

1) a) $6 + 5 \cdot 3^x - 3^{2x} = 0$
 $-(3^x)^2 + 5 \cdot 3^x + 6 = 0$

pongo $z = 3^x$

$-z^2 + 5z + 6 = 0 \quad / \cdot (-1)$

$z^2 - 5z - 6 = 0$

$\Delta = 25 + 24 = 49$

$z_{1,2} = \frac{5 \pm \sqrt{49}}{2} = \frac{5 \pm 7}{2} \begin{matrix} -1 \\ 6 \end{matrix}$

$z_1 = -1 = 3^x$ IMPOSSIBILE ($3^x > 0 \quad \forall x \in \mathbb{R}$)

$z_2 = 6 = 3^x \Rightarrow x = \log_3 6$

a) bis $6 + 5 \cdot 3^x - 3^{x+2} = 0$

$6 + 5 \cdot 3^x - 3^{x+2} = 0$

$3^{x+2} - 5 \cdot 3^x = 6$

$3^x (9 - 5) = 6$

$3^x = \frac{6}{4}, \quad 3^x = \frac{3}{2}$

$x = \log_3 \frac{3}{2}$

$x = \log_3 3 - \log_3 2$

$x = 1 - \log_3 2$

b) $\frac{8^x \cdot 2}{2^{x+3}} = \frac{2^{x+1}}{2^{2x+2}}$

$\frac{(2^3)^x \cdot 2}{2^{x+3}} = \frac{2^{x+1}}{2^{2x+2}}$

$\frac{2^{3x+1}}{2^{x+3}} = \frac{2^{x+1}}{2^{2x+2}}$

$2^{(3x+1)-(x+3)} = 2^{(x+1)-(2x+2)}$

$2^{2x-2} = 2^{-x-1} \Leftrightarrow 2x-2 = -x-1$

exp è iniettiva $3x = 1$

$x = \frac{1}{3}$

c) $3^{2x} - \left(\frac{1}{3}\right)^{x+1} = 0$

$3^{2x} - (3^{-1})^{x+1} = 0$

$3^{2x} = 3^{-x-1} \Leftrightarrow 2x = -x-1$

$x = -\frac{1}{3}$

d) $\log_{10} (10-2x) = \log_{10} (5-x) - \log_{10} 4$

$\log_{10} (10-2x) = \log_{10} \frac{5-x}{4}$

\log_{10} è iniettiva \Uparrow

$10-2x = \frac{5-x}{4}$

$40-8x = 5-x$

$7x = 35, \quad x = 5$

e) $\log_{1/4} (x^2-6) - \log_{1/4} (x-3) = 0$

$\log_{1/4} (x^2-6) = \log_{1/4} (x-3)$

$\Leftrightarrow x^2-6 = x-3$

$x^2-x-3 = 0$

$\Delta = 1+12=13$

$x_{1,2} = \frac{1 \pm \sqrt{13}}{2}$

2) a) $5^{2(x-2)} (5^{2(x-1)})^{x+1} > 125^{x-1}$

$5^{2(x-2)} \cdot 5^{2(x-1)(x+1)} > 5^{3(x-1)}$

$5^{2(x-2) + 2(x-1)(x+1)} > 5^{3(x-1)}$

$5 > 1 \Rightarrow 5^x$ è strettamente crescente

$\Leftrightarrow 2(x-2) + 2(x-1)(x+1) > 3(x-1)$

$2(x^2-1) + 2(x-2) - 3(x-1) > 0$

$2x^2 - 2 + 2x - 4 - 3x + 3 > 0$

$2x^2 - x - 3 > 0$

$\Delta = 1 + 24 = 25$

$x_{1,2} = \frac{1 \pm \sqrt{25}}{4} \begin{matrix} -1 \\ 3/2 \end{matrix}$

$\Rightarrow x < -1 \quad \text{opp.} \quad x > 3/2$

b) $\log_3 \log_3 (4x+6) < 0$

$3 > 1 \Rightarrow \log_3 x$ è strettamente crescente

$0 = \log_3 1$

$\log_3 \log_3 (4x+6) < \log_3 1$

$\Leftrightarrow \log_3 (4x+6) < 1$

$\log_3 (4x+6) < 3^1$

$4x+6 < 3$

$x < -\frac{3}{4}$

c) $\log_{10} (x^2-13x-14) > 2$

$\Leftrightarrow 10^{\log_{10} (x^2-13x-14)} > 10^2$

$\Leftrightarrow x^2-13x-14 > 100$

$x^2-13x-114 > 0$

$\Delta = 169 + 4 \cdot 114 = 625, \quad \sqrt{\Delta} = 25$

$x_{1,2} = \frac{13 \pm 25}{2} \begin{matrix} -6 \\ 19 \end{matrix}$

$\Rightarrow x < -6 \quad \text{opp.} \quad x > 19$

e) $\log_{1/2} (x) \geq -1$

$(1/2)^x$ è strettamente decrescente
 perché $1/2 < 1$

$\Leftrightarrow (1/2)^{\log_{1/2} (x)} \leq (1/2)^{-1}$

$x \leq 2$

f) $\left(\frac{\sqrt{2}}{2}\right)^{9x} > \left(\frac{\sqrt{2}}{2}\right)^{1/x}$

$\frac{\sqrt{2}}{2} < 1 \Rightarrow \left(\frac{\sqrt{2}}{2}\right)^x$ è strettamente decrescente

$\Leftrightarrow 9x < 1/x \quad : \text{se } x > 0$

$9x^2 < 1 \Leftrightarrow 0 < x < 1/3$

se $x < 0 \quad 9x^2 > 1$

$9x^2 - 1 > 0$

$x^2 - \frac{1}{9} > 0 \Leftrightarrow -\frac{1}{3} < x < 0$

h) $2 \log_5 x \geq 3$

$\log_5 x \geq \frac{3}{2}$

$5^{\log_5 x} \geq 5^{3/2}$

$x \geq 5^{3/2} = \sqrt{125}$