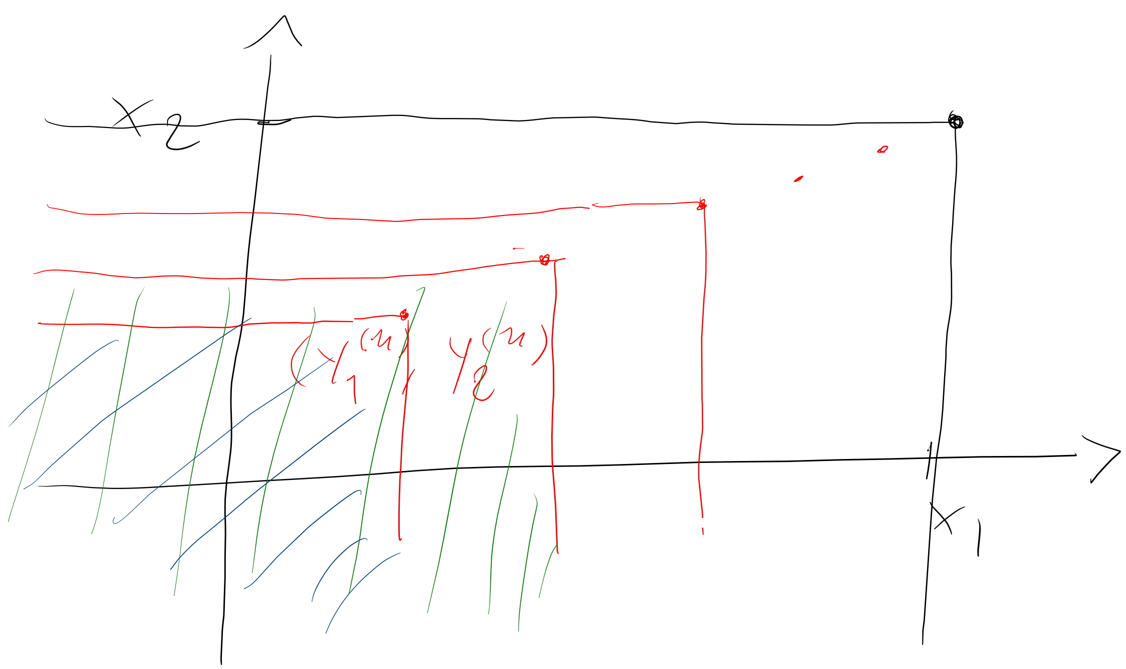
X_1, X_2 $\qquad (x_1, x_2)$ FISSATO

$$\lim_{y_1 \uparrow x_1,\ y_2 \uparrow x_2} F(y_1, y_2) = \Pr(X_1 < x_1, X_2 < x_2)$$

SIA $y_1^{(n)}, y_2^{(n)}$ CON $y_1^{(n)} \uparrow x_1$, $y_2^{(n)} \uparrow x_2$

$y_1^{(n)} < y_1^{(n+1)}$ E $\lim_{n \to +\infty} y_1^{(n)} = x_1$ SIMILE PER $y_2^{(n)}$



$$A_n = \{X_1 \le y_1^{(n)},\ X_2 \le y_2^{(n)}\}$$

$A_n \subset A_{n+1}$

MONOTONA NON DECRESCENTE

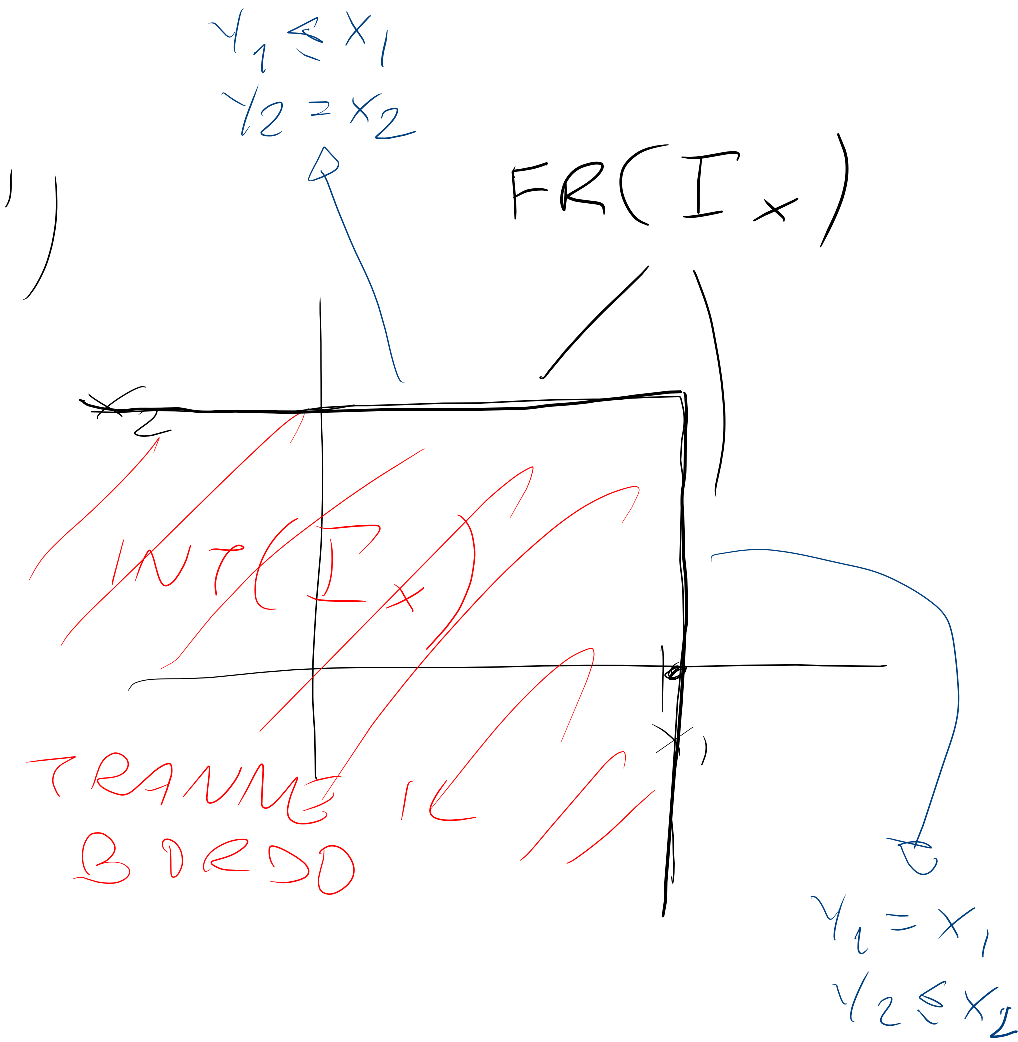
$$P\Big(\underbrace{\lim A_n}\Big) = \lim_{n\to+\infty} \underbrace{P(A_n)}$$

$$\underbrace{\bigcup A_n} \qquad\qquad F\big(Y_1^{(n)}, Y_2^{(n)}\big)$$

$$\underbrace{\bigcup_n \left\{X_1 \le Y_1^{(n)}, X_2 \le Y_2^{(n)}\right\}}$$

$$\underbrace{\left\{X_1 < x_1 \; X_2 < x_2\right\}}$$

$$(x_1, x_2) \in \operatorname{int}\big(I_{(x_1,x_2)}\big)$$



$$FR\left(I_{(x_1,x_2)}\right) = \left\{ (y_1,y_2) \;\middle|\; \begin{array}{l} y_1 = x_1,\ y_2 \leq x_2 \quad \text{o} \\ y_1 \leq x_1,\ y_2 = x_2 \end{array} \right\}$$

$$F(x_1,x_2) - \lim_{y_1 \uparrow x_1,\, y_2 \uparrow x_2} F(y_1,y_2) = \Pr\left((X_1,X_2) \in FR(I_{x_1,x_2})\right) =$$

MONOTONIA DELLA PROB

$$= \Pr\left((X_1 = x_1, X_2 \leq x_2) \text{ o } (X_1 \leq x_1, X_2 = x_2)\right)$$

$\underline{x} = (x_1, \ldots x_n)$

$$\{\underline{X} \in (x_1 - \varepsilon, x_1] \times \ldots \times (x_n - \varepsilon, x_n]\} = A_\varepsilon$$

$$\{\underline{X} = \underline{x}\} \subset A_\varepsilon$$

$$P(X = \underline{x}) \leq P(A_\varepsilon) = \Delta \ldots \Delta F_{\underline{X}}$$

$\underline{n = 2}$

$$F(x_1, x_2) - \underbrace{F(x_1 - \varepsilon, x_2)}_{\to F(x_1, x_2)} - \underbrace{F(x_1, x_2 - \varepsilon)}_{\to F(x_1, x_2)} + \underbrace{F(x_1 - \varepsilon, x_2 - \varepsilon)}_{\to F(x_1, x_2)}$$

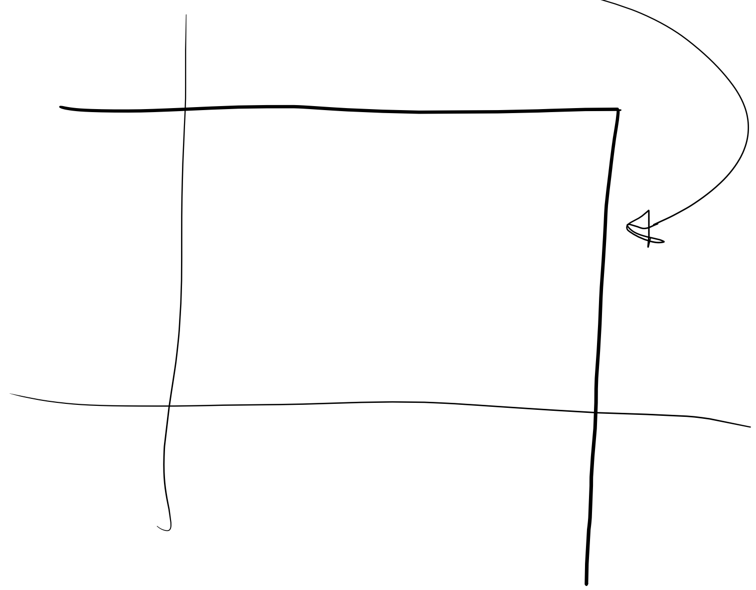
$\lim_{\varepsilon \to 0}$

$P(A_\varepsilon) \to 0$ SE F CONTINUA

SE F DISCONTINUA IN (x_1, x_2)

DALLA (6), (7)

$$P\big(\underbrace{\underline{X} \in Fr(I_{(x_1,x_2)})}\big) > 0$$



MA POTREBBE ESSERE $P(\underline{X} = \underline{x}) = 0$ $\forall\, \underline{x}$

CASO $n = 1$ È DIVERSO

NEL TEOREMA, 2ª PARTE, SE ABBIAMO F CHE SODDISFA (2), (3), (4)

PARTE 1 DEL TEOREMA

$\Omega = \mathbb{R}^n$, $\mathcal{F} = \mathcal{B}^n$, $P = \mathbb{P}$

$\underline{X} : \Omega \longrightarrow \mathbb{R}^n$ $\quad \underline{X}(\omega) = \omega$ (IDENTITÀ)

ALLORA LA LEGGE DI $\underline{X}$ È PROPRIO $\mathbb{P}$

$$P(\underline{X} \in B) = \mathbb{P}(B)$$

CASO MULTIVARIATO

$$1 - F(x_1 \ldots x_n) \neq P(X_1 > x_1 \ldots X_n > x_n)$$

CASO UNIVARIATO

$$I_x = (-\infty, x]$$

$$\mathrm{INT}(I_x) = (-\infty, x) \qquad \mathrm{FR}(I_x) = \{x\}$$

$$X = Y \quad \text{q.c.} \qquad P(X = Y) = 1$$

$$F_X(x) = P(X \leq x) = P\big((X \leq x) \cap (X = Y \cup X \neq Y)\big)$$

$$= P(X \leq x, X = Y) + \underbrace{P(X \leq x, X \neq Y)}_{=0}$$

$$= P(Y \leq x, X = Y)$$

$$= P(Y \leq x, X = Y) + \overbrace{P(Y \leq x, X \neq Y)}$$

$$= P(Y \leq x) = F_Y(x)$$

$X = 1_A \qquad Y = 1_{\bar{A}}$

CON $P(A) = \frac{1}{2}$

$P(X = Y) = 0$

MA $F_X = F_Y \qquad P(A) = P(\bar{A}) = \frac{1}{2}$

(VEDREMO FRA POCO)

$$\underline{X} = (X_1, X_2, X_3)^T \qquad F_{\underline{X}}$$

$$F_{(X_1,X_2)}(x_1,x_2) = \lim_{x_3 \to +\infty} F_{\underline{X}}(x_1, x_2, x_3)$$

$$F_{X_2}(x_2) = \lim_{\substack{x_1 \to +\infty \\ x_3 \to +\infty}} F_{\underline{X}}(x_1, x_2, x_3)$$