Artificial Intelligence for Cyber-Physical Systems

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Lecture 5: Stochastic (Hybrid) Systems

Probabilistic Models

- Models for components that we studied so far were either deterministic or nondeterministic.
- The goal of models is to represent computation or time-evolution of a physical phenomenon.
- These models *do not* do a great job of capturing uncertainty.
- We can usually model uncertainty using probabilities, so probabilistic models allow us to account for likelihood of environment behaviors
- Machine learning/AI algorithms also require probabilistic modelling!

Stochastic Differential Equation Model

- State Dynamics with Process Noise:

$$\dot{\boldsymbol{x}} \ = \boldsymbol{A}\boldsymbol{x} + B\boldsymbol{u} + \dot{\mathbf{w}} \text{(Process Model)}$$

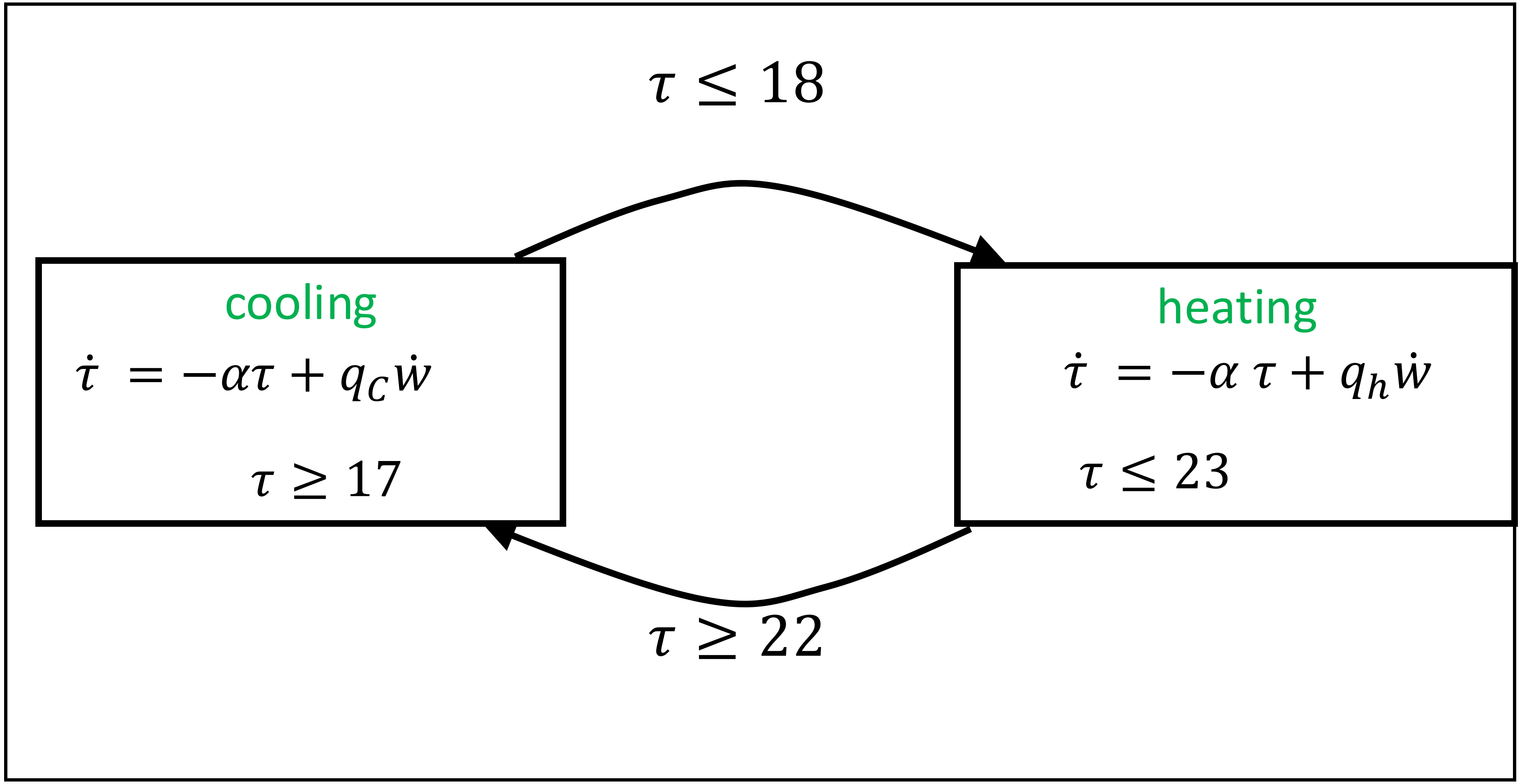
where $\boldsymbol{w}$ is process noise (white noise).

- Output Equation with Sensor Noise:

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + D\boldsymbol{u} + \boldsymbol{\eta} \ \text{(Measurement Model)}$$

where $\boldsymbol{\eta}$ is sensor/measurement noise (white noise).

Example



$\tau \leq 18$
cooling
$\dot{\tau} = -\alpha\tau + q_C\dot{w}$
$\tau \geq 17$
heating
$\dot{\tau} = -\alpha\,\tau + q_h\dot{w}$
$\tau \leq 23$
$\tau \geq 22$

Stochastic Difference Equation Models

- We assume that the plant (whose state we are trying to estimate) is a stochastic discrete dynamical process with the following dynamics:

$$\mathbf{x}_k = A\mathbf{x}_{k-1} + B\mathbf{u}_k + \mathbf{w}_k \text{ (Process Model)}$$

$$\boldsymbol{y}_k = H\mathbf{x}_\mathrm{k} + \mathbf{v}_k \text{ (Measurement Model)}$$

$\mathbf{x}_k, \mathbf{x}_{k-1}$	State at time k,$k-1$
$\mathbf{u}_k$	Input at time k
$\mathbf{w}_k$	Random vector representing noise in the plant, $\mathbf{w} \sim N(\mathbf{0}, Q_k)$
$\mathbf{v}_k$	Random vector representing sensor noise, $\mathbf{v} \sim N(\mathbf{0}, R_k)$
$\boldsymbol{y}_k$	Output at time k

The Family of Markov Models

Markov Models		*Do we have control over the state transitions?*	
		No	*Yes*
Are the states completely observable?	*Yes*	**MC** *Markov Chain*	**MDP** *Markov Decision Process*
	No	**HMM** *Hidden Markov Model*	**POMDP** *Partially Observable Markov Decision Process*

The Memoryless Property

$$\boxed{p(s_{t+1}|s_{1:t}) \;=\; p(s_{t+1}|s_t).}$$

The knowledge of the state s_t captures the complete information capturing all the relevant information about the present and the past of the system necessary for predicting its future evolution