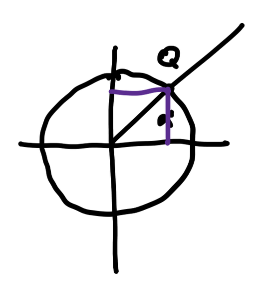
Lezione 15 ottobre 2024

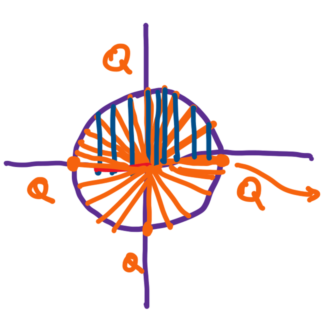
martedì 15 ottobre 2024 14:05

FUNZIONI TRIGONOMETRICHE



$Q = (\cos\alpha, \sin\alpha)$

raggio $= 1$



$\to (1,0)$

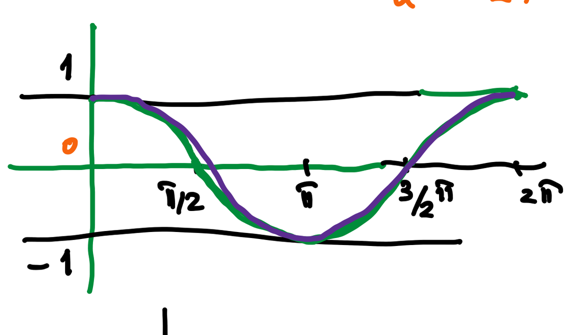
$\alpha = 0$ $\cos 0 = 1$

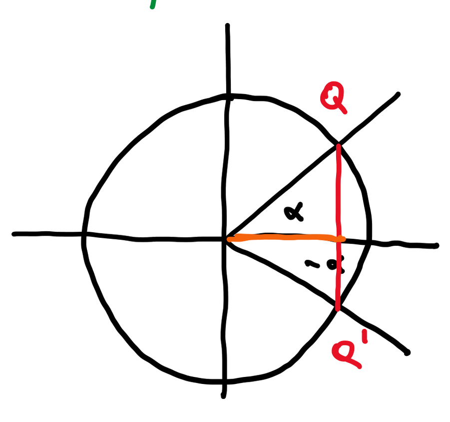
$\alpha = \pi/2$ $\cos \pi/2 = 0$

$\alpha = \pi$ $\cos \pi = -1$

$\alpha = \frac{3}{2}\pi$ $\cos \frac{3}{2}\pi = 0$

$\alpha = 2\pi$ $\cos 2\pi = 1$





cos x è pari? sì

è dispari? no

$\cos(\alpha)$ $\cos(-\alpha)$

$\cos(\alpha) = \cos(-\alpha)$

Sin α :

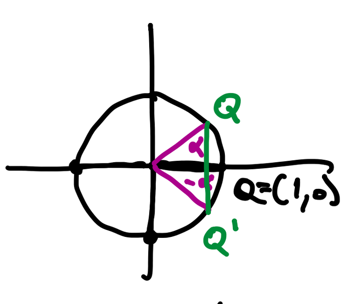
$\sin 0 = 0$

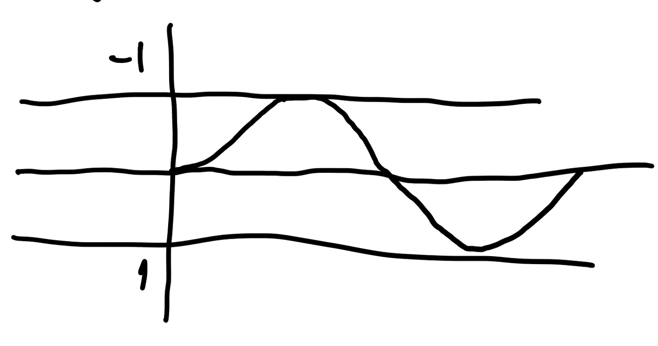
$\sin \pi/2 = 1$

$\sin \pi = 0$

$\sin \frac{3}{2}\pi = -1$

$\sin 2\pi = 0$





$\sin(-\alpha) = -\sin(\alpha)$

$\Rightarrow$ sin è DISPARI

Teo Fourier: $f(x)$ continua e periodica di periodo T: si può approssimare con una somma del tipo (POLINOM. TRIG.)

$T_1 = \frac{2\pi}{\omega}$ $T_2 = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{T_1}{2}$ ($T_1 = 2T_2$) $T_k = \frac{2\pi}{k\omega} = \frac{T_1}{k}$ ($T_1 = kT_k$)

$$f(x) \sim a_0 + a_1 \sin(\omega x) + a_2 \sin(2\omega x) + \dots + a_k \sin(k\omega x)$$
$$+ b_1 \cos(\omega x) + b_2 \cos(2\omega x) + \dots + b_h \cos(h\omega x)$$

PART. FUNZ. PERIODICHE

Fuz. Sinusoidali

$$f(x) = a \cdot \cos(\omega \cdot (x - x_0)) + b$$

i valori sono sempre compresi tra

$b - a$ e $b + a$

$a =$ ampiezza dell'oscillazione

$\omega =$ frequenza angolare

Ricordiamo: $\sin(\omega x)$, $\cos(\omega x)$ ha periodo $T = \frac{2\pi}{\omega}$

$\sin(\omega x)$ ha periodo $T_1 = \frac{2\pi}{\omega}$

$= f_1(x)$ $f_1(x + T_1) = f_1(x)$

$\sin(2\omega x) = f_2(x)$ $f_2(x + T_2) = f_2(x)$

$= f_2(x + \frac{T_1}{2}) = f_2(x)$

$= f_2(x + \frac{T_1}{2} + \frac{T_1}{2}) = f_2(x)$

$= f_2(x + T_1) = f_2(x)$

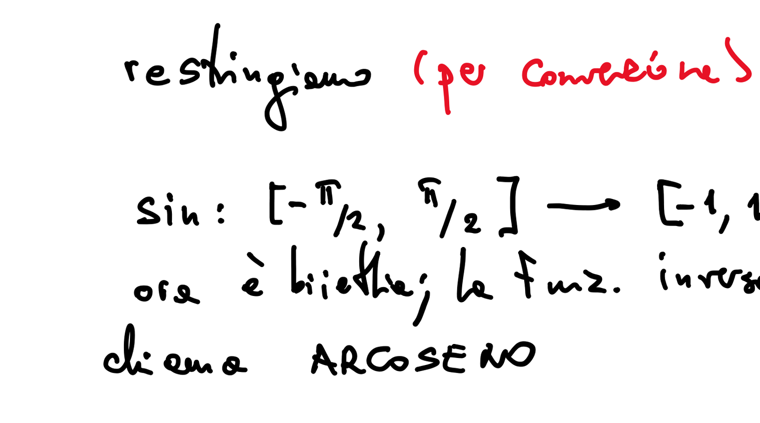
$f_3(x) = \sin(3\omega x)$, $T_3 = \frac{2\pi}{3\omega} = \frac{T_1}{3}$

$\Rightarrow a_0 + a_1 \sin(\omega x) + a_2 \sin(2\omega x) + \dots + a_k \sin(k\omega x)$ ha periodo $T_1 = \frac{2\pi}{\omega}$

~ . ~ . ~

FUNZ. TRIG. INVERSE:

$\sin: \mathbb{R} \to \mathbb{R}$



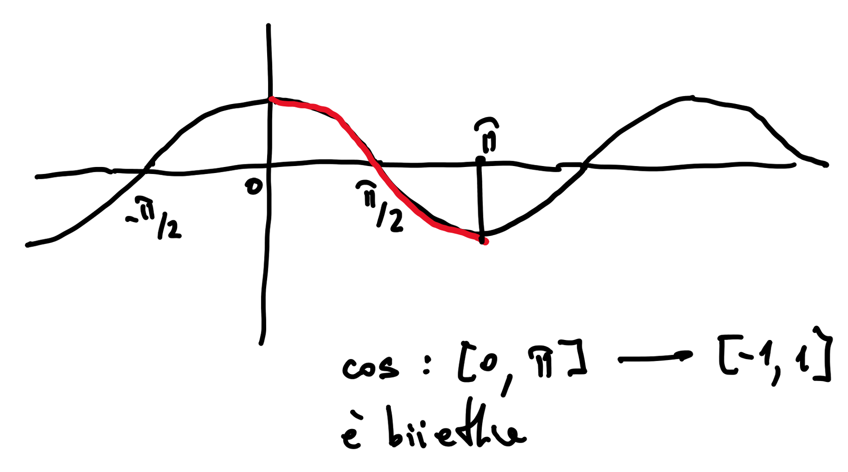
restringiamo (per convenzione)

$\sin: [-\pi/2, \pi/2] \to [-1, 1]$

ora è biiettiva; la funz. inversa si chiama ARCOSENO

$\arcsin: [-1, 1] \to [-\pi/2, \pi/2]$

Analog. per $\cos: \mathbb{R} \to \mathbb{R}$



$\cos: [0, \pi] \to [-1, 1]$

è biiettiva

$\Rightarrow$ la funz. inversa ARCO COSENO

$\arccos: [-1, 1] \to [0, \pi]$

FORMULE DI ADDIZIONE:

- $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \sin\beta \cos\alpha$

  $\sin(2\alpha) = \sin(\alpha + \alpha) = 2 \sin(\alpha)$

$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \sin\beta \cos\alpha$

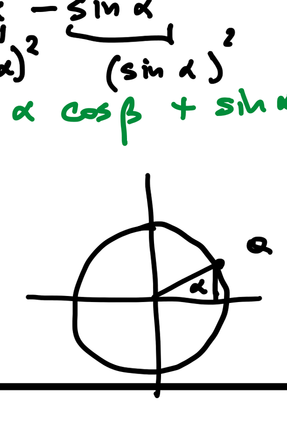
$(= \sin(\alpha + (-\beta)))$

- $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

  $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$, $(\cos\alpha)^2$ $(\sin\alpha)^2$

  $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

REL. FOND.:

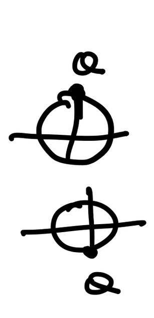


$$\sin^2\alpha + \cos^2\alpha = 1$$

FUNZIONE TANGENTE

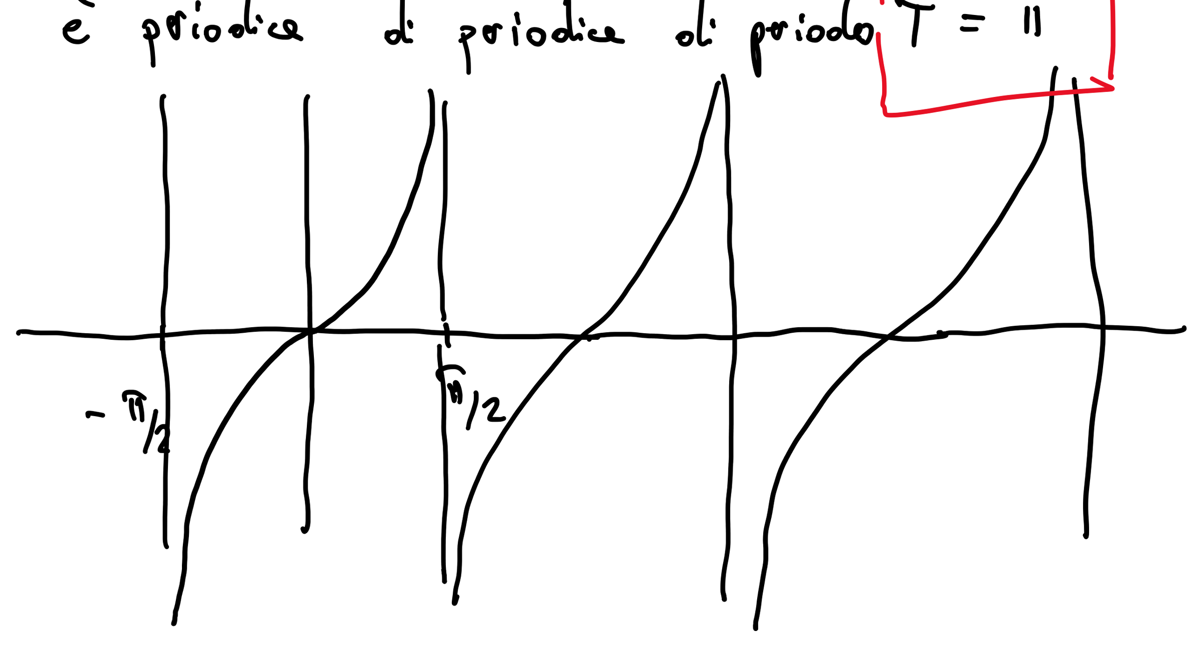
$\tan(\alpha) = tg(\alpha) := \frac{\sin\alpha}{\cos\alpha}$

$\tan: \mathbb{R} \setminus \{\alpha : \alpha = \frac{\pi}{2} + 2k\pi,\ \alpha = \frac{3}{2}\pi + 2k\pi\}$



$\tan: \mathbb{R} \setminus \{\alpha : \alpha = \frac{\pi}{2} + k \cdot \pi\} \to \mathbb{R}$

è periodica di periodo di periodo $T = \pi$



Se restring: $\tan: (-\pi/2, \pi/2) \to \mathbb{R}$

biiettiva

ammette inversa: ARCO TANGENTE

arctan, arctg $: \mathbb{R} \to (-\pi/2, \pi/2)$