$n = 2 \quad A_1, A_2 \quad P(A_1 \cap A_2) = P(A_1)P(A_2)$

$n = 3 \quad A_1, A_2, A_3$

$$\left.\begin{aligned}
P(A_1 \cap A_2) &= P(A_1)P(A_2)\\
P(A_1 \cap A_3) &= P(A_1)P(A_3)\\
P(A_2 \cap A_3) &= P(A_2)P(A_3)\\
P(A_1 \cap A_2 \cap A_3) &= P(A_1)P(A_2)P(A_3)
\end{aligned}\right\} 4$$

$n = 4 \quad A_1, A_2, A_3, A_4$

$$\left.\begin{aligned}
&\binom{4}{2} + \binom{4}{3} + \binom{4}{4}\\
&= 6 + 4 + 1 = 11
\end{aligned}\right\} 11$$

$n \quad A_1, A_2 \ldots, A_n$

$2^n - n - 1$

$$A_1 = \{abc, acb, \underline{aaa}\}$$

$$A_2 = \{cab, bac, \underline{aaa}\} \qquad A_3 = \{cba, bca, \underline{aaa}\}$$

$$P(A_i) = \frac{3}{9} = \frac{1}{3}$$

$$P(A_1 \cap A_2) = P(\{aaa\}) = \frac{1}{9} = \frac{1}{3} \cdot \frac{1}{3} = P(A_1)P(A_2)$$

$$P(A_1 \cap A_2 \cap A_3) = P(\{aaa\}) = \frac{1}{9} \neq \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = P(A_1)P(A_2) \cdot P(A_3)$$

$$\Omega = \{1C, \ldots, 10C, JC, QC, KC,$$
$$1Q, \quad , 10Q, JQ, QQ, KQ,$$
$$1F$$
$$1P \qquad \}$$

$$\mathcal{F} = 2^{\Omega}$$

$$P(\{\omega\}) = \frac{1}{52}$$

$$P(\{SEME = C\}) = \frac{13}{52} = \frac{1}{4}$$

$$P(\{RANGO = 10\}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\{10C\}) = \frac{1}{52} = \frac{1}{13} \cdot \frac{1}{4} =$$

$$= P(\{CUORI\}) \cdot P(\{DIECI\})$$

2 DADI $\quad \Omega = \{(i,j) \mid 1 \le i,j \le 6\}$

$A =$ " SOMMA $= 7$ "

$X_1 =$ PRIMO DADO $\quad \{X_1 = 3\} \quad \{X_1 = i\} \quad 1 \le i \le 6$

$$P(A \cap \{X_1 = i\}) = P(\{(i, 7-i)\}) = \frac{1}{36} =$$

$$= \frac{1}{6} \cdot \frac{1}{6} = P(A)\,P(\{X_1 = i\})$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(\{X_1 = i\}) = \frac{1}{6}$$

MAZZO DI CARTE

R = RANGO S = SEME

$$\{ \{R=1\}, \ldots, \{R=K\} \} = \mathcal{A}_1$$

$$\{ \{S=C\}, \{S=Q\}, \{S=F\}, \{S=P\} \} = \mathcal{A}_2$$

LANCIO DI n DADI

$\mathbb{P}_1, \ldots, \mathbb{P}_n$

$\mathbb{P}_1, \mathbb{P}_3, \mathbb{P}_7$

$D_{1,1} \in \mathbb{P}_1 \quad D_{6,3} \in \mathbb{P}_3 \quad D_{5,7} \in \mathbb{P}_7$

$$P(D_{1,1} \cap D_{6,3} \cap D_{5,7}) = P(D_{1,1})P(D_{6,3}) \cdot P(D_{5,7})$$

A EVENTO COSTRUITO PARTENDO DAGLI EVENTI IN $\mathbb{P}_1, \mathbb{P}_3$

B // // // // //
// $\mathbb{P}_7$

A, B INDIPENDENTI

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= P(A)$$

SE A DIPENDE DA $\mathbb{P}_1, \mathbb{P}_3$

B " " $\mathbb{P}_3, \mathbb{P}_7$

$$P(A|B) = P(A)$$

$\mathcal{A}_1, \mathcal{A}_2\ \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5$ INSIEMI DI EVENTI INDIPENDENTI

$I = \{1, 2, \ldots 5\}$

$I_1 = \{1, 3\}\quad I_2 = \{4, 5\}\quad I_3 = \{2\}$

$\mathcal{F}_1 = \sigma(\mathcal{A}_1 \cup \mathcal{A}_3)\qquad \mathcal{F}_2 = \sigma(\mathcal{A}_4 \cup \mathcal{A}_5)\qquad \mathcal{F}_3 = \sigma(\mathcal{A}_2)$

$\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$ INDIPENDENTI

LANCI INFINITI DI UNA MONETA

$$P(E_1^1 \cap E_2^1 \cap \dots \cap E_n^1) = P(E_1^1) \cdot \dots \cdot P(E_n^1)$$

ESTENSIONE

$$= \frac{1}{2^n}$$

$$P(\{\omega\}) = P(E_1^1 \cap E_2^1 \cap \dots \cap E_n^1 \cap \dots) = 0$$

L ALTERNO T CON C INFINITE VOLTE

$$P(\{TCTCTC\dots\})$$

$$\{\underbrace{TCTC\ldots\ldots}_{\downarrow}\} \subset \{\underbrace{TCTC\ldots TC}_{n \text{ VOLTE}}\} = A_n$$

$$\underbrace{P(\{\omega\})}_{=0!} \leq \frac{1}{2^n} \quad (\text{MONOTONIA}) \quad \forall n$$

$$\frac{1}{2^n} \to 0 \text{ SE } n \to +\infty$$

$$A_n \supset A_{n+1} \qquad \lim A_n = \bigcap_{n=1}^{\infty} A_n = \{\omega\}$$

$$P(\lim A_n) = P(\{\omega\}) = \lim_{n\to+\infty} \underbrace{P(A_n)}_{\frac{1}{2^n}} = 0$$

$A = \{\text{TESTA ESCE PRIMA O POI}\}$

$\overline{A} = \{CCC \ldots C \ldots\}$ $\quad P(\overline{A}) = 0$

$$A = E_1 \cup (\overline{E_1} \cap E_2) \cup (\overline{E_1} \cap \overline{E_2} \cap E_3) \cup \ldots$$

$$\ldots \cup \underbrace{(\overline{E_1} \cap \overline{E_2} \cap \ldots \cap \overline{E_{n-1}} \cap E_n)}_{\text{ESCE T PER LA PRIMA VOLTA AL COLPO } n} \cup \ldots$$

$$P(A) = P(E_1) + P(\overline{E_1} \cap E_2) + \ldots + P(\overline{E_1} \cap \ldots \cap \overline{E_{n-1}} \cap E_n)$$
$$+ \ldots = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = 1$$

B = "ESCONO K TESTE CONSECUTIVE PRIMA O POI"

($K \geq 1$ FISSATO) $\quad P(B) = 1$

$P(\overline{B}) = 0 \quad \overline{B}$ = "K TESTE CONSECUTIVE NON ESCONO MAI"

D_n = "K TESTE CONSECUTIVE NON ESCONO MAI NEI PRIMI n LANCI"

$$\overline{B} \subset D_n \quad \forall n$$

F_m = "K TESTE CONSECUTIVE NON ESCONO NEI PRIMI K LANCI, NEI SECONDI K LANCI, – –, NEGLI m-ESIMI K LANCI"

$$\underbrace{1\quad 2\quad \dots\quad k}_{1^\circ\ \text{BLOCCO}}\ \Big|\ \underbrace{k+1\quad \dots\quad 2k}_{2^\circ\ \text{BLOCCO}}\ \Big|\ \dots\ \Big|\ \underbrace{(m-1)k+1\quad \dots\quad mk}_{m^\circ\ \text{BLOCCO}}\ \Big|$$

$$F_m$$

$$\overline{B} \subset D_{mk} \subset F_m \quad \forall m$$

IMPACCHETTAMENTO,
ESTENSIONE

$$P(F_m) = P(F_m^1 \cap \dots \cap F_m^m) = P(F_m^1) \cdots P(F_m^m) =$$

F_m^j = "NON ESCONO K TESTE CONSECUTIVE NEL BLOCCO J"

$$= \underbrace{\left(1-\frac{1}{2^k}\right)\cdot\left(1-\frac{1}{2^k}\right)\cdots\left(1-\frac{1}{2^k}\right)}_{m \text{ VOLTE}}$$

$$= \left(1-\frac{1}{2^k}\right)^m$$

$$P(\overline{B}) \underset{\text{MONOTONIA}}{\leq} \underbrace{\left(\overbrace{1-\frac{1}{2^k}}^{<1}\right)^m}_{} \xrightarrow[m\to+\infty]{} 0 \qquad \forall m$$

$$\omega = L_1 L_2 L_3 \ldots L_n \ldots \qquad L_n \begin{cases} T \\ C \end{cases}$$

$$X_n(\omega) = \begin{cases} 1 & \text{SE } L_n = T \\ 0 & \text{SE } L_n = C \end{cases}$$

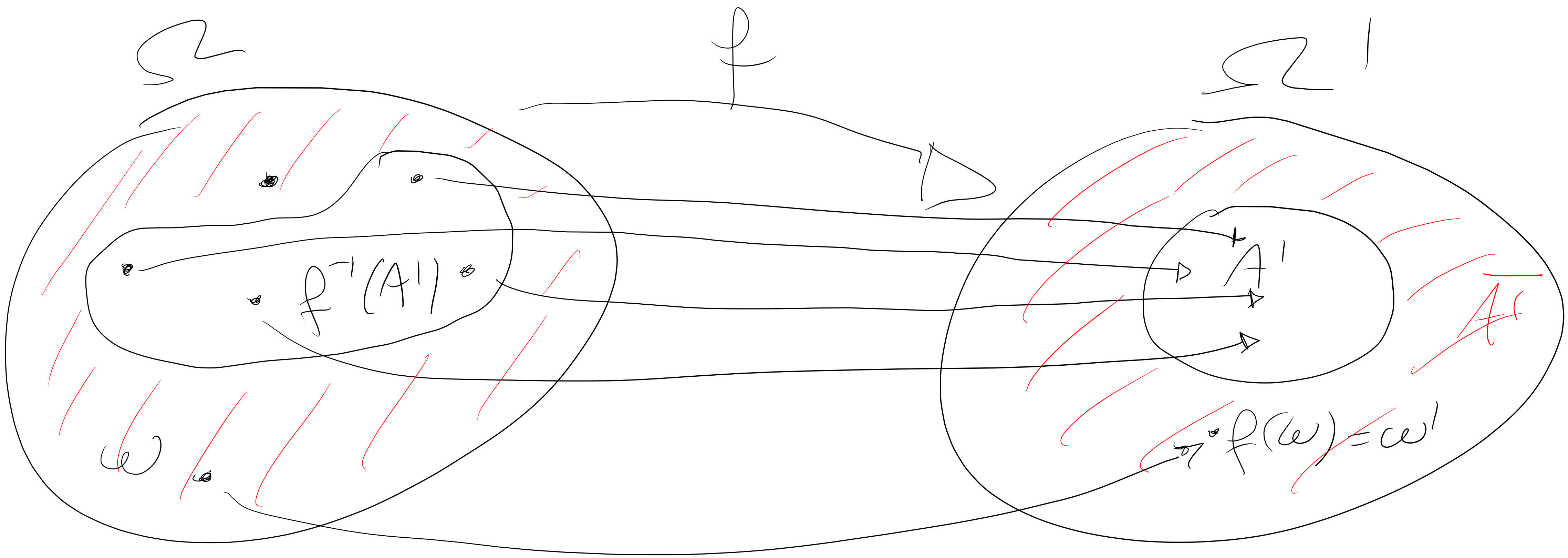
FISSATO

$$T_n(\omega) = \begin{cases} 0 & \text{SE } L_1 = L_2 = \ldots = L_n = C \\ 1 & \text{1 T TRA } L_1 \ldots L_n \text{, LE ALTRE SONO C} \\ 2 & \\ \vdots & \\ n & \text{SE } L_1 = L_2 = \ldots = L_n = T \end{cases}$$

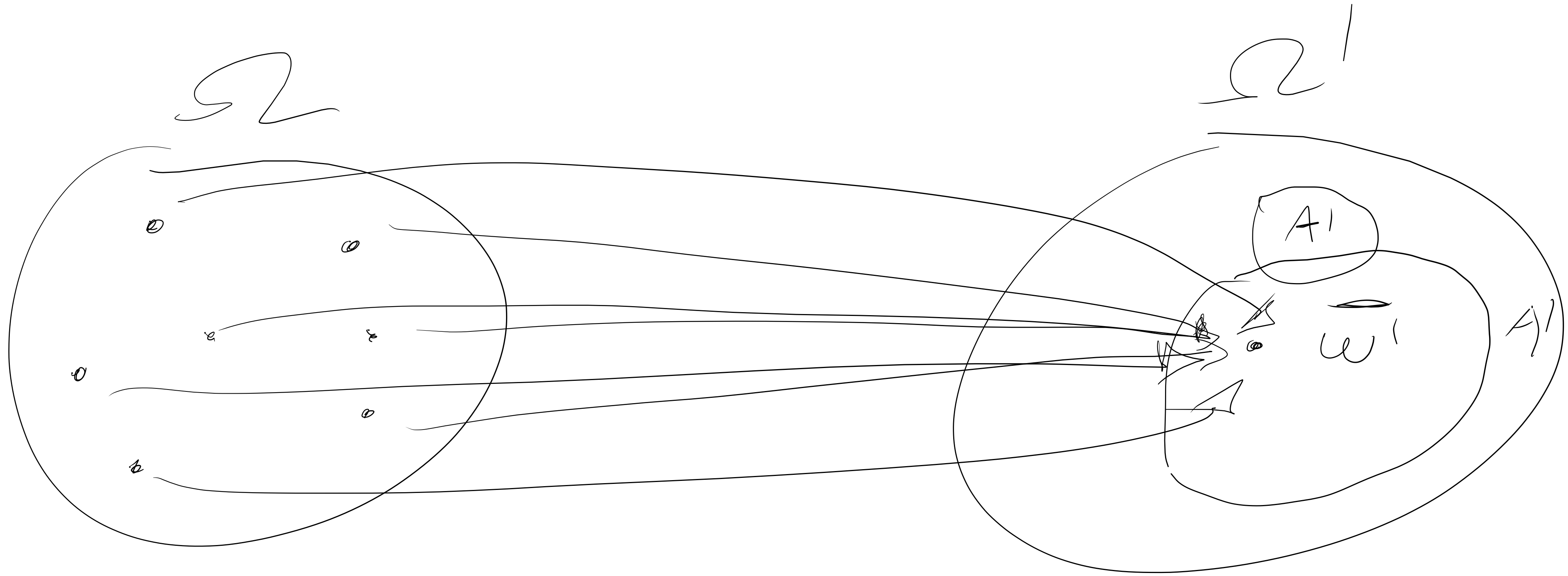
$$D_n(\omega) = T_n(\omega) - (n - T_n(\omega)) = 2T_n(\omega) - n$$

$$C_n(\omega)$$

$$C_{10}(\underbrace{TT}_{2}C\underbrace{T}_{1}C\underbrace{TTTT}_{4}C\ldots) = 4$$



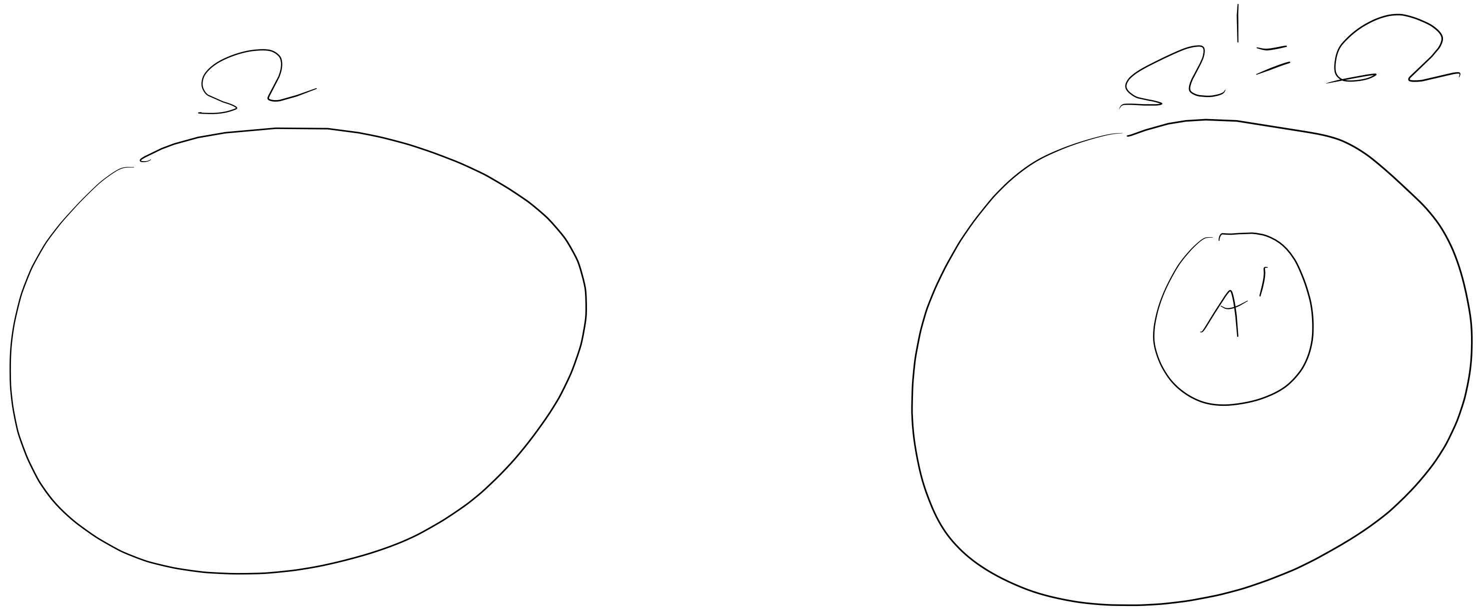
$f^{-1}(A') \subset \Omega$



$A' \in \mathcal{F}'$

$$f^{-1}(A') = \begin{cases} \phi \in \mathcal{F} & \text{si } \bar{\omega}' \notin A' \\ \Omega \in \mathcal{F} & \text{si } \bar{\omega}' \in A' \end{cases}$$

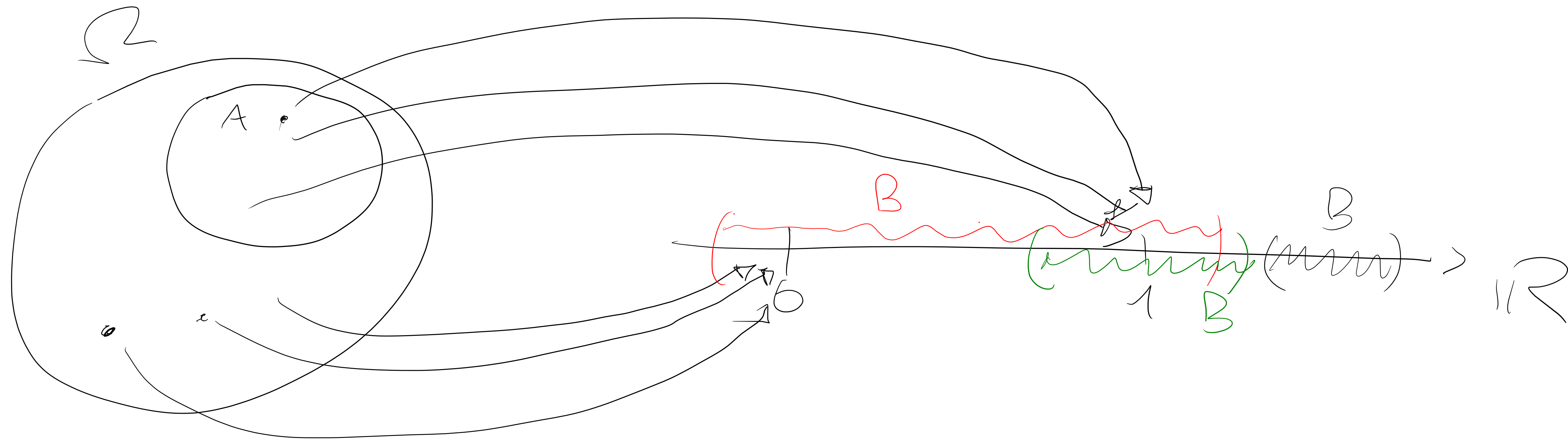
$\Omega = \Omega'$ $\quad f(\omega) = \omega$



$A' \in \mathcal{F}'$ $\quad f^{-1}(A') = A' \in \mathcal{F}$ (PER ESSERE MISURABILE)

$\mathcal{F} \supset \mathcal{F}'$

$(\Omega, \mathcal{F})$ $\quad A \subset \Omega \quad 1_A$ MISURABILE $\iff A \in \mathcal{F}$



$B \in \mathcal{B} \qquad 1_A^{-1}(B) = \emptyset \in \mathcal{F}$ SE $0, 1 \notin B$

$= \Omega \in \mathcal{F}$ SE $0, 1 \in B$

$= A \in \mathcal{F} \qquad 0 \notin B, 1 \in B$

$= \overline{A} \in \mathcal{F} \qquad 0 \in B, 1 \notin B$

$$\mathbb{1}_{\bar{A}} = 1 - \mathbb{1}_A$$

$$\mathbb{1}_{A \cap B} = \mathbb{1}_A \cdot \mathbb{1}_B$$

$$\mathbb{1}_{A \cup B} = \mathbb{1}_A + \mathbb{1}_B - \mathbb{1}_A \mathbb{1}_B$$

$$X(\Omega) = \{x_1, \ldots, x_n\}$$

$$X = x_1 \cdot 1_{\{X = x_1\}} + x_2 1_{\{X = x_2\}} + \ldots + x_n \cdot 1_{\{X = x_n\}}$$