Cyber-Physical Systems

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Lecture 12: Automata and Temporal Logic

[Many Slides due to J. Deshmukh, USC, LA,USA]

$\square_{[1,3]}(x > 0) \wedge \lozenge_{[0,0.001]}(y < 0) \Rightarrow (x > 1) \vee (x < -1)$

$p_1\, \mathcal{U}_{(a_1,b_1)}\, (p_2\, \mathcal{U}_{(a_2,b_2)}\, (p_3\, \mathcal{U}_{(a_3,b_3)}\, (p_4\, \mathcal{U}_{(a_4,b_4)}\, \mathcal{G}\, p_5)))$

$\square_{[1,3]}(x > 0) \wedge \lozenge_{[0,0.001]}(y < 0) \Rightarrow (x > 1) \vee (x < -1)$

$\square_{[1,3]}(x > 0) \vee (x < -1)$

$\square_{[1,3]}(x > 0) \Rightarrow \lozenge_{[1,3]}((y > 0) \wedge \lozenge_{[0,0.001]}(y < 0) \Rightarrow (x > 1) \vee (x < -1)$

Requirements

- Requirements describe desirable properties of system behaviors
- High assurance/safety-critical, or mission-critical systems must use formal requirements
- Behavioral requirements: requirement can be evaluated on individual system behaviors
- Requirements met by the whole system if *all* behaviors satisfy requirements
- There needs to be a clear separation between requirements (what needs to be implemented) and the design (how should it be implemented)
- Unfortunately, this is not often obeyed

Rigor in Requirements

- Informal requirements: implicit or stated in natural language
  - If an obstacle is sensed by the car, it should stop if it is safe to do so
- Formal requirements: explicit and mathematically precise
  - If the vision system, with a probability > 0.8,
    - labels an object $(x + d_{\text{safe}})$ meters from the car as a stationary obstacle,
  - then as long as the current velocity u of the car is less than $\sqrt{2xb(u)}$,
    - the vehicle should execute an emergency stop maneuver within 10 ms.

Here, the maximum braking deceleration that the car can produce at velocity u is $-b_{\max}(u)$, and d_{safe} is a safe stopping distance between vehicles

Types of Specifications/Requirements

- Hard Requirements: Violation leads to endangering safety-criticality or mission-criticality
  - **Safety** Requirements: system never does something bad
  - **Liveness** Requirements: from any point of time, system eventually does something good
- Soft Requirements: Violations lead to inefficiency, but are not critical
  - (Absolute) Performance Requirements: system performance is not worst than a certain level
  - (Average) Performance Requirements: average system performance is at a certain level

Requirement Formalisms

- Languages and Logics to describe *mathematically precise* requirements

- Examples:
  - Automata, State Machines
  - Propositional Logic, Temporal Logic, Regular Expressions
  - Structured language/grammar-based requirements

Temporal Logic

- Temporal Logic (literally logic of time) allows us to specify infinite sequences of states using logical formulae
- Amir Pnueli in 1977 used a form of temporal logic called Linear Temporal Logic (LTL) for requirements of reactive systems: later selected for the 1996 Turing Award
- Clarke, Emerson, Sifakis in 2007 received the Turing Award for the model checking algorithm, originally designed for checking Computation Tree Logic (CTL) properties of distributed programs

What is a logic in context of today's lecture?

- **Syntax**: A set of operators that allow us to construct formulas from specific ground terms
- **Semantics**: A set of rules that assign meanings to well-formed formulas obtained by using above syntactic rules
- Simplest form is Propositional Logic

Propositional Logic

- Simplest form of logic with a set of:
  - atomic propositions:
    $$AP = \{p, q, r, \dots\}$$
  - Boolean connectives:
    $$\wedge, \vee, \neg, \Rightarrow, \equiv$$

- Syntax recursively gives how new formulae are constructed from smaller formulae

Syntax of Propositional Logic

$\varphi ::=$	$true$	the true formula
	p	p is a prop in AP
	$\neg\varphi$	Negation
	$\varphi \wedge \varphi$	Conjunction
	$\varphi \vee \varphi$	Disjunction
	$\varphi \Rightarrow \varphi$	Implication
	$\varphi \equiv \varphi$	Equivalence

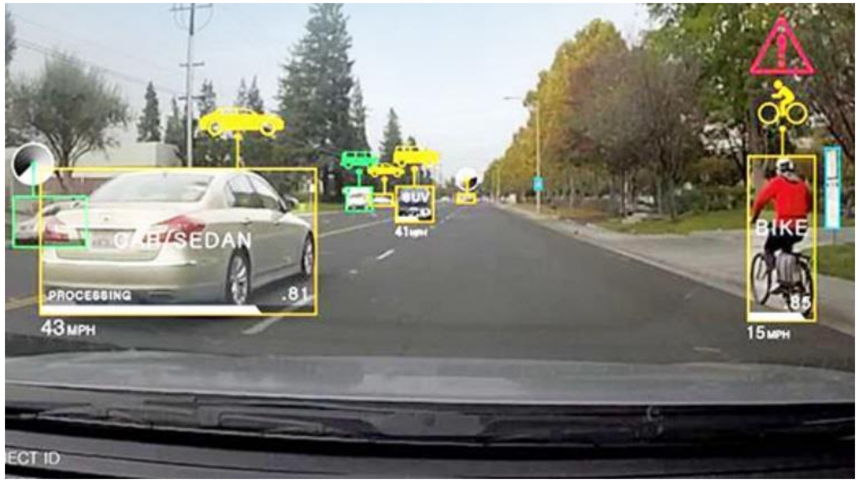
Semantics

- Semantics (i.e. meaning) of a formula can be defined recursively
- Semantics of an atomic proposition defined by a ***valuation*** function ν
- Valuation function assigns each proposition a value 1 (true) or 0 (false), always assigns the $true$ formula the value 1, and for other formulae is defined recursively

Semantics of Prop. Logic	
$\nu(true)$	1
$\nu(p)$	1 if $\nu(p) = 1$
$\nu(\neg\varphi)$	1 if $\nu(\varphi) = 0$ 0 if $\nu(\varphi) = 1$
$\nu(\varphi_1 \wedge \varphi_2)$	1 if $\nu(\varphi_1) = 1$ and $\nu(\varphi_2) = 1$, 0 otherwise
$\varphi_1 \vee \varphi_2$	$\nu(\neg(\neg\varphi_1 \wedge \neg\varphi_2))$
$\varphi_1 \Rightarrow \varphi_2$	$\nu(\neg\varphi_1 \vee \varphi_2)$
$\varphi_1 \equiv \varphi_2$	$\nu\big((\varphi_1 \Rightarrow \varphi_2) \wedge (\varphi_2 \Rightarrow \varphi_1)\big)$

Examples

- p : There is an upright bicycle in the middle of the road
- r: the bicycle has a rider
- $p \Rightarrow r$: If there is an upright bicycle in the middle of the road, the bicycle has a rider
- q : There is car in the field of vision
- o_i: Car i is in the intersection
- $(o_1 \land \neg o_2) \lor (\neg o_1 \land o_2)$



Interpreting a formula of prop. logic

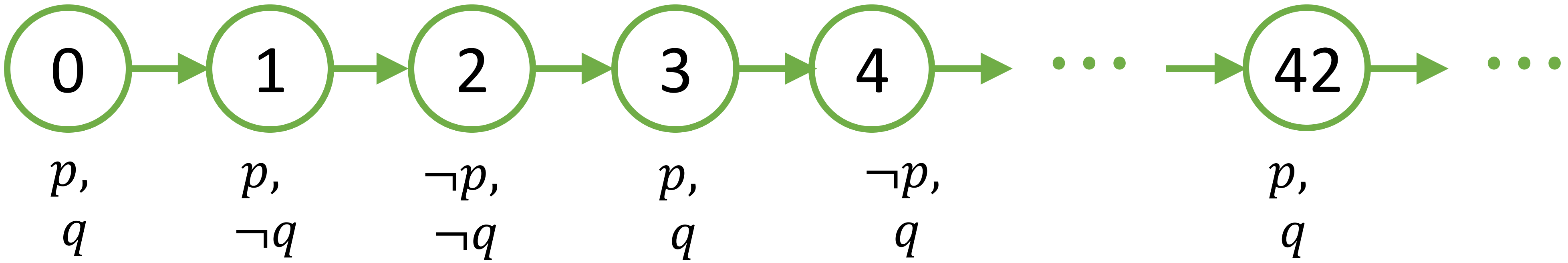
- $v: p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 0$. What is $v\big((p_1 \land p_2) \Rightarrow p_3\big)$?
- $v\big((p_1 \land p_2) \Rightarrow p_3\big)$ =1

- $v: p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 0$. What is $v\big((p_1 \Rightarrow p_3) \land (p_2 \Rightarrow p_3)\big)$
- $v\big((p_1 \Rightarrow p_3) \land (p_2 \Rightarrow p_3)\big)$ =0

- Is this true? $v\left(\underline{(p_1 \land p_2) \Rightarrow p_3} \equiv \underline{(p_1 \Rightarrow p_3) \land (p_2 \Rightarrow p_3)}\right) = 1$? (For all valuations)?

Temporal Logic = Prop. Logic + Temporal Operators

- Propositional Logic is interpreted over valuations to atoms
- Temporal Logic is interpreted over traces/sequences/strings
- Trace is an infinite sequence of valuations
- ρ:



- Can also write as: (0,1,1), (1,1,0), (2,0,0), (3,1,1),(4,0,1),… ,(42,1,1), …

LTL

Linear Temporal Logic

- LTL is a logic interpreted over infinite traces
- Temporal logic with a view that time evolves in a linear fashion
  - Other logics where time is branching!
- Assumes that a trace is a discrete-time trace, with equal time intervals
- Actual interval between time-points does not matter : similar to rounds in synchronous reactive components
- LTL can be used to express safety and liveness properties!

LTL Syntax

- LTL formulas are built from propositions using:
  - Boolean connectives
  - Temporal Operators
- Only shown $\land$ and $\neg$, but can define $\lor, \Rightarrow, \equiv$ for convenience

Syntax of LTL			
$\varphi ::=$	p	|	p is a prop in AP
	$\neg\varphi$	|	Negation
	$\varphi \land \varphi$	|	Conjunction
	$\mathbf{X}\varphi$	|	NeXt Step
$\Diamond$	$\mathbf{F}\varphi$	|	Some Future Step
$\Box$	$\mathbf{G}\varphi$	|	Globally in all steps
	$\varphi\ \mathbf{U}\ \varphi$	|	In all steps Until in some step

LTL Semantics

- Semantics of LTL is defined by a valuation function that assigns to each proposition at each time-point in the trace a truth value (0 or 1)
- We use the symbol $\vDash$ (read models) to show that a trace-point satisfies a formula
- $\rho, n \vDash \varphi$: Read as trace ρ at time n satisfies formula φ
- If we omit n, then the meaning is time 0. I.e. $\rho \vDash \varphi$ is the same as $\rho, 0 \vDash \varphi$
- Semantics is defined recursively over the formula
- Base case: Propositional formulas, Recursion over structure of formula

Recursive semantics of LTL: I

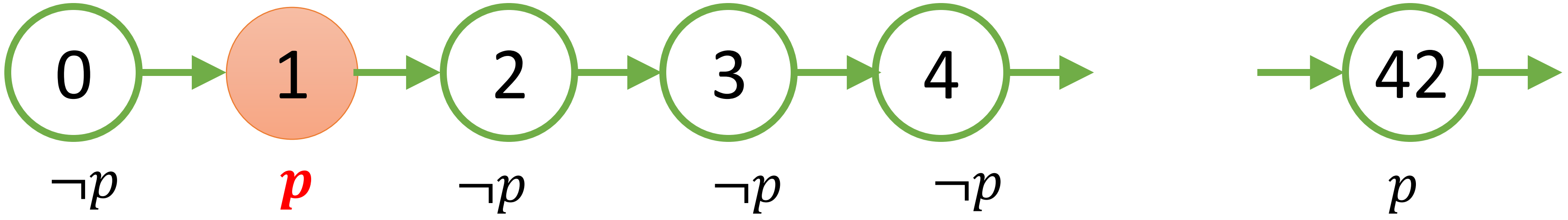
- $\rho, n \vDash p$ if $v_n(p) = 1$,
  - i.e. if p is true at time n
- $\rho, n \vDash \neg\varphi$ if $\rho, n \nvDash \varphi$,
  - i.e. if φ is ***not*** true for the trace starting time n
- $\rho, n \vDash \varphi_1 \wedge \varphi_2$ if $\rho, n \vDash \varphi_1$ and $\rho, n \vDash \varphi_2$
  - i.e. if φ_1 and φ_2 ***both hold*** starting time n

Recursive semantics of LTL: II

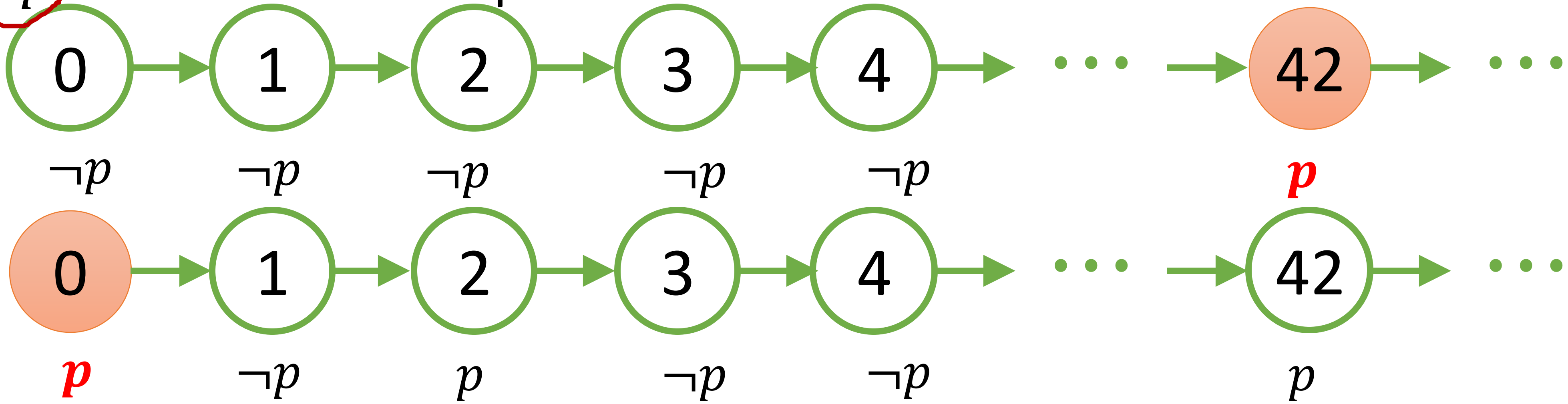
- $\rho, n \vDash \mathbf{X}\varphi$ if $\rho, n+1 \vDash \varphi$
  - i.e. if φ holds starting at the next time point
- $\rho, n \vDash \mathbf{F}\,\varphi$ if $\exists m \geq n$ such that $\rho, m \vDash \varphi$
  - i.e. φ is true starting now, or there is some future time-point m from where φ is true

Visualizing the temporal operators

- $\mathbf{X}p$: Ne**X**t Step



- $\mathbf{F}p$: Some **F**uture step

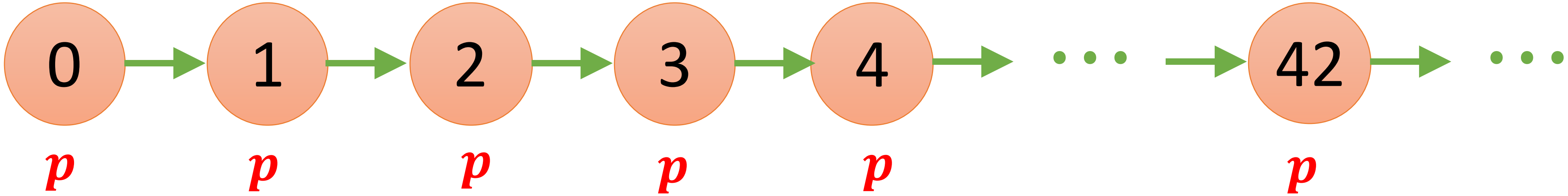


Recursive semantics of LTL: II

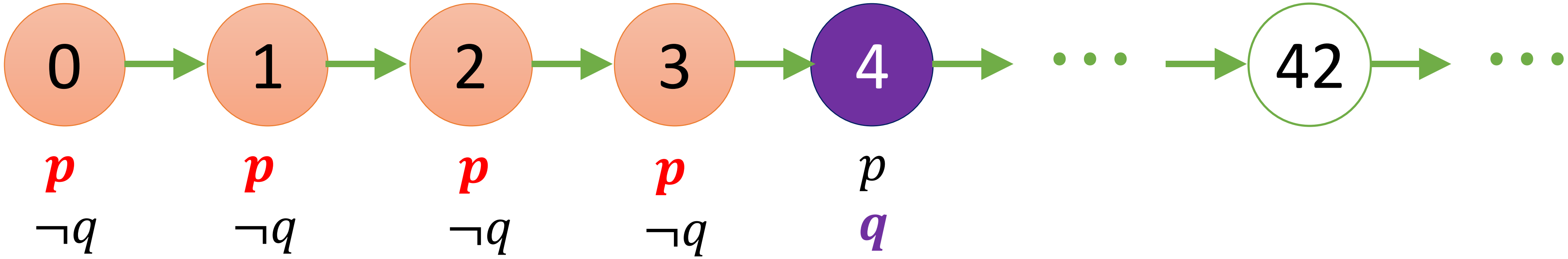
- $\rho, n \models \mathbf{G}\, \varphi$ if $\forall m \geq n : \rho, m \models \varphi$
  - i.e. φ is true starting now, and for all future time-points m, φ is true starting at m
- $\rho, n \models \varphi_1 \mathbf{U} \varphi_2$ if $\exists m \geq n$ s.t. $\rho, m \models \varphi_2$ and $\forall \ell$ s.t. $n \leq \ell < m, \rho, \ell \models \varphi_1$
  - i.e. φ_2 eventually holds, and for all positions till φ_2 holds, φ_1 holds

Visualizing the temporal operators

- $\mathbf{G}p$: **G**lobally p holds

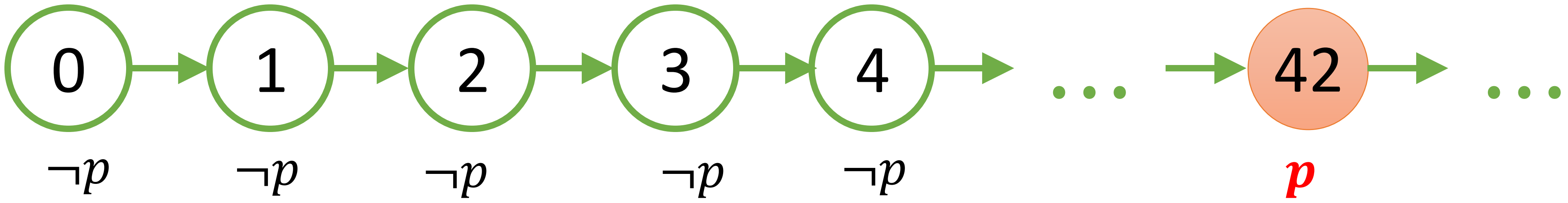


- $p\ \mathbf{U}\ q$: p holds Until q holds

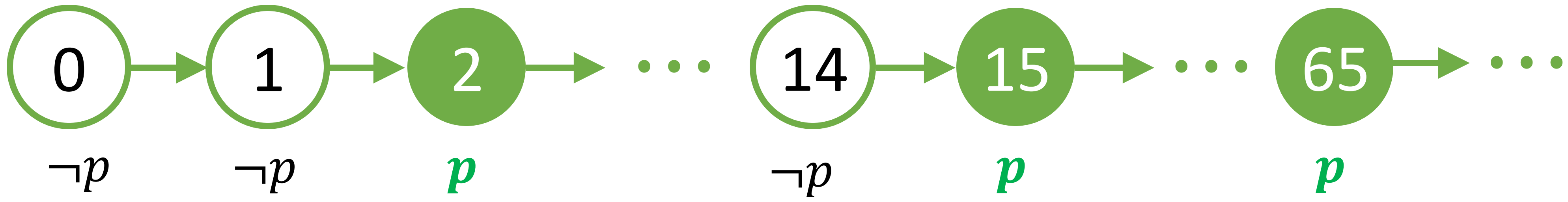


You can nest operators!

- What does $\boxed{\mathbf{XF}\ p}$ mean?
  - Trace satisfies $\mathbf{XF}p$ (at time 0) if at time 1, $\mathbf{F}p$ holds. I.e. p holds at some point strictly in the future

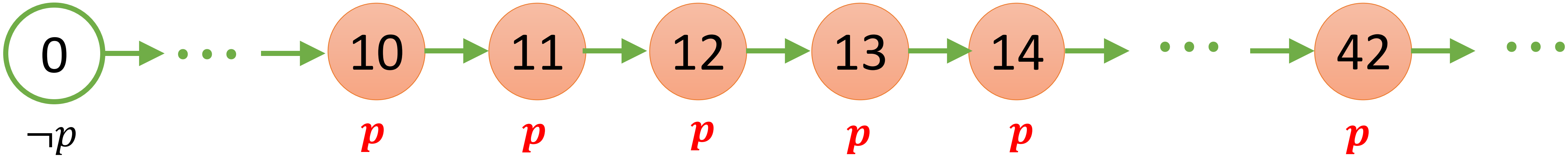


- What does $\boxed{\mathbf{GF}\ p}$ mean?
  - Trace satisfies $\mathbf{GF}p$ (at time 0) if at n, there is always a $\boldsymbol{p}$ in the future

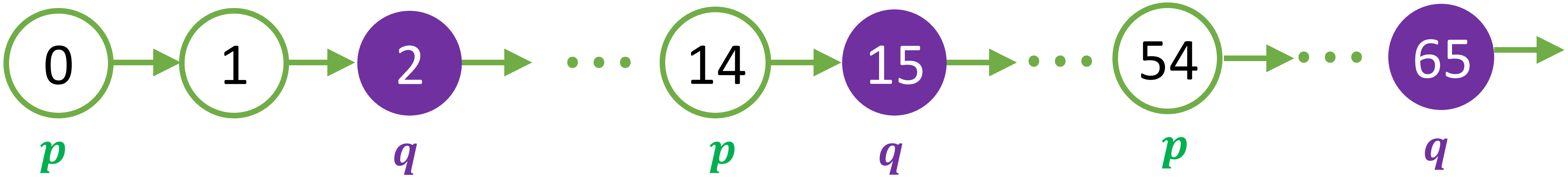


More operator fun

- What does $\mathbf{FG}p$ mean?

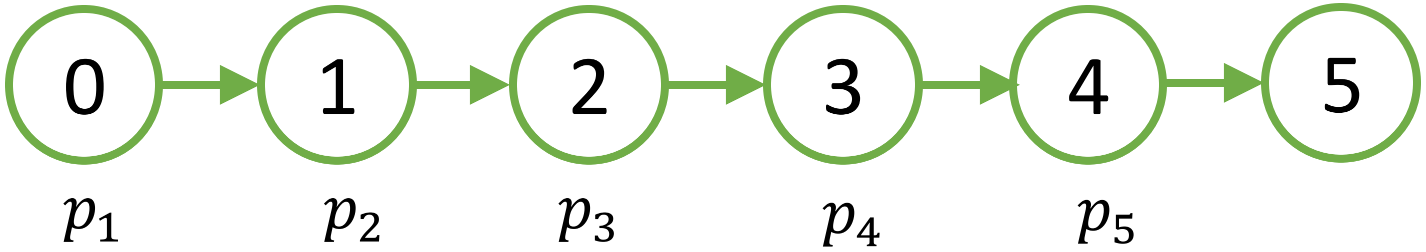


- What does $\mathbf{G}(p \Rightarrow \mathbf{F}q)$ mean?

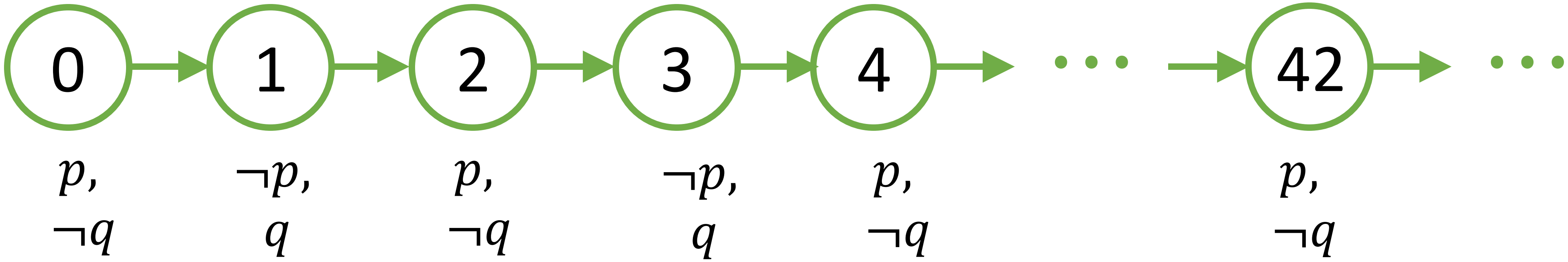


More, more operator fun

- What does the following formula mean: $p_1 \wedge \mathbf{X}\big(p_2 \wedge \mathbf{X}(p_3 \wedge \mathbf{X}(p_4 \wedge \mathbf{X}p_5))\big)$?



- Is this true? $\boxed{\mathbf{F}(p \wedge q)}$ is the same as $\boxed{\mathbf{F}p \wedge \mathbf{F}q}$?



Linear Temporal Logic (LTL) specification

It is a logic interpreted over infinite discrete-time traces

E.g. It is always true that the highest temperature will be below 75 degree and the lowest temperature will be above 60 degree

$\mathbf{G}(p \wedge q)$ $p = T<75$, $q=T>60$

Linear Temporal Logic (LTL) specification

It is a logic interpreted over infinite discrete-time traces

E.g. **For the next 3 days** the highest temperature will be below 75 degree and the lowest temperature will be above 60 degree

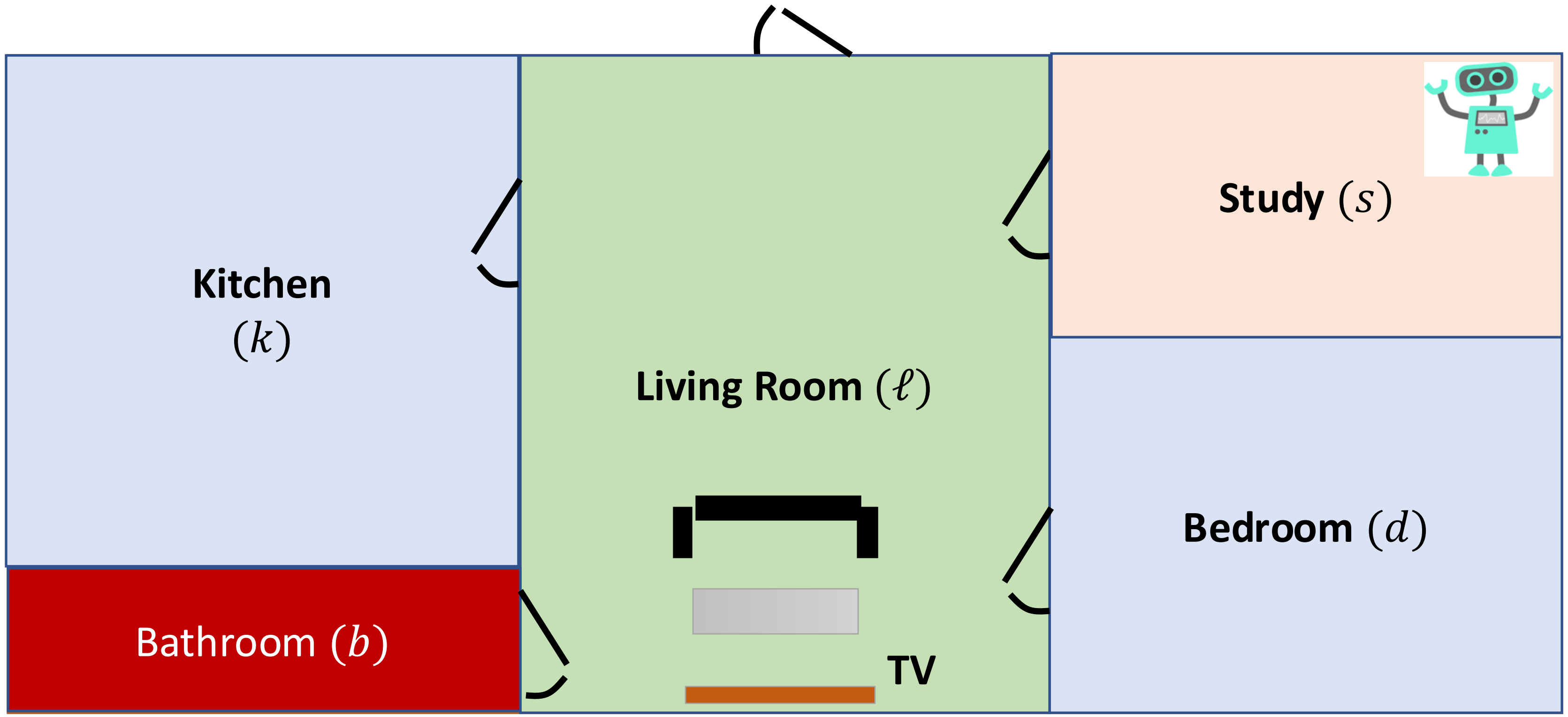
$X\,(p \wedge q) \wedge X\,X\,(p \wedge q) \wedge X\,X\,X\,(p \wedge q)$ with $p = T<75$, $q=T>60$

Operator duality and identities

- $\mathbf{F}\varphi \equiv \neg\mathbf{G}\neg\varphi$
- $\mathbf{GF}\varphi \equiv \neg\mathbf{FG}\neg\varphi$
- $\mathbf{F}(\varphi \vee \psi) \equiv \mathbf{F}\varphi \vee \mathbf{F}\psi$
- $\mathbf{G}(\varphi \wedge \psi) \equiv \mathbf{G}\varphi \wedge \mathbf{G}\psi$
- $\mathbf{FF}\varphi \equiv \mathbf{F}\varphi$
- $\mathbf{GG}\varphi \equiv \mathbf{G}\varphi$
- $\mathbf{FGF}\varphi \equiv \mathbf{GF}\varphi$
- $\mathbf{GFG}\varphi \equiv \mathbf{FG}\varphi$

Example specifications in LTL

- Suppose you are designing a robot that has to do a number of missions



- Whenever the robot visits the kitchen, it should visit the bedroom after.

$$\mathbf{G}(k_r \Rightarrow \mathbf{F}\, d_r)$$

- Robot should never go to the bathroom.

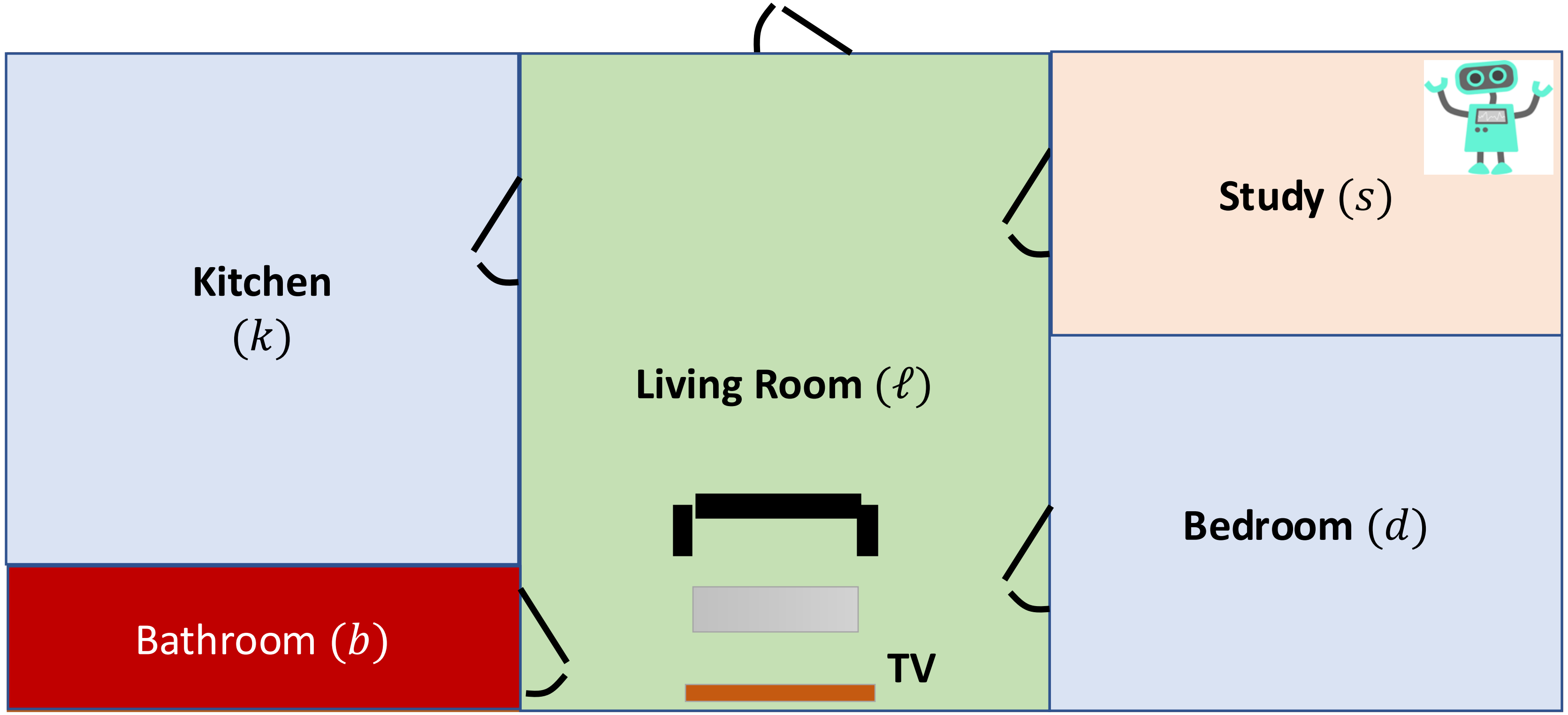
$$\mathbf{G}\neg b_r$$

- The robot should keep working until its battery becomes low

$$working\ \mathbf{U}\ low\_battery$$

Example specifications in LTL

- Suppose you are designing a robot that has to do a number of missions



- The robot should repeatedly visit the living room

$$\mathbf{GF}\ \ell$$

- Whenever the TV is on and the living room has no person in it, then within three steps, the robot should turn off the TV

$o(r)$: room occupied by a person

$$\boxed{\mathbf{G}}\Big((\neg o(\ell) \wedge TV_{on}) \Rightarrow \mathbf{F}^{\leq 3}(TV_{off})\Big)$$

$$\mathbf{F}^{\leq 3}\varphi \equiv \varphi \vee \mathbf{X}\varphi \vee \mathbf{XX}\varphi \vee \mathbf{XXX}\varphi$$

(Hard) Requirements

- **Safety** and **liveness** requirements require fundamentally different classes of model checking algorithms
- **safety** requirement: “system never does something bad”

  “if something bad happens on an infinite run, then it happens already on some finite prefix”

  Counterexamples no reachable ERROR state

- **liveness** requirement: “system eventually does something good “

  “no matter what happens along a finite run, something good could still happen later”

  Infinite-length counterexamples, loop

Requirements example

- It cannot happen that both processes are in their critical sections simultaneously

- Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter.

- Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

- The elevator will arrive within 30 seconds of being called

- Patient’s blood glucose never drops below 80 mg/dL

Requirements example

- It cannot happen that both processes are in their critical sections simultaneously **S**
- Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter. **S**
- Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually. **L**
- The elevator will arrive within 30 seconds of being called **S** (observe the finite prefix of all computation steps until 30 seconds have passed, and decide the property, therefore safety)
- Patient’s blood glucose never drops below 80 mg/dL **S**

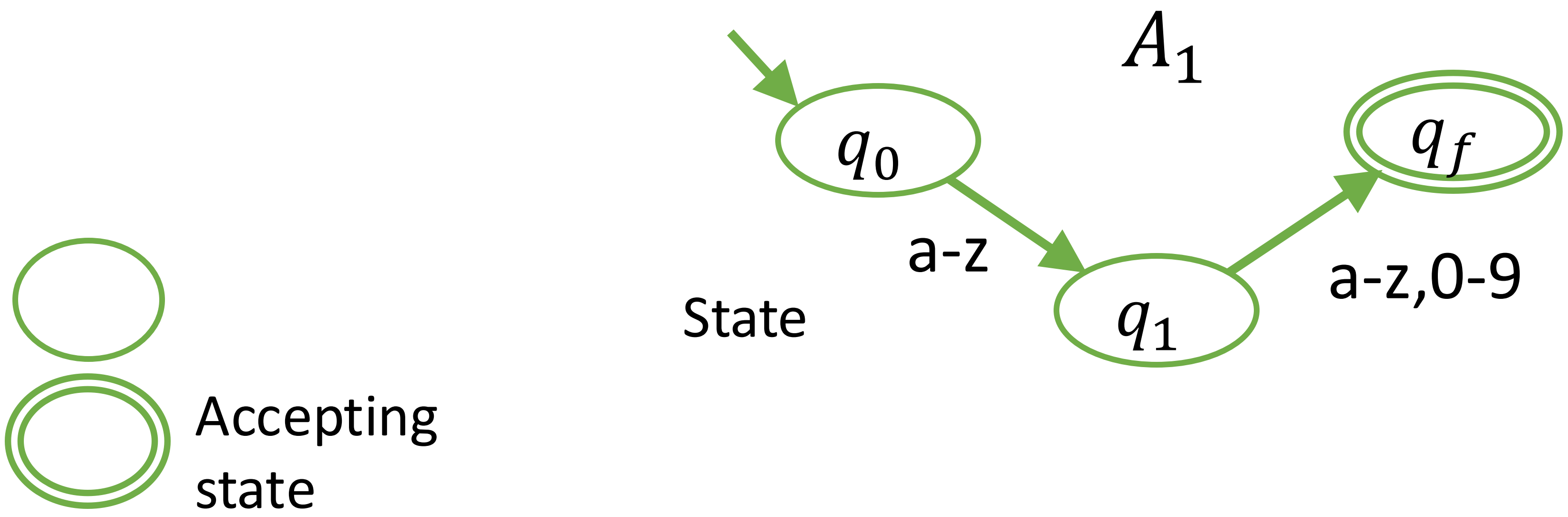
Detour to automata and formal languages

- Most programmers have used regular expressions
- Regular Expressions (RE) are sequences of characters that specify (acceptable) pattern of ***finite*** length
- Example:
  - [a-z][a-z 0-9] : strings starting with a lowercase letter (a-z) followed by ***one*** lowercase letter or number
  - [a-z][0-9]*[a-z] : strings starting with a lowercase letter, followed by ***finitely many*** numbers followed by a lowercase letter

Finite State Automata (FSA)

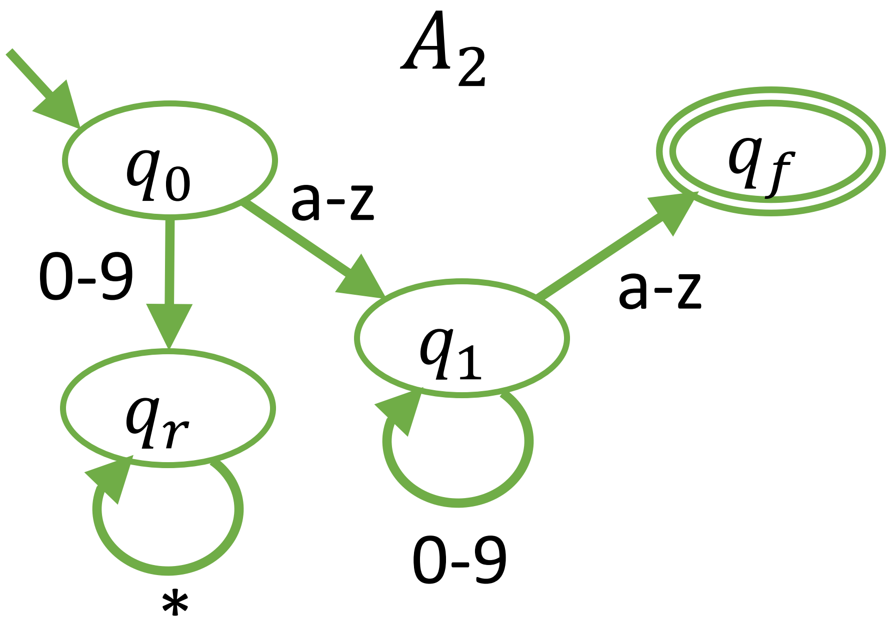
Famous equivalence between FSA and regular expressions:

- For every regular expression R_i , there is a corresponding FSA A_i that accepts the set of strings generated by R_i .
- For every FSA A_i there is a corresponding regular expression that generates the set of strings accepted by A_i .



[a-z][a-z 0-9]

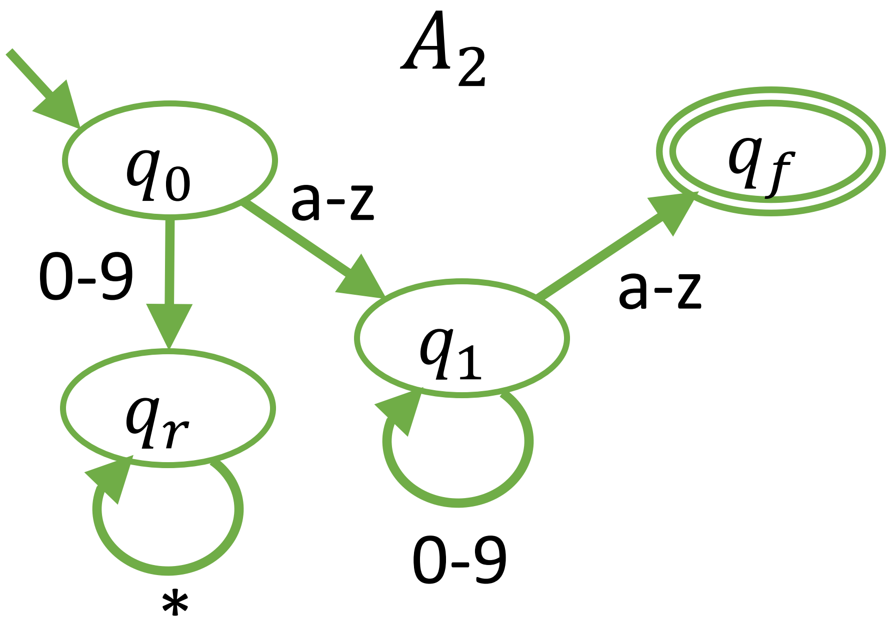
How does a Finite State Automaton work?



[a-z][0-9]*[a-z]

- Starts at the initial state q_0
- In q_0, if it receives a letter in a-z, goes to q_1
  else, it goes to q_r
- In q_1, if it receives a number in 0-9, it stays in q_1
  else, it goes to q_f (as it received a-z)
- In q_r, no matter what it gets, it stays in q_r
- q_f is an accepting state where computation halts
- Any string that takes the automaton from q_0 to q_f is ***accepted*** by the automaton

Language of a finite state automaton



- What strings are accepted by A_2?
  - ab, zy, s2r, q123s, u3123123v, etc.
- What strings are not accepted by A_2?
  - 2b, 334a, etc.
- The set of all strings accepted by A_2 is called its ***language***
- The language of a finite state automaton consists of strings, each of which can be arbitrarily long, ***but finite***

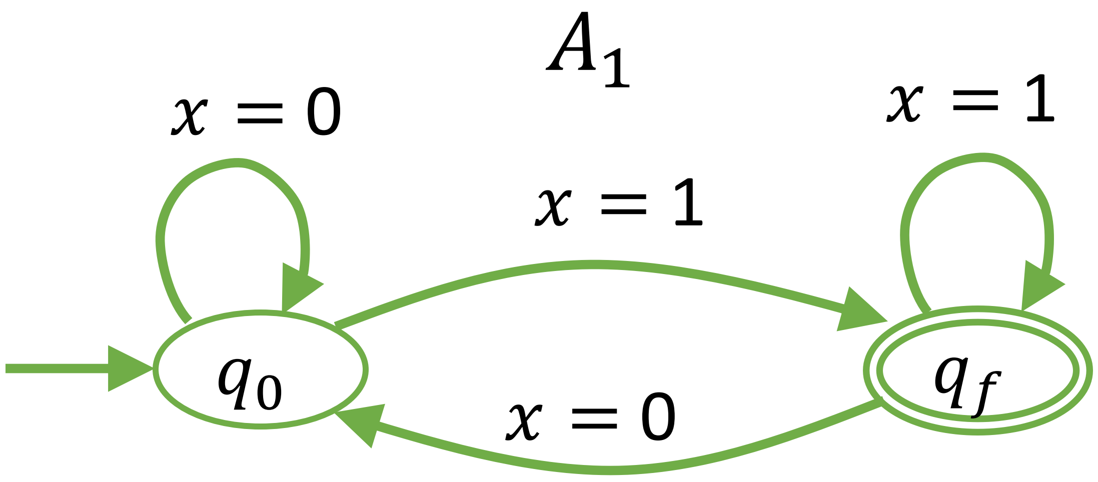
LTL Monitors

- A safety monitor classifies system behaviors into good and bad
- Can we use a monitor to classify **infinite** behaviors into good or bad?
- Yes, using theoretical model of Büchi automata proposed by J. Richard Büchi in 1960

Büchi Automata

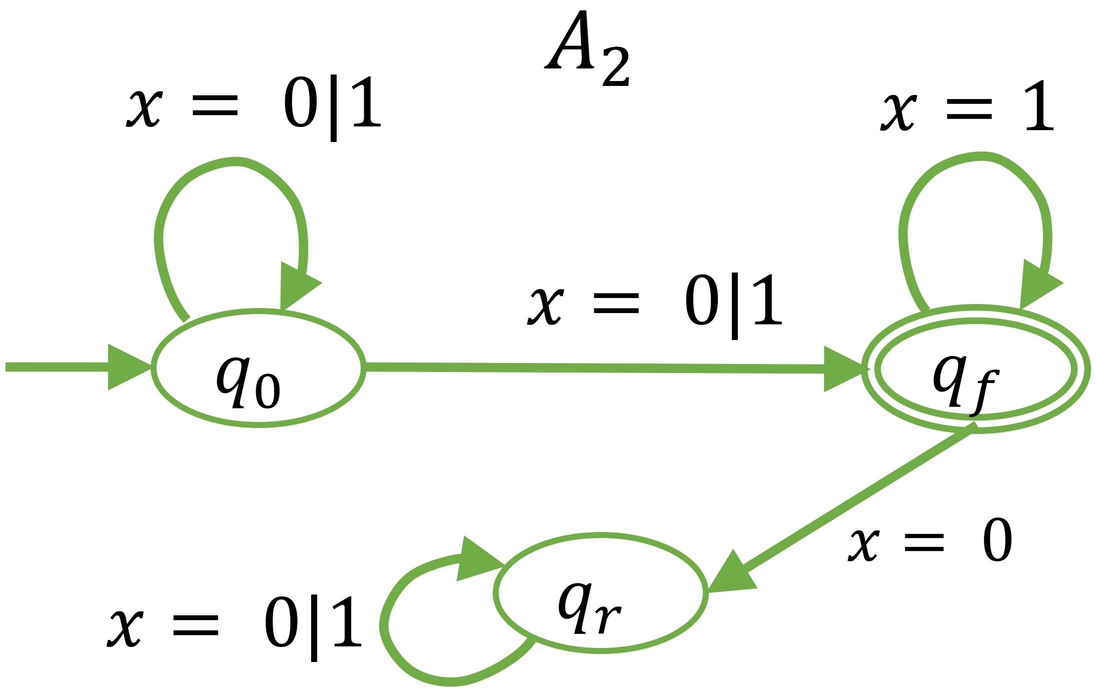
Büchi automaton Example 1

- Extension of finite state automata to accept infinite strings



- States Q: $\{q_0, q_f\}$
- Input variable x with domain Σ: $\{0,1\}$
- Final state: $\{q_f\}$
- Transitions: (as shown)
- Given trace ρ (infinite sequence of symbols from Σ), ρ is accepted by A_1, if q_f appears inf. often
- What is the language of A_1?
  - LTL formula $\mathbf{GF}(x = 1)$

Büchi automaton Example 2

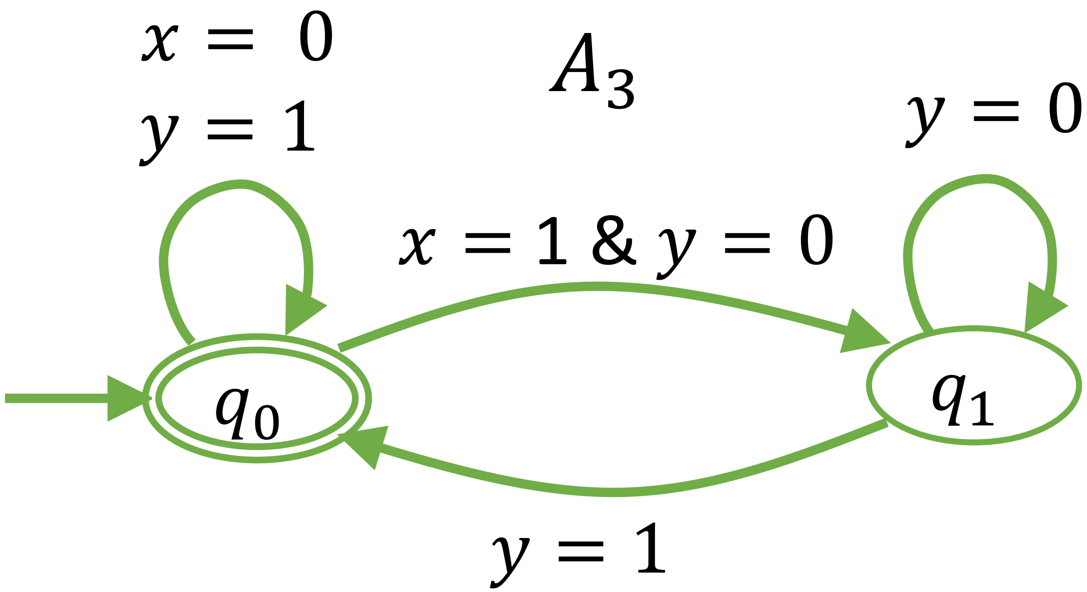


- Note that this is a nondeterministic Büchi automaton
- A_2 accepts ρ if ***there exists a path*** along which a state in F appears infinitely often
- What is the language of A_2?
  - LTL formula $\mathbf{FG}(x = 1)$

- Q: $\{q_0, q_f\}$, Σ: $\{0,1\}$, F: $\{q_f\}$
- Transitions: (as shown)

Fun fact: there is no deterministic Büchi automaton that accepts this language

Büchi automaton Example 3



- $Q: \{q_0, q_1\}$, $\Sigma: \{0,1\}$, $F: \{q_f\}$
- Transitions: (as shown)

- What is the language of A_3?
  - LTL formula:
    $\mathbf{G}\big((x = 1) \Rightarrow \mathbf{F}(y = 1)\big)$
  - I.e. always when $(x = 1)$, in some future step, $(y = 1)$
  - In other words, $(x = 1)$ must be followed by $(y = 1)$

Using Büchi monitors

- Theoretical result: Every LTL formula φ can be converted to a Büchi monitor/automaton A_φ
- Size of A_φ is generally exponential in the size of φ; blow-up unavoidable in general

Büchi monitors for runtime monitoring

- Runtime monitoring: return a verdict based on only a finite portion of the trace
- Some kinds of formulas can be monitored on finite traces
  - $\mathbf{F}\ (p \lor q)$
  - $\mathbf{G}\ (\neg \mathrm{p}\ )$
- Finitely satisfiable: $\mathbf{F}(p \lor q)$
- Finitely refutable: $\mathbf{G}(\neg p)$
- Some formulas can never return a verdict on finite traces
  - $\mathbf{GF}\ p, \mathbf{FG}\ q, \mathbf{G}(p \Rightarrow \mathbf{F}q)$

Reachability, MC, Monitoring and SMC

- **Monitoring**: computing β for a single trace $\mathbf{x} \in trace\ M$

- **Model checking (MC)** is an algorithmic method for determining if a system satisfies a formal specification expressed in temporal logic

$$M \vDash \phi \Leftrightarrow \forall \mathbf{x} \in trace\ (M)\ \ \beta(\ \varphi, \mathbf{x}, 0\ ) = 1$$

Type equation here.

- **Statistical Model Checking (SMC)**: "doing statistics" on $\beta\ (\varphi, \mathbf{x}, 0)$ for a finite-subset of $trace\ (M)$

- **Reachability** analysis is the process of computing the set of reachable states for a system