G(\mathcal{C})}_{A'_1 \cap \ldots \cap A'_n} \subset \sigma(\mathcal{C})$

QUINDI $\sigma(\mathbb{P}_G(\mathcal{C})) \subset \sigma(\mathcal{C})$ (MINIMALITÀ)

$(-\infty, a]$ $(-\infty, a)$ $a \in \mathbb{R}$

$[b, +\infty)$ $(b, +\infty)$ $b \in \mathbb{R}$

$(a, b)$ $(a, b]$ $[a, b)$ $[a, b]$

$a \leq b$

$\emptyset$ $\Omega = \mathbb{R} = (-\infty, +\infty)$

$\mathcal{C}$ $\underline{\underline{\mathcal{B} = \sigma(\mathcal{C})}}$

$$\mathcal{C}_1 = \{ \underline{(-\infty, a)} \mid a \in \mathbb{R} \}$$

$$\mathcal{C}_1 \subset \mathcal{C} \qquad \sigma(\mathcal{C}_1) \subset \sigma(\mathcal{C}) = \mathcal{B}$$

OGNI ALTRO INTERVALLO SI OTTIENE DA $(-\infty, a)$ CON LE SOLITE OPERAZIONI

$$\overline{(-\infty, a)} = [a, +\infty)$$

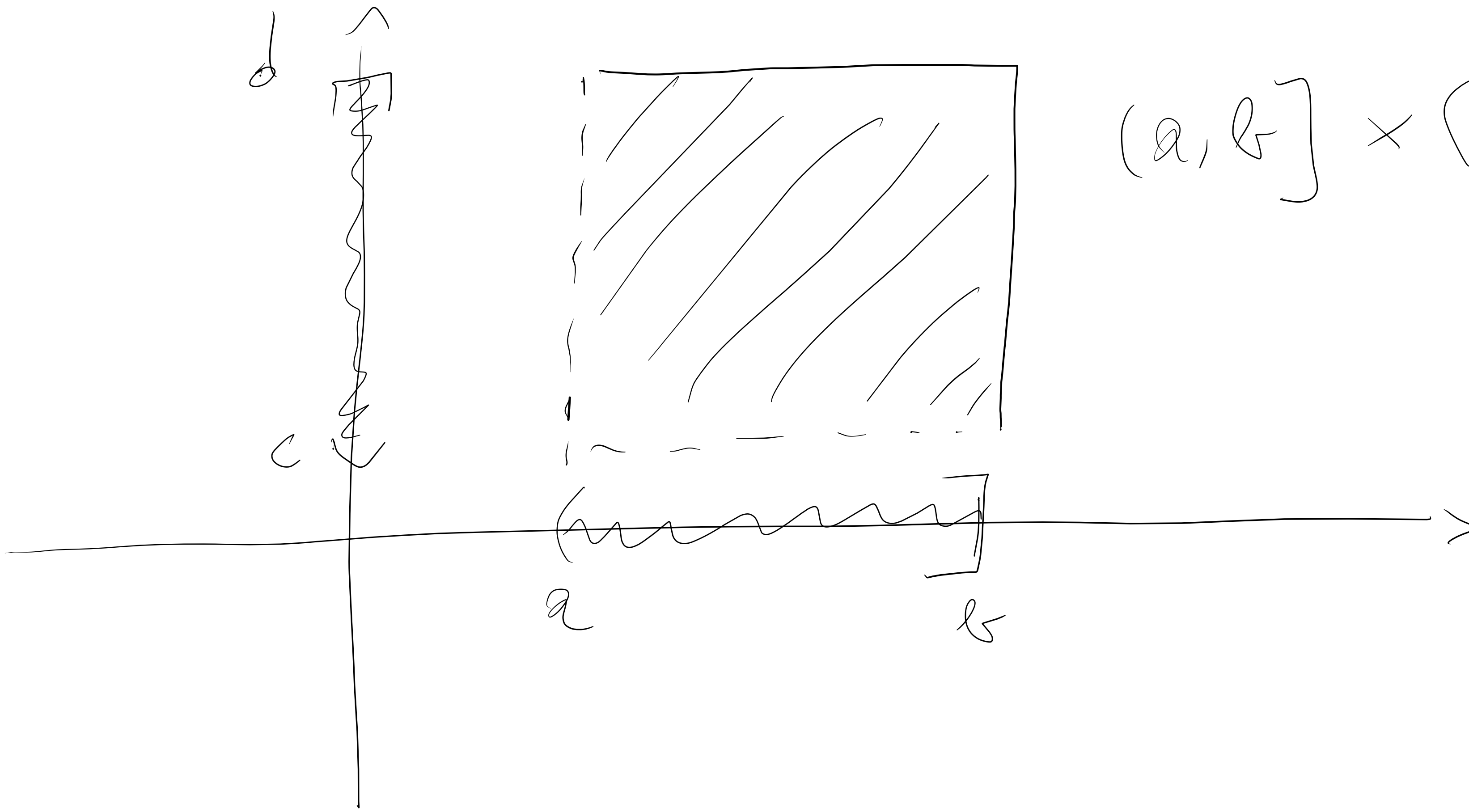
$$a < b \qquad (-\infty, b) - (-\infty, a) = [a, b)$$

$$(-\infty, a] = (-\infty, a+1) \cap \left(-\infty, a+\frac{1}{2}\right) \cap \left(-\infty, a+\frac{1}{3}\right) \cap$$

$$\cap \dots \cap \left(-\infty, a+\frac{1}{n}\right) \cap \dots$$

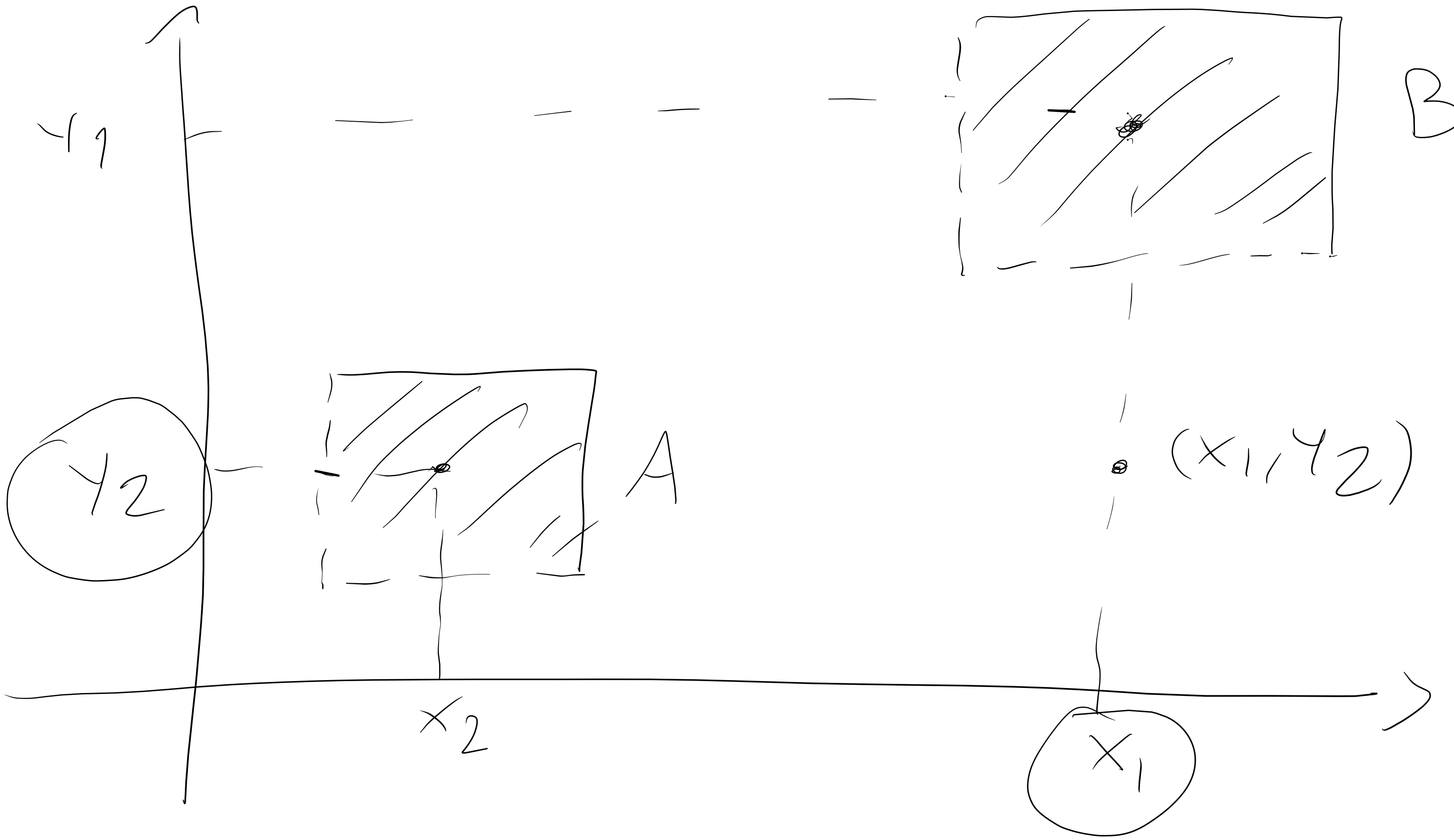
$\mathbb{R}^n = \{(x_1 \ldots x_n) \mid x_i \in \mathbb{R}\}$

<u>$n=2$</u>




$(a,b] \times (c,d]$

$\mathcal{B}^2$




$(x_1, y_2) \notin A \cup B$

$A \cup B$ È UN PRODOTTO CARTESIANO?

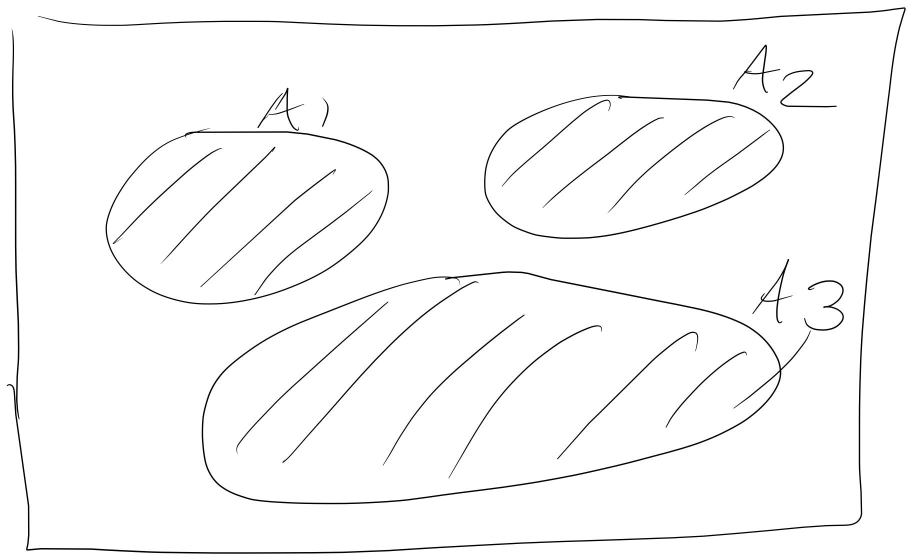
$= A_1 \times A_2$ ?

$A_1 \in \mathcal{B}$

$A_2 \in \mathcal{B}$

$A_1 \dots A_n$ A DUE A DUE DISGIUNTI

$P(A_1 \cup \dots \cup A_n) = P(A_1) + \dots + P(A_n)$ FINITA ADDITIVITÀ

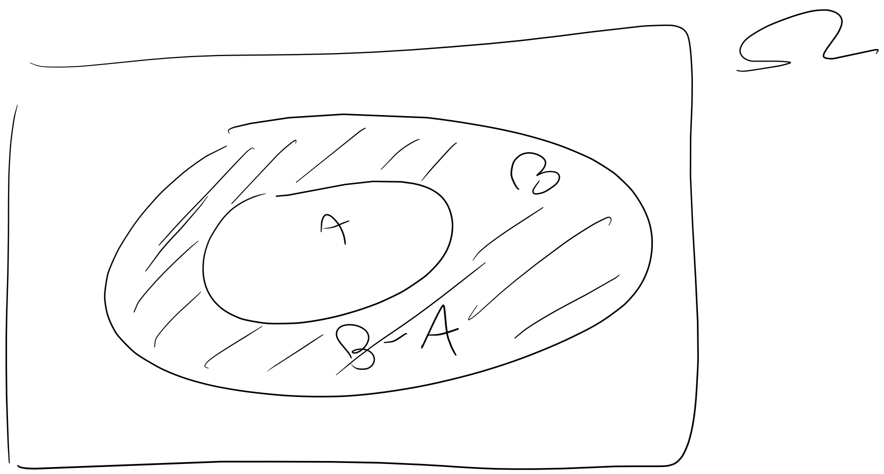



$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1 \cup \dots \cup A_n \cup \emptyset \cup \emptyset \cup \dots)$

$= P(A_1) + P(A_2) + \dots + P(A_n) + \underbrace{P(\emptyset)}_{=0} + \underbrace{P(\emptyset)}_{=0} + \dots$

$$\underbrace{P(A \cup \bar{A})}_{P(A) + P(\bar{A})} = P(\Omega) = 1$$

$(A, \bar{A}$ DISGIUNTI$)$



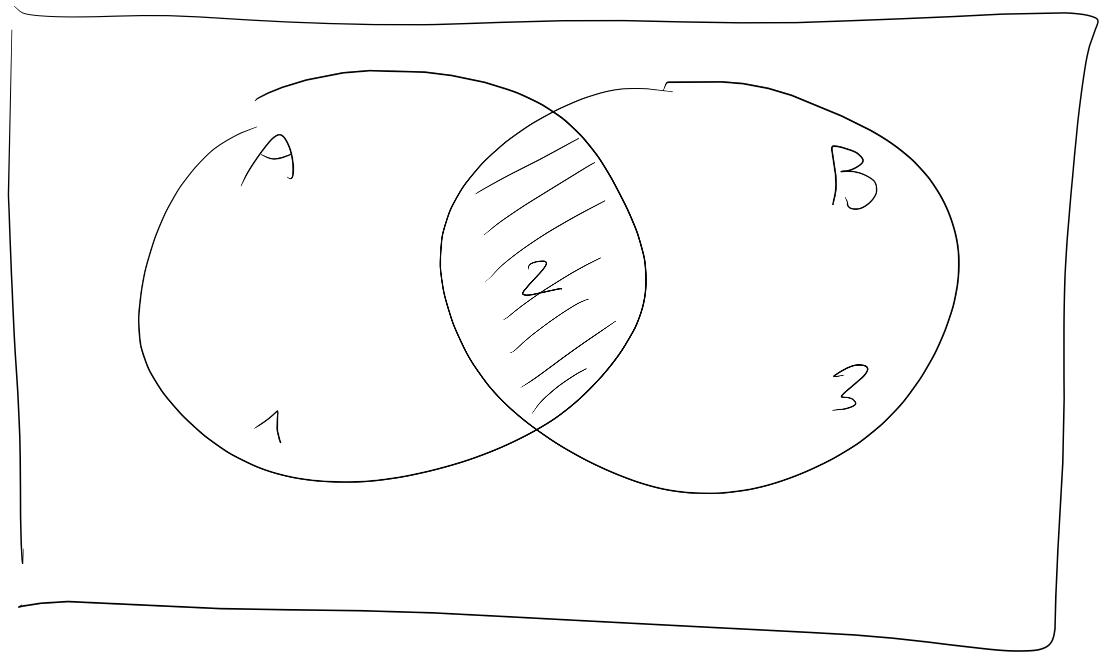

$A \subset B$

$$P(B-A) = P(B) - P(A)$$

$$B = A \cup (B-A)$$

$A, B-A$ DISGIUNTI

$$P(B) = P(A) + P(B-A)$$




$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\underbrace{A \cup B}) = P((A - B) \cup (A \cap B) \cup (B - A)) =$$

$$\underbrace{(A - B)}_{1} \cup \underbrace{(A \cap B)}_{2} \cup \underbrace{(B - A)}_{3}$$ A DUE A DUE DISGIUNTI

$$= \underbrace{P(A-B) + P(A\cap B)}_{*} + P(B-A)$$

$$- P(A\cap B) + P(A\cap B) \qquad \boxtimes$$

$*$ $\quad P(A-B) + P(A\cap B) = P\big((A-B)\cup(A\cap B)\big) = P(A)$

$\boxtimes$ $\quad P(B-A) + P(A\cap B) = \_\_\_\_ = P(B)$

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$$
$$- P(A_1 \cap A_2) - P(A_1 \cap A_3)$$
$$- P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$