The structure and evolution of stars

Lecture 6: Nuclear reactions in stellar interiors

Fred Hoyle

Learning Outcomes

The student will learn

- The main nuclear processes in stellar interiors
- The relative importance of each
- The temperature dependence of the the processes and likely sites of occurrence
- The relation to the cosmic abundance pattern

Introduction

We have seen that the 4 equations of stellar structure must be supplemented with expressions for P, ε, κ

- ε will be defined by the nuclear energy source in the interiors. We need to develop a theory and understanding of nuclear physics and reactions
- κ will be determined by the atomic physics of the stellar material
- P will be given by the equation of state of the stellar matter
- This lecture will explore the physics of the nuclear energy reactions

The binding energy of the atomic nucleus

The general description of a nuclear reaction is

$$I(A_i, Z_i) + J(A_j, Z_j) \leftrightarrow K(A_k, Z_k) + L(A_l, Z_l)$$

Where A_i = *the baryon number, or nucleon number (nuclear mass)*

And Z_i = *the nuclear charge*

The nucleus of any element is uniquely defined by the two integers A_i and Z_i

Recall that in any nuclear reaction the following must be conserved:

1. The baryon number – protons, neutrons and their anti-particles
2. The lepton number – light particles, electrons, positrons, neutrinos, and anti-neutrinos.
3. Charge

Note also that the anti-particles have the opposite baryon/lepton number to their particles.

The total mass of a nucleus is known to be less than the mass of the constituent nucleons. Hence there is a decrease in mass if a companion nucleus is formed from nucleons, and from the Einstein mass-energy relation $E=mc^2$ the mass deficit is released as energy. This difference is known as the *binding energy* of the compound nucleus. Thus if a nucleus is composed of Z protons and N neutrons, it's binding energy is

$$Q(Z,N) \equiv \left[Z m_p + N m_n - m(Z,N) \right] c^2$$

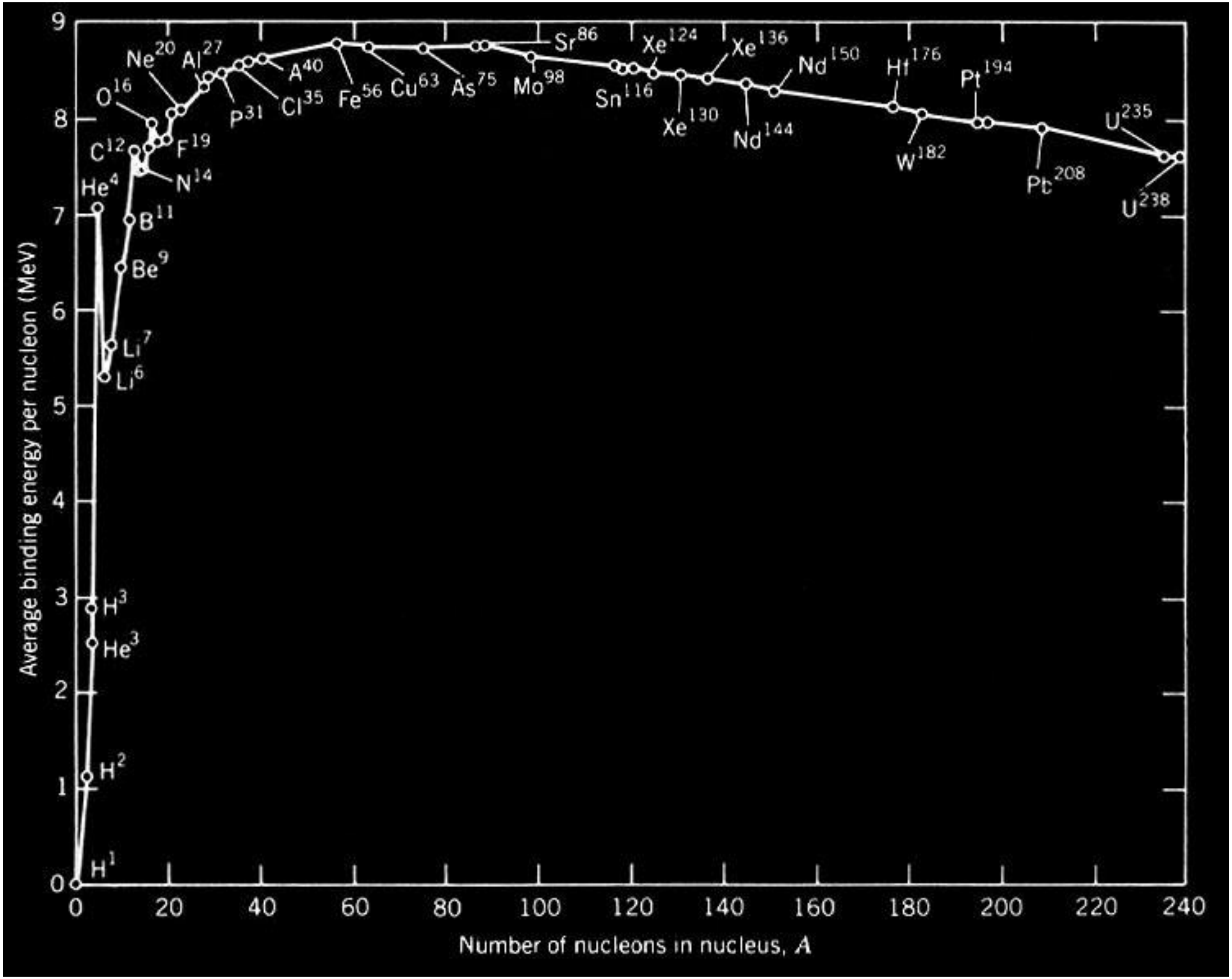
For our purposes, a more significant quantity is the total *binding energy per nucleon*. And we can then consider this number relative to the hydrogen nucleus

$$\frac{Q(Z,N)}{A}$$

The binding energy per nucleon

The variation of binding energy per nucleon with baryon number *A*

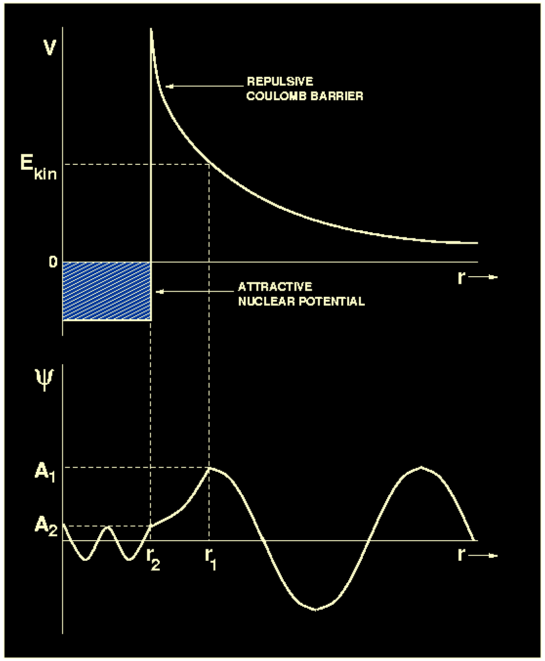
- General trend is an increase of *Q* with atomic mass up to *A*= 56 (Fe). Then slow monotonic decline
- There is steep rise from H through ^{2}H, ^{3}He, to ^{4}He → fusion of H to He should release larger amount of energy *per unit mass* than say fusion of He to C
- Energy may be gained by *fusion* of light elements to heavier, up to iron
- Or from *fission* of heavy nuclei into lighter ones down to iron.


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120
140
160
180
200
220
240
H1
H2
He3
H3
Li6
Li7
Be9
B11
He4
N14
C12
F19
O16
Ne20
Al27
P31
Cl35
A40
Fe56
Cu63
As75
Sr86
Mo98
Sn116
Xe124
Xe130
Xe136
Nd144
Nd150
Hf176
W182
Pt194
Pb208
U235
U238

Occurrence of fusion reactions

Now will discuss the conditions under which fusion can occur – and whether such conditions exist in stellar interiors

- Nuclei interact through four forces of physics – only electromagnetic and strong nuclear important here
- Two positively charged nuclei must overcome coulomb barrier (long range force $\propto 1/r^2$), to reach separation distances where strong force dominates (10^{-15} m, typical size of nucleus)



Schematic plots

- *V* (potential energy) vs nuclei separation distance
- Wave function representing penetration of a potential barrier by nucleus with kinetic energy of approach E_{kin} (below barrier height).

To fuse nuclei, must surmount the coulomb barrier. Height of barrier is estimated by:

$$\frac{z_1 z_2 e^2}{4\pi\varepsilon_0 r}$$

z_1, z_2 = number of protons in each nuclei

e= charge on electron=1.6 10^{-19} C

ε_0= permittivity of free space = 8.85 10^{-12} $C^2 N^{-1} m^{-2}$

Class task:

Calculate the potential energy required for fusion of 2 H nuclei (set r= 10^{-15} m)

Compare this to the average kinetic energy of a particle ($3kT/2$: Lecture 4)

What T do we require for fusion ?

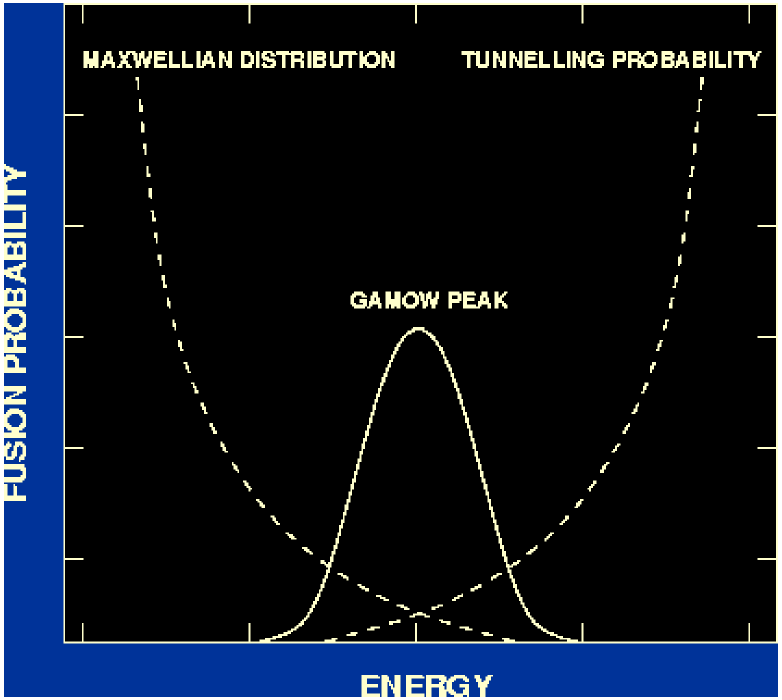
How does this compare with the minimum mean T of sun (Lecture 3) ?

Any comments on these two temperatures ?

The Gamow peak

Schematically this is plotted, and the fusion most likely occurs in the energy window defined as the Gamow Peak.

The Gamow peak is the product of the Maxwellian distribution and tunnelling probability. The area under the Gamow peak determines the reaction rate.



The higher the electric charges of the interacting nuclei, the greater the repulsive force, hence the higher the E_{kin} and T before reactions occur.

Highly charged nuclei are obviously the more massive, so reactions between light elements occur at lower T than reactions between heavy elements.

Quantum Tunnelling

As derived in your Quantum Mechanics courses, there is a finite probability for a particle to penetrate the Coulomb barrier as if "tunnel" existed.

Quantum effect discovered by George Gamow (1928) in connection with radioactivity.

Penetration probability (calculated by Gamow) is given as:

- tunneling probability: $P = \exp(-2\pi\eta)$

$$\text{with } \eta = \frac{Z_x Z_y e^2}{\hbar v}$$

$$\Rightarrow 2\pi\eta = 31.29 \cdot Z_x Z_y \left(\frac{A}{E}\right)^{\frac{1}{2}}$$

where A is the reduced mass in units of m_u and E is in keV

Hence this increases with *v* (particle velocity), but we know *v* will be Maxwellian distribution for ideal gas. Hence fusion probability is product

$$prob(fusion) \propto e^{\frac{-\pi Z_1 Z_2 e^2}{\varepsilon_0 h v}} e^{-\frac{mv^2}{2kT}}$$

The Gamow peak

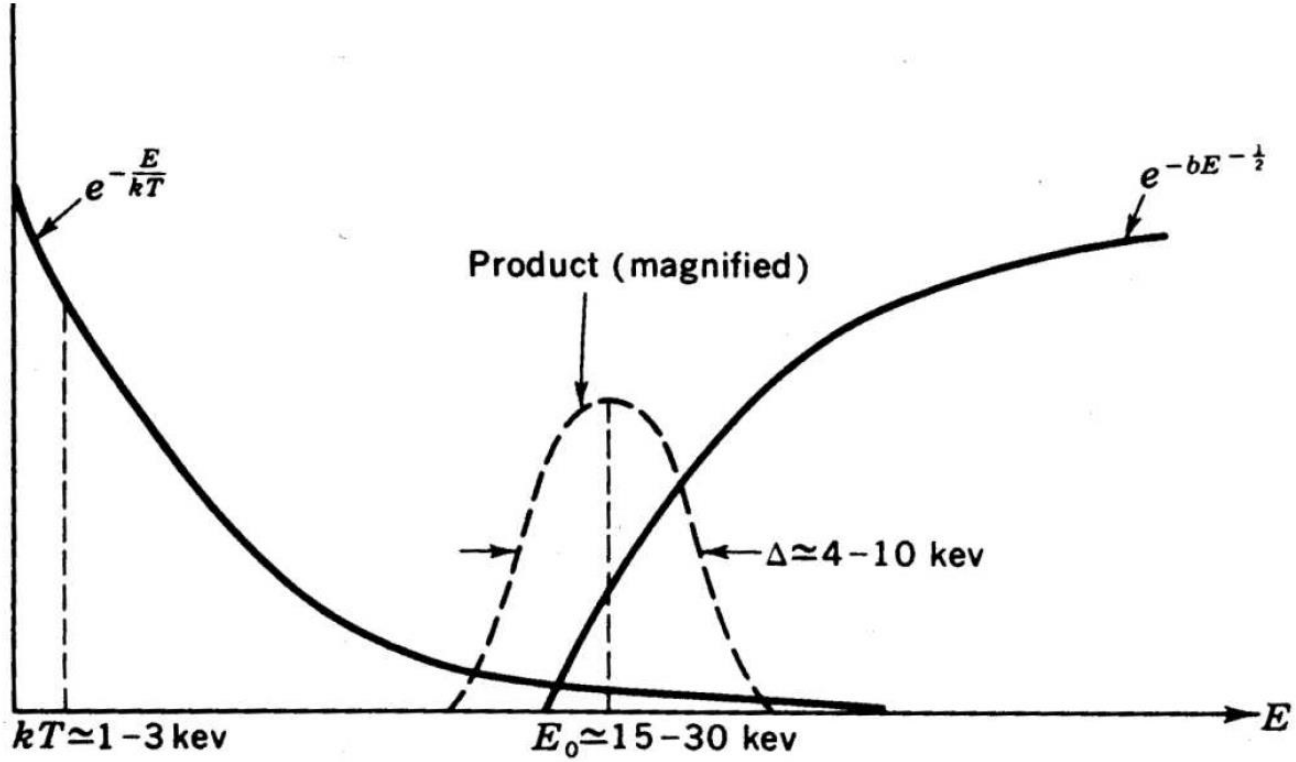


Figure 2.4: Dominant energy dependent factors in thermonuclear reactions. Most reactions occur in the overlap between the high-E tail of the Maxwell-Boltzmann distribution, giving a factor $\exp(-E/kT)$, and the probablity of tunneling through the Coulomb barrier, giving a factor $\exp(-b/\sqrt{E})$. Their product gives a fairly sharp peak called the **Gamow peak** at an energy E_0 which is generally much larger than kT. Figure from CLAYTON.

C. Nuclear reaction rate

Definition: nuclear reaction rate $\boxed{r_{xy} := N_x \cdot N_y \cdot v \cdot \sigma(v)}$

with: N_x, N_y number density of particles x, y (i.e., particles per cm^3)
v relative velocity between x and y
$\sigma(v)$ cross section
$[r]$ = reactions per cm^3 per s = cm^{-3} s^{-1}

- in stellar gas: Maxwell-Boltzmann distribution of velocities $\Phi(v)$

$\Rightarrow$ $r_{xy} = N_x N_y \langle \sigma v \rangle$

with $\langle \sigma v \rangle := \int_0^\infty \Phi(v) v \sigma(v) dv$

$$\Phi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) = f(T)$$

with $m = \dfrac{m_x m_y}{m_x + m_y}$ = "reduced mass"

$E = \frac{1}{2} m v^2$

$$\Rightarrow \boxed{\langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{\frac{1}{2}} \frac{1}{(kT)^{\frac{3}{2}}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE}$$

Astrophysical S Factor

Cross section (**B**) $\Rightarrow \sigma(E) \sim \pi\lambda^2 \sim 1/E$

Tunnel effect (**D**) $\Rightarrow \sigma(E) \sim \exp(-2\pi\eta), \ \eta \sim 1/\sqrt{E}$

$\Rightarrow$ define $S(E)$ such that

$$\sigma(E) = \frac{1}{E}\exp(-2\pi\eta)\cdot S(E)$$

$$\Rightarrow \langle\sigma v\rangle = \left(\frac{8}{\pi m}\right)^{1/2}\frac{1}{(kT)^{3/2}}\int_0^\infty S(E)\exp\left[-\frac{E}{kT}-\frac{b}{\sqrt{E}}\right]dE$$

with $b := (2m)^{1/2}\pi e^2 Z_x Z_y/\hbar$

$= 0.989 Z_x Z_y A^{1/2} \quad [(\text{MeV})^{1/2}]$

(b^2 = "Gamow energy")

- often: S(E) varies slowly with E
  - $\rightarrow$ **Gamow-peak** at energy $E_0 > kT$
  - $\rightarrow$ for narrow T-range: $S(E) \simeq S(E_0) = \text{const.}$

$$\Rightarrow \langle\sigma v\rangle = \left(\frac{8}{\pi m}\right)^{1/2}\frac{1}{(kT)^{3/2}}S(E_0)\int_0^\infty \exp\left(-\frac{E}{kT}-\frac{b}{\sqrt{E}}\right)dE$$

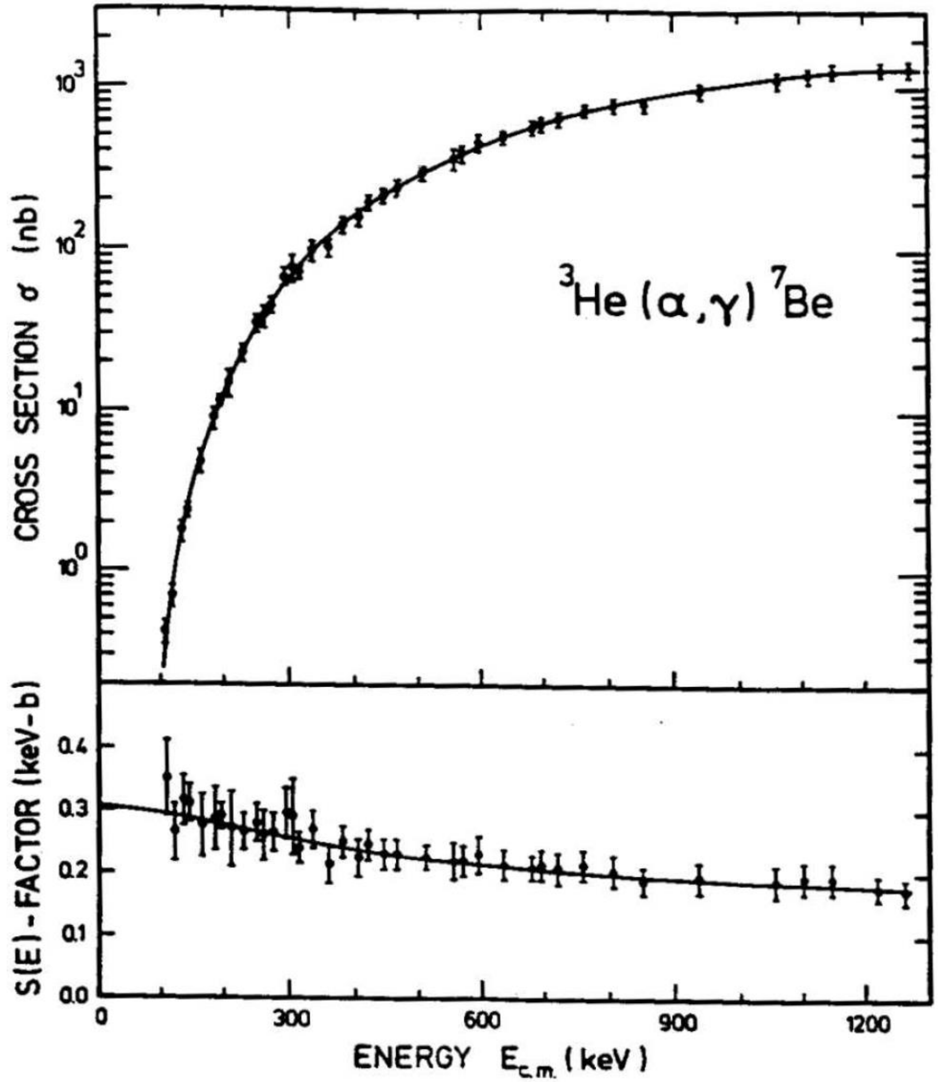


Figure 2.3: Energy dependence of the measured cross section (top) of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction. An extrapolation to $E < 50\,\mathrm{keV}$, which is relevant in astrophysical environments, appears treacherous. However, the S-factor (bottom) is only weakly dependent on energy, and therefore much easier to extrapolate (solid line). Figure from Rolfs & Rodney (1988).

Hydrogen and helium burning

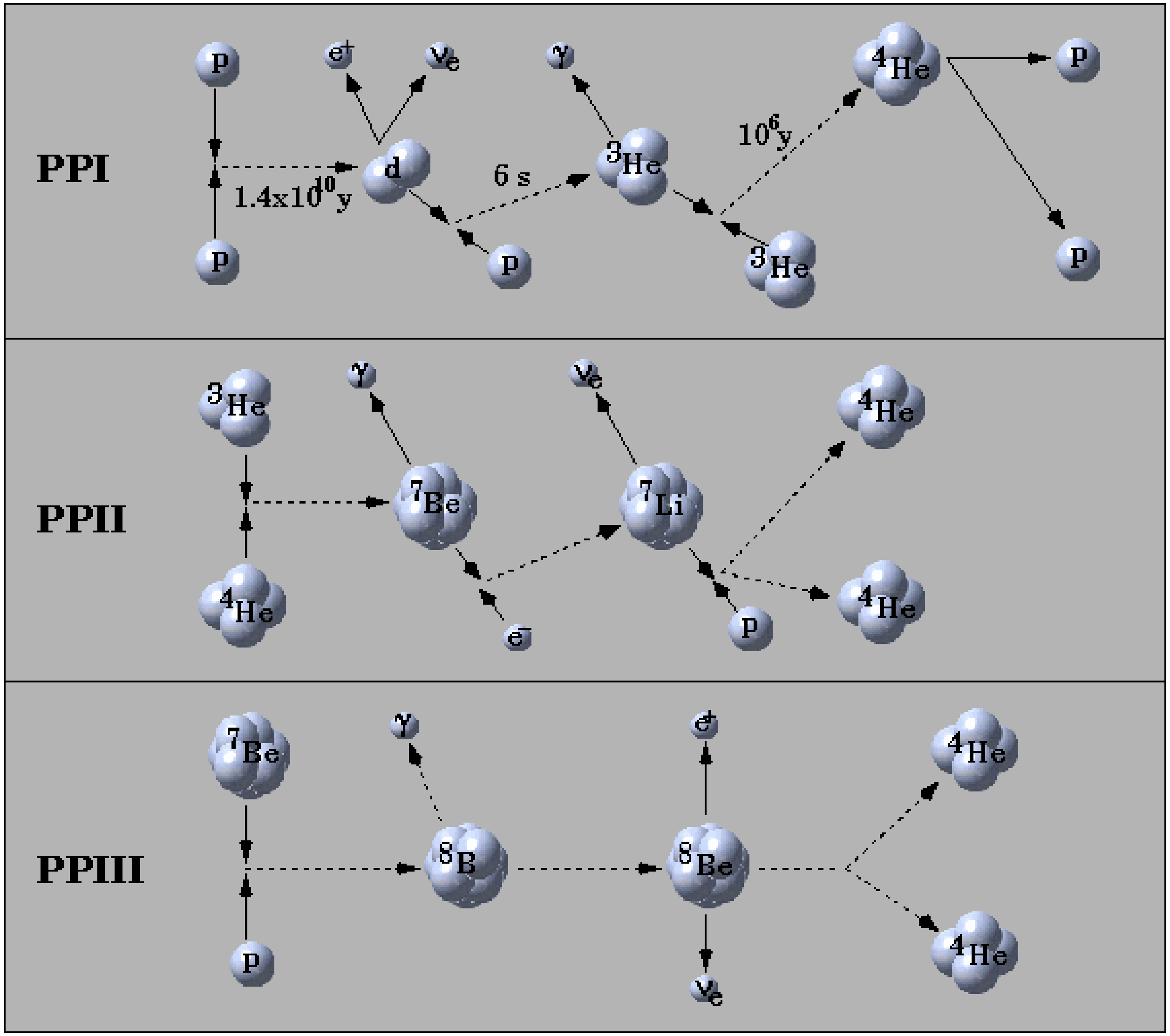
The most important series of fusion reactions are those converting H to He (H-burning). As we shall see this dominates ~90% of lifetime of nearly all stars.

- Fusion of 4 protons to give one ^{4}He is completely negligible
- Reaction proceeds through steps – involving close encounter of 2 particles
- We will consider the main ones: the **PP-chain** and the **CNO cycle**

The PP Chain

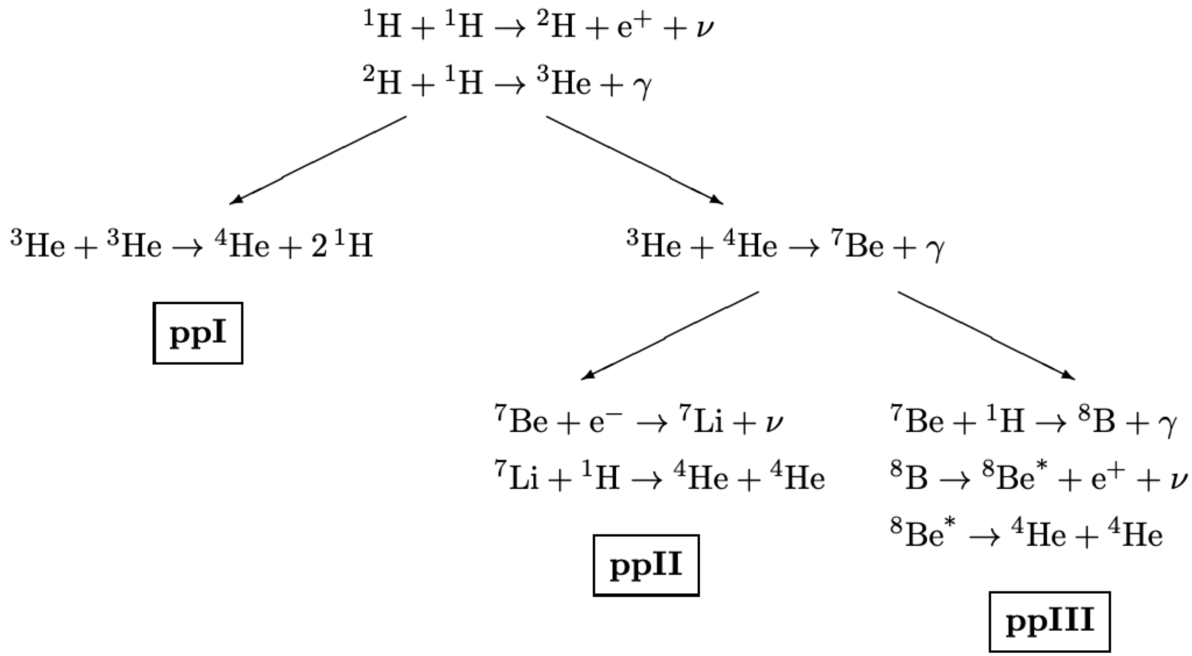
The PP chain has three main branches called the PPI, PPII and PPIII chains.

PPI Chain	PPII Chain	PPIII Chain
1 $p + p \rightarrow d + e^+ + \nu_e$	3' $^3He + {}^4He \rightarrow {}^7Be + \gamma$	4'' $^7Be + p \rightarrow {}^8B + \gamma$
2 $d + p \rightarrow {}^3He + \gamma$	4' $^7Be + e^- \rightarrow {}^7Li + \nu_e$	5'' $^8B \rightarrow {}^8Be + e^+ + \nu_e$
3 $^3He + {}^3He \rightarrow {}^4He + 2p$	5' $^7Li + p \rightarrow {}^4He + {}^4He$	6'' $^8Be \rightarrow 2\,{}^4He$



Relative importance of PPI and PPII chains (*branching ratios*) depend on conditions of H-burning (*T, ρ* , abundances). The transition from PPI to PPII occurs at temperatures in excess of 1.3 10^7 K.

Above 3 10^7 K the PPIII chain dominates over the other two, but another process takes over in this case.



reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	dS/dE (barn)	τ (yr)
$^1H(p, e^+\nu)^2H$	1.442	0.265	3.94×10^{-22}	4.61×10^{-24}	10^{10}
$^2H(p, \gamma)^3He$	5.493		2.5×10^{-4}	7.9×10^{-6}	10^{-8}
$^3He(^3He, 2p)^4He$	12.860		5.18×10^{3}	-1.1×10^{1}	10^{5}
$^3He(\alpha, \gamma)^7Be$	1.587		5.4×10^{-1}	-3.1×10^{-4}	10^{6}
$^7Be(e^-, \nu)^7Li$	0.862	0.814			10^{-1}
$^7Li(p, \alpha)^4He$	17.347		5.2×10^{1}	0	10^{-5}
$^7Be(p, \gamma)^8B$	0.137		2.4×10^{-2}	-3×10^{-5}	10^{2}
$^8B(e^+\nu)^8Be^*(\alpha)^4He$	18.071	6.710			10^{-8}

ppI summary:

- D is *destroyed* (in fact already on the pre-main sequence)
- ^{3}He is destroyed in the center, enriched above the core of the Sun
- the pp-reaction is the slowest, and (when ^{3}He is in equilibrium) sets the pace of the overall rate of the ppI chain

Comparing ppI, ppII and ppIII

- ppII, III start with $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, but the α-particle is used only as a catalyst: it is liberated in the end, by $^7\mathrm{Li}(\mathrm{p},\alpha)^4\mathrm{He}$ or by $^8\mathrm{B}(\alpha)^4\mathrm{He}$
- ppII, III need the (slow) pp-reaction only *once* to produce one ^{4}He nucleus, whereas ppI needs *two* pp-reactions per ^{4}He nucleus $\Rightarrow$ at high temperature: ppII, III may dominate

Energy production and neutrino emission

Energy released in the formation of an alpha particle by fusion of four protons. Is essentially given by the difference of the mass excesses of four protons and one α particle.

$$Q_{p-p} = \left[4\Delta M(^{1}H) - \Delta M(^{4}He)\right]c^{2} = 26.7 \text{ MeV}$$

Since any reaction branch that completes this must turn 2 protons in 2 neutrons, two neutrinos are also emitted, which carry energy away from the reaction site.

It is these neutrinos that *directly* confirm the occurrence of nuclear reactions in the interior of the Sun. No other *direct* observational test of nuclear reactions is possible.

The mean neutrino energy flux is ~0.26MeV for d creation (PPI/II) and ~7.2MeV for B decay (PPIII). But as PPIII is negligible, the energy released for each He nucleus assembled is ~26MeV (or $6 \ 10^{14}$ JKg^{-1})

Exercise

Estimate the total number of neutrinos generated in the Sun per second, and the flux of solar neutrinos on the Earth (i.e., the number of neutrinos per unit area and per unit time).

Assume that the cross section for reaction between a neutrino and a nucleus is 10^{46} cm^2.

Estimate the total number of neutrino reactions per year in the body of a typical student.

(EXTRA NEUTRINOS)

The CNO Cycle

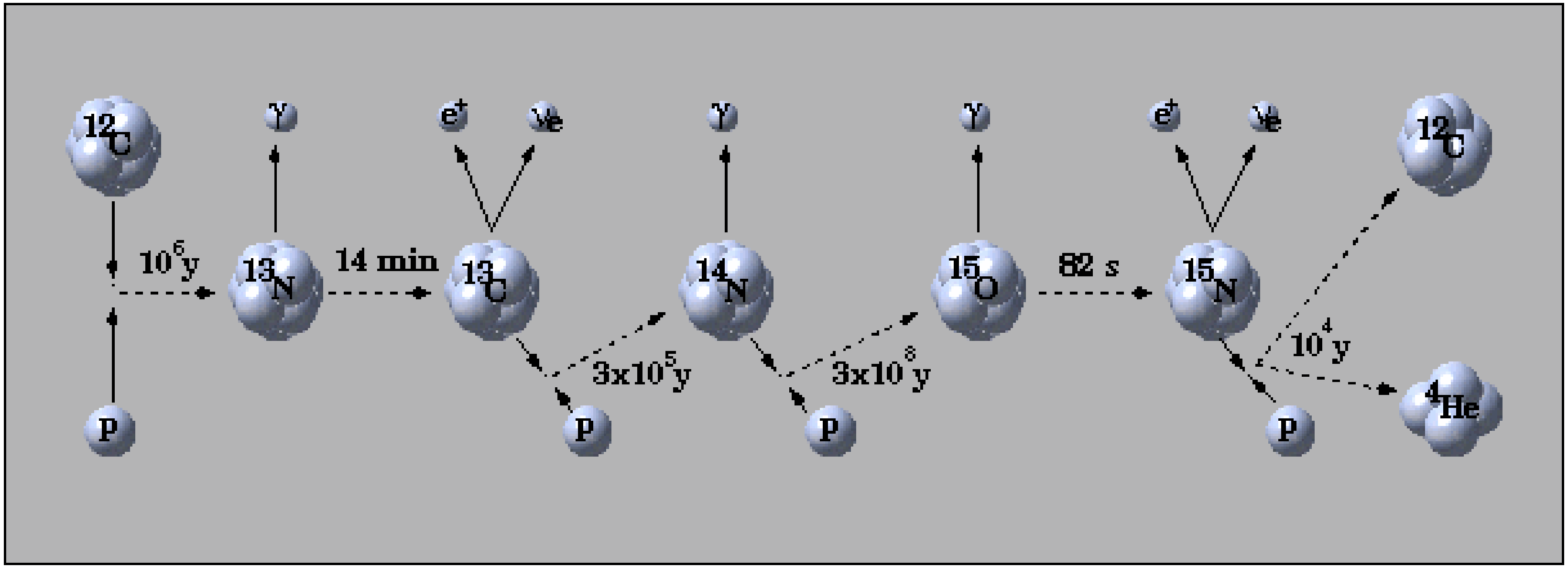
At birth stars contain a small (2%) mix of heavy elements, some of the most abundant of which are carbon, oxygen and nitrogen (CNO). These nuclei may induce a chain of H-burning reactions in which they act as catalysts.

The process is known as the CNO Cycle. There are alternative names that you may come across :

- The CNO bi-cycle
- The CNOF cycle
- The CN and NO cycles
- The CN and NO bi-cycles

In this course we will just refer to it all as the CNO cycle – and discuss the branches.

The main branch



1 $^{12}C + p \rightarrow \; ^{13}N + \gamma$

2 $^{13}N \rightarrow \; ^{13}C + e^{+} + v_{e}$

3 $^{13}C + p \rightarrow \; ^{14}N + \gamma$

4 $^{14}N + p \rightarrow \; ^{15}O + g$

5 $^{15}O \rightarrow \; ^{15}N + e^{+} + v_{e}$

6 $^{15}N + p \rightarrow \; ^{12}C + \; ^{4}He$

In the steady state case, the abundances of isotopes must take values such that the isotopes which react more slowly have higher abundance. The slowest reaction is p capture by ^{14}N .

Hence most of ^{12}C is converted to ^{14}N.

CN nucleosynthesis

Lifetimes: $^{15}\mathrm{N} : {}^{13}\mathrm{C} : {}^{12}\mathrm{C} : {}^{14}\mathrm{N} \simeq 1 : 45 : 190 : 26\,000$
$\Rightarrow$ CN-equilibrium abundance distribution:

$$^{15}\mathrm{N} : {}^{13}\mathrm{C} : {}^{12}\mathrm{C} : {}^{14}\mathrm{N} \simeq \frac{1}{26\,000} : \frac{1}{600} : \frac{1}{140} : 1$$

independent of the initial composition

Comparison with solar system composition:

$$^{15}\mathrm{N} : {}^{13}\mathrm{C} : {}^{12}\mathrm{C} : {}^{14}\mathrm{N} = \frac{1}{1136} : \frac{1}{114} : \frac{4}{5} : \frac{1}{5}$$

$\Rightarrow$ ^{15}N, ^{13}C and $^{12}\mathrm{C} \downarrow \quad {}^{14}\mathrm{N} \uparrow$
$\Rightarrow$ ^{14}N is the major nucleosynthesis product of the CN-cycle (except for ^{4}He).

The CNO bi-cycle

Not all proton captures on $^{15}\mathrm{N}$ lead to $^{12}\mathrm{C} + \alpha$:

$$^{15}\mathrm{N} + \mathrm{p} \rightarrow {}^{16}\mathrm{O}^* \begin{array}{ll} \nearrow {}^{12}\mathrm{C} + \alpha & \text{for} \sim 99.9\% \\ \searrow {}^{16}\mathrm{O} + \gamma & \text{for} \sim 0.1\% \end{array}$$

This initiates a second cycle (Fig. 4.10):

$$^{15}\mathrm{N}\,(\mathrm{p},\gamma)\,^{16}\mathrm{O}\,(\mathrm{p},\gamma)\,^{17}\mathrm{F}\,(\mathrm{e}^+\nu)\,^{17}\mathrm{O}\,(\mathrm{p},\alpha)\,^{14}\mathrm{N}\,(\mathrm{p},\gamma)\,^{15}\mathrm{O}\,(\mathrm{e}^+\nu)\,^{15}\mathrm{N} \quad \text{(CNO-II)}$$

ratio of 1:1000 $\Rightarrow$

- second cycle *not* relevant for energy generation
- but: very relevant for **nucleosynthesis**:
  it makes the $^{16}\mathrm{O}$-reservoir available to flow into the CN-cycle.

Table 4.4: Reactions of the CNO bi-cycle. Cross section factors are taken from Bahcall (1989). The last column gives the lifetime of the CNO nuclei for $T_6 = 20$.

reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (MeV barn)	dS/dE (barn)	τ (yr)
$^{12}C(p, \gamma)^{13}N$	1.944		1.45×10^{-3}	2.45×10^{-3}	6.6×10^3
$^{13}N(e^+\nu)^{13}C$	2.220	0.707			863 s
$^{13}C(p, \gamma)^{14}N$	7.551		5.50×10^{-3}	1.34×10^{-2}	1.6×10^3
$^{14}N(p, \gamma)^{15}O$	7.297		3.32×10^{-3}	-5.91×10^{-3}	9.3×10^5
$^{15}O(e^+\nu)^{15}N$	2.754	0.997			176 s
$^{15}N(p, \alpha)^{12}C$	4.965		7.80×10^1	3.51×10^2	3.5×10^1
$^{15}N(p, \gamma)^{16}O$	12.127		6.4×10^{-2}	3×10^{-2}	3.9×10^4
$^{16}O(p, \gamma)^{17}F$	0.600		9.4×10^{-3}	-2.3×10^{-2}	7.1×10^7
$^{17}F(e^+\nu)^{17}O$	2.761	0.999			93 s
$^{17}O(p, \alpha)^{14}N$	1.192		resonant reaction		1.9×10^7

Temperature dependence of PP chain and CNO Cycle

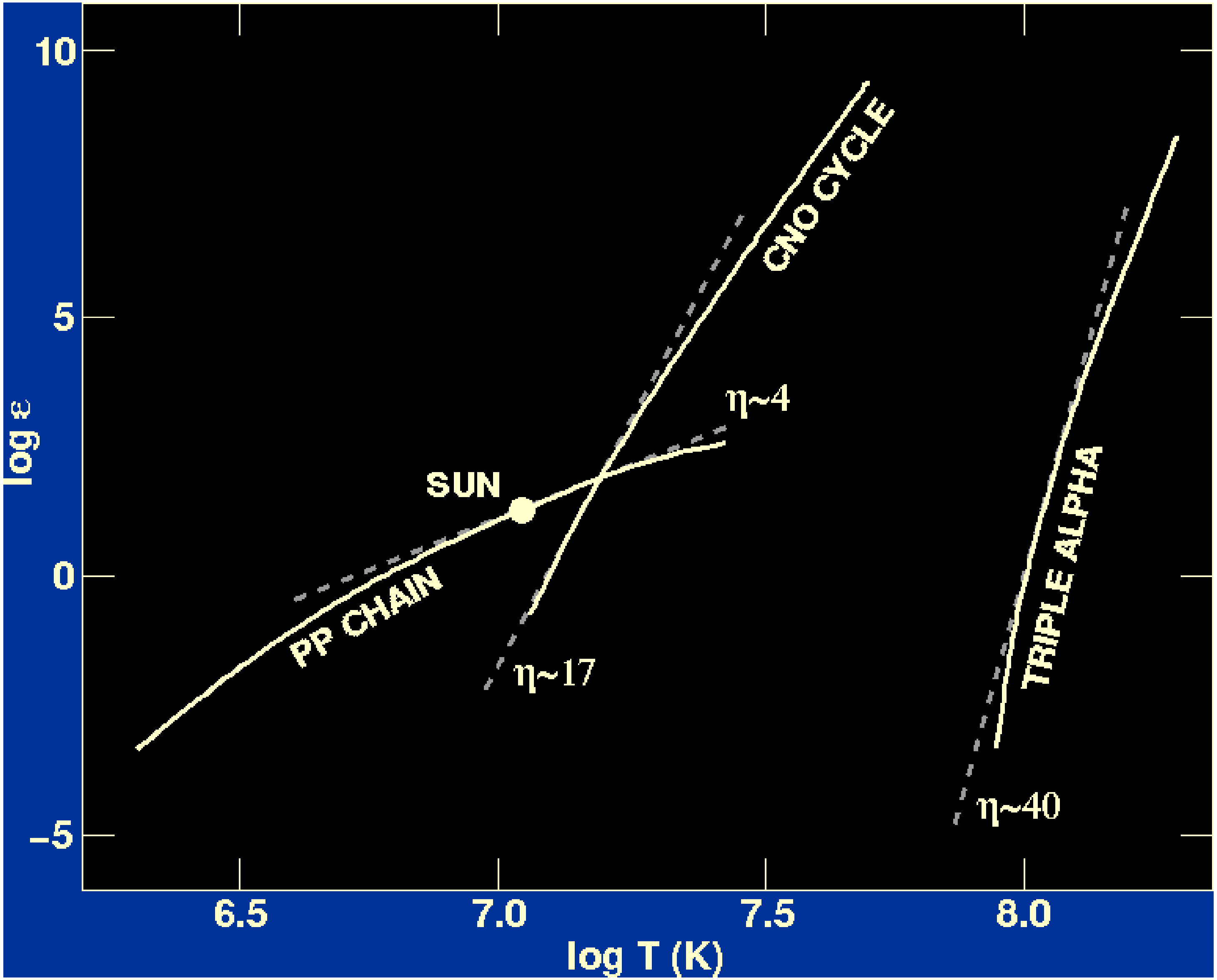
The two processes have very different temperature dependences. The rate of energy production in each:

$$\varepsilon_{PP} = \varepsilon_0 \rho X_H^2 \left(\frac{T}{T_0} \right)^{4.6} \qquad \varepsilon_{CNO} = \varepsilon_0 \rho X_H X_{CNO} f_N \left(\frac{T}{25 \times 10^6} \right)^{16.7}$$

Equating this two gives the *T* at which they produce the same rate of energy production:

$$T \approx 1.7 \times 10^7 \left(\frac{X_H}{50 X_{CN}} \right)^{\frac{1}{12.1}} \text{K}$$

Below this temperature the PP chain is most important, and above it the CNO Cycle dominates. This occurs in stars slightly more massive than the sun e.g. 1.2-1.5$M_\odot$.


10
5
0
−5
log ε
SUN
PP CHAIN
CNO CYCLE
TRIPLE ALPHA
η~4
η~17
η~40
6.5
7.0
7.5
8.0
log T (K)

Helium Burning: the triple-α reaction.

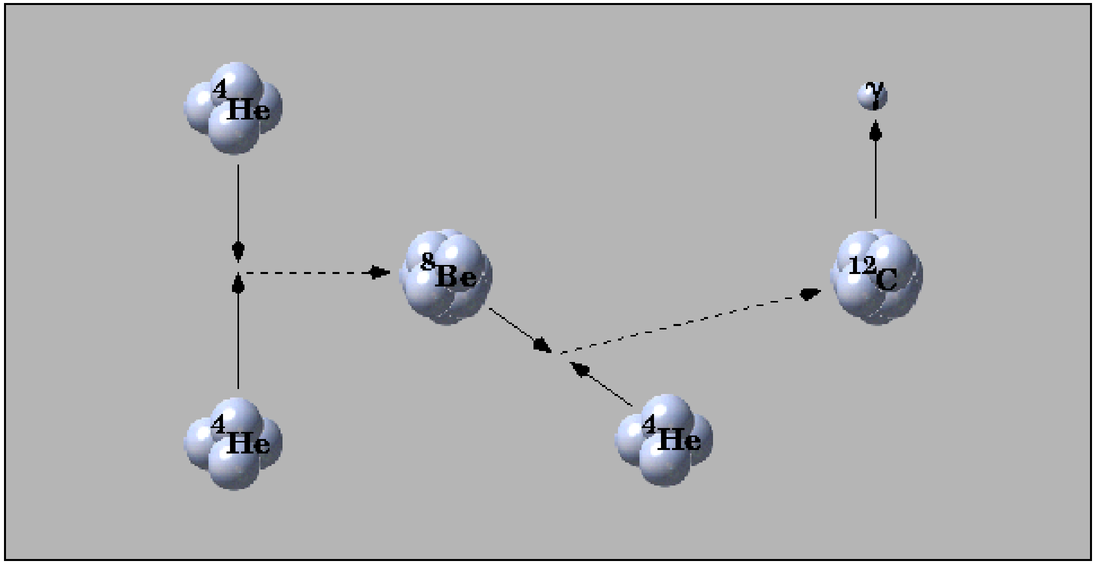
Simplest reaction in a helium gas should be the fusion of two helium nuclei. There is no stable configuration with *A=8.* For example, the beryllium isotope ^{8}Be has a lifetime of only 2.6 10^{-16} s

$^{4}He + ^{4}He \rightarrow ^{8}Be$

But a third helium nucleus can be added to ^{8}Be before decay, forming ^{12}C by the "triple-alpha" reaction

$^{4}He + ^{4}He \rightarrow ^{8}Be$

$^{8}Be + ^{4}He \rightarrow ^{12}C + \gamma$



Helium Burning: the triple-α reaction.

Fred Hoyle (1952-54) suggested this small probability of alpha-capture by short lived ^{8}Be would be greatly enhanced if the C nucleus had an energy level close to the combined energies of the reacting ^{8}Be and ^{4}He nuclei. The reaction would be a faster "resonant" reaction.

This resonant energy level of ^{12}C was not experimentally known at the time. Hoyle's prediction led to nuclear experiment at Caltech, and resonant level discovered.

Thus helium burning proceeds in a 2-stage reaction, and energy released is

$$Q_{3\alpha} = \left[3\Delta M(^{4}He) - \Delta M(^{12}C)\right]c^{2} = 7.275\,\mathrm{MeV}$$

In terms of energy generated per unit mass = 5.8 10^{13} $\mathrm{JKg^{-1}}$ (I.e. 1/10 of energy generated by H-burning). But the *T* dependence is astounding:

$$\varepsilon_{3\alpha} \propto \rho^{2} T^{40}$$

Further reactions:

$\boxed{^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}}$	very important
$^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$	less important (only at very high T)

Remark: the $^{12}\mathrm{C}(\alpha,\gamma)$-rate is not very well known today.
$\Rightarrow$ unclear wether main product of He-burning is carbon or oxygen!

Secondary nucleosynthesis during helium burning:

$$^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^{+})^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne} \quad \text{(already at } T \simeq 10^{8}\,\mathrm{K})$$

$\Rightarrow$ production of $^{18}\mathrm{O}$ and $^{22}\mathrm{Ne}$

$$^{22}\mathrm{Ne}(\alpha,\mathrm{n})^{25}\mathrm{Mg} \quad (\text{for } T \geq 3\cdot 10^{8}\,\mathrm{K})$$

$\Rightarrow$ **n**-production !! $\Rightarrow$ s-process

Carbon burning

Carbon burning (fusion of 2 C nuclei) requires temperatures above 5 10^8 K

Interactions of C and O nuclei are negligible – as at the intermediate temperatures required by the coulomb barrier the C nuclei are quickly destroyed by interacting with themselves

Starting composition: ashes of helium burning, mainly ^{12}C, ^{16}O
no light particles (p, n, α) available initially!

Main reaction of carbon burning: $^{12}\mathbf{C} + {}^{12}\mathbf{C}$

There are two main exit channels:

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}^* \rightarrow {}^{20}\mathrm{Ne} + \alpha + 4.6\,\mathrm{MeV} \quad (\sim 50\%)$$

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}^* \rightarrow {}^{23}\mathrm{Na} + \mathrm{p} + 2.2\,\mathrm{MeV} \quad (\sim 50\%)$$

$\Rightarrow$ production of p and α particles; those are immediately captured $\Rightarrow$

many side reactions, e.g.: $^{23}\mathrm{Na}(\mathrm{p},\alpha)^{20}\mathrm{Ne}$, $^{20}\mathrm{Ne}(\alpha,\gamma)^{24}\mathrm{Mg}$ ...
also $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ ...

Composition after carbon burning: ^{16}O, ^{20}Ne, ^{24}Mg (together: 95%)

Ne burning

First expectation: oxygen burning should follow carbon burning, as ^{16}O is the lightest remaining nucleus, **but:**

^{16}O is doubly-magic nucleus (magic n and magic p number!)
$\Rightarrow$ extremely stable: α-separation energy: 7.2 MeV
^{20}Ne is much less stable: α-separation energy: 4.7 MeV

Consequence: $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ occurs at lower temperatures than $^{16}\mathrm{O} + {}^{16}\mathrm{O}$

Main reactions of neon burning:

$$^{20}\mathrm{Ne} + \gamma \longleftrightarrow {}^{16}\mathrm{O} + \alpha$$

$$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

Effectively: $2\,{}^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg}$ $\Rightarrow$ effective energy generation > 0!

Composition after neon burning: ^{16}O, ^{24}Mg (together: 95%)

O burning

Starting composition: ashes of neon burning, mainly ^{16}O, ^{24}Mg
no light particles (p, n, α) available initially!

Main reaction of oxygen burning: $^{16}\mathrm{O} + {}^{16}\mathrm{O}$

There are two main exit channels:

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* \rightarrow {}^{28}\mathrm{Si} + \alpha + 9.6\,\mathrm{MeV} \quad (\sim 60\%)$$

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* \rightarrow {}^{31}\mathrm{P} + \mathrm{p} + 7.7\,\mathrm{MeV} \quad (\sim 40\%)$$

$\Rightarrow$ production of p and α particles (like in carbon burning) which are immediately captured $\Rightarrow$

many side reactions, e.g.: $^{31}\mathrm{P}(\mathrm{p},\alpha)^{28}\mathrm{Si}$, $^{28}\mathrm{Si}(\alpha,\gamma)^{32}\mathrm{S}$ ...
also $^{24}\mathrm{Mg}(\alpha,\gamma)^{28}\mathrm{Si}$, ... but $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ is blocked!

Composition after oxygen burning: ^{28}Si, ^{32}S (together: 90%)

Silicon burning: nuclear statistical equilibrium

Main ash of oxygen burning, and lightest remaining isotope: $^{28}\mathrm{Si}$
but: $^{28}\mathrm{Si} + {}^{28}\mathrm{Si}$ is prohibited by the high Coulomb-barrier

instead: $^{28}\mathrm{Si}(\gamma,\alpha)^{24}\mathrm{Mg}$ occurs for $T > 3 \cdot 10^9$ K
$\Rightarrow$ Si-burning occurs similar to Ne-burning: through (γ,α) and (α,γ)-reactions:

$$^{28}\mathrm{Si}\,(\gamma,\alpha)\,^{24}\mathrm{Mg}\,(\gamma,\alpha)\,^{20}\mathrm{Ne}\,(\gamma,\alpha)\,^{16}\mathrm{O}\,(\gamma,\alpha)\,^{12}\mathrm{C}\,(\gamma,\alpha)\,2\alpha$$

and

$$^{28}\mathrm{Si}\,(\alpha,\gamma)\,^{32}\mathrm{S}\,(\alpha,\gamma)\,^{36}\mathrm{Ar}\,(\alpha,\gamma)\,^{40}\mathrm{Ca}\,(\alpha,\gamma)\,^{44}\mathrm{Ti}\,(\alpha,\gamma)\,^{48}\mathrm{Cr}\,(\alpha,\gamma)\,^{52}\mathrm{Fe}\,(\alpha,\gamma)\,^{56}\mathrm{Ni}$$

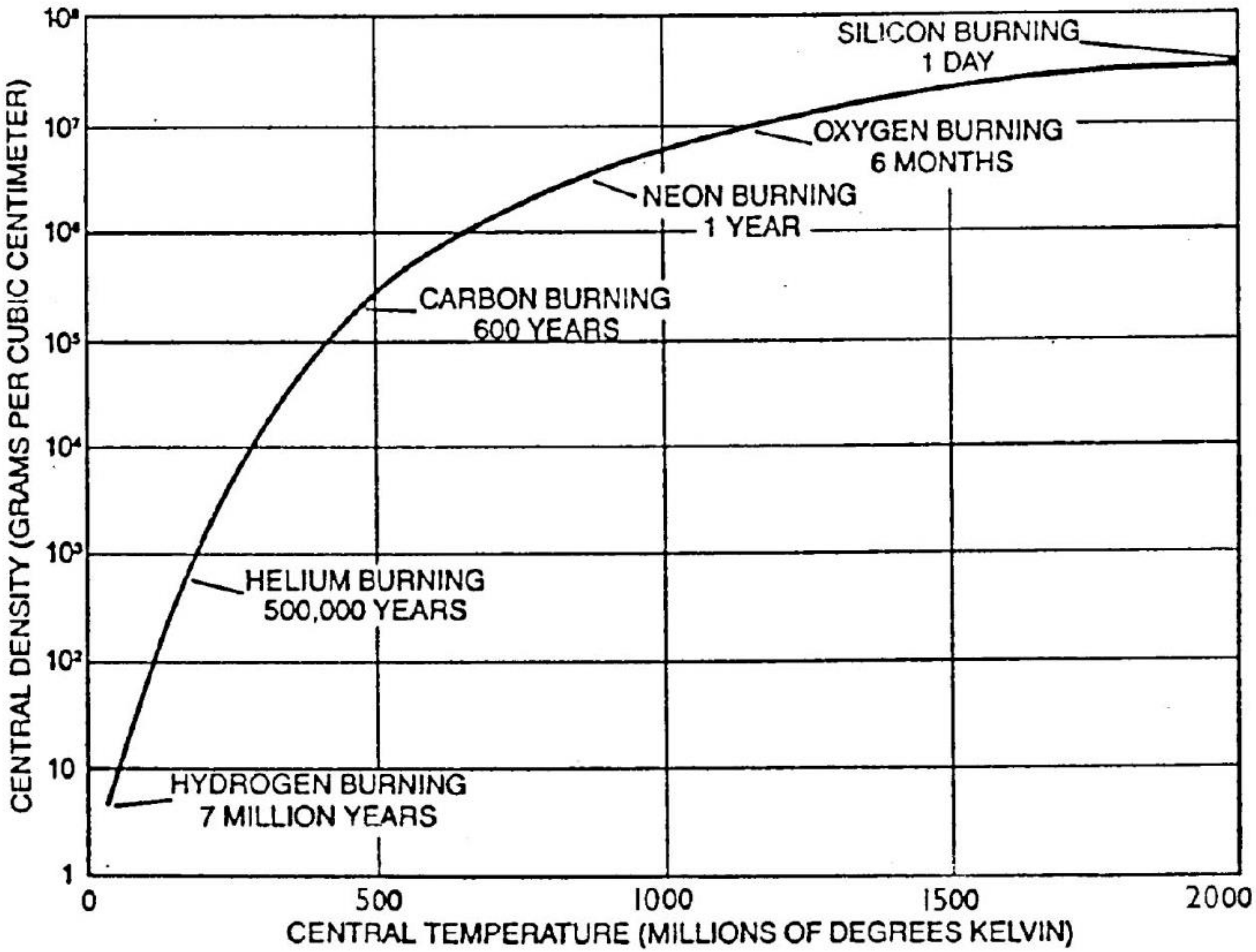
For $T > 4 \cdot 10^9$ K: almost nuclear statistical equilibrium (NSE) may be reached
As in the matter $\mathrm{p/n} < 1$ due to β-decays and possibly e^--captures: final composition may be mostly $^{56}\mathrm{Fe}$.

Major nuclear burning processes

Common feature is release of energy by consumption of nuclear fuel. Rates of energy release vary enormously. Nuclear processes can also absorb energy from radiation field, we shall see consequences can by catastrophic.

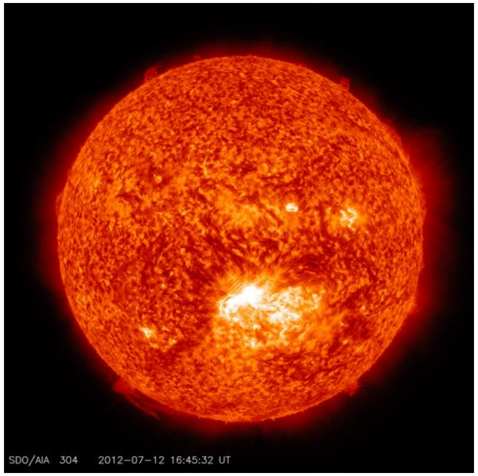
Evolutionary Time Scales for a 15 M_{sun} Star

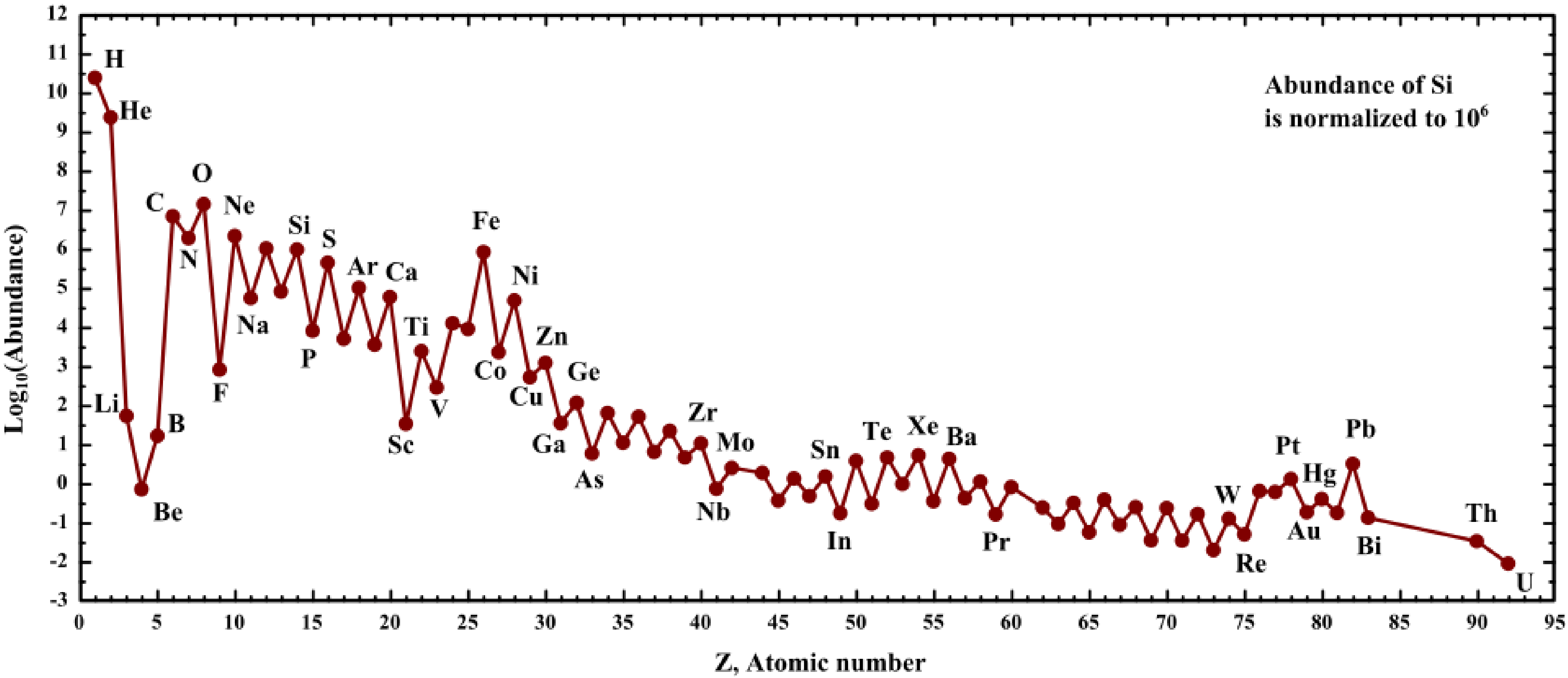
Fused	Products	Time	Temperature
H	^{4}He	10^{7} yrs.	4 X 10^{6} K
^{4}He	^{12}C	Few X 10^{6} yrs	1 X 10^{8} K
^{12}C	^{16}O, ^{20}Ne, ^{24}Mg, ^{4}He	1000 yrs.	6 X 10^{8} K
^{20}Ne +	^{16}O, ^{24}Mg	Few yrs.	1 X 10^{9} K
^{16}O	^{28}Si, ^{32}S	One year	2 X 10^{9} K
^{28}Si +	^{56}Fe	Days	3 X 10^{9} K
^{56}Fe	Neutrons	< 1 second	3 X 10^{9} K



SILICON BURNING
1 DAY
OXYGEN BURNING
6 MONTHS
NEON BURNING
1 YEAR
CARBON BURNING
600 YEARS
HELIUM BURNING
500,000 YEARS
HYDROGEN BURNING
7 MILLION YEARS
CENTRAL DENSITY (GRAMS PER CUBIC CENTIMETER)
CENTRAL TEMPERATURE (MILLIONS OF DEGREES KELVIN)

Composizione chimica del Sole





The *s*-process and *r*-process

Interaction between nuclei and free neutrons (neutron capture) – the neutrons are produced during C, O and Si burning.

Neutrons capture by heavy nuclei is not limited by the Coulomb barrier – so could proceed at relatively low temperatures. The obstacle is the scarcity of free neutrons. If enough neutrons available, chain of reactions possible:

$$I(A, Z) + n \rightarrow I_1(A+1, Z)$$

$$I_1(A+1, Z) + n \rightarrow I_2(A+2, Z)$$

$$I_2(A+2, Z) + n \rightarrow I_3(A+3, Z)$$

If a radioactive isotope is formed it will undergo beta-decay, creating new element.

$$I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- + \bar{\nu}$$

if new element stable, it will resume neutron capture, otherwise my undergo series of beta-decays

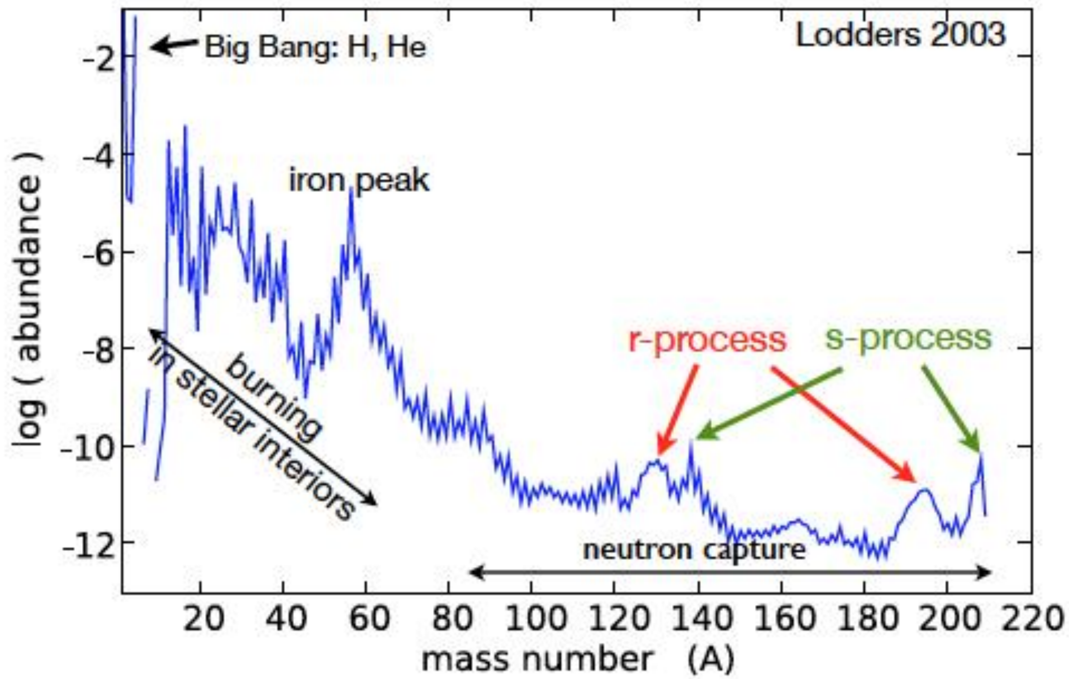
$$J(A+N, Z+1) \rightarrow K(A+N, Z+2) + e^- + \bar{\nu}$$

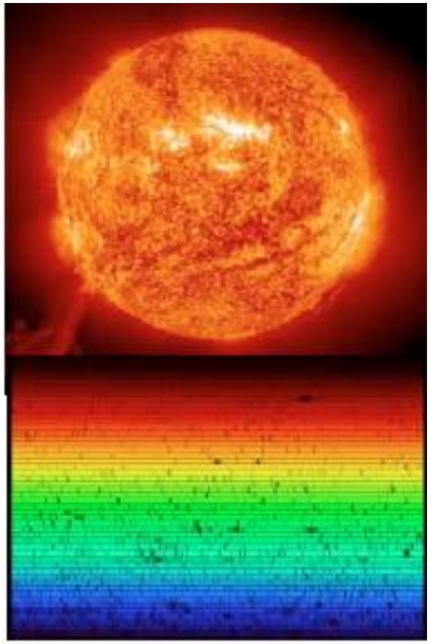
$$K(A+N, Z+2) \rightarrow L(A+N, Z+3) + e^- + \bar{\nu}$$

Solar system abundances

Solar photosphere and meteorites:
chemical signature of gas cloud where the Sun formed

Contribution of all nucleosynthesis processes





s-process:
slow neutron capture
r-process:
rapid neutron capture

abundance = mass fraction / mass number

Neutron capture elements: r-s process

The elements beyond the iron peak (A>60) are manly formed through
neutron capture on seed nuclei (iron and silicon).
Two cases:

s-process

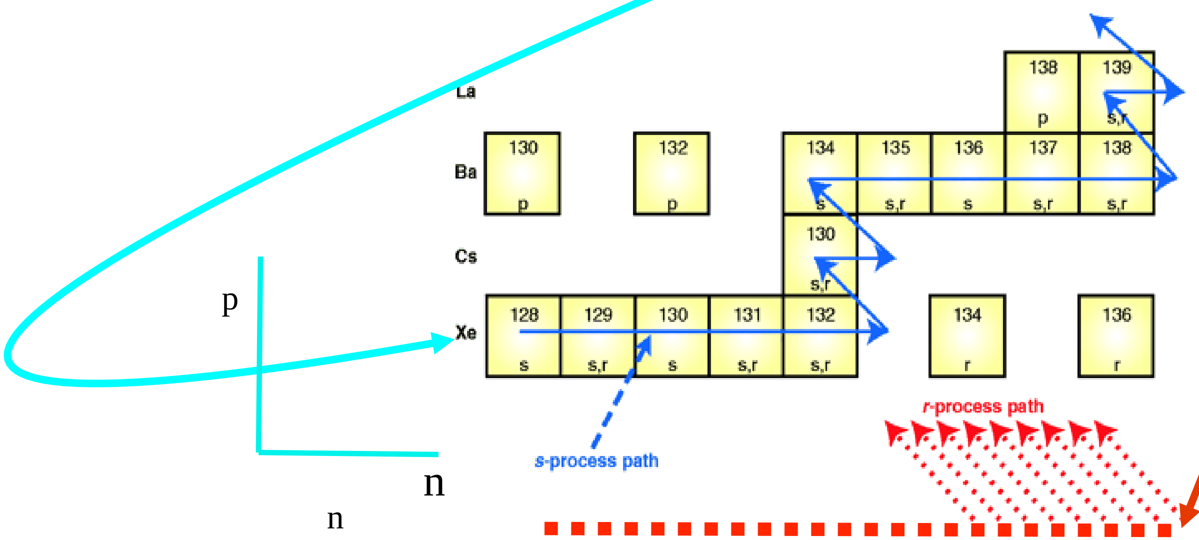
r-process

Different Timescale of the neutron capture

$\tau_\beta << \tau_c$

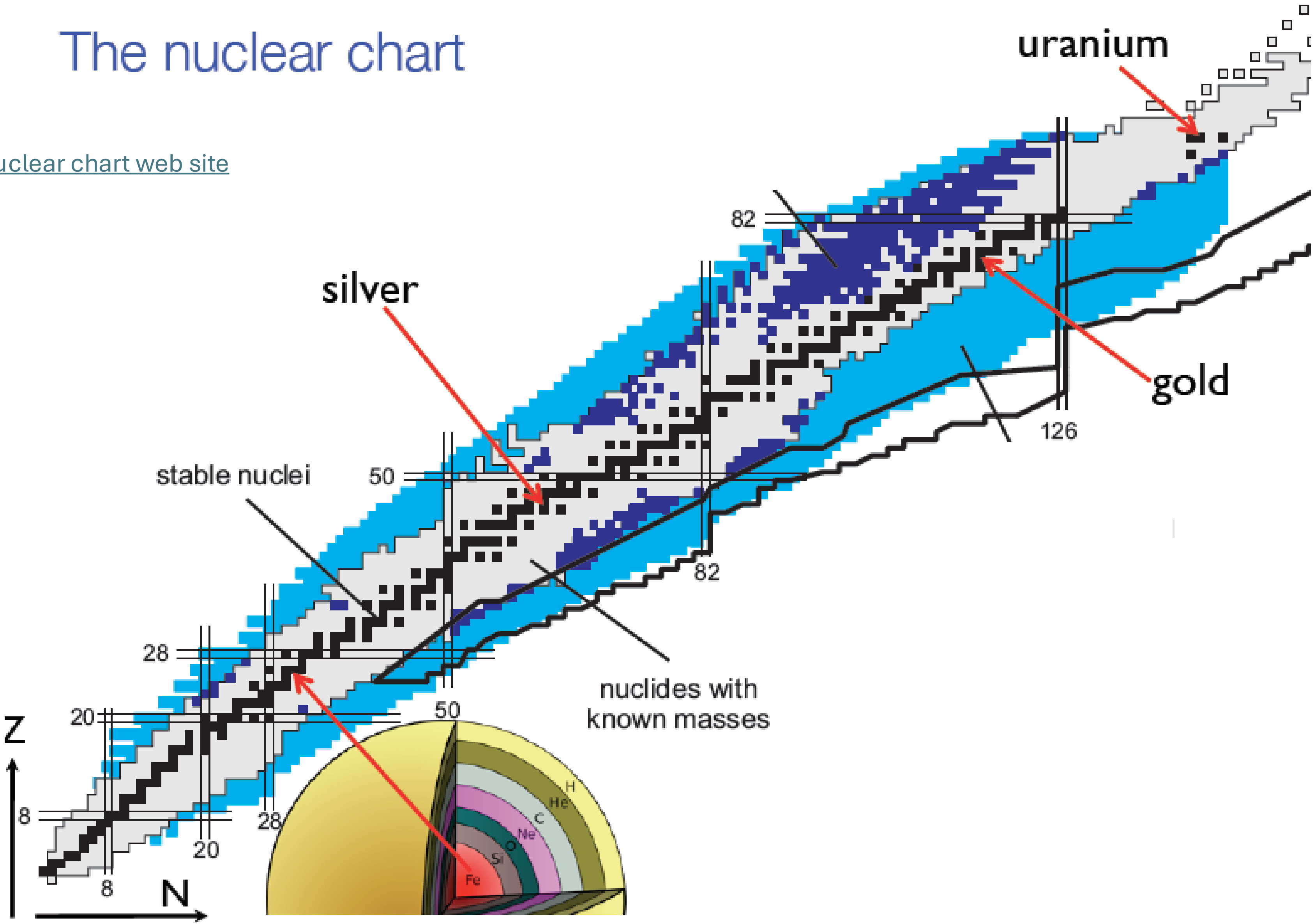
$\tau_\beta >> \tau_c$

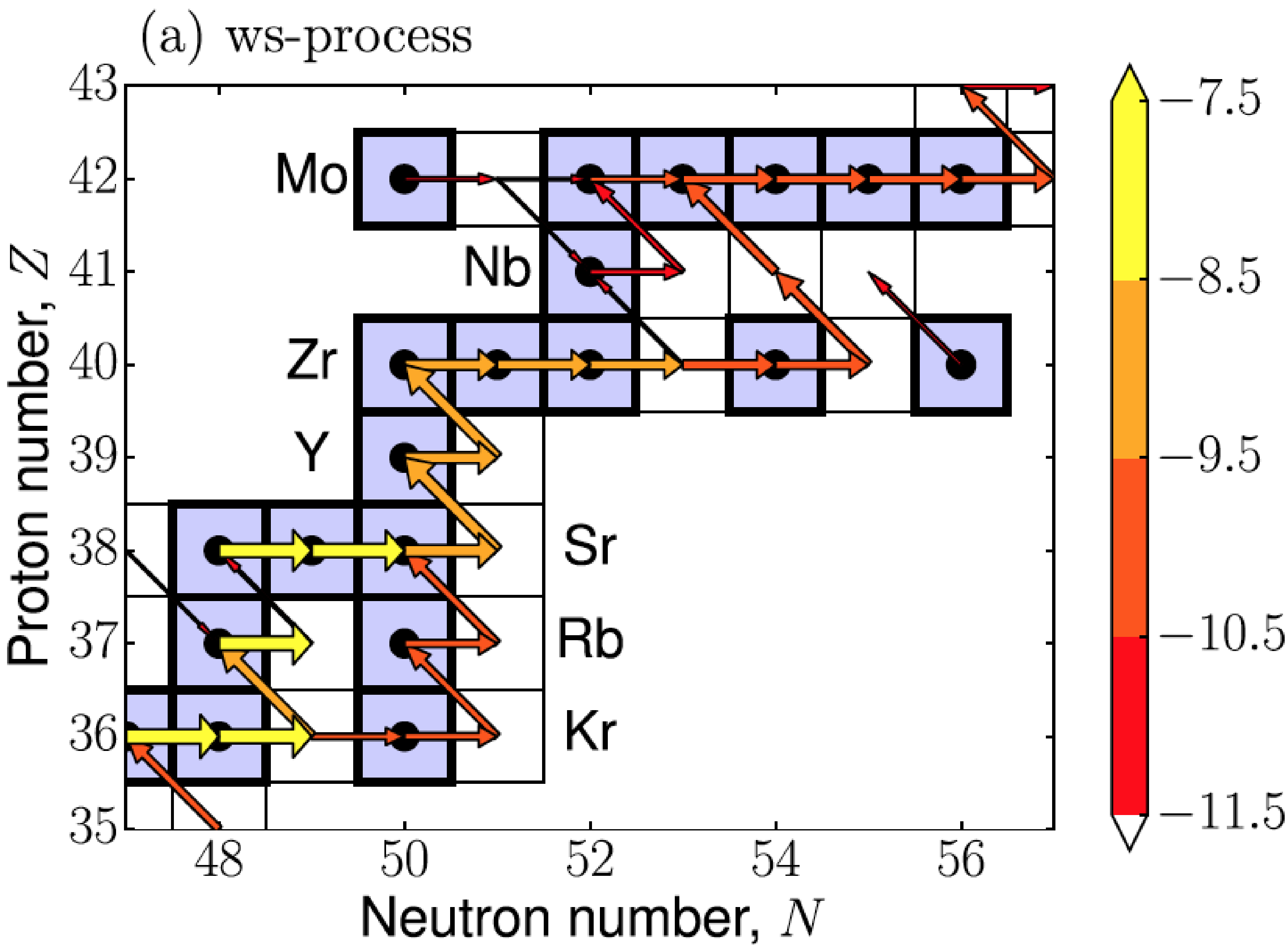
Different process path



The nuclear chart

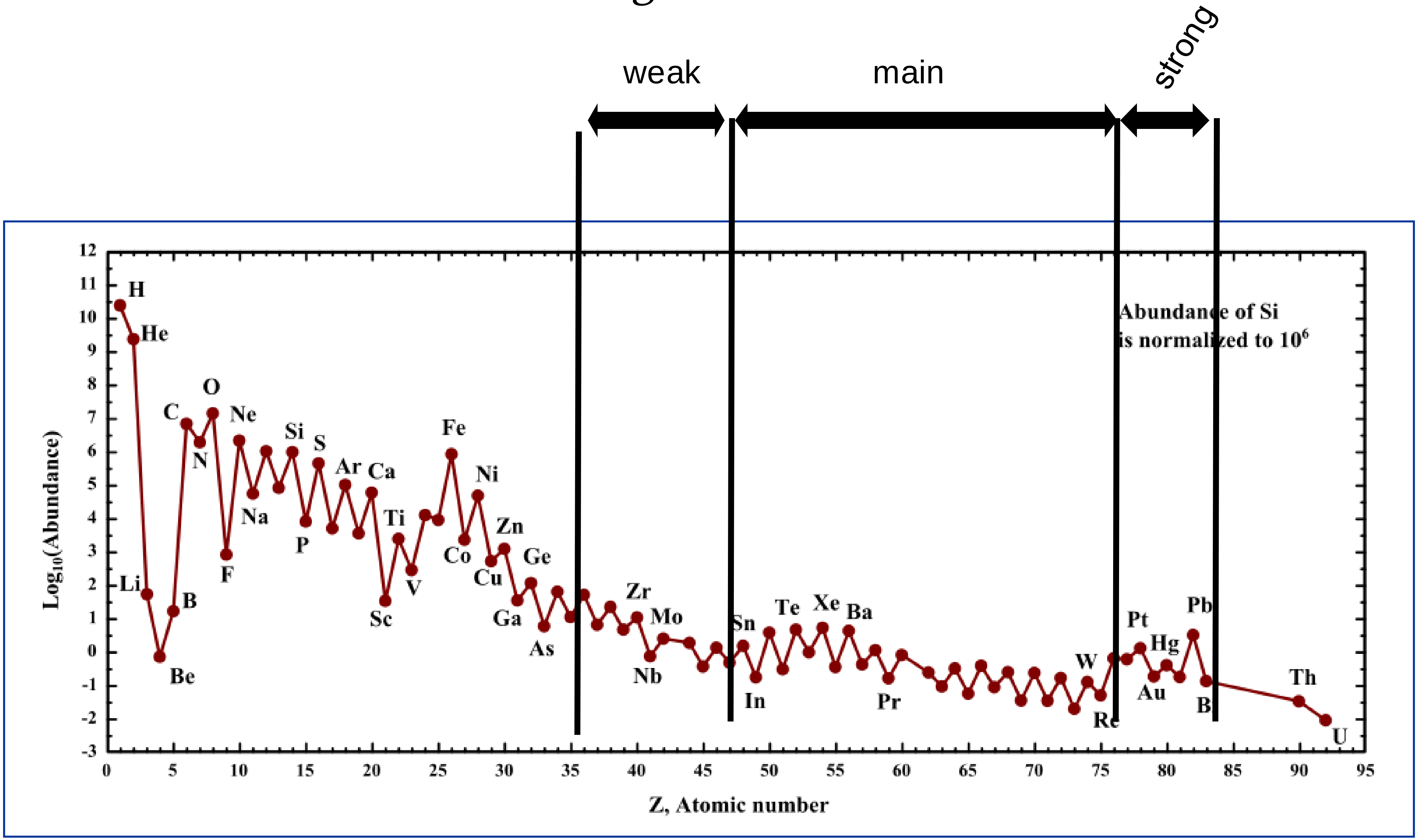
nuclear chart web site




(a) ws-process
Mo
Nb
Zr
Y
Sr
Rb
Kr
Proton number, Z
Neutron number, N
43
42
41
40
39
38
37
36
35
48
50
52
54
56
Flux, $\log_{10}(f)$
−7.5
−8.5
−9.5
−10.5
−11.5

On the s-process

3 components: weak, main and strong



Summary and conclusions

Recalling that we wanted to determine the physics governing will be defined by the nuclear energy source in the interiors. We needed to develop a theory and understanding of nuclear physics and reactions

- We have covered the basic principles of energy production by fusion
- The PP chain and CNO cycle have been described,
- He burning by the triple-alpha reaction was introduced
- Later burning stages of the heavier elements (C,O, Si) discussed
- The r- and s-processes – origin of the elements heavier than Fe