Image Processing for Physicists

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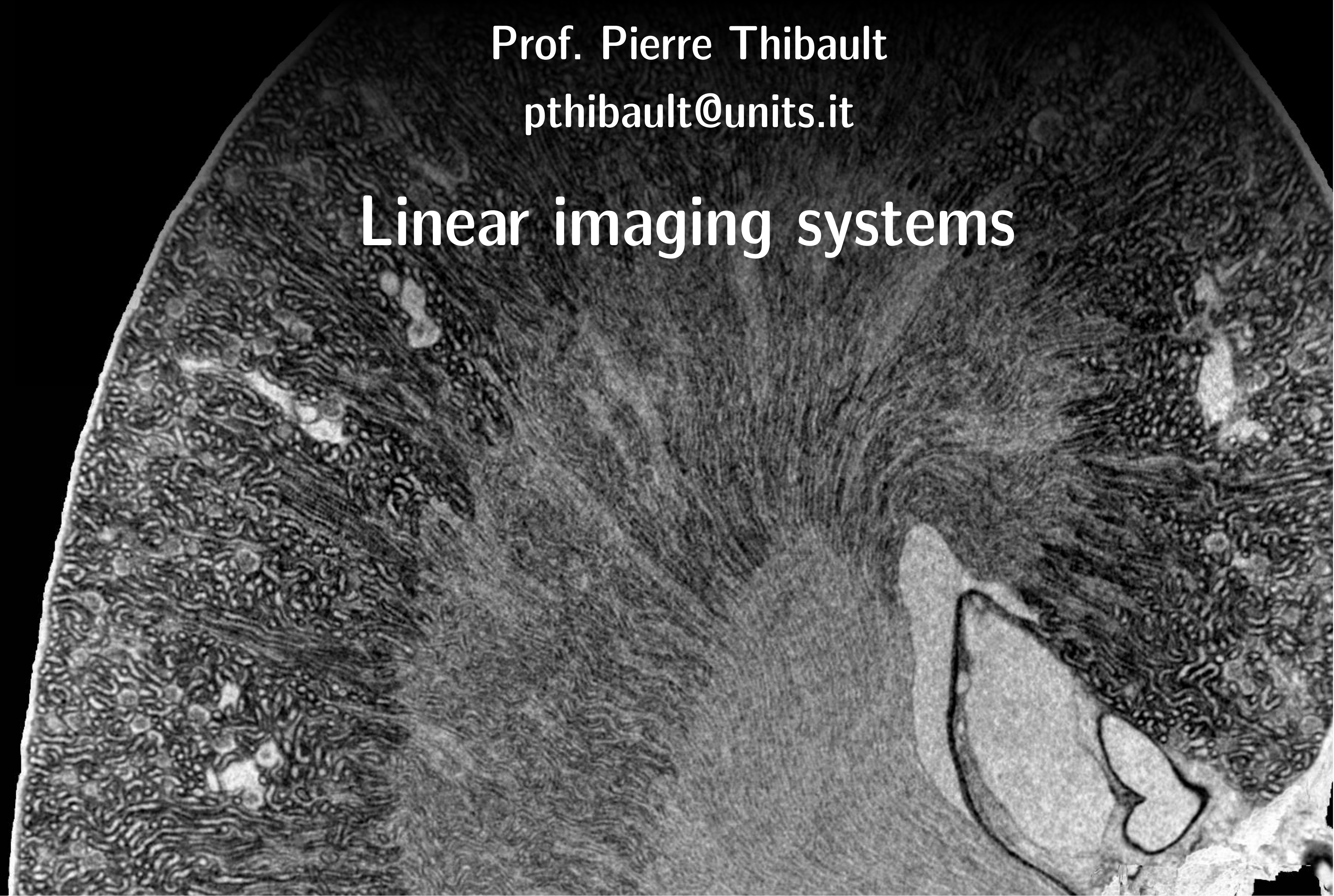
Linear imaging systems



Overview

- Definition of resolution
- Imaging systems:
  - Linear transfer model
  - Noise

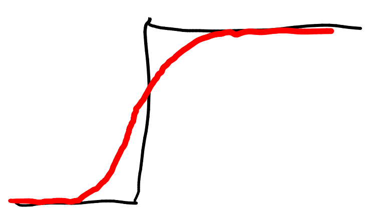
Resolution

"the smallest detail that can be distinguished"

- No unique definition
  - Numerical aperture ← microscopy, photography, astronomy
  - Pixel size ← detector-limited
  - Other criteria (PSF, MTF)
- What is "detail"?
- What is "distinguish"?

* separation of two points
↳ Rayleigh criterion

* how is a detail burred by the imaging system?



Resolution

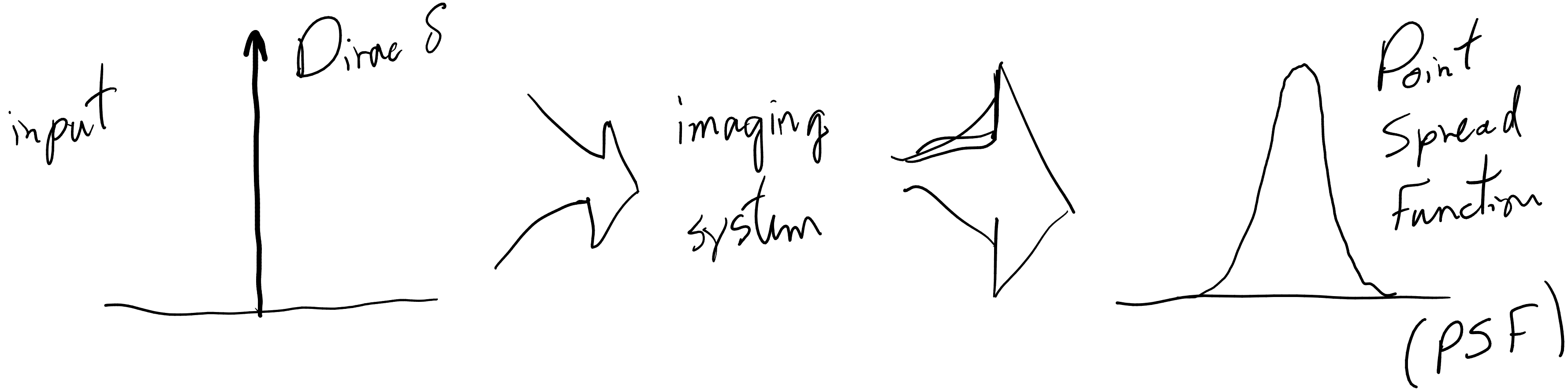
1280 x 1280

640 x 640

- **not** simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear translation-invariant systems

- Point spread function ("impulse response")



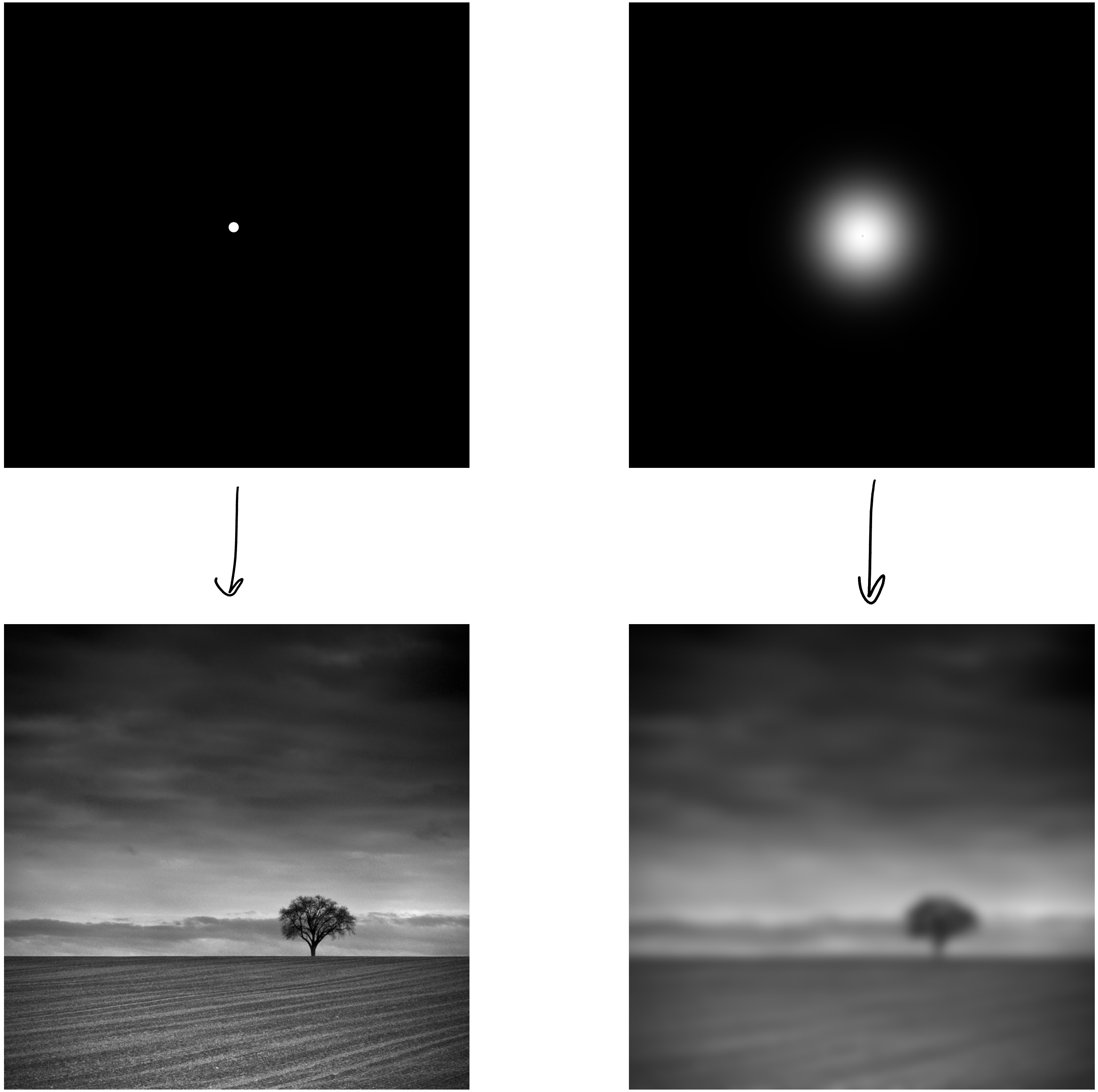
- LTI system: convolution with PSF

$$f(x,y) = \int dx'dy'\, f(x',y')\, \underbrace{\delta(x-x')\,\delta(y-y')}$$

$\downarrow$ Imaging system

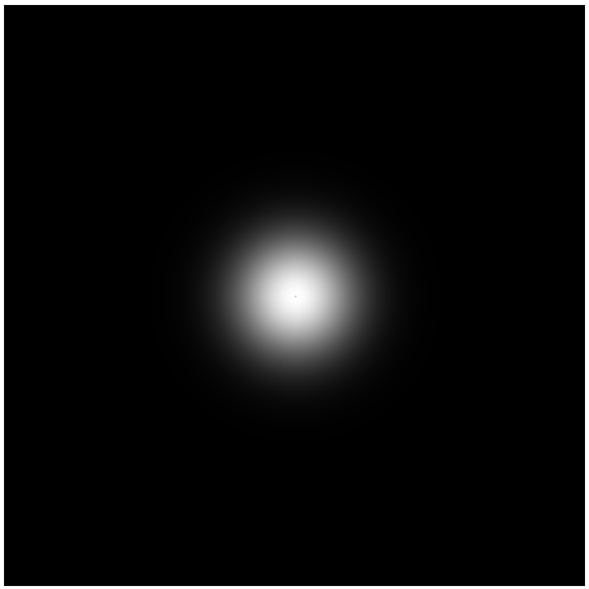
$$\int dx'dy'\, f(x',y')\, h(x-x',y-y') = f * h$$

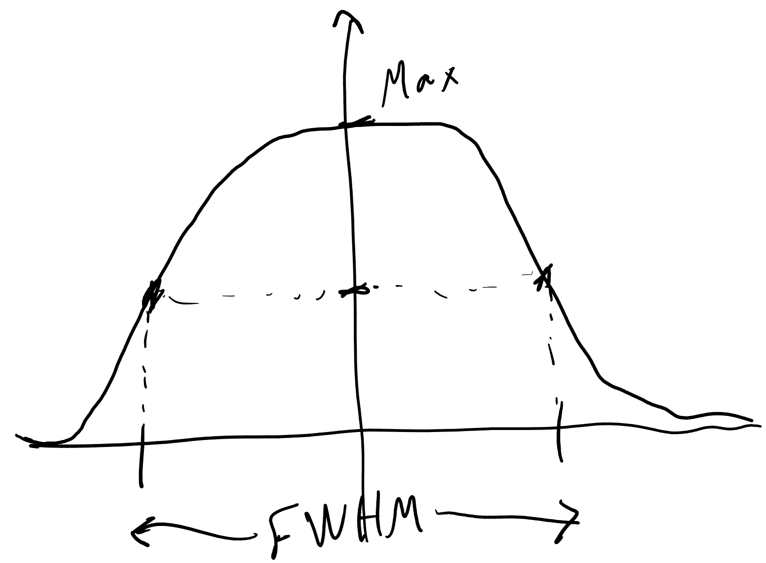
Point spread function



PSF and resolution

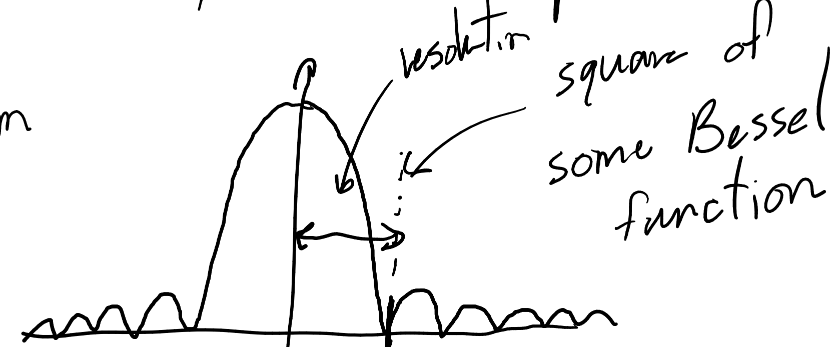
* Common way to describe resolution as a function of PSF is with the "Full Width at Half Maximum" (FWHM)





* Another definition of resolution is Rayleigh criterion
  ↳ applies to PSF generated by circular aperture

resolution: position of first minimum



Measurement of the PSF

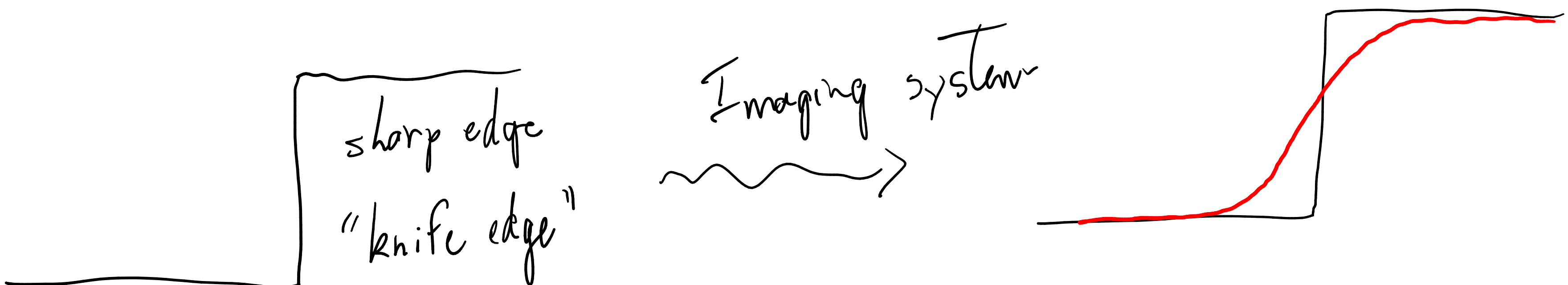
- Direct measurement from impulse

Generate a sharp point $\rightarrow$ output = PSF

astronomy: just pick a star!

Edge

- ~~Line~~-spread function

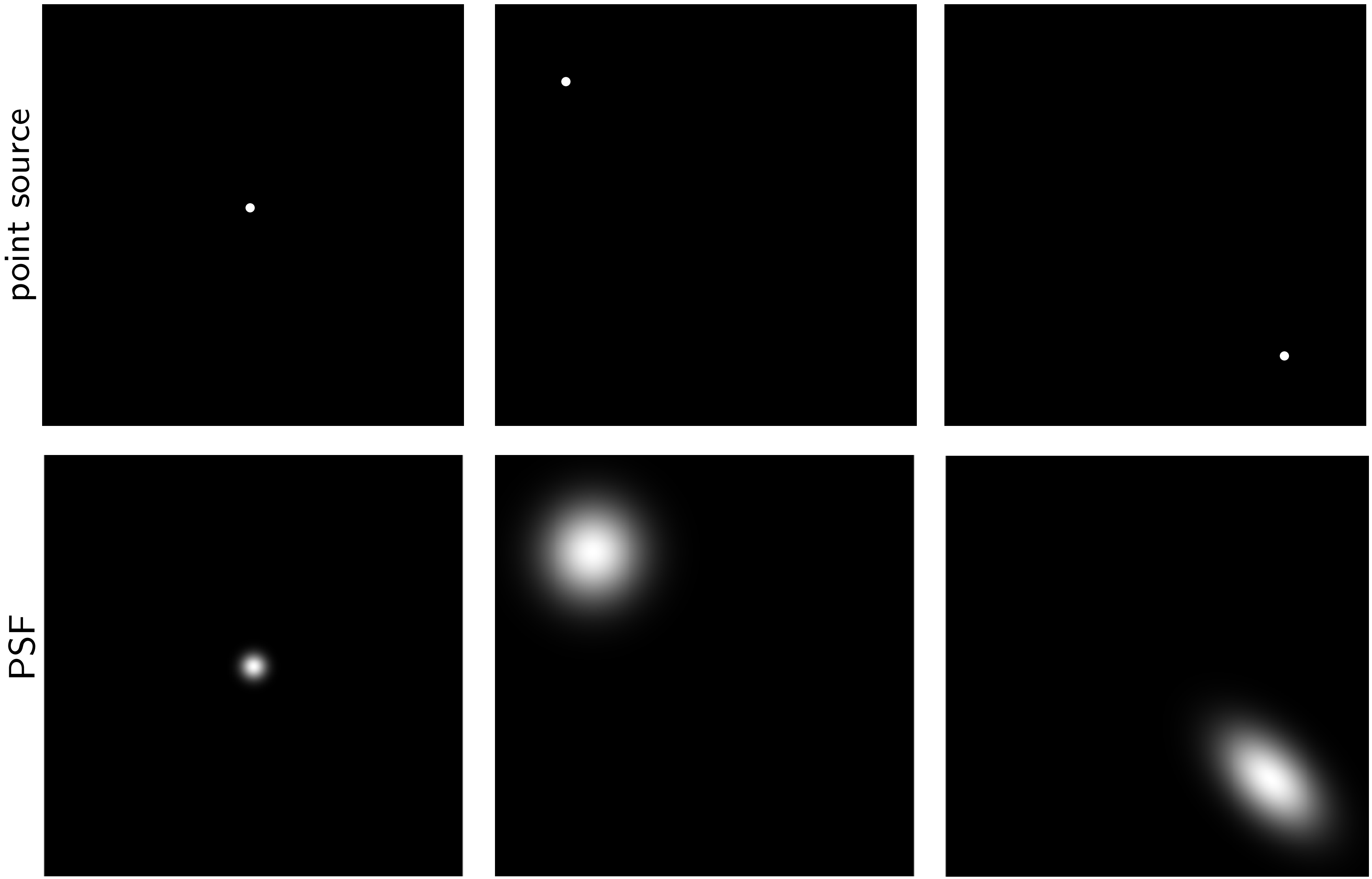


input : $H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$

$$\frac{\partial H}{\partial x} = \delta(x)$$

PSF = derivative of edge-spread function

PSF and translation invariance



Not translation invariant!

- Not translation invariant → PSF depends on position → not a convolution
- Useful to model system imperfections, lens aberrations, ...

The Fourier picture

$$\mathcal{F}\{f * h\} = F(u) \cdot H(u)$$

Fourier transform of PSF

=

"Optical Transfer Function"

OTF

Consider a single spatial frequency (u_0)

$$\underbrace{A}_{\text{original amplitude}}^{F(u_0)} e^{2\pi i u_0 x} \xrightarrow{\text{Imaging system}} \underbrace{H(u_0)\,A}_{\text{modulated amplitude}}\, e^{2\pi i u_0 x}$$

$H(u_0) \leq 1$

* observation: pure oscillations of the form $e^{2\pi i u_0 x}$ are eigenfunctions of a LTI imaging system

Optical transfer function

Response of a system to an oscillating signal with well-defined frequency

$$OTF(u) = \mathcal{F}\{PSF(x)\}$$

complex valued in general

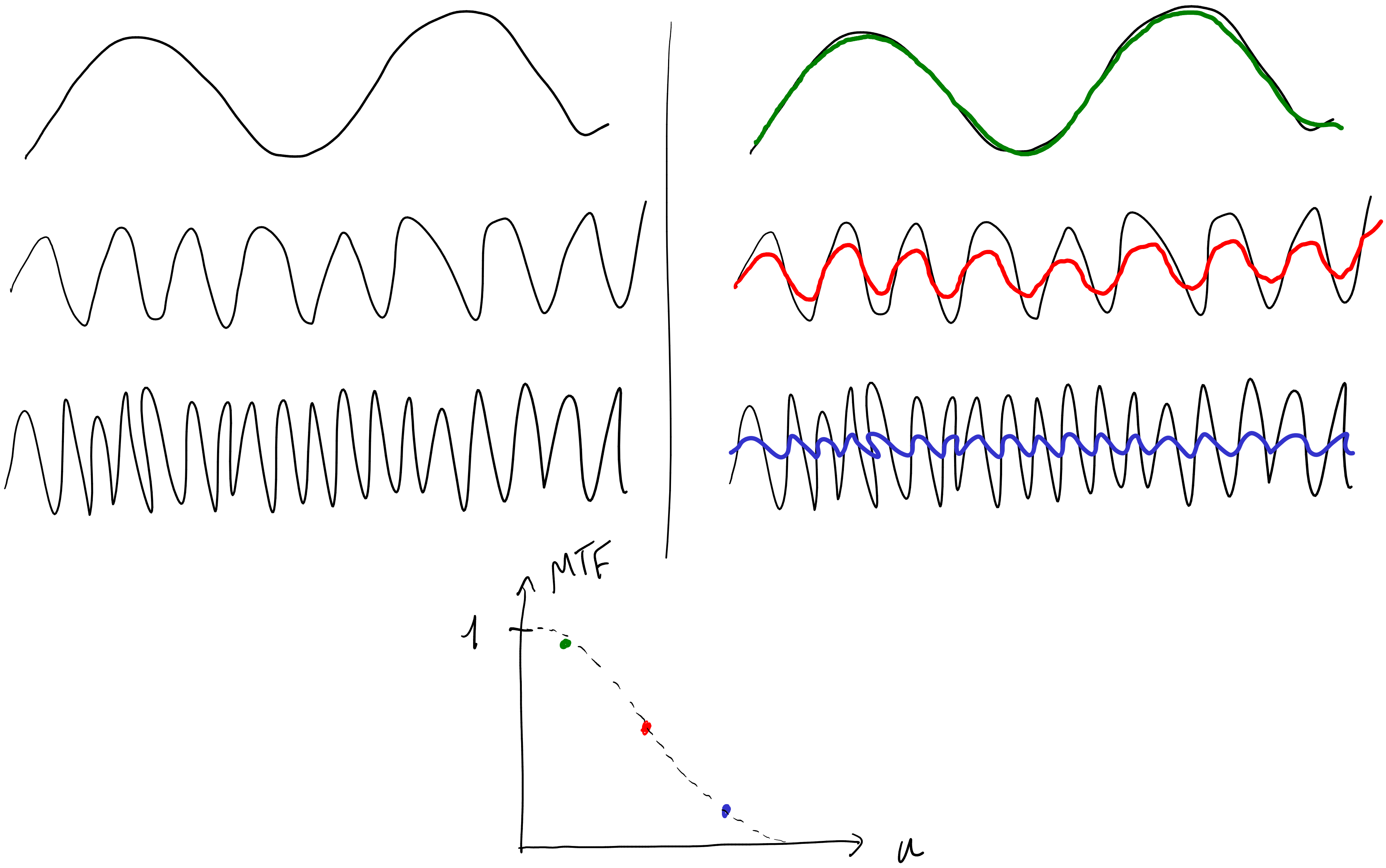
Amplitude $|OTF| = MTF$ "modulation transfer function"

Phase: $\arg\{OTF\} = PTF$ "phase transfer function"

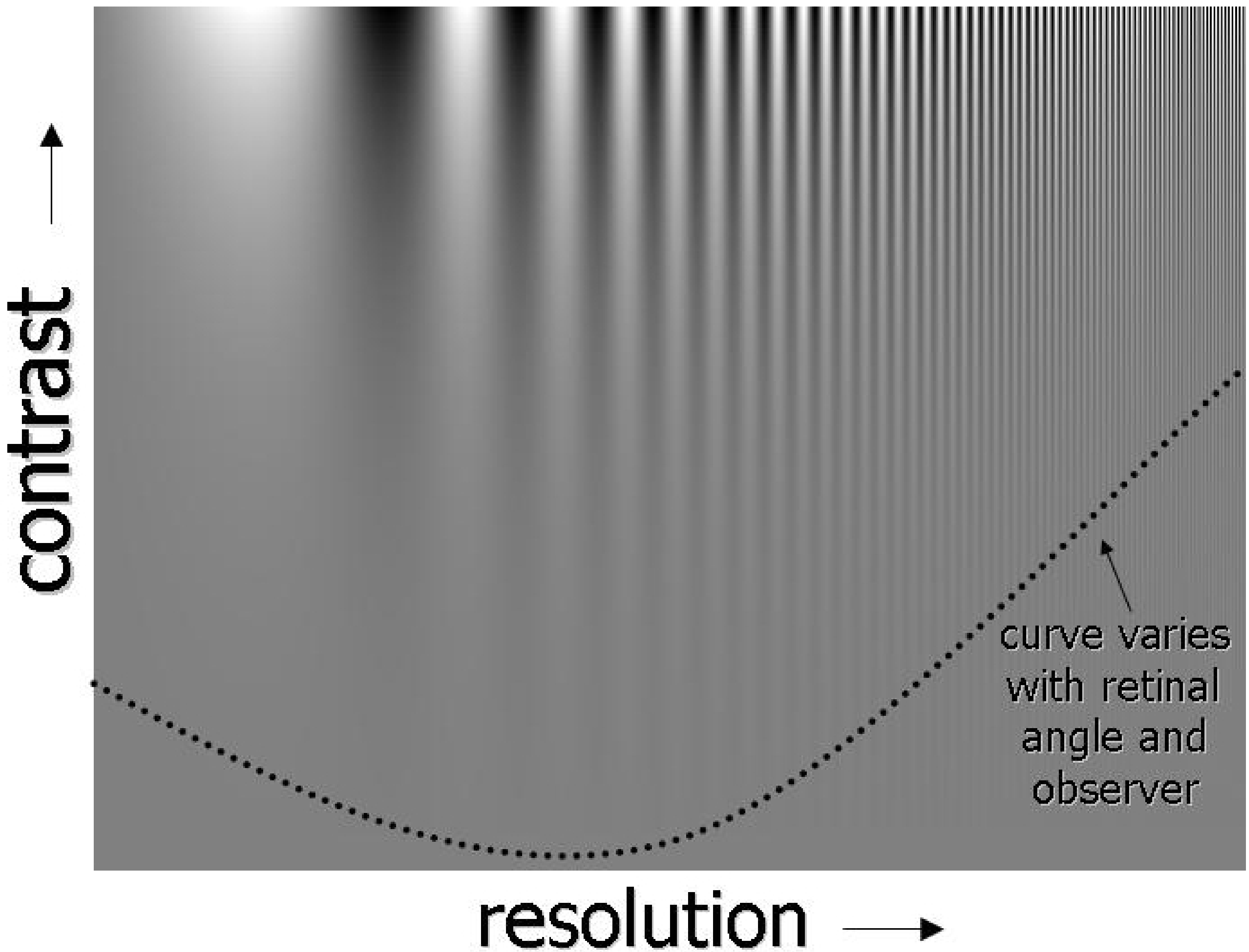
$$OTF = MTF\, e^{iPTF}$$

Modulation transfer function

Amplitude change of an oscillating signal for a given frequency



Eye MTF


contrast
curve varies with retinal angle and observer
resolution

Campbell-Robson curve

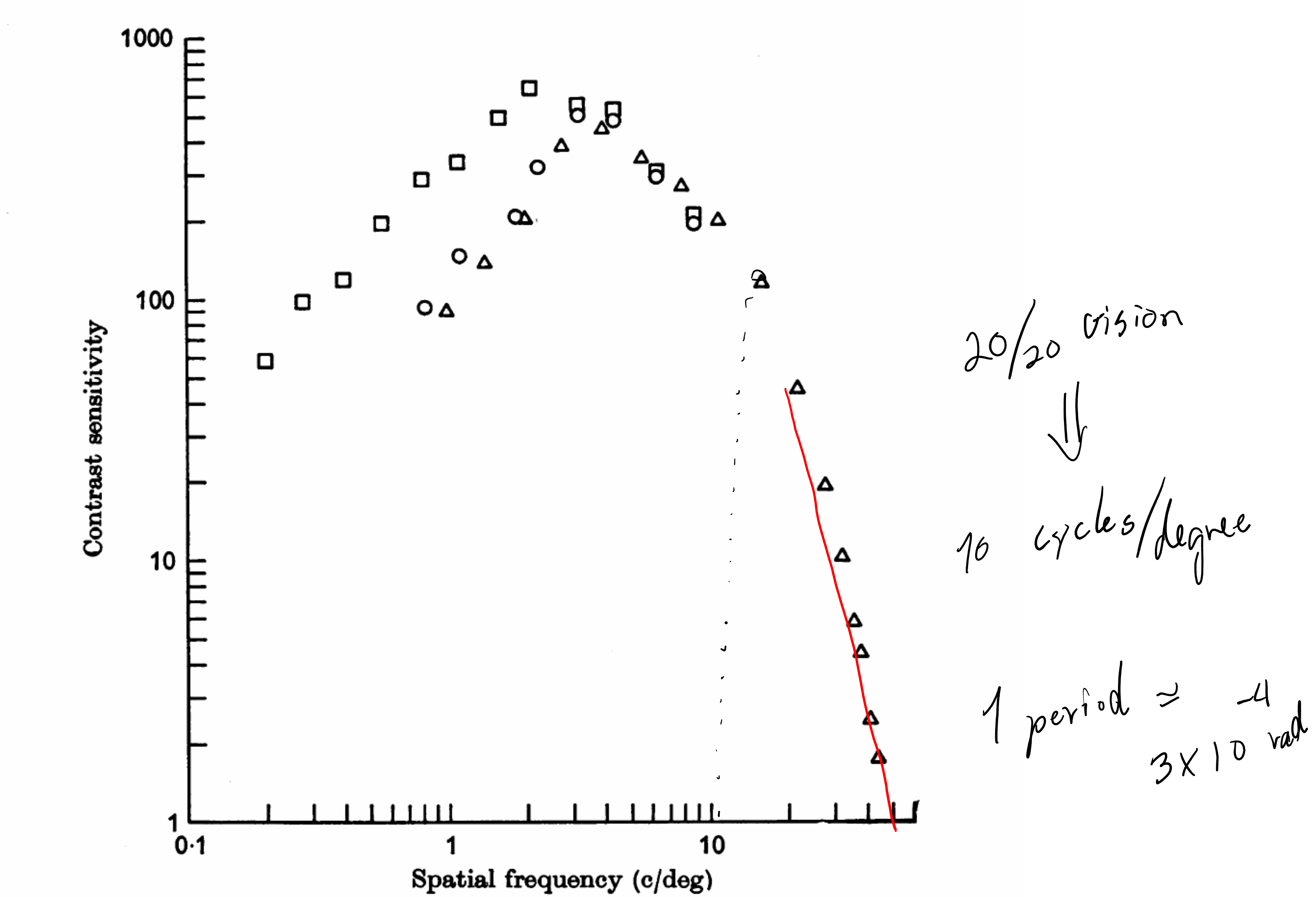
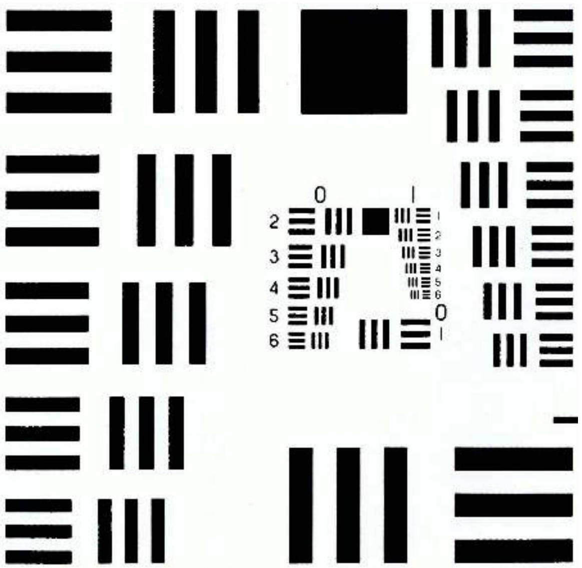
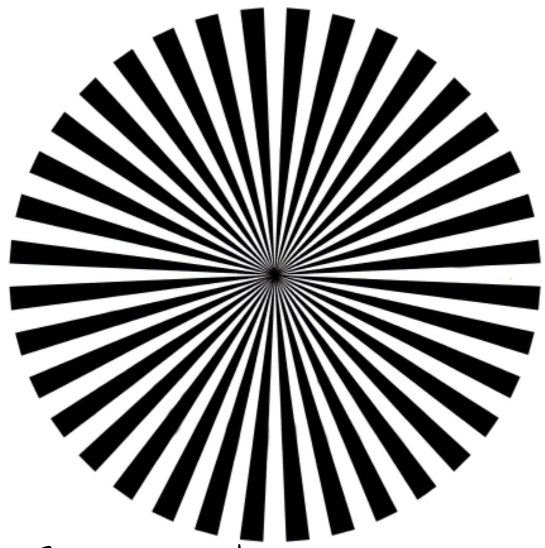


Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture 2° × 2°, △; viewing distance 57 cm, aperture 10° × 10°, □; viewing distance 57 cm, aperture 2° × 2°, ○.

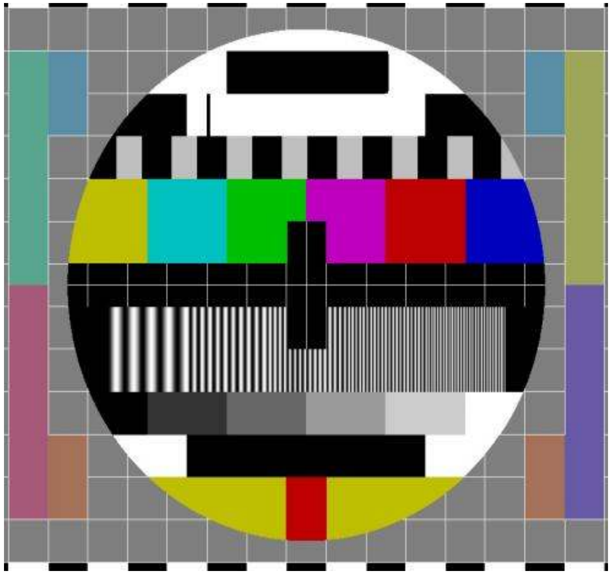
Measurement of MTF



USAF
resolution
target



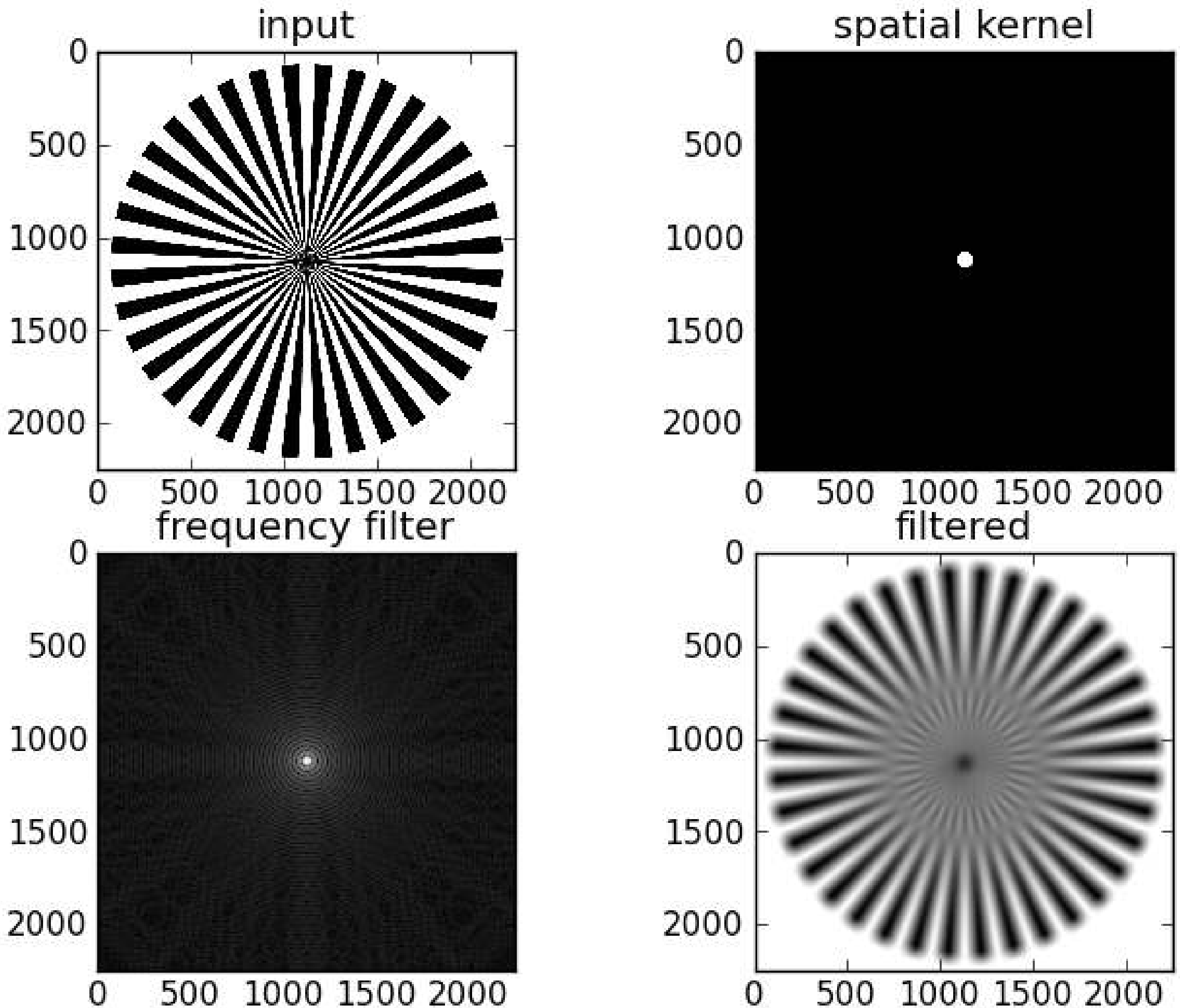
Siemens star



source: http://fotomagazin.de

Phase transfer function

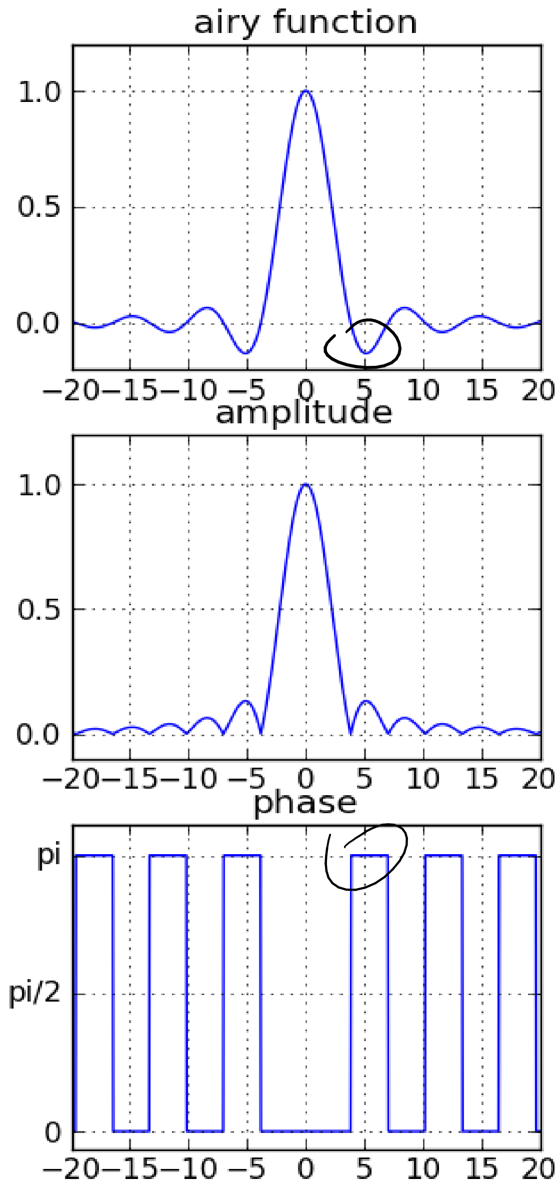
describes how an oscillating signal changes in phase due to system

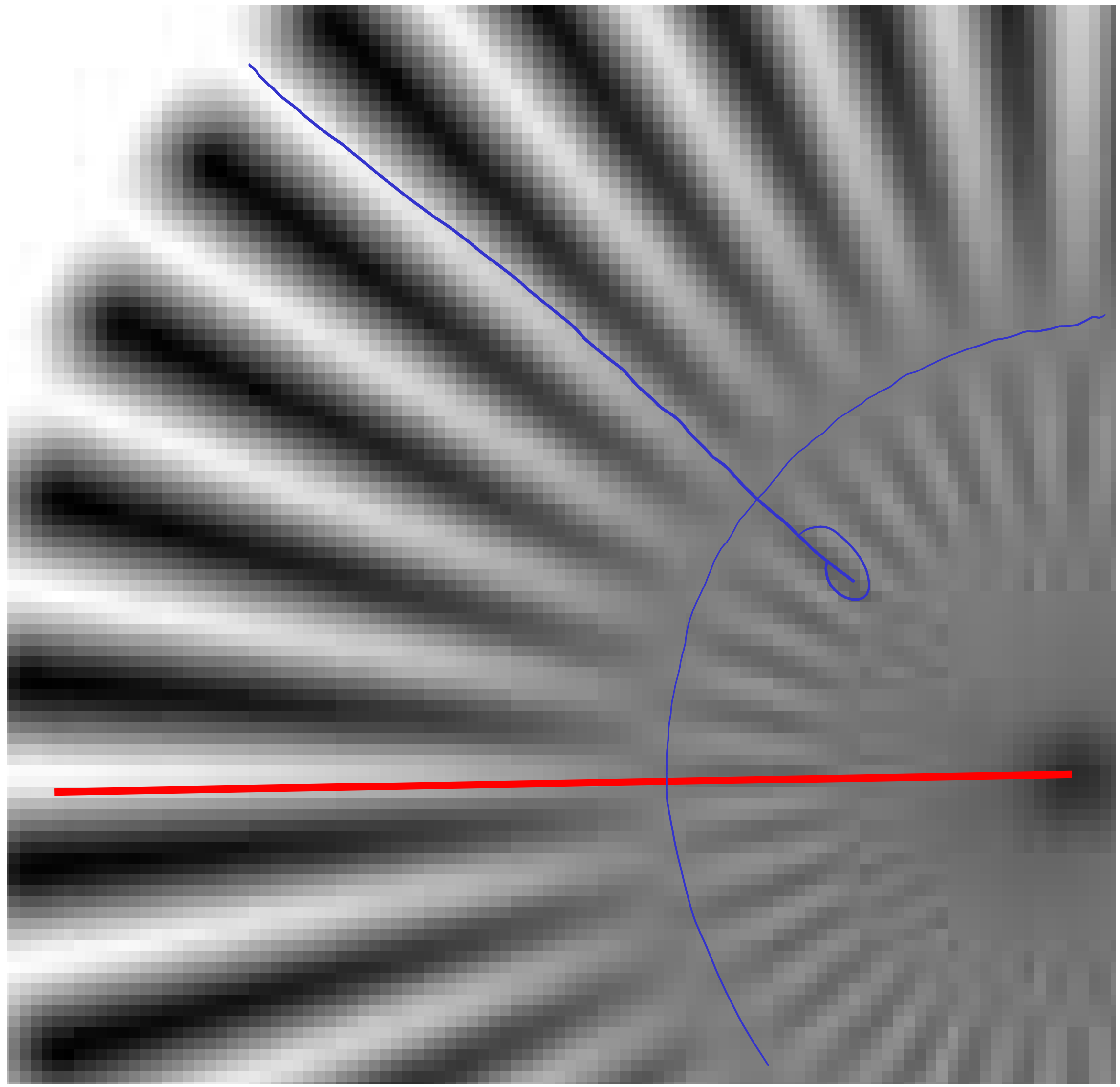


occurs in very specific situations

Phase transfer function

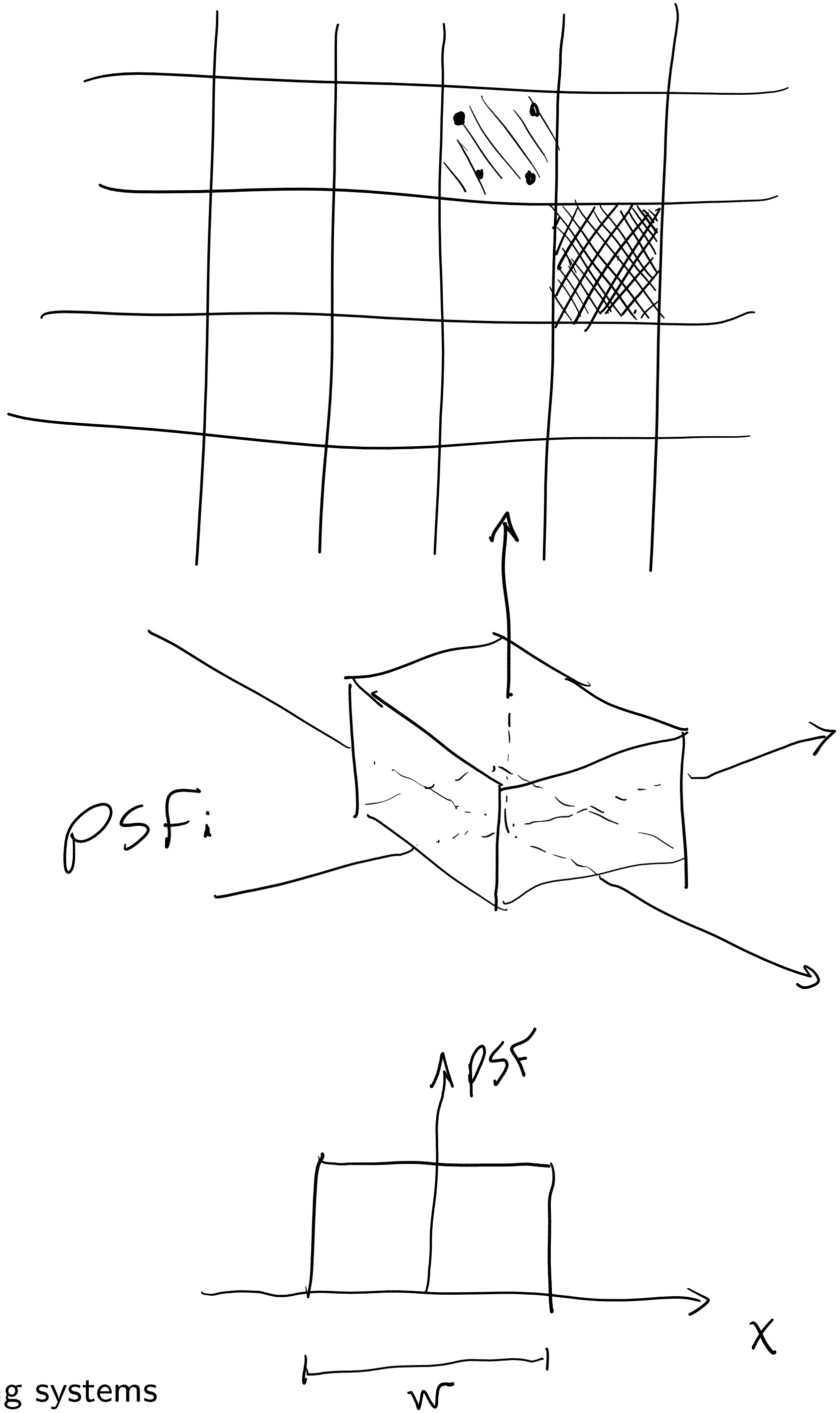
describes how an oscillating signal changes in phase due to system

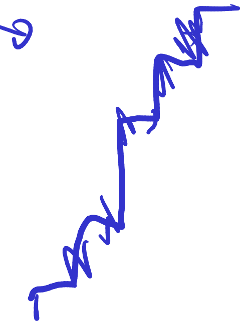


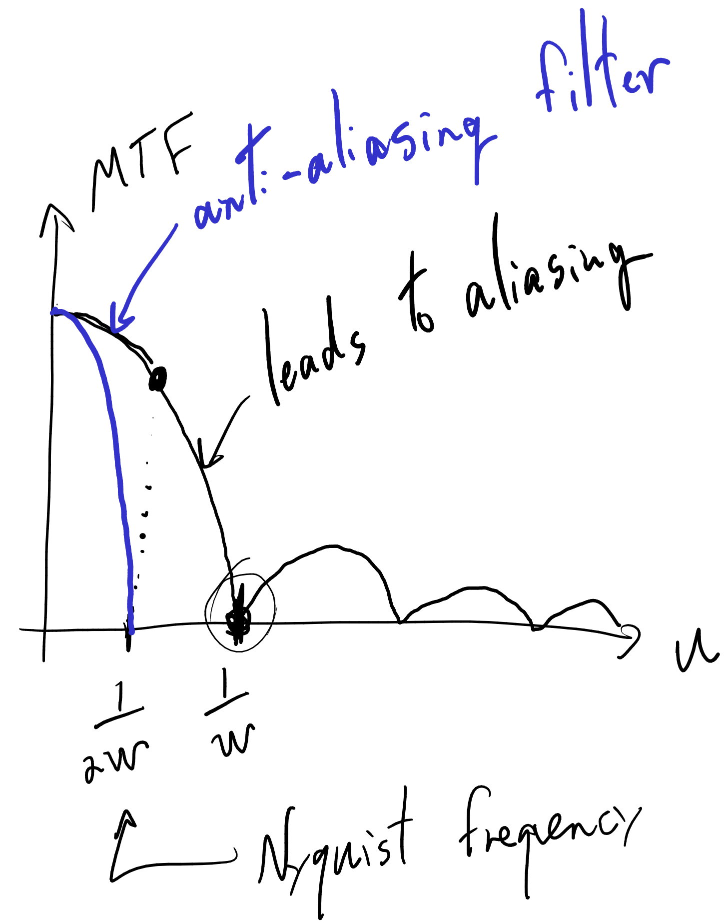


MTF of an ideal pixel

square

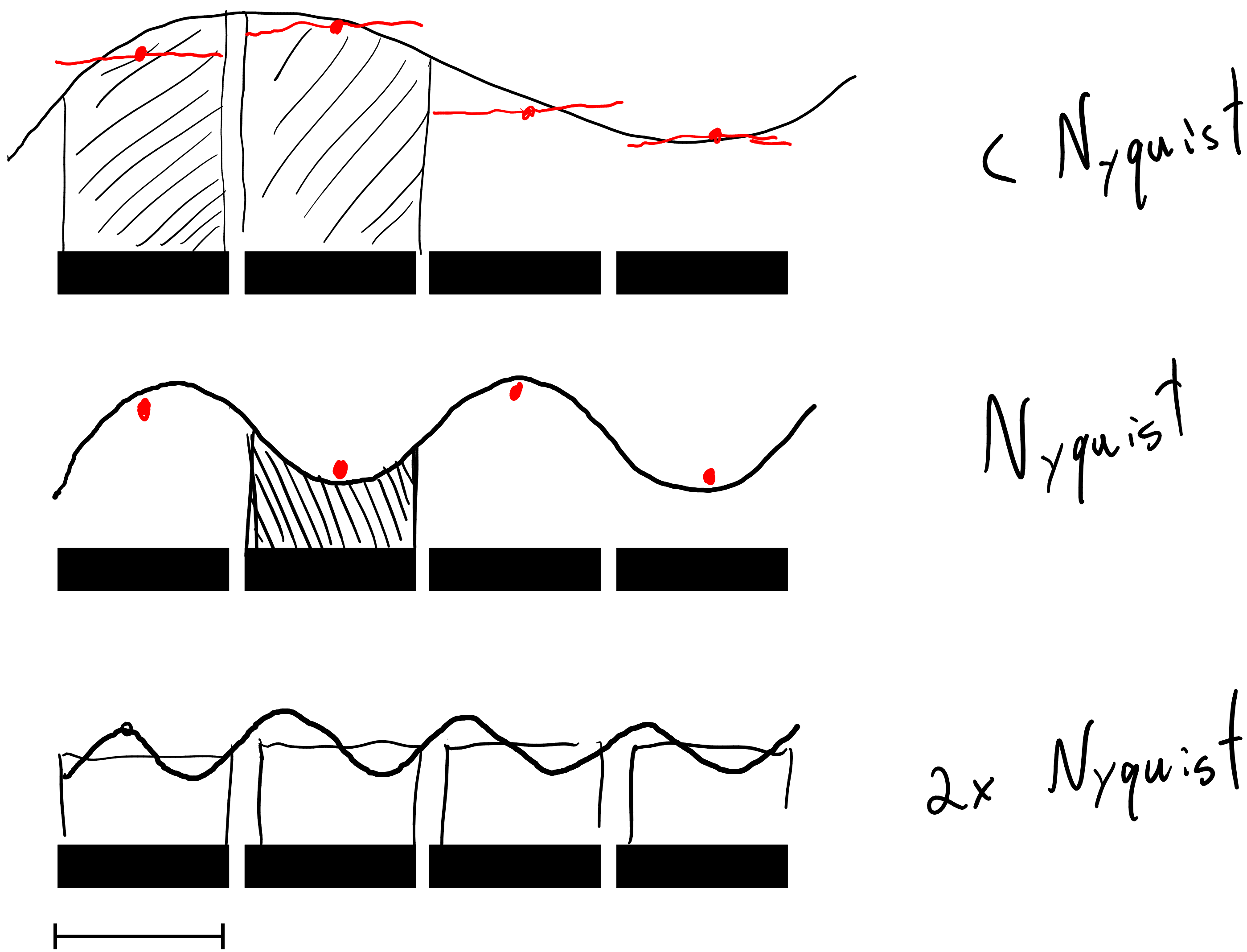






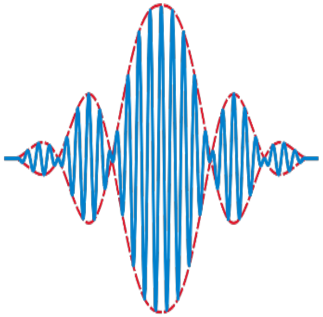
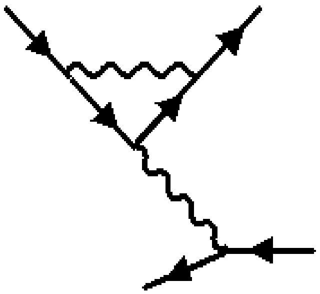
Pixel MTF

Modulation transfer function of a single detector pixel



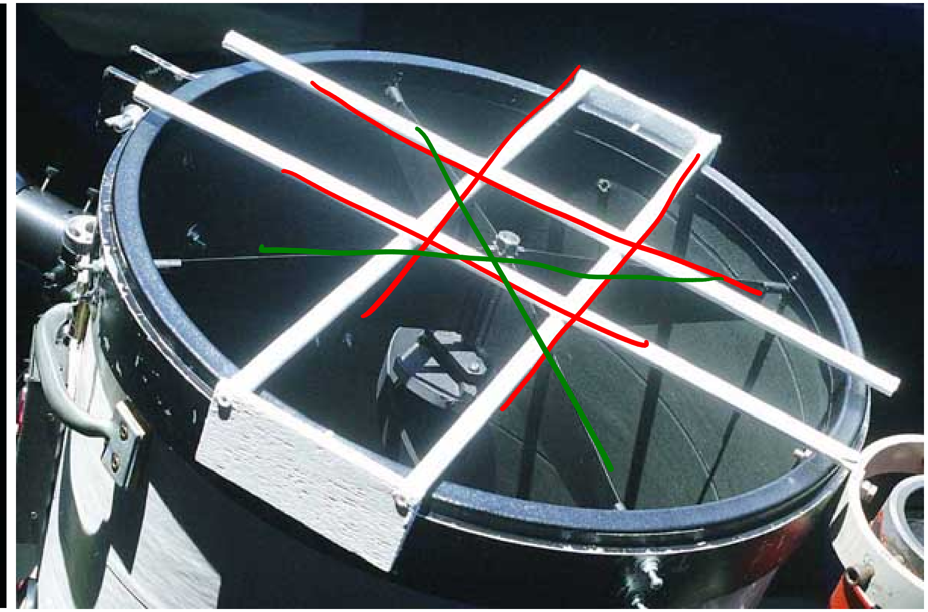
Imaging as a linear filter

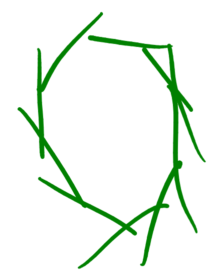
$$\text{Output}(u) = \text{Input}(u) \cdot \underbrace{\text{MTF}_{\text{optics}}(u) \cdot \text{MTF}_{\text{detector}}(u) \left(\cdot \text{MTF}_{\text{algorithm}}(u) \cdot \ldots \right)}_{\text{effective MTF}}$$

PSF examples

- isolated stars are essentially PSFs

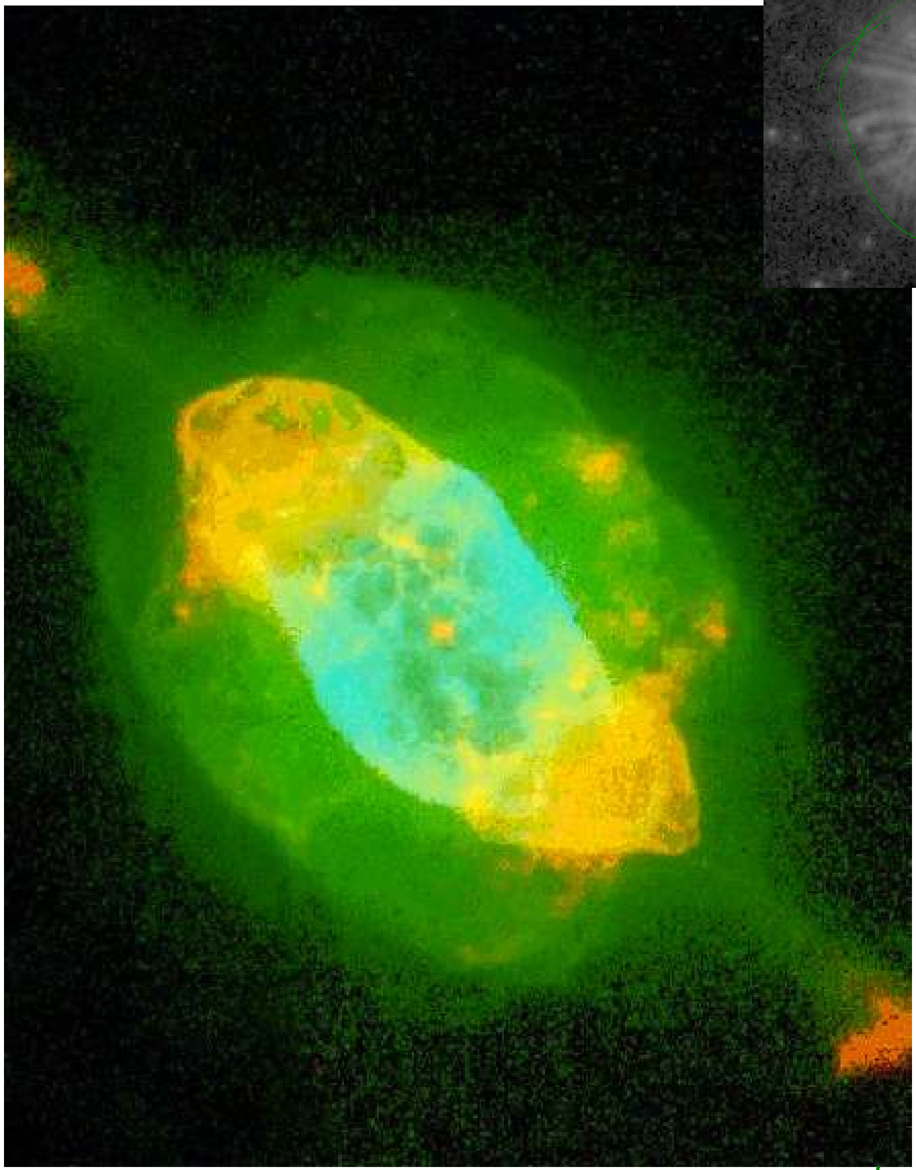


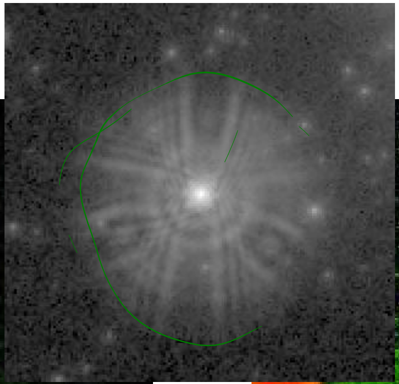


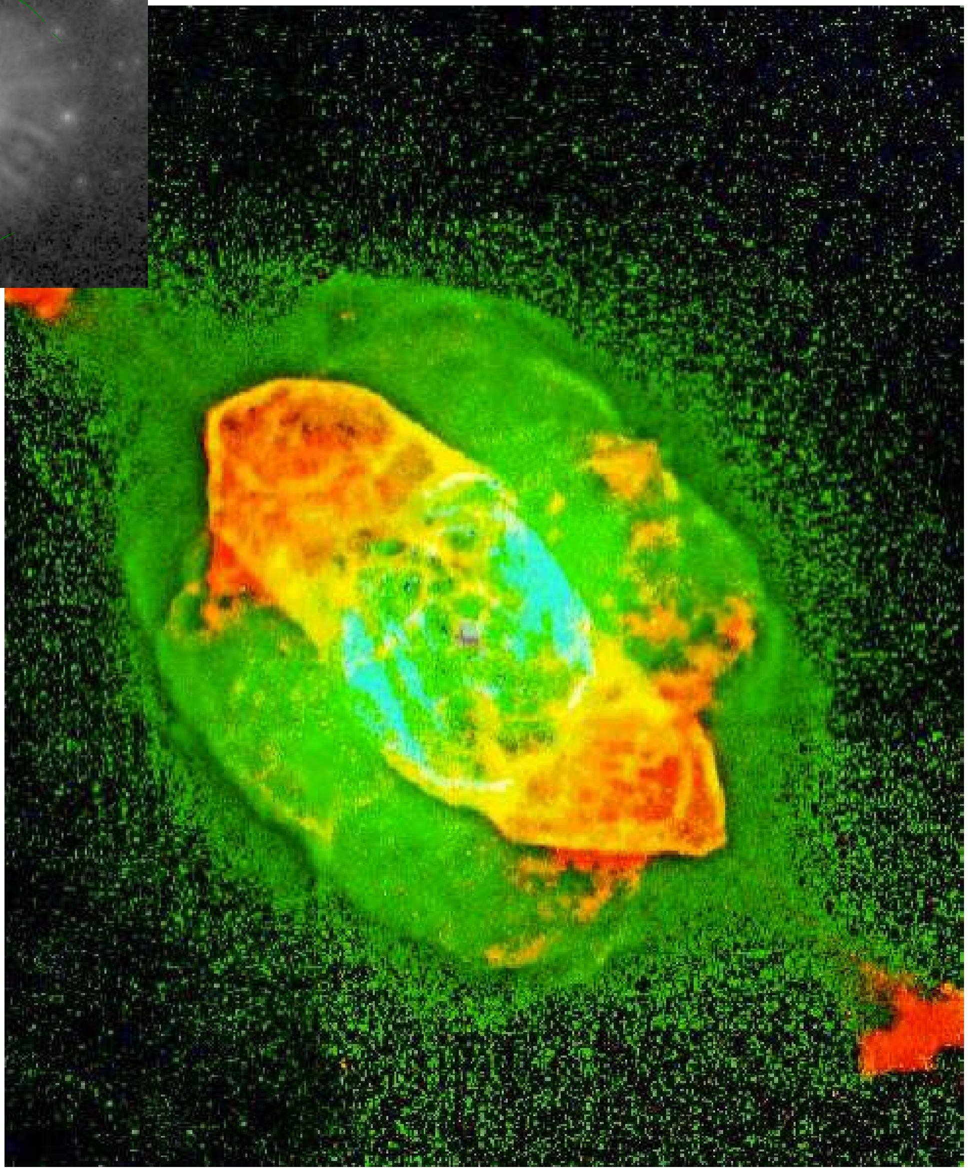
source: www.apod.nasa.gov

PSF examples

Hubble flawed mirror deconvolution (correction for spherical aberration)





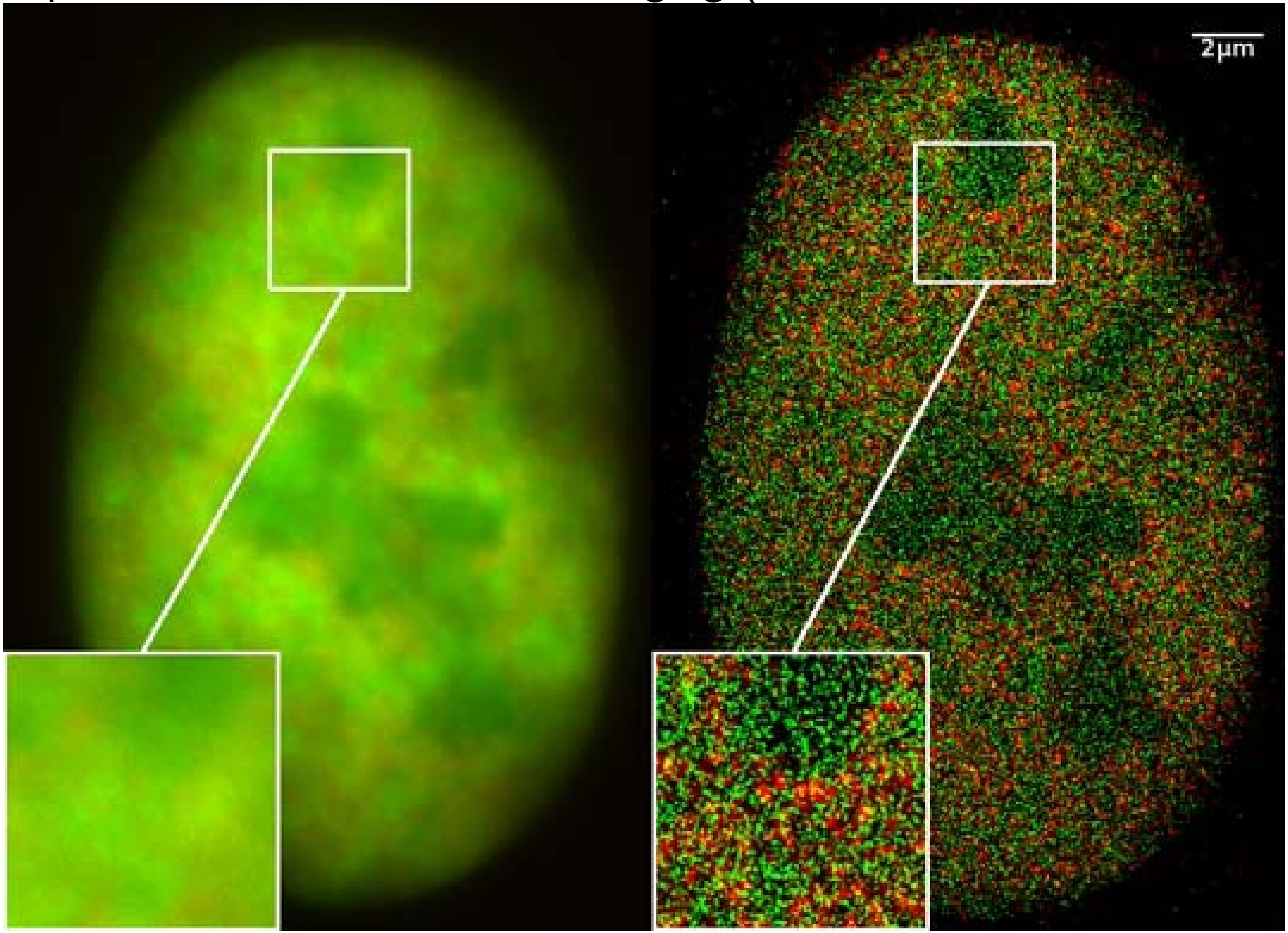


source: www.wikipedia.org

deconvolution

PSF examples

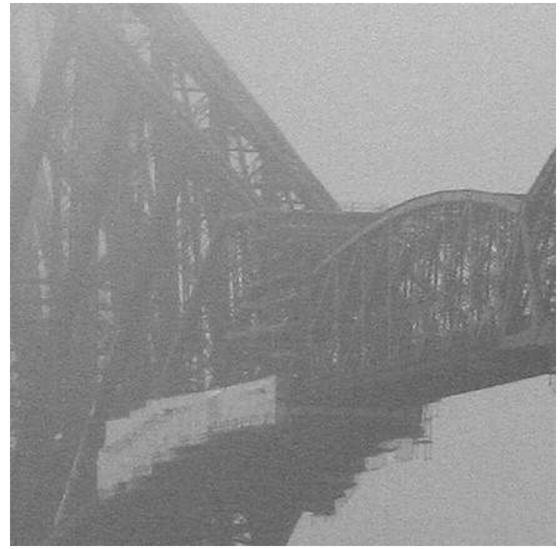
Super-resolution fluorescence imaging (STORM, STED, PALM, ...)

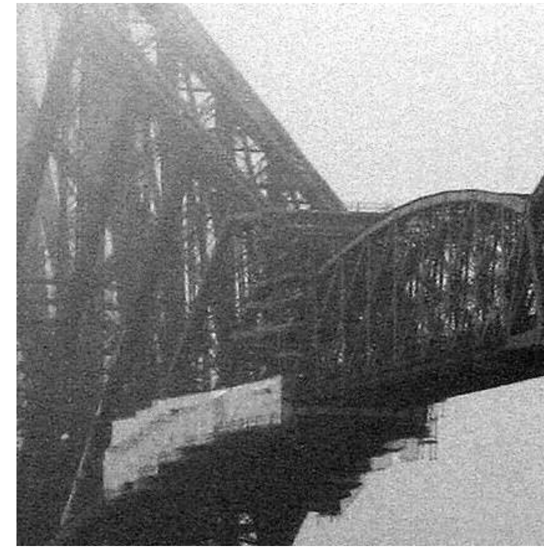


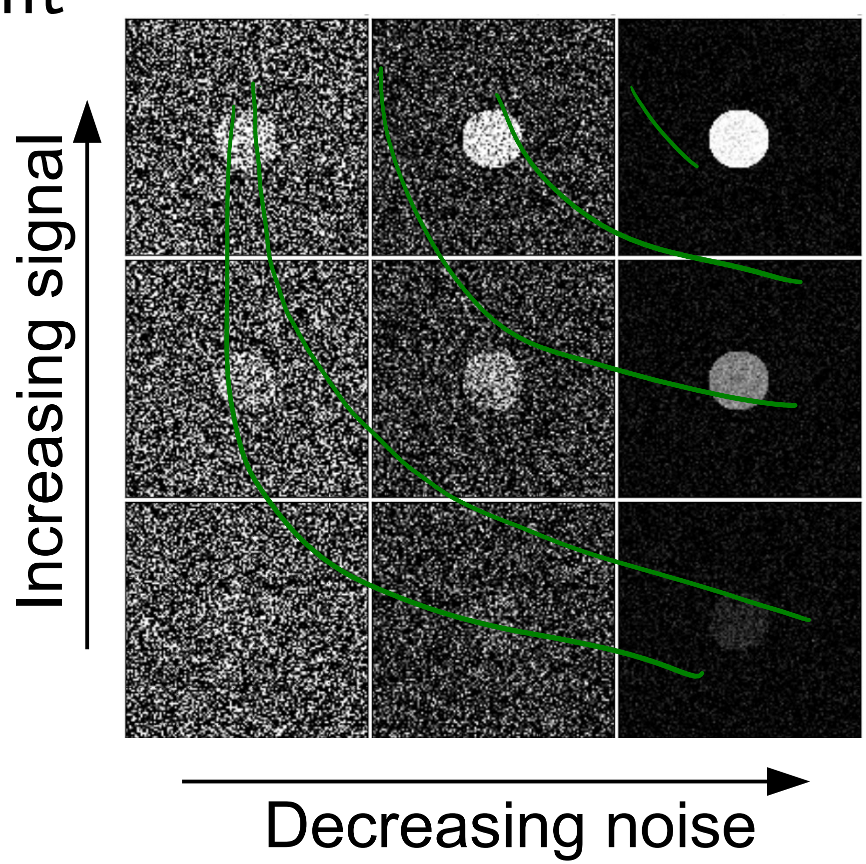
source: www.wikipedia.org

Contrast and noise

- Intensity operation: higher contrast, higher noise
- Contrast-to-noise remains constant







lines of equal contrast to noise

Random variables

- random variable, sample space

  X $\qquad$ Ω $\qquad$ probability of measuring x : $p(x)$

  $$p(\Omega) = 1$$

- probability density function

  $$p(a < x < b) = \int_a^b \rho(x)\,dx$$

  probability density

- expectation value

  $$\langle f \rangle = \int_\Omega f(x)\,\rho(x)\,dx$$

  special cases: $\langle x \rangle = \int_\Omega x\,\rho(x)\,dx$ "μ"

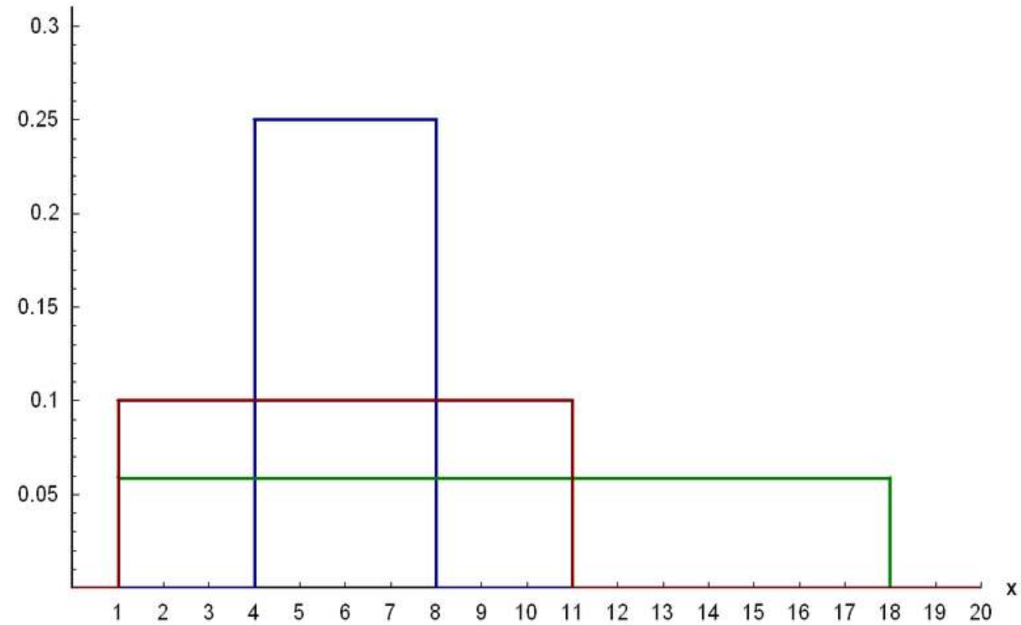
- variance

  $$\mathrm{var}(x) = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

Uniform distribution

- probability density function

$$\rho(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



- expectation value

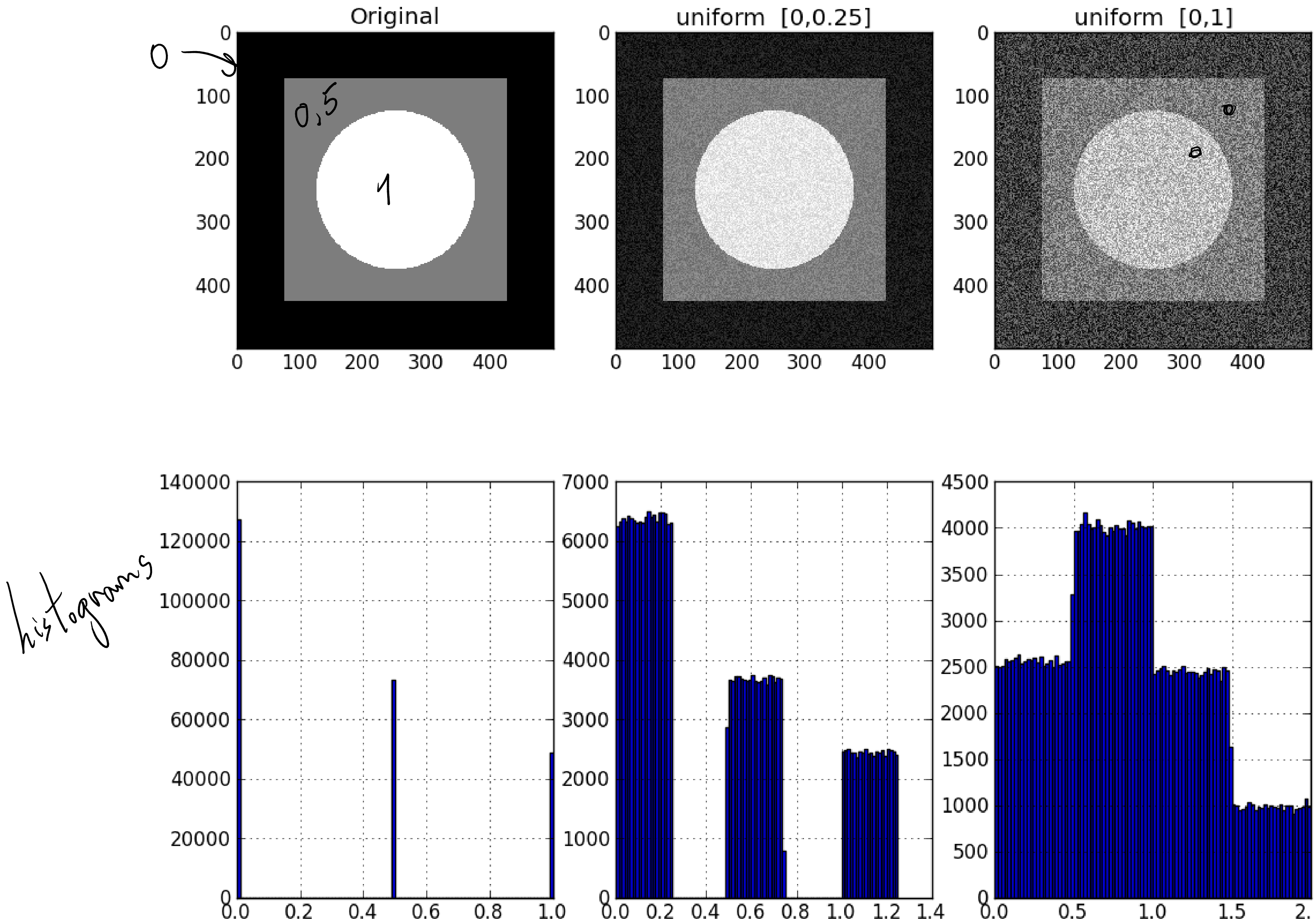
$$\langle x \rangle = \frac{1}{2}(a+b)$$

- variance

$$\operatorname{var} x = \frac{(b-a)^2}{12}$$

- occurrence $\longrightarrow$ not very common in imaging, but useful to build other distributions

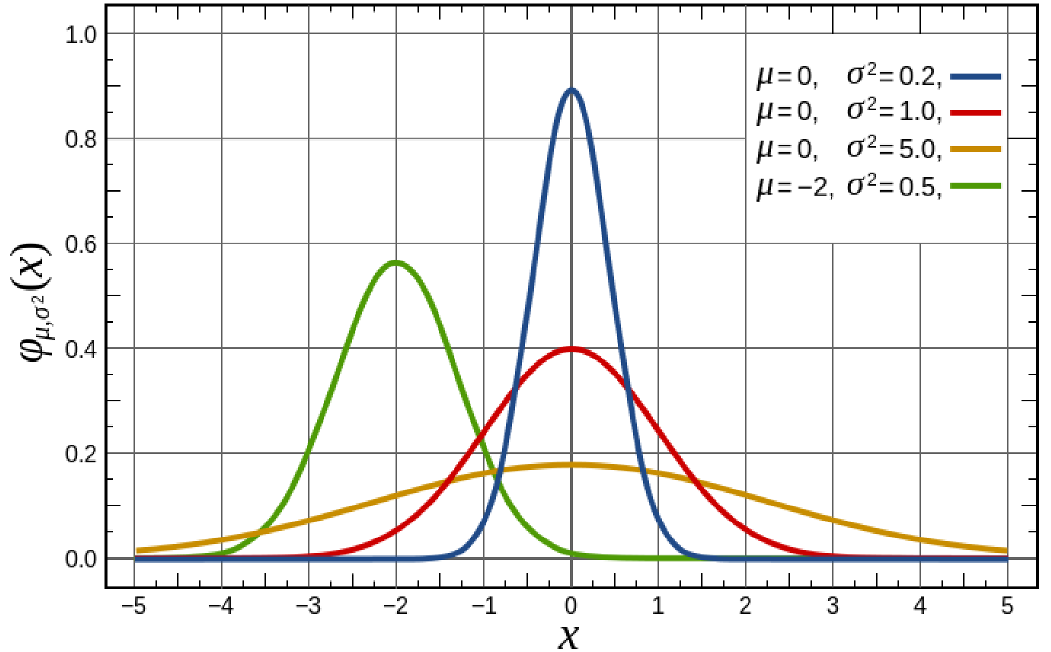
Uniform distribution



Gaussian distribution

- probability density function

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



- expectation value

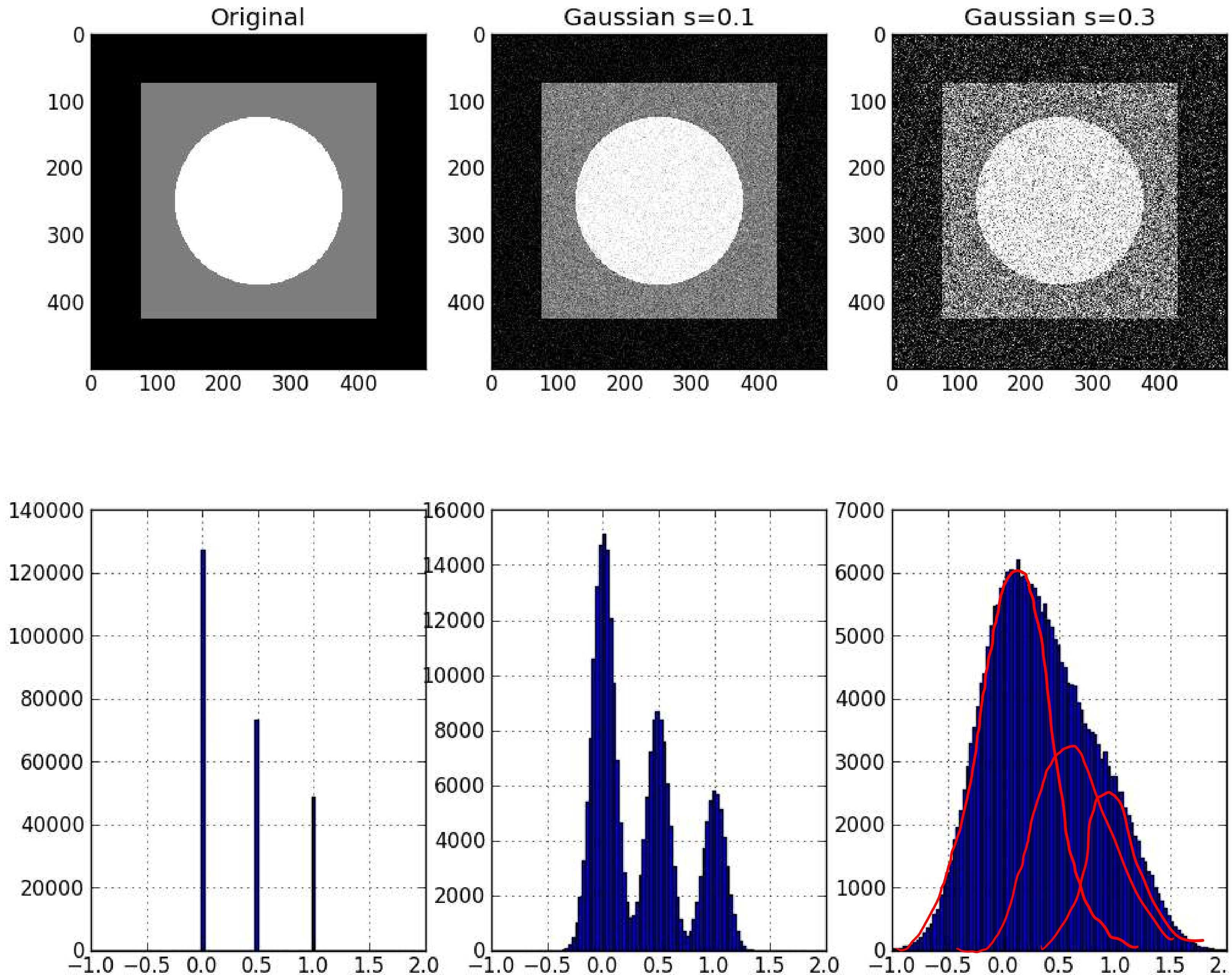
$$\langle x \rangle = \mu$$

- variance

$$\mathrm{var}\, x = \sigma^2$$

- occurrence   very common

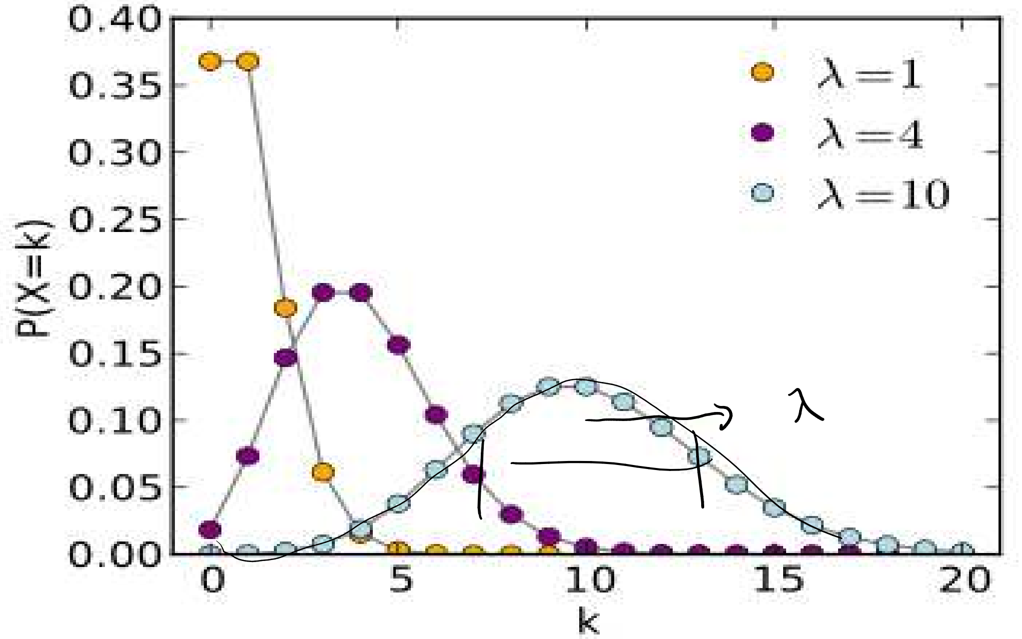
Gaussian distribution



Poisson distribution

- probability mass function

$$p(n) = \frac{1}{n!} \lambda^n e^{-\lambda}$$



- expectation value

$$\langle n \rangle = \lambda$$

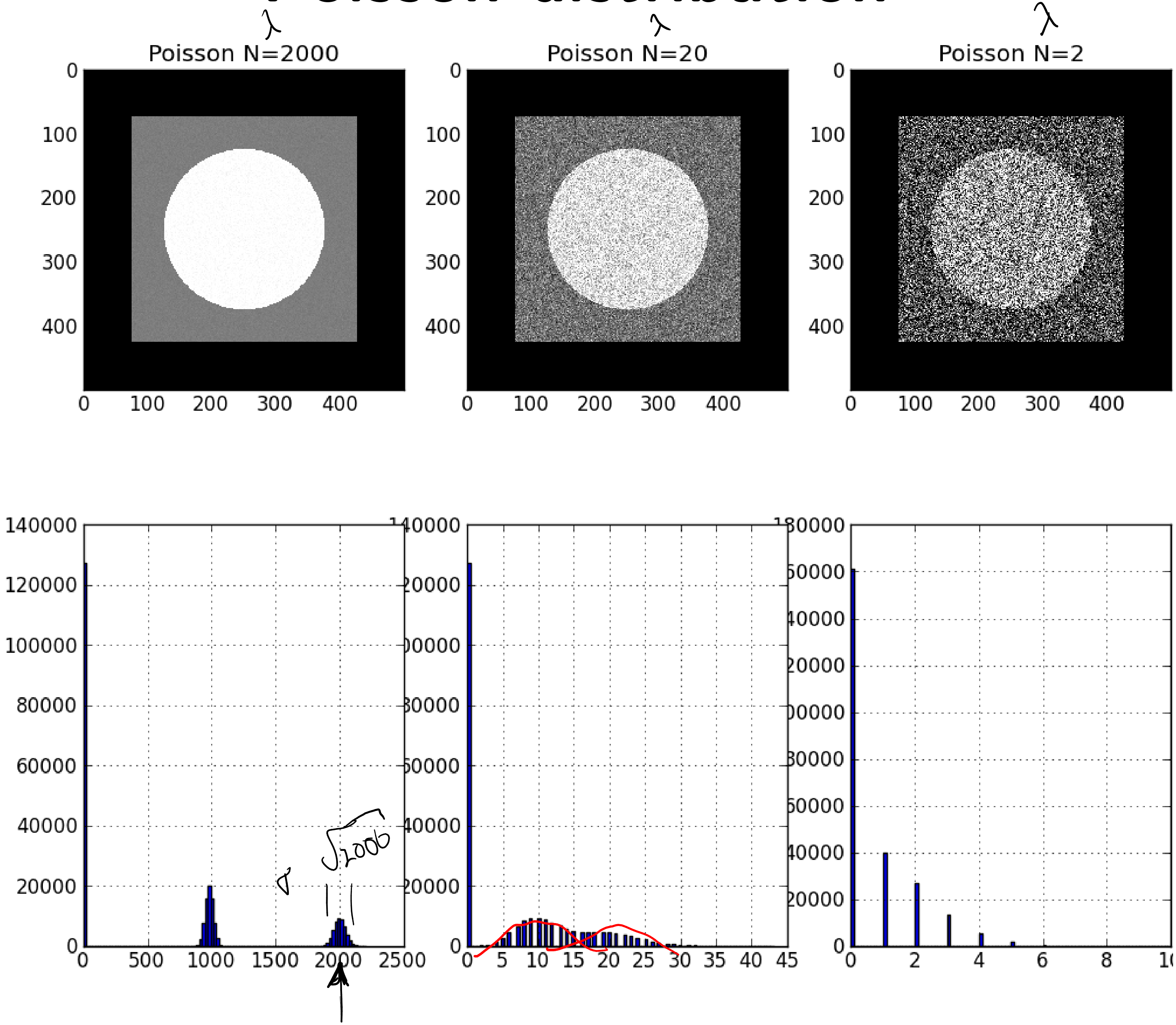
- variance

$$\operatorname{var} n = \lambda$$

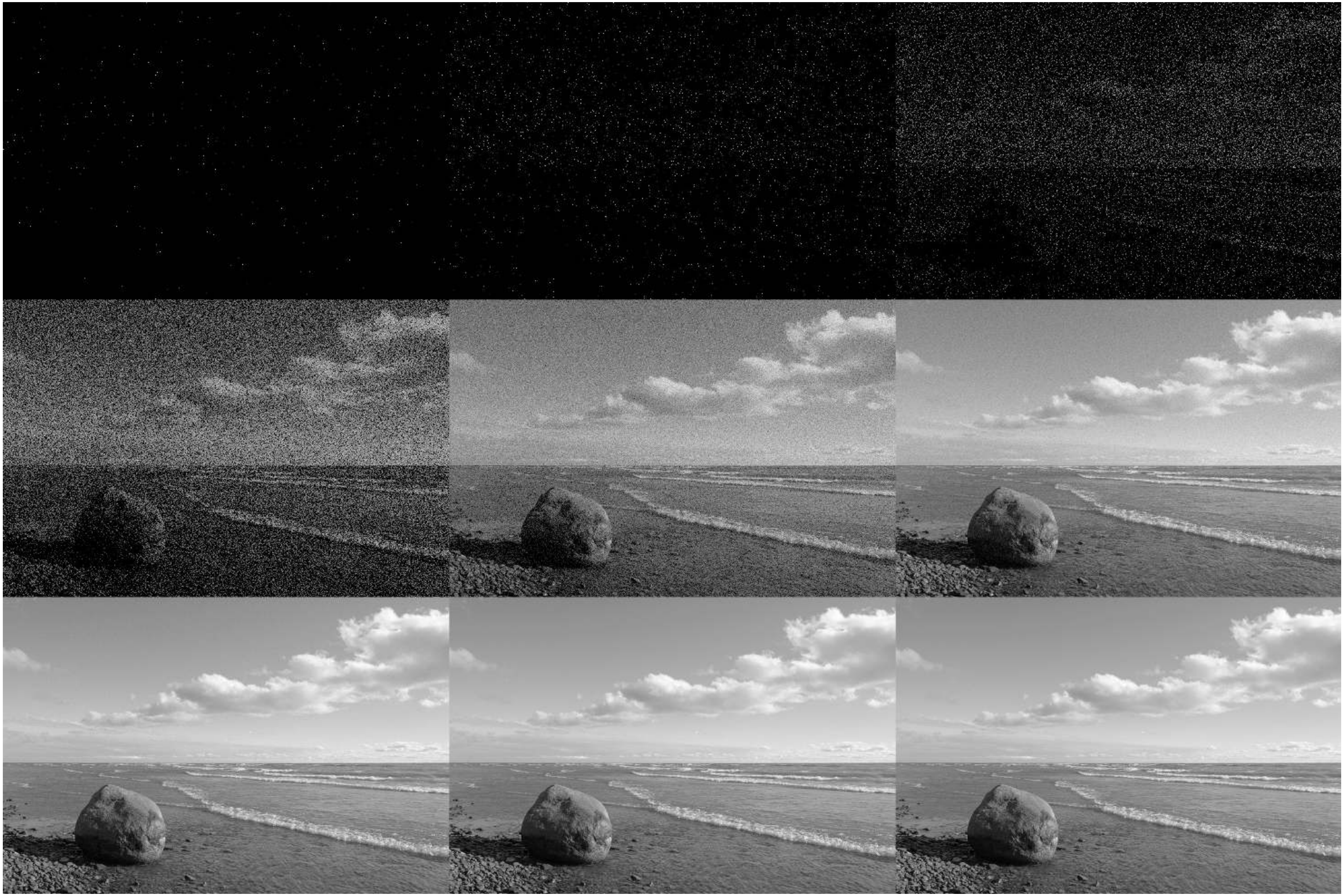
- occurrence   counting process   (photons, electrons, ...)

"shot noise"

Poisson distribution

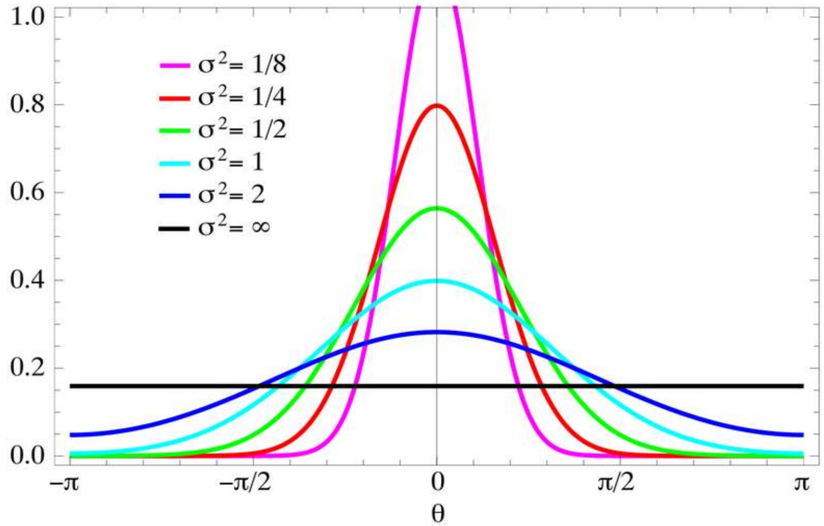


Poisson distribution

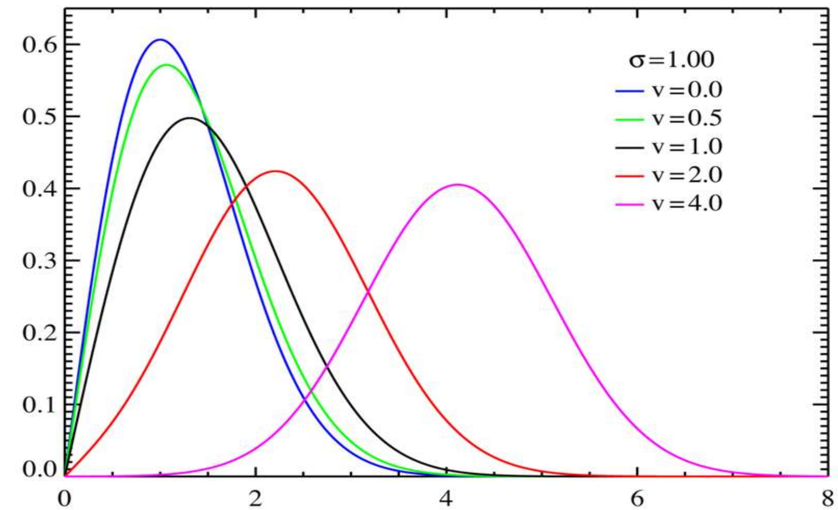


Many other distributions

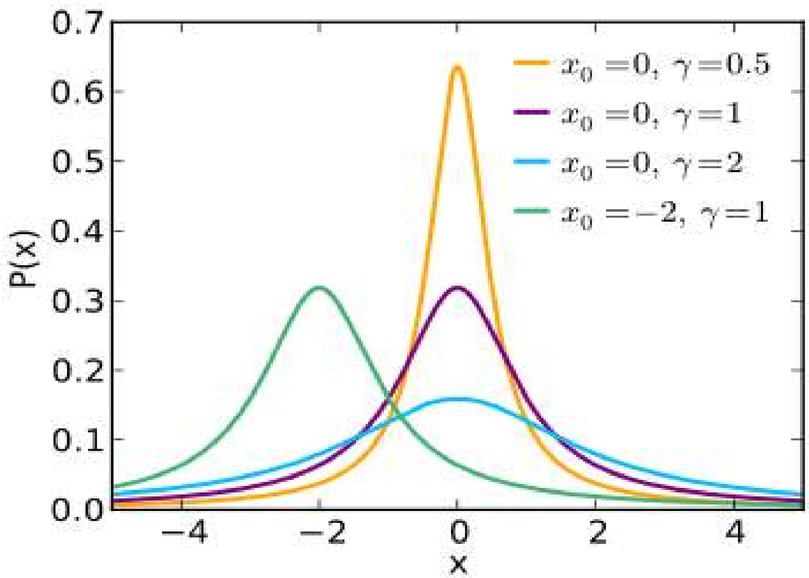
Wrapped normal distribution



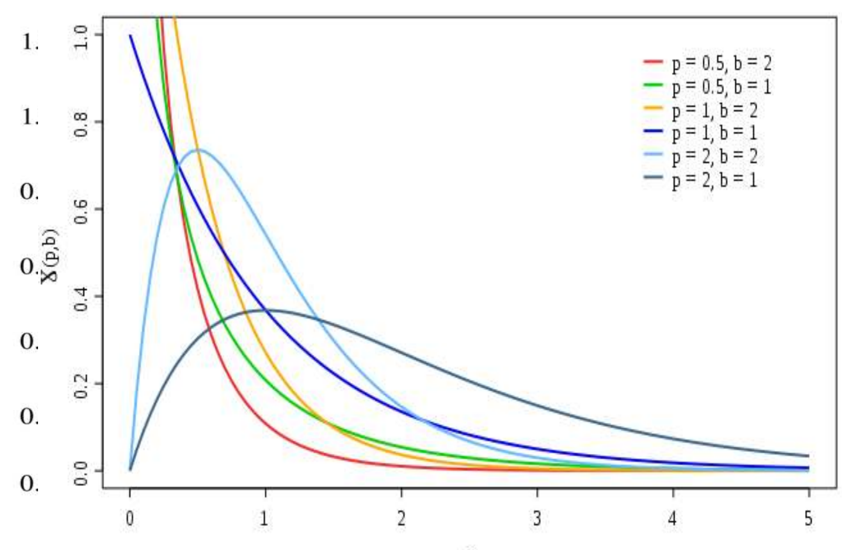
Rice distribution



Lorentz distribution



Gamma distribution

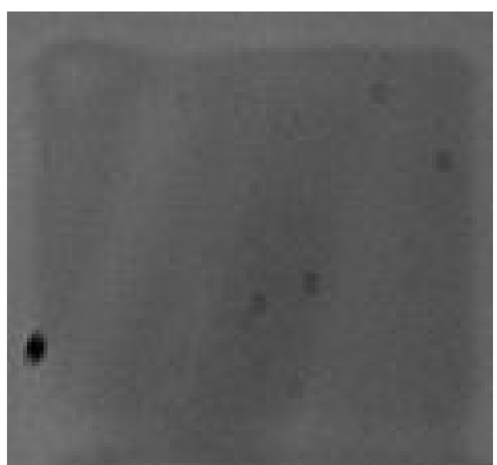


Detector noise (CCD)

- Various sources:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gauss)
  - many other imperfections ...
- dark frame measures detector noise, hot pixels, dead pixels
- bright frame measures gain differences and imperfections (dust, etc)

dark frame

bright frame



raw image

calibrated image

source: H. Raab, Johannes-Kepler-Observatory, Linz

Correlation & Convolution

* Convolution: $f * g = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx'$

* Convolution theorem: $\mathcal{F}\{f * g\} = F \cdot G$

* Correlation $f \circledast g = \int_{-\infty}^{\infty} f^*(x')\, g(x+x')\, dx'$

$$f \circledast g = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} F^*(u)\, e^{-2\pi i u x'} du \int_{-\infty}^{\infty} G(u')\, e^{2\pi i u'(x+x')} du'$$

$$= \iint_{-\infty}^{\infty} du\, du'\, F^*(u)\, G(u')\, e^{2\pi i u' x} \underbrace{\int_{-\infty}^{\infty} dx'\, e^{2\pi i x'(u'-u)}}_{\delta(u'-u)}$$

$$= \int_{-\infty}^{\infty} du\, F^*(u)\, G(u)\, e^{2\pi i u x} = \mathcal{F}^{-1}\{F^* G\}$$

Noise power spectrum

- power spectrum of pure noise image

$$NPS = \langle | \mathcal{F}\{ n(x,y) \} |^2 \rangle$$

ensemble average

random variable (multivariate)

$$N(u,v) = \mathcal{F}\{ n(x,y) \}$$

- connection to auto-correlation

$$|N(u,v)|^2 = N^*(u,v)\, N(u,v)$$

$$\mathcal{F}^{-1}\{ NPS \} = \langle \mathcal{F}^{-1}\{ |N(u,v)|^2 \} \rangle = \langle n \circledast n \rangle$$

autocorrelation of n

Measuring NPS

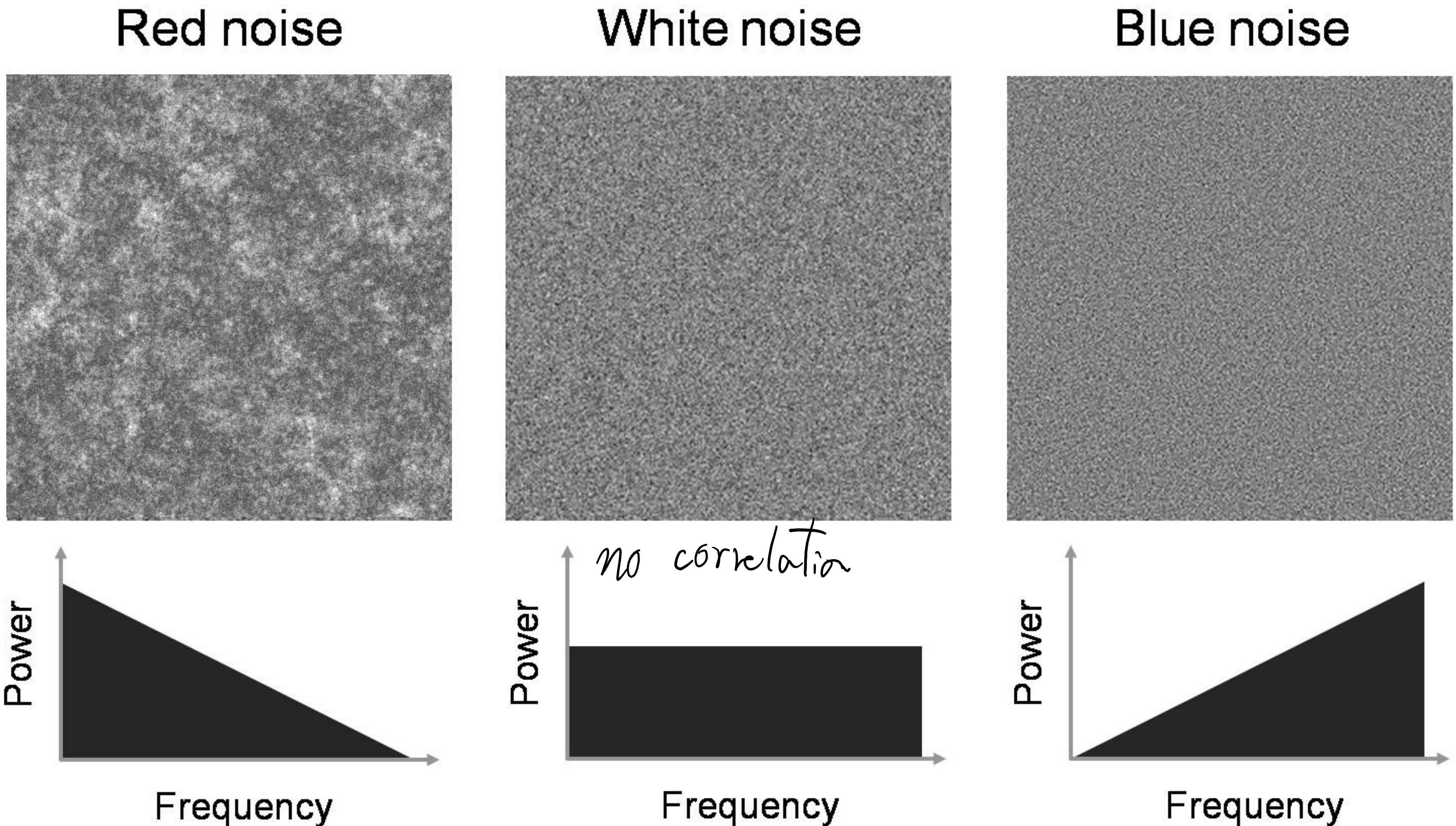
1) measure multiple realizations of random variable $n(x,y)$
(sample the ensemble)

2) $N_i(u,v) = \mathcal{F}\{n_i(x,y)\}$

3) $\langle |N(u,v)|^2 \rangle = \frac{1}{M} \sum_i |N_i(u,v)|^2 \rightarrow$ approximation for NPS

4) $\mathcal{F}^{-1}\{NPS\}$: estimate of noise autocorrelation

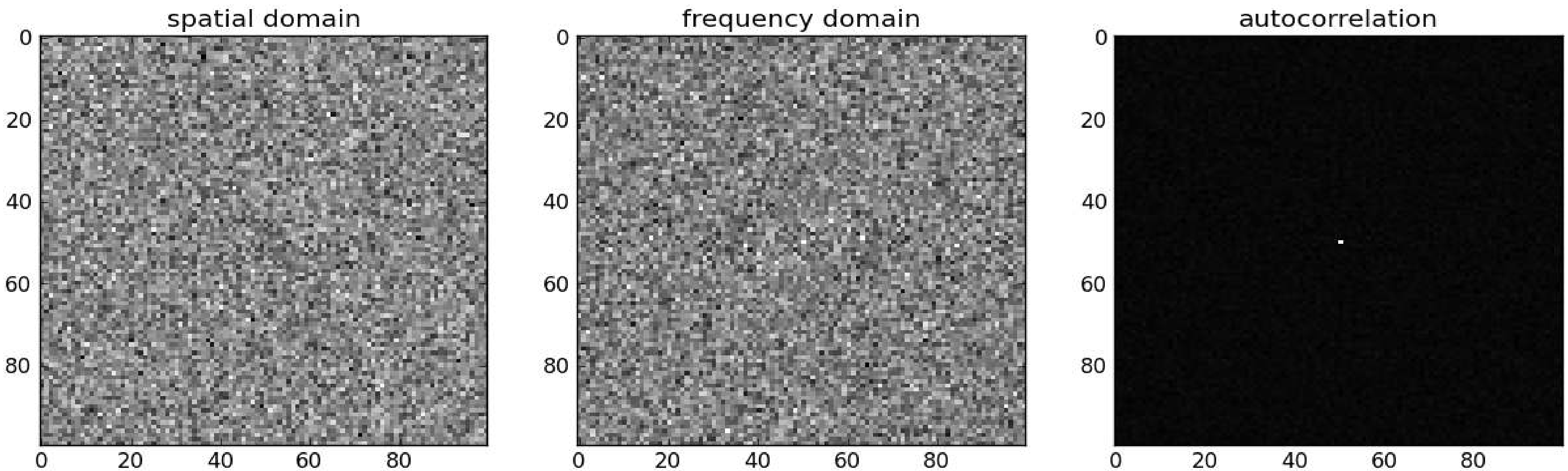
Noise power spectrum



radial averages

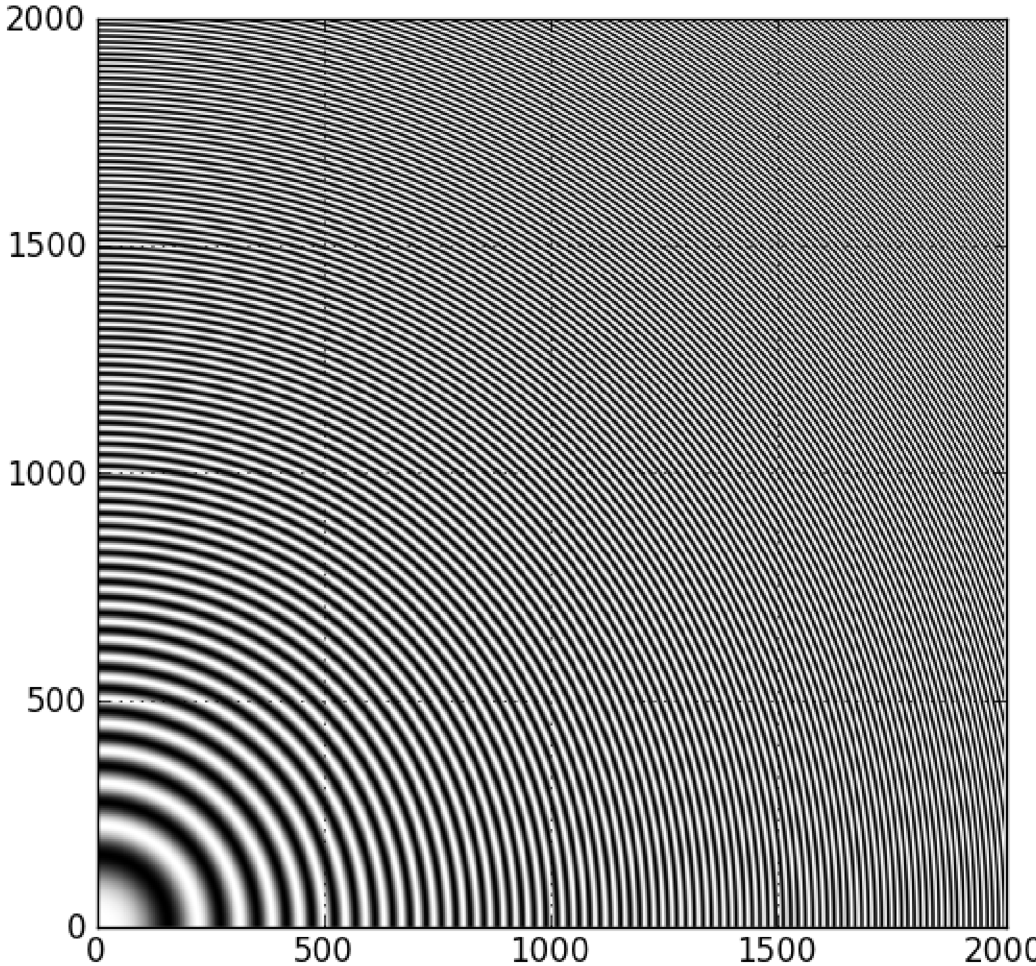
source: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

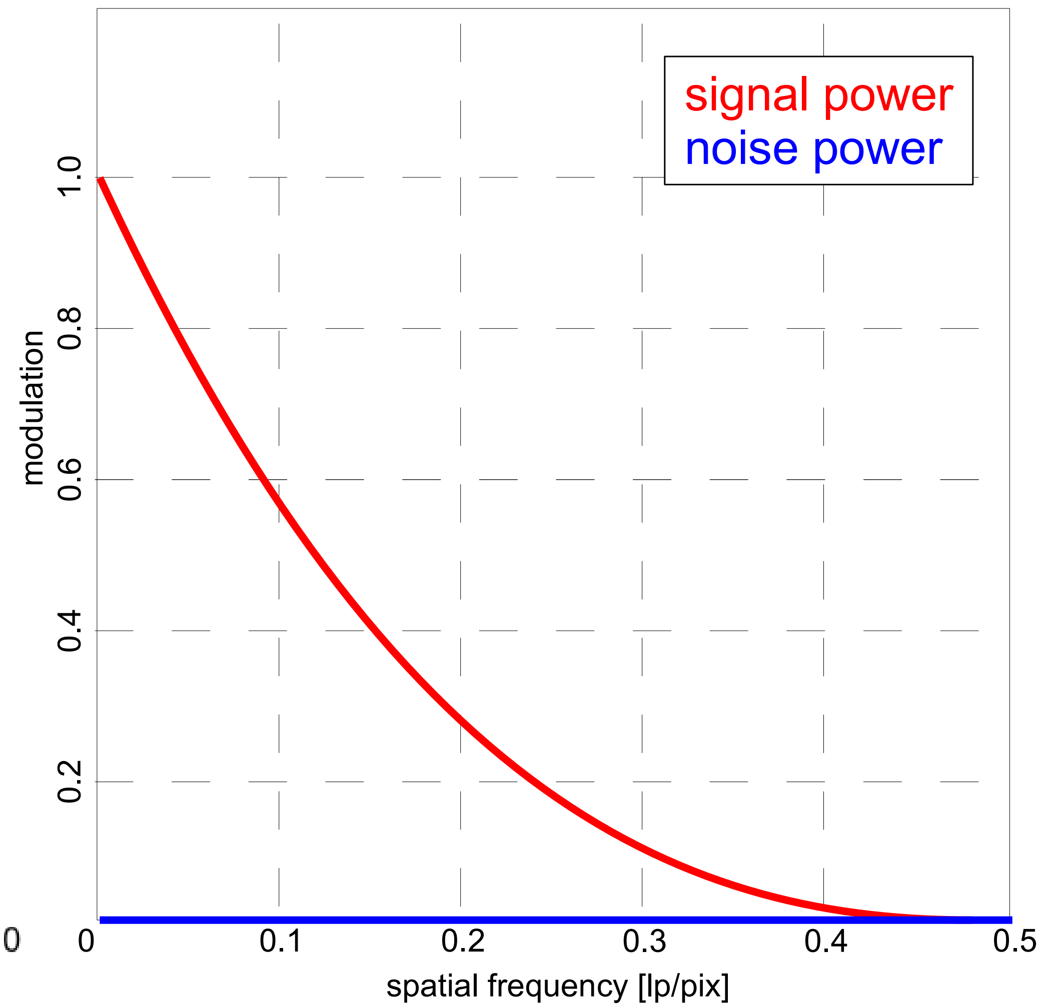
White noise



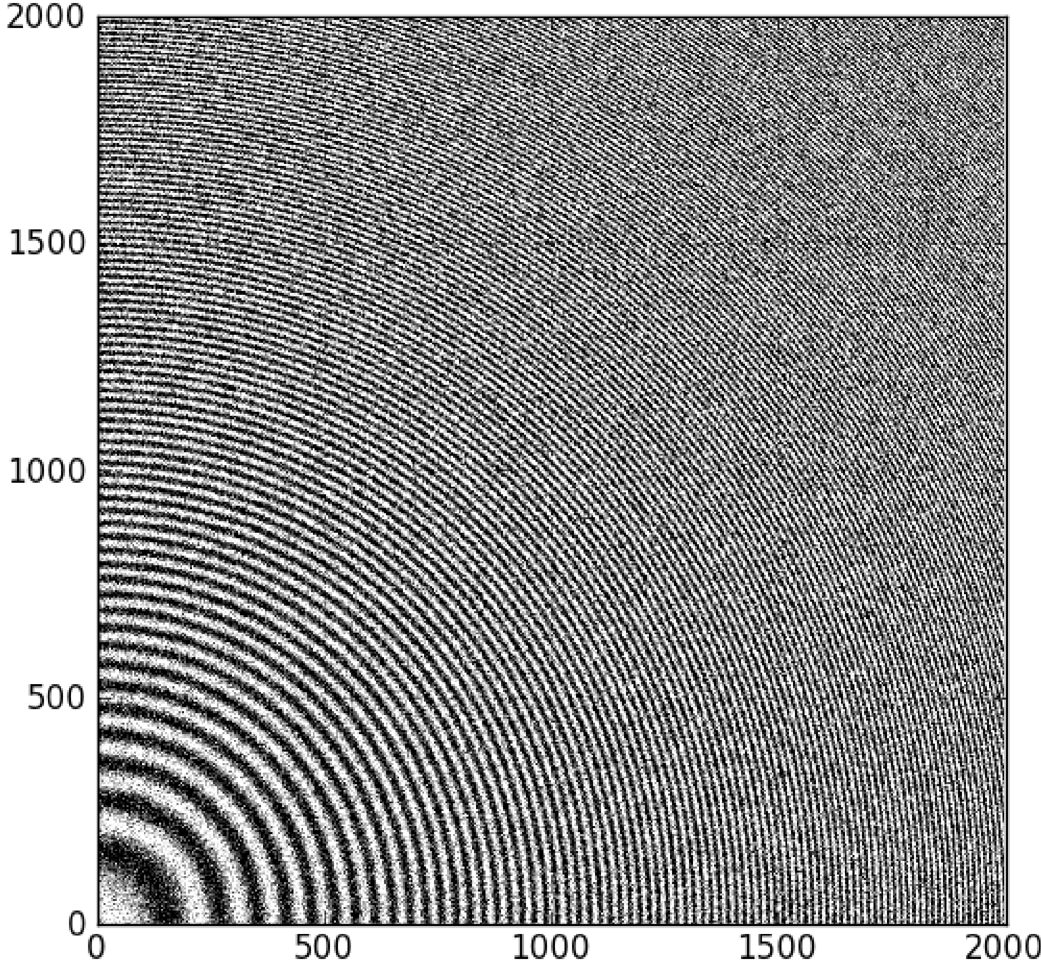
- white noise in spatial domain equals white noise in frequency domain
- white noise is perfectly uncorrelated
- all other types of noise are correlated to some degree
- white noise is an idealization

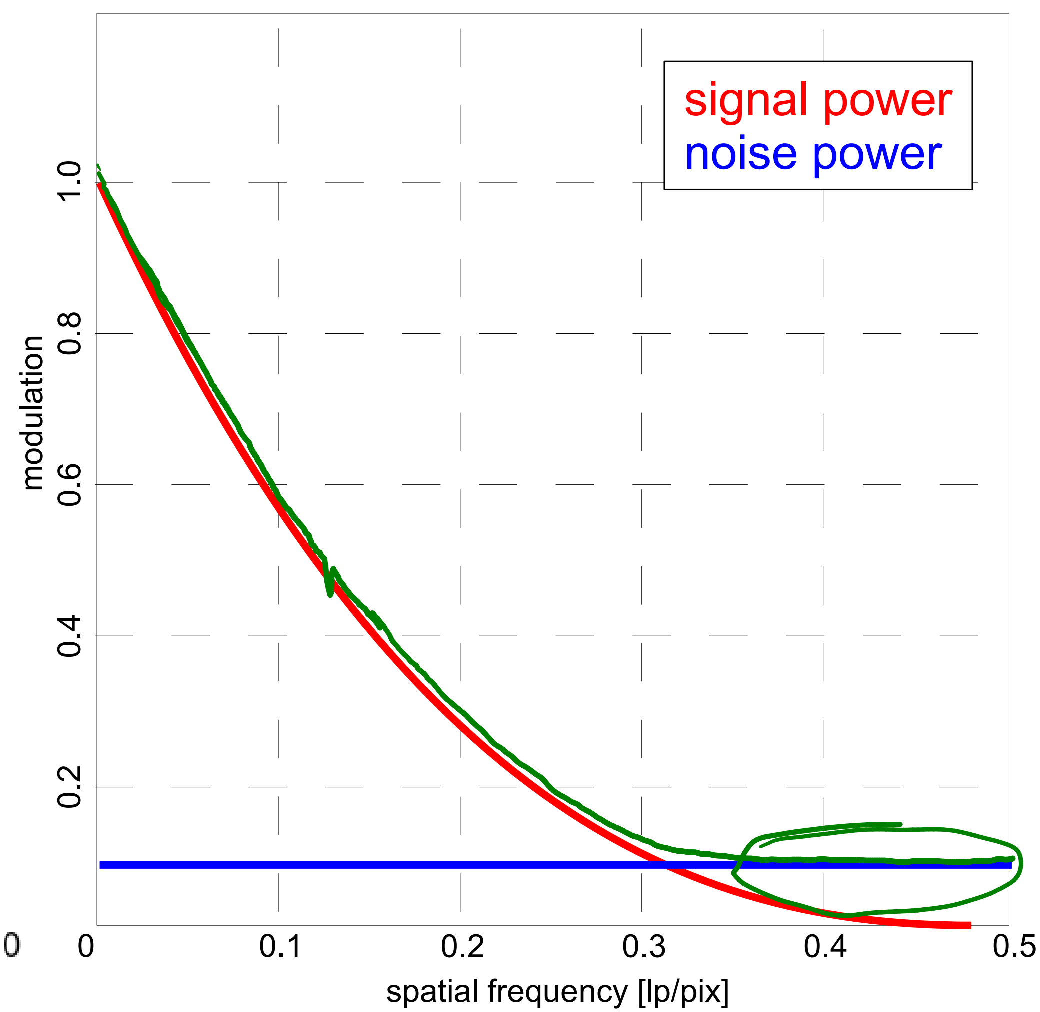
Signal power vs. noise power



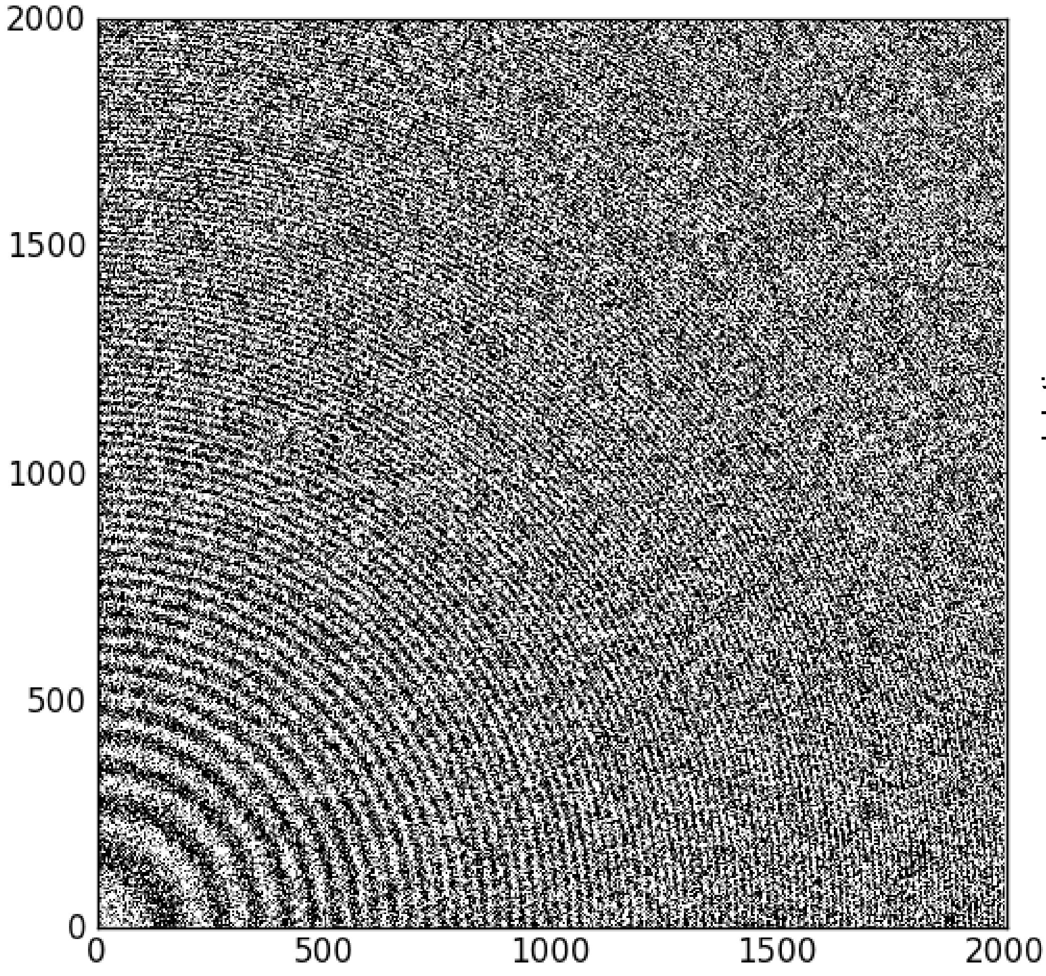


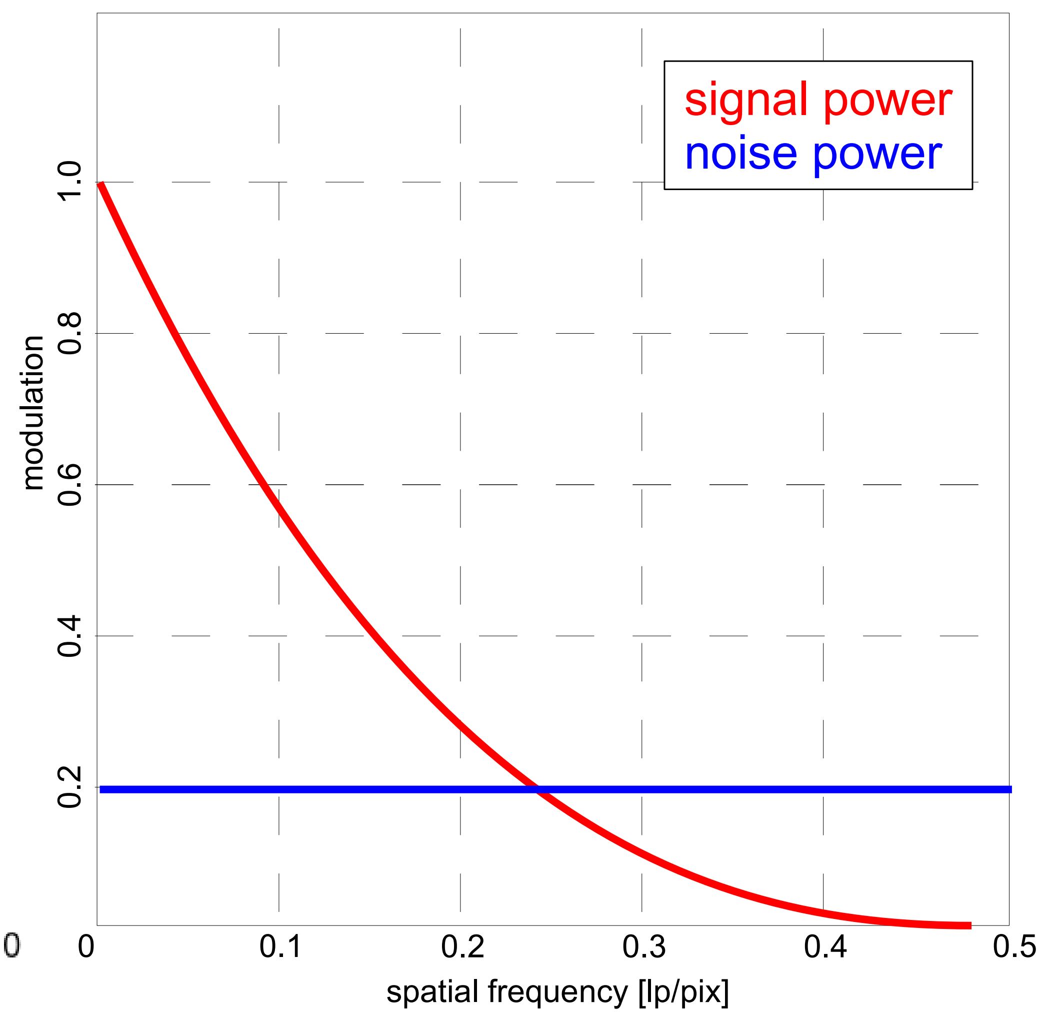
Signal power vs. noise power



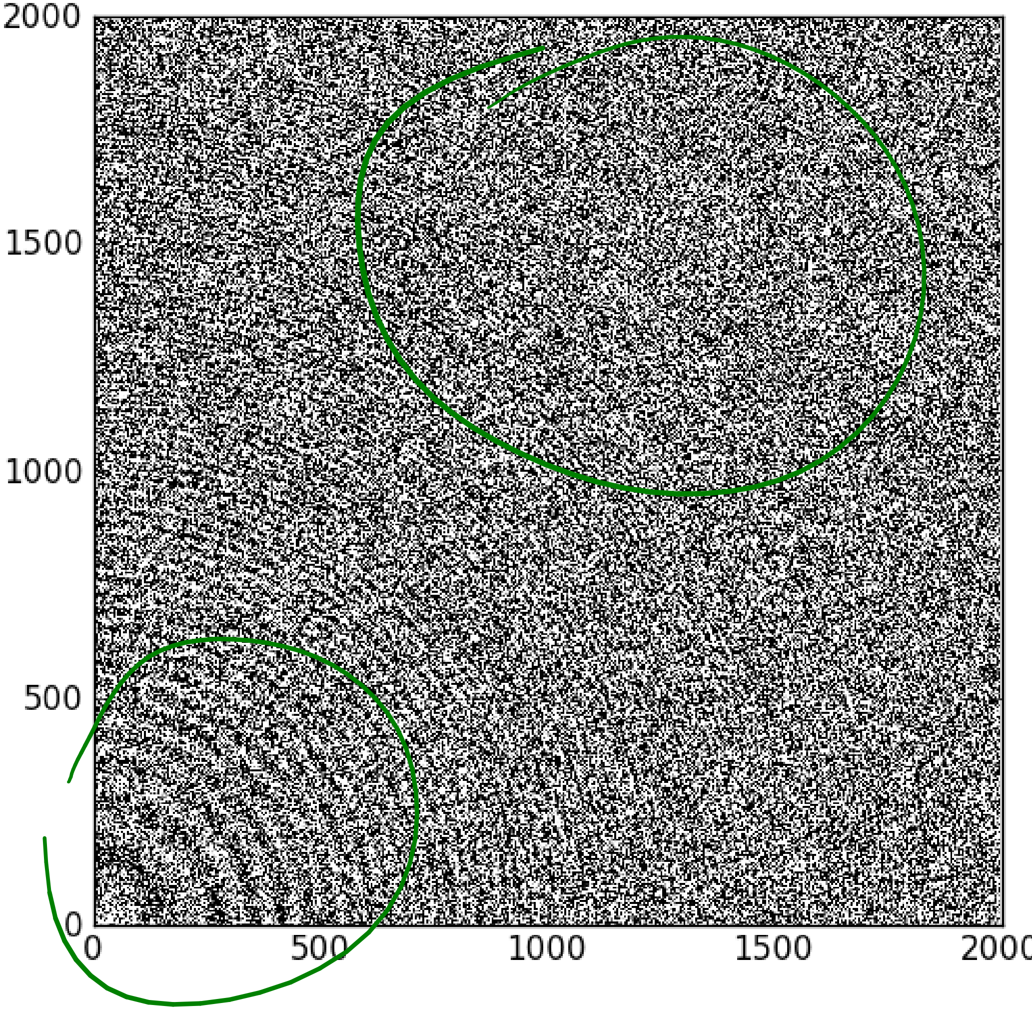


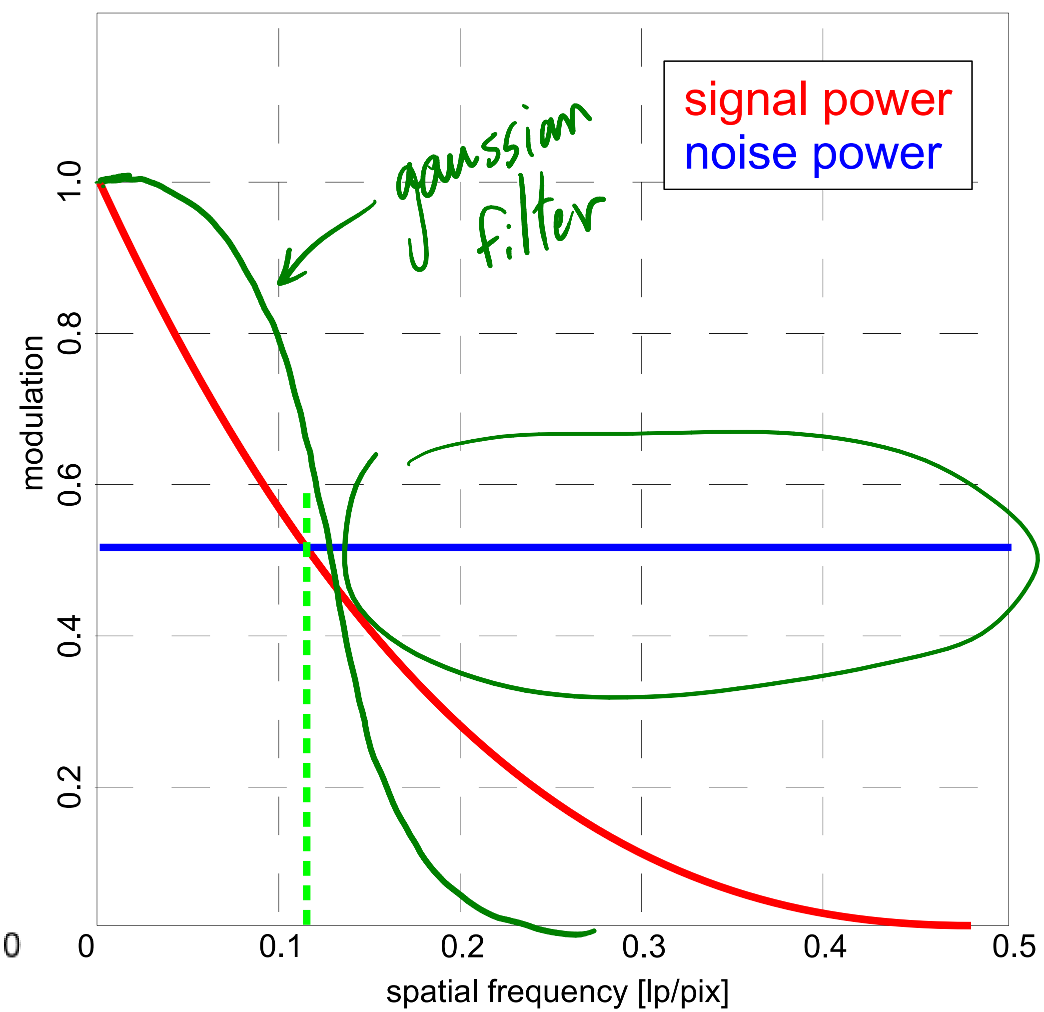
Signal power vs. noise power





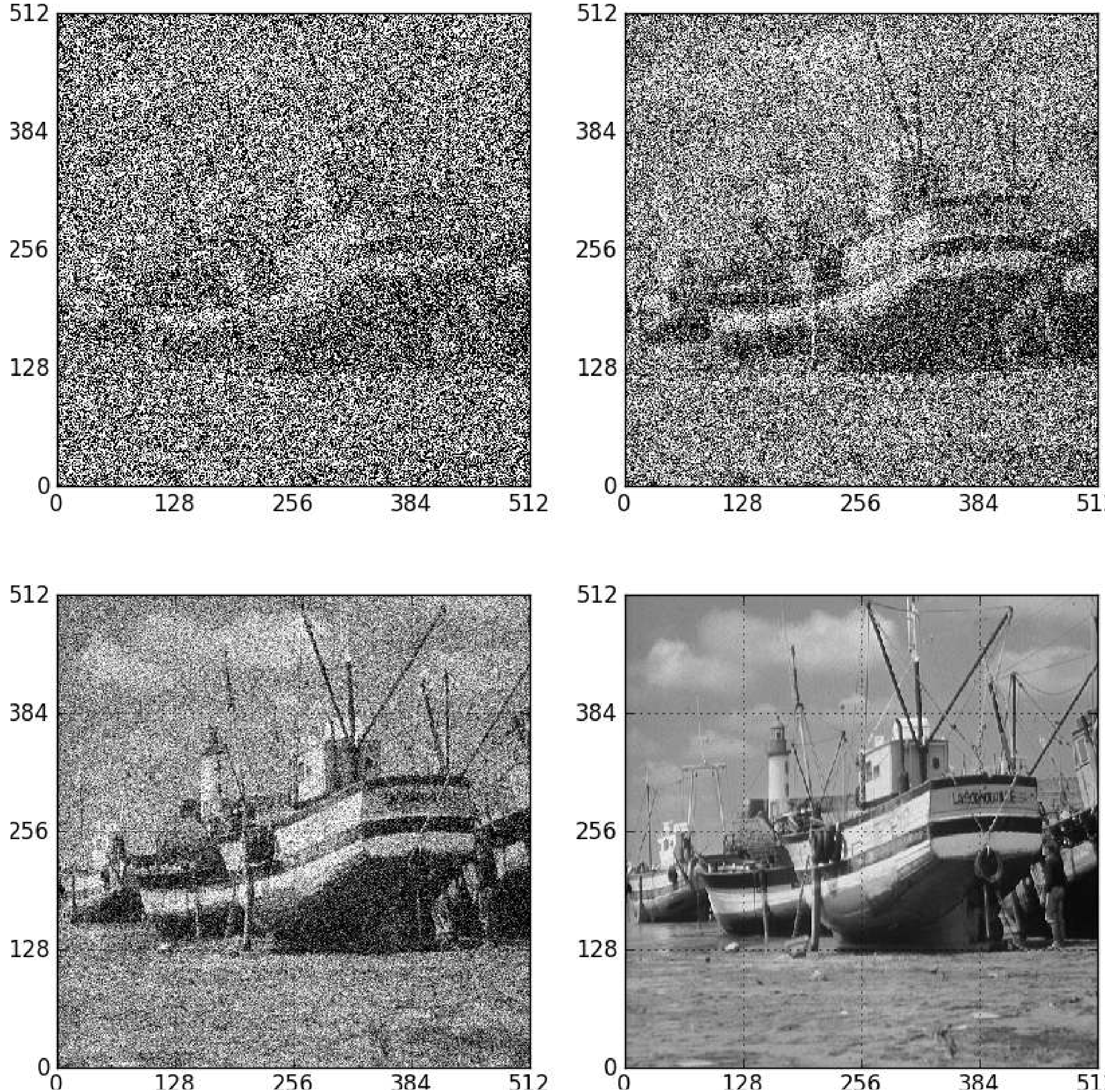
Signal power vs. noise power





- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

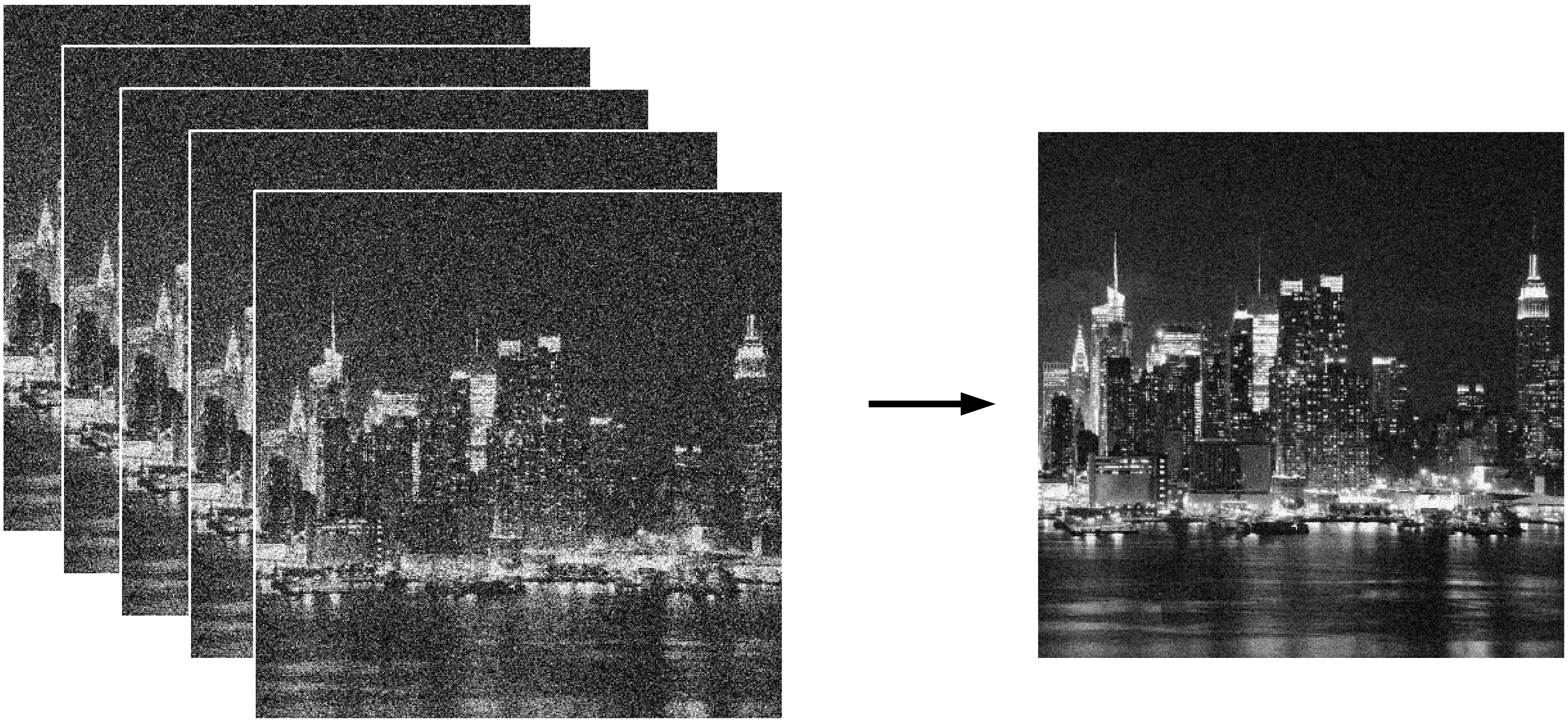
Signal power vs. noise power



+ noise normally means lower resolution

Noise reduction by averaging

- Average multiple images



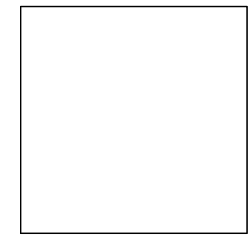
- requirement: additive noise, zero mean

Denoising by linear filtering

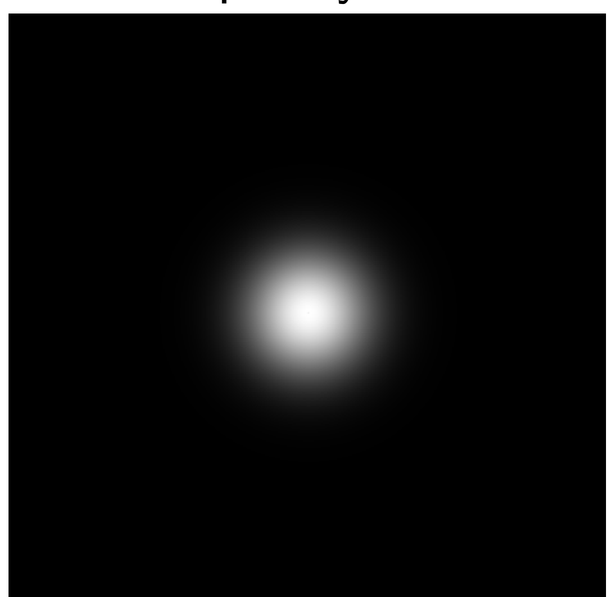
- use spatial convolution or frequency filtering to reduce noise
- noise reduction possible, but at cost of sharpness
- trade-off between noise reduction and resolution
- need fancier methods

original

convolution kernel

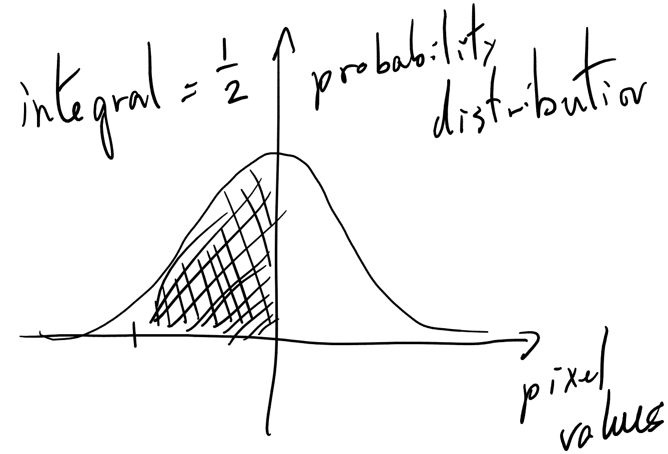


frequency filter

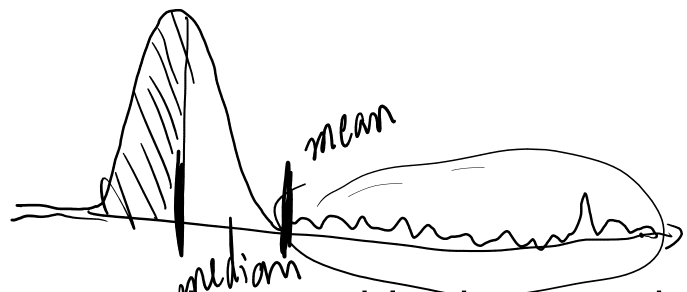


Resulting image

Median filtering



- Use median as estimator for fat tail distributions



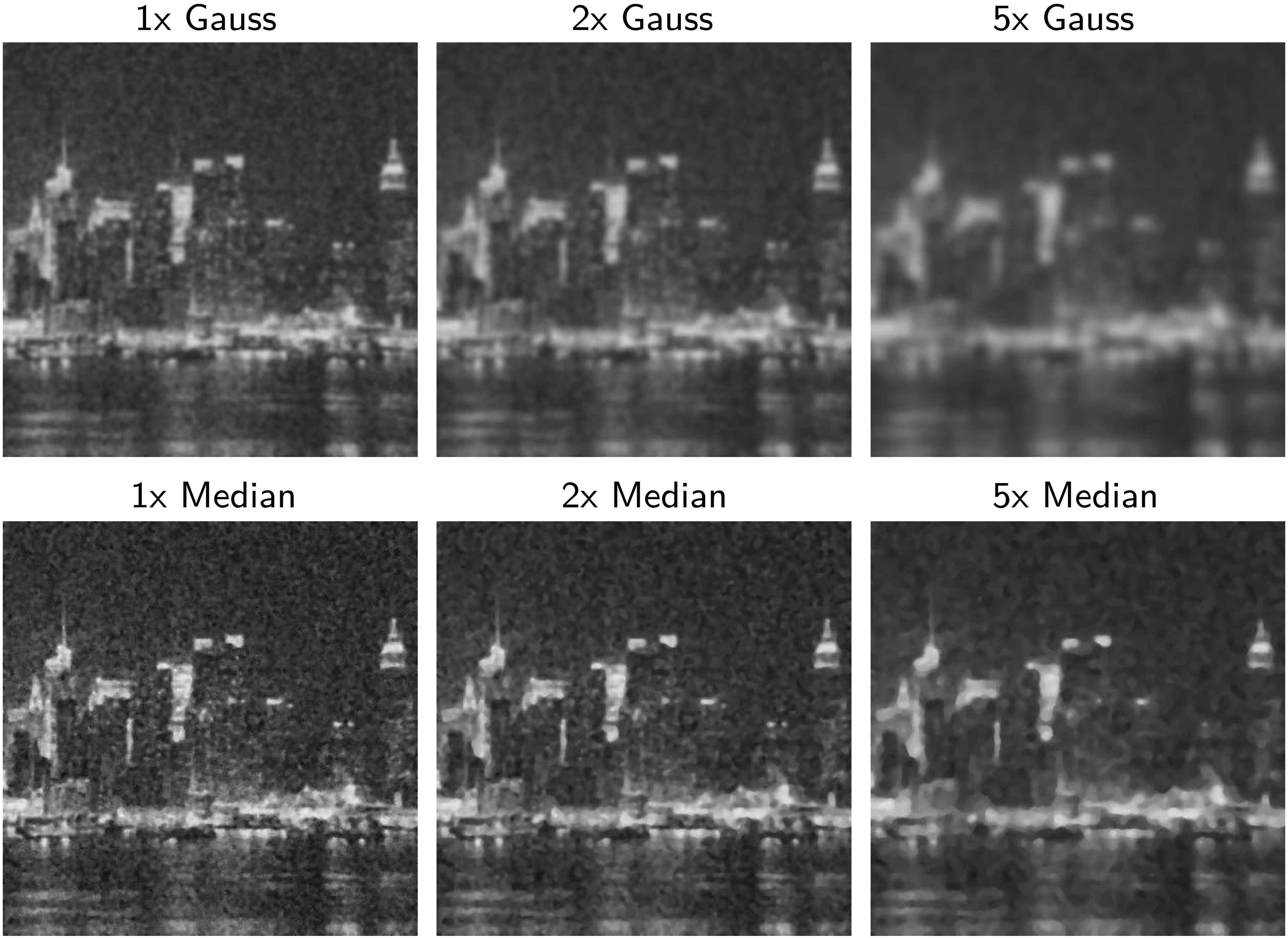
- less sensitive to outliers in pixel ensemble, better edge preservation

Salt and pepper noise

Gauss sigma=1 pixel

Median 1 pixel

Median filtering



Common abbreviations

Abbreviation	Name	Definition
IRF	Impulse response function	Linear operator map of delta function
PSF	Point spread function	Image of point object (optical IRF)
OTF	Optical transfer function	Fourier transform of PSF
PTF	Phase transfer function	Phase part of OTF
MTF	Modulation transfer function	Amplitude of OTF
CTF	Contrast transfer function	MTF for non-sinusoidal objects
PDF	Probability density function	Probability distribution for a given random variable
SPS	Signal power spectrum	Amplitude squared of signal F.T.
NPS	Noise power spectrum	Amplitude squared of noise F.T.
SNR	Signal to noise ratio	Mean signal / mean noise
CNR	Contrast to noise ratio	Mean contrast / mean noise