Esercitazioni di "Geometria"
Foglio 4

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16 ottobre 2024

"La pratica è la verifica della teoria

Esercizio 1. Dimostrare che:

$$\forall n \in \mathbb{Z}, \alpha, z \in \mathbb{C}, \overline{\alpha \cdot z^n} = \overline{\alpha} \cdot \overline{z}^n.$$

Esercizio 2. Svolgere i seguenti calcoli:

$$-2i(1-3i, 1+3i) + 4(-2i+3, i+5), (-4+3i)(i,2) + 5(2+i, i-1), -7i(1,i) + 7\left(1, -\frac{2}{7}i\right) \in \mathbb{C}^2,$$

$$\mathbb{C}^3 \ni (-3+2i)(1-i, -i, 3+2i) + i(3,2,1), \pi\left(\frac{1+i}{2}, \frac{1-i}{3}, \frac{i-1}{6}\right) - \pi\left(\frac{3(1-i)}{2}, \frac{5(1+i)}{3}, \frac{7(i+1)}{6}\right),$$

$$\frac{4}{5}i\left(10i-5, 10+5i, \sqrt{25}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right) + \frac{2}{3}\left(\sqrt{9}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right), 27i-6, 18-9i\right),$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix}, (2i-4)\begin{pmatrix} -\frac{1+i}{2} & \frac{3-4i}{2} \\ \frac{1}{2(i-1)} & -\frac{3+4i}{2i^3} \end{pmatrix} + (4-2i)\begin{pmatrix} \frac{3i-9}{4} & -\frac{5+15i}{4} \\ \frac{1}{4(1-i)} & -\frac{(-1-i)^4}{4i^4} \end{pmatrix},$$

$$i\sqrt[3]{2}\begin{pmatrix} \sqrt[3]{4} & -i\frac{1}{\sqrt[3]{2}} \\ -i\frac{1}{\sqrt[3]{2}} & -\sqrt[3]{4} \end{pmatrix} - \sqrt[3]{2}\begin{pmatrix} \frac{i}{\sqrt[3]{2}} & -\sqrt[3]{4} \\ \sqrt[3]{4} & \frac{i}{\sqrt[3]{2}} \end{pmatrix} \in \mathbb{C}^2_2, \begin{pmatrix} 1+i & -1+2i & 0 \\ 0 & i-1 & (i-1)^3 \\ \frac{-1}{3-4i} & 0 & i^{360} \end{pmatrix} +$$

$$+\begin{pmatrix} -1+i & 0 & i-1 \\ (-1+i)^2 & 0 & (i-1)^2 \\ (-1+i)^{-2} & 0 & (i-1)^{-2} \end{pmatrix} \in \mathbb{C}^3_3, \begin{pmatrix} (2i-1)^{-1} & -(2-i)^{-1} & 0 \\ 0 & (1+i)^2 & -(1-i)^2 \end{pmatrix} +$$

$$+\begin{pmatrix} -(4+3i)^{-1} & 0 & (4-3i)^{-1} \\ \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 & 0 & -\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 \end{pmatrix} \in \mathbb{C}^3_2, \begin{pmatrix} \frac{-\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6}\right)i}{\frac{1}{\sqrt{2}}(1+i)^3} & 0 \\ 0 & \frac{\frac{4+3i}{4-3i}}{\frac{1}{\sqrt{2}}(1-i)^3} \end{pmatrix} +$$

$$+\begin{pmatrix} \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)i\right) & \frac{1}{\sqrt{2}}\left(\cos\left(\frac{5\pi}{4}\right)+\sin\left(\frac{5\pi}{4}\right)i\right) \\ \frac{1}{\sqrt{2}}\left(\cos\left(\frac{7\pi}{4}\right)+\sin\left(\frac{7\pi}{4}\right)i\right) & \frac{1}{\sqrt{2}}\left(\cos\left(\frac{3\pi}{4}\right)+\sin\left(\frac{3\pi}{4}\right)i\right) \\ -1 & 1 \end{pmatrix} \in \mathbb{C}^2_3.$$

Esercizio 3. Date le seguenti matrici:

$$A = \begin{pmatrix} \overline{-2i+1} & 0 & \frac{1}{10}|6i-8|(3+i) \\ \frac{5}{1-2i} & i^2 & 3+i \end{pmatrix} \in \mathbb{C}^3_2,\ B = \begin{pmatrix} 2i-1 & 0 \\ -3+2i & i-3 \\ -\frac{1}{\sqrt{32}}|4+4i| & \frac{3i-4i}{8+6i} \end{pmatrix} \in \mathbb{C}^2_3,$$

$$C = \begin{pmatrix} 0 & i & 1 \\ -1 & 0 & 1+i \\ -i & -1+i & 0 \end{pmatrix} \in \mathbb{C}^3_3,\ D = \begin{pmatrix} i & 0 \\ -i & 0 \end{pmatrix},\ E = \begin{pmatrix} 0 & 0 \\ -i & i \end{pmatrix} \in \mathbb{C}^2_2;$$

calcolare i seguenti prodotti di matrici: $AB, BA, AC, BD, BE, CB, CC, DD, EE, DE, ED, ABD$.

Esercizio 4. Dato il seguente polinomio $p(x) = x^2 + i \in \mathbb{C}[x]$.

a) Scomporre $p(x)$ sul campo $\mathbb{C}$.

b) Scrivere in coordinate polari gli zeri di $p(x)$.

Esercizio 5. Calcolare la matrice inversa di ognuna delle seguenti matrici:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{pmatrix} \in \mathbb{C}^2_2.$$

Esercizio 6. Risolvere i seguenti sistemi di equazioni lineari.

$$\begin{cases} x+y+2z=4 \\ z=2 \\ 2z=y+3 \end{cases}, \begin{cases} 2x+3y+3=z \\ z+x=0 \\ 2x+2z+y+t-1=0 \end{cases}, \begin{cases} 4y=1 \\ x+z=y \\ 2z=1+3y \end{cases}, \begin{cases} 1+x+2y+z+t=0 \\ y+t=0 \\ t=2+2y+2z \end{cases},$$

$$\begin{cases} t=2+2z+y \\ 1+x+2y+z+t=0 \\ t=y \end{cases}, \begin{cases} z-y=0 \\ x+z=1 \\ 2z-1=0 \end{cases}, \begin{cases} a+e+f=2c \\ b+2e=c+d \\ c+d+f=e \end{cases}.$$