Image Processing for Physicists

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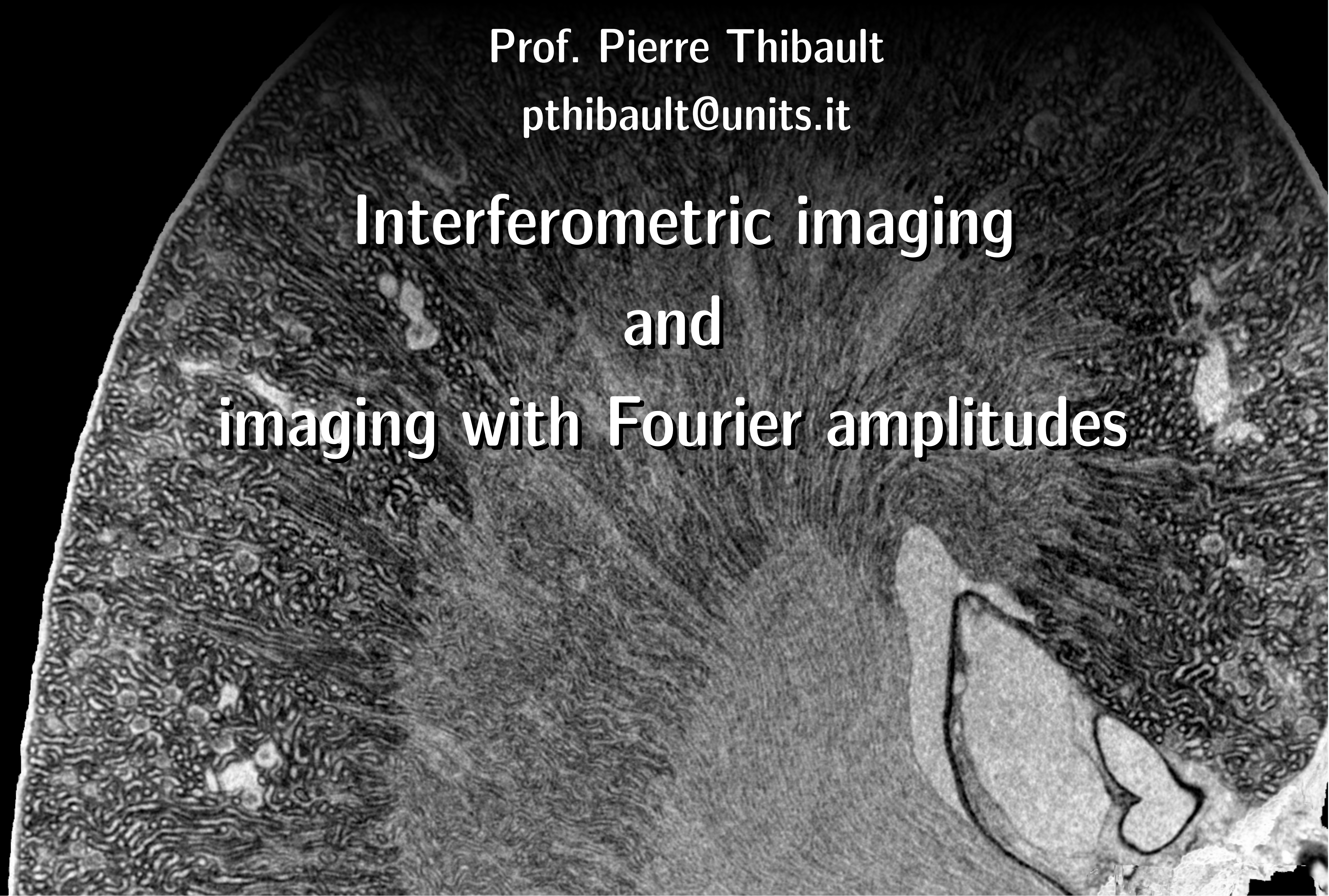
Interferometric imaging

and

imaging with Fourier amplitudes



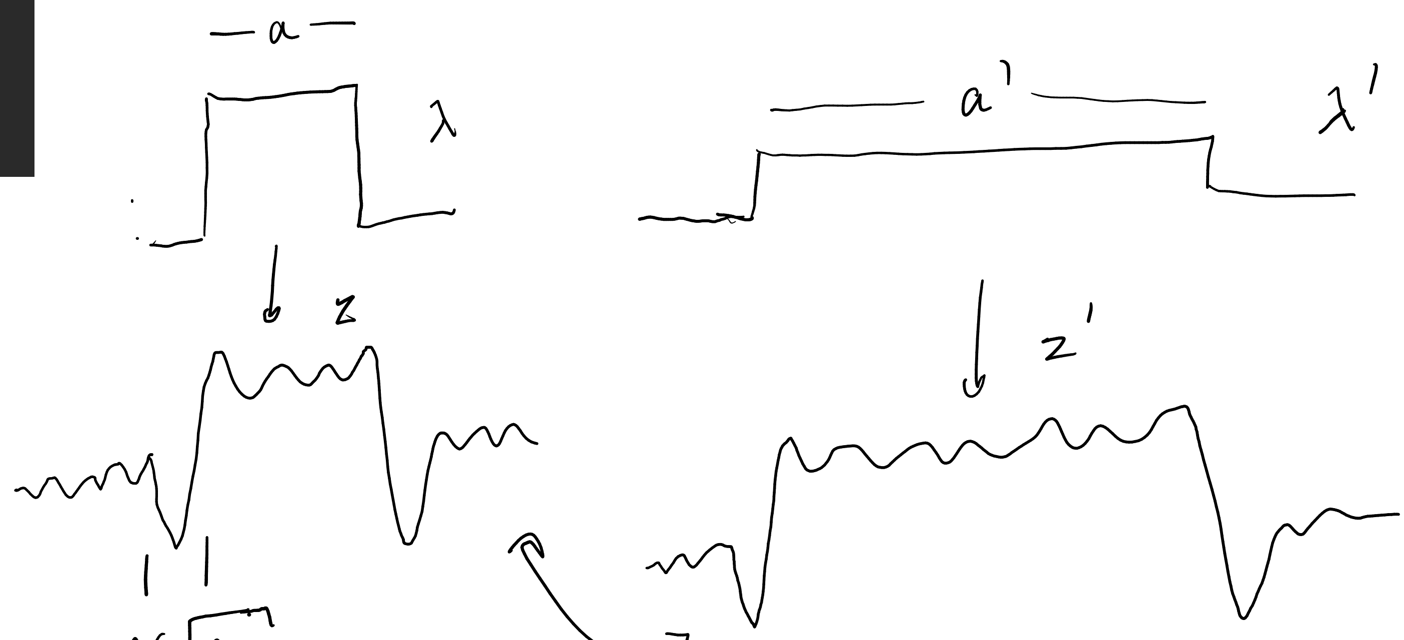
Overview

- The phase problem
- Holography: on/off-axis
- Grating interferometric imaging
- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

Wave propagation

far-field / near-field

observation: $\exp(i\pi \underbrace{u^2 \lambda z}_{\text{unitless number}})$



$\frac{a^2}{\lambda z} = f$

"Fresnel number"

$f \ll 1$: far-field

$f \gg 1$: near-field

$\sim \sqrt{\lambda z}$

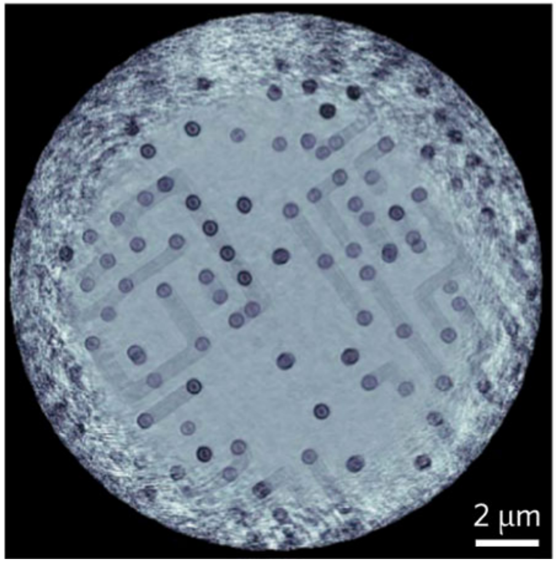
Fresnel zone length

identical if $\frac{1}{a^2} \lambda z = \frac{1}{a'^2} \lambda' z'$

$\sqrt{\lambda z}$ = characteristic length

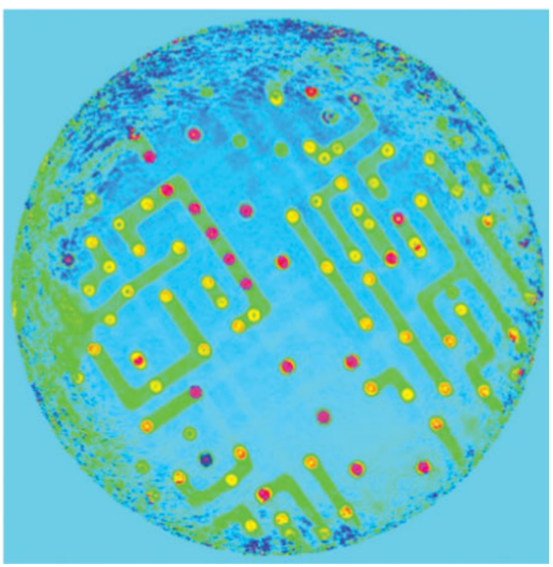
Complex-valued images

X-ray transmission image



Amplitude

attenuation of the wave



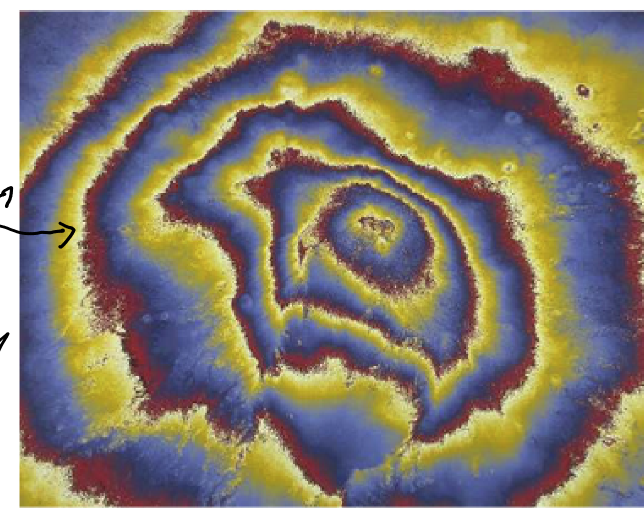
phase

delay in the phase (due to refraction)

sythetic aperture radar

phase unwrapping

same phase

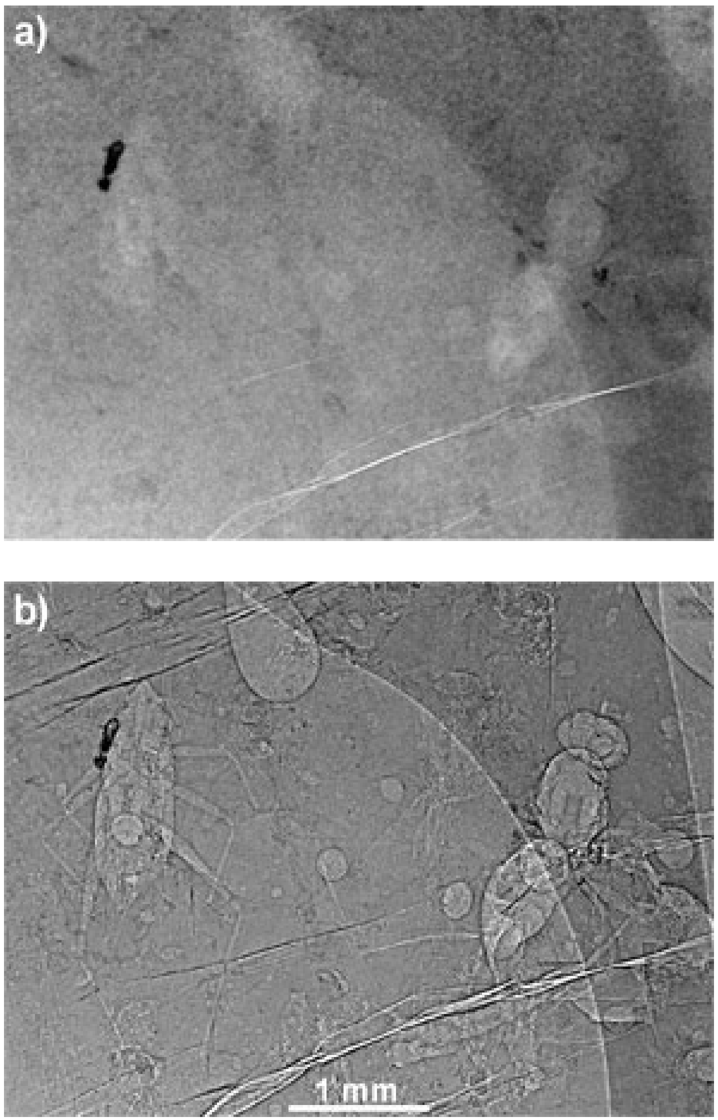


phase

Phase-contrast

Visible light: see Zernike

Hard X-ray propagation-based phase contrast

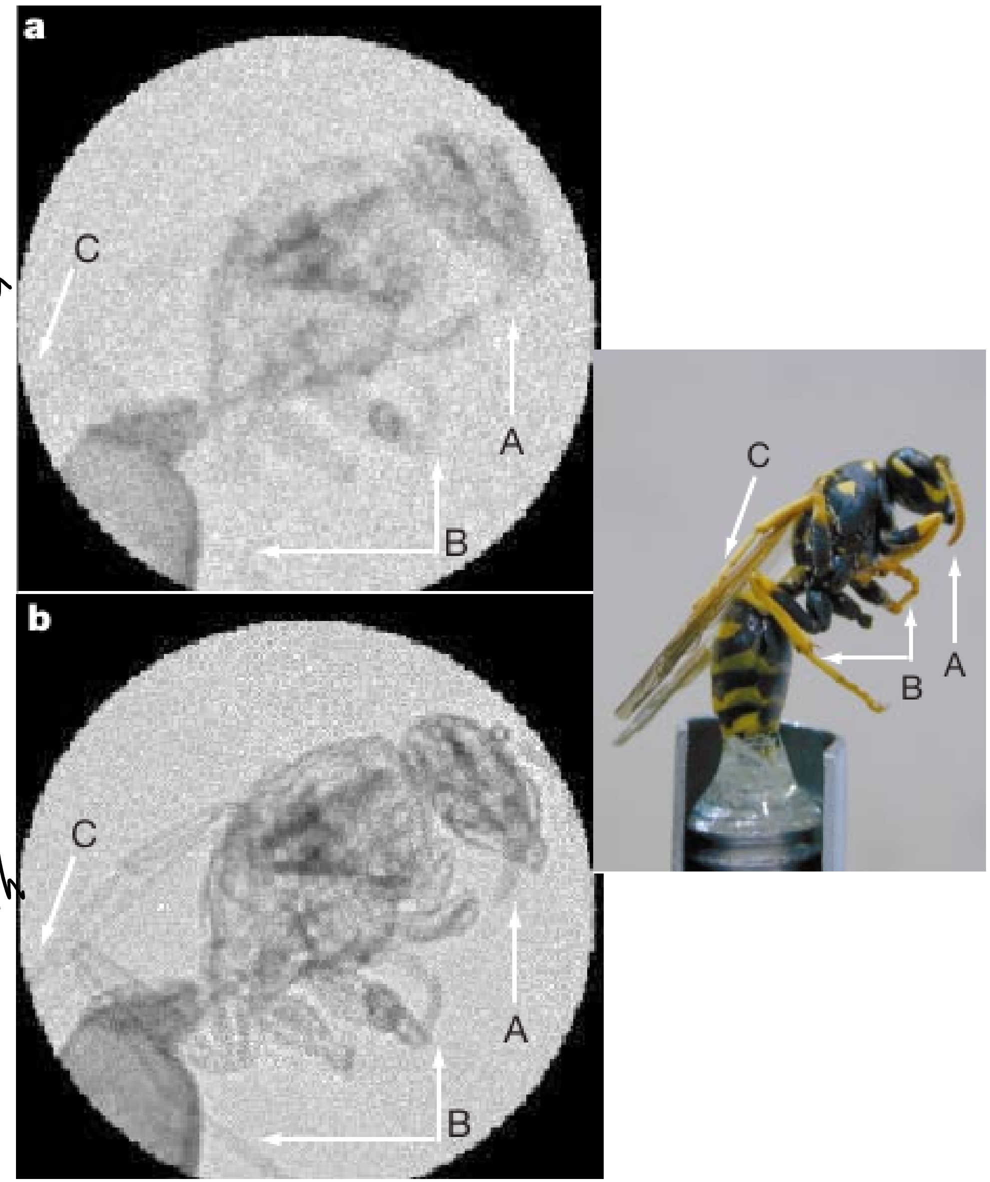


Source: www.esrf.eu/news/general/amber/amber/

short camera length ~ contact images

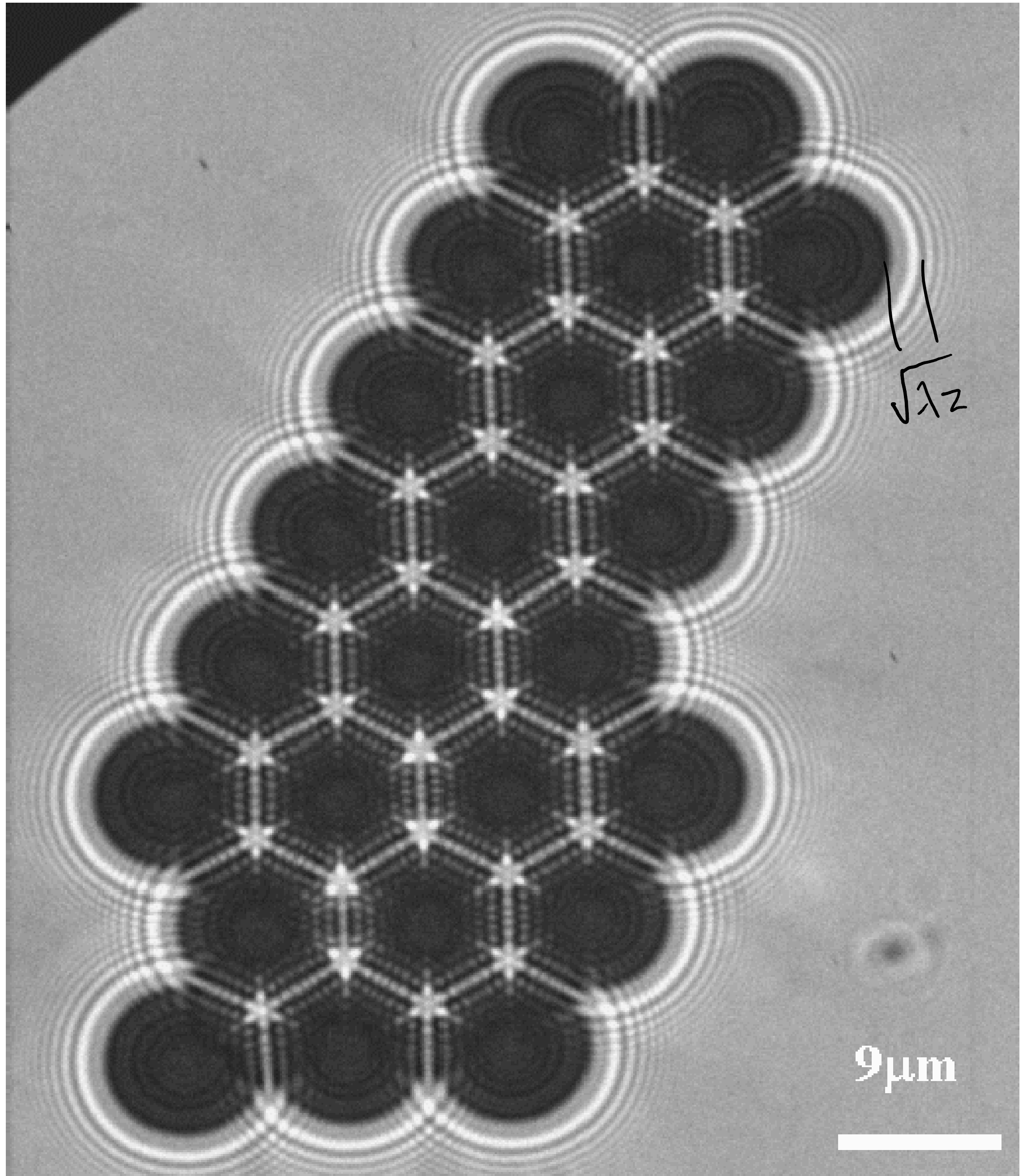
longer camera length (≈ detector distance)

Neutron phase contrast



Source: Allman et al. Nature **408** (2000).

Inline holography



Source: Mayo et al. Opt Express **11** (2003).

Inline holography

Measure $\quad I(\vec{r}) = |\psi(r;z)|^2$

* plane monochromatic wave

* weak transmission of imaged object

small perturbation

$$\psi(\vec{r}, z=0) = A\,(1 + \varepsilon(\vec{r}))$$

constant (plane wave)

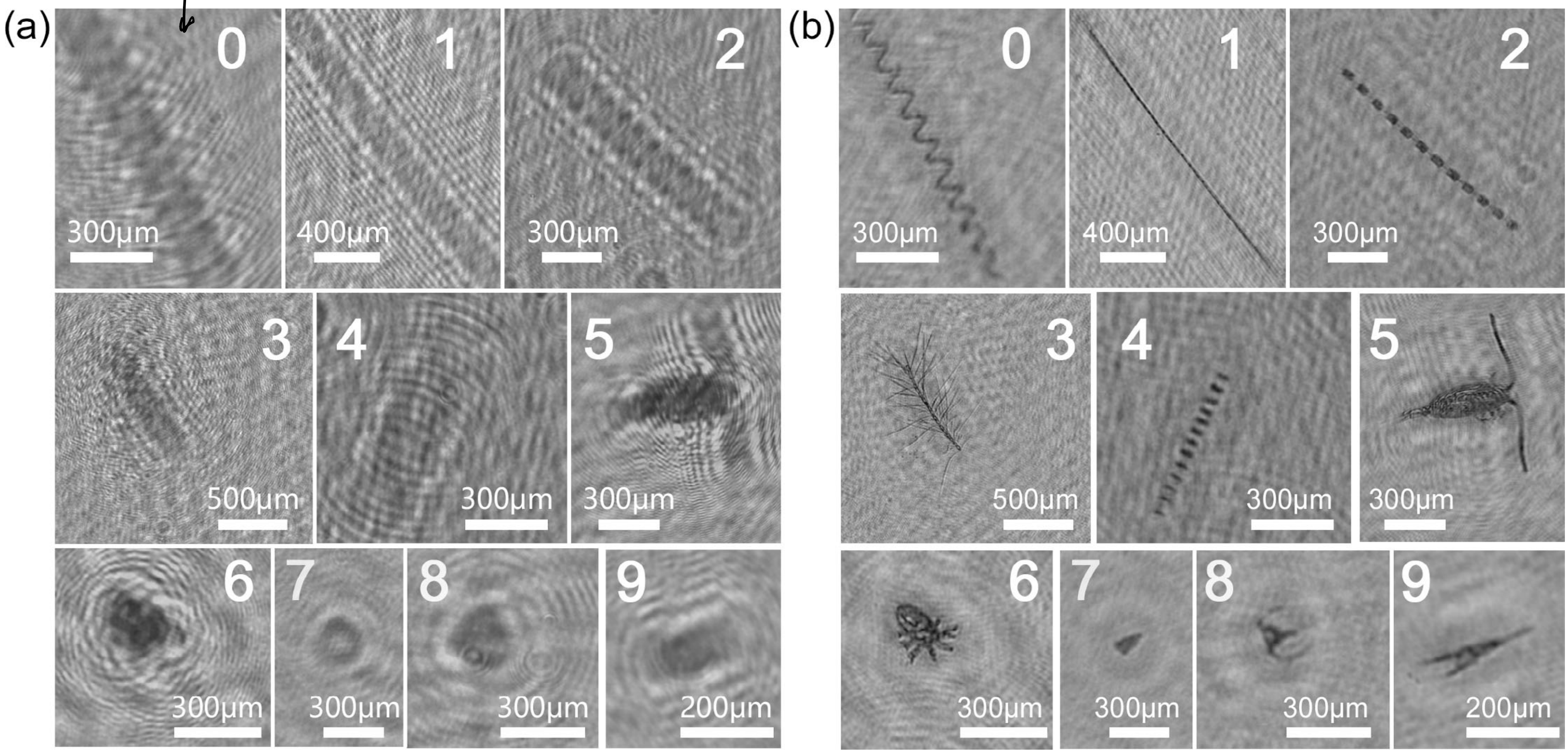
$$I(\vec{r}) = |A\,(1+\varepsilon(\vec{r};z))|^2 = |A|^2\left(1 + \varepsilon(\vec{r};z) + \varepsilon^*(r;z) + \cancel{|\varepsilon(\vec{r};z)|^2}\right)$$

$$= |A|^2\left[1 + \underbrace{\varepsilon(\vec{r};z)}_{\text{propagated by } z} + \underbrace{\varepsilon^*(\vec{r};z)}_{\text{propagated by } -z}\right]$$

"twin image problem"

Digital inline holography

$|\psi|^2$



Trick: apply Fresnel (angular spectrum) propagator!

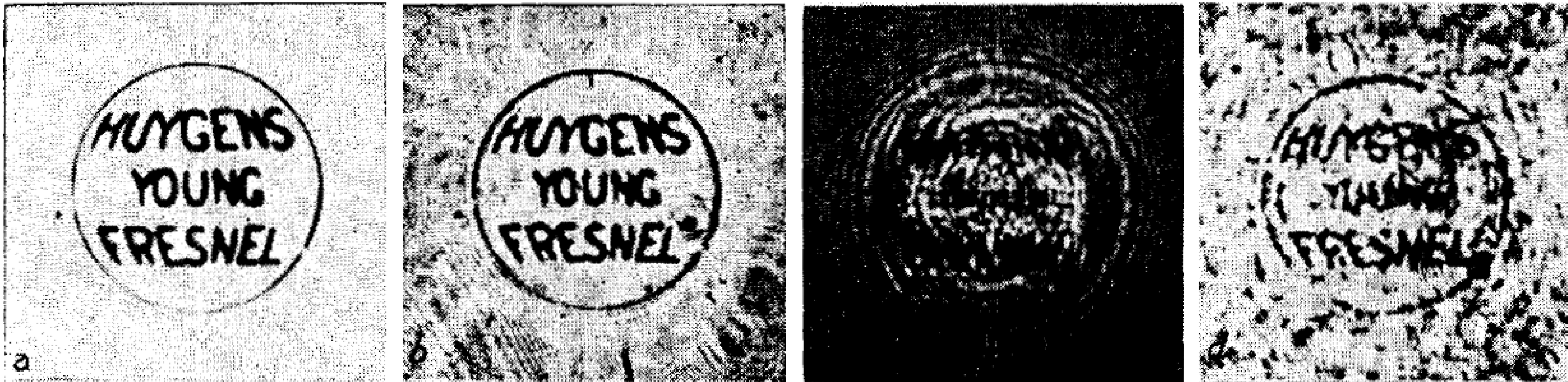
The phase problem

We always measure $|\psi|^2$. phases are lost.

We need to recover the phase part of the wavefield

* sometimes the phases are interesting
* most often: the phase are an auxiliary quantity for proper interpretation of the wavefield

In-line holography



D. Gabor, Nature **161**, 777-778 (1948).

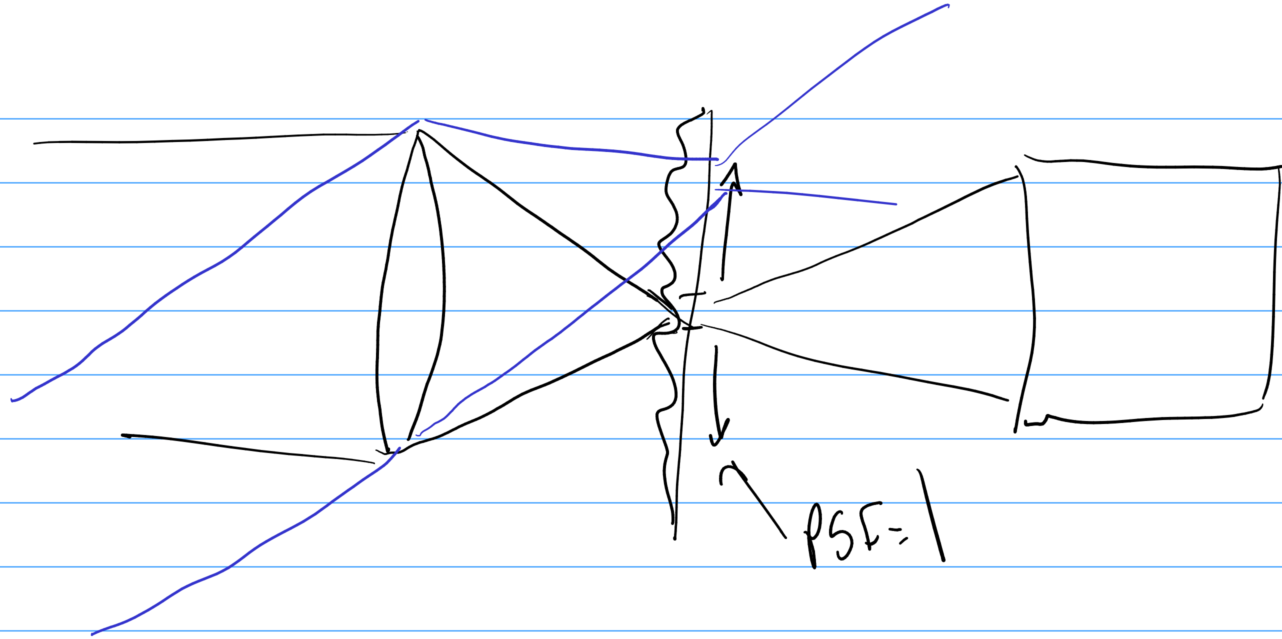
mask

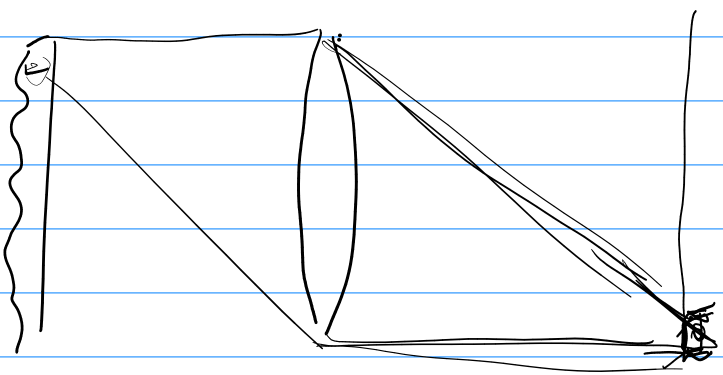
"in focus"

measurement
after propagation

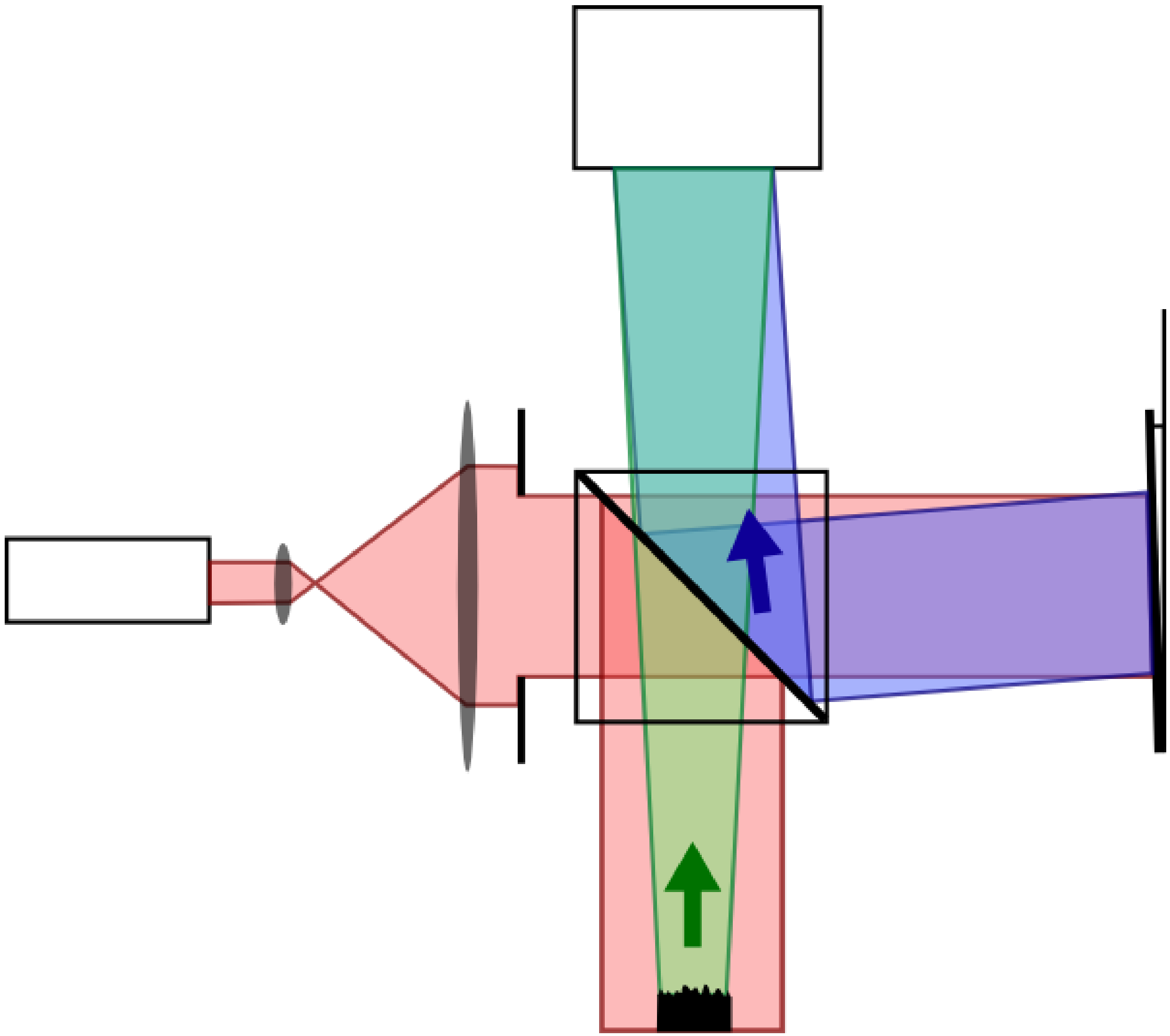
propagated

problem: twin image


PSF = 1

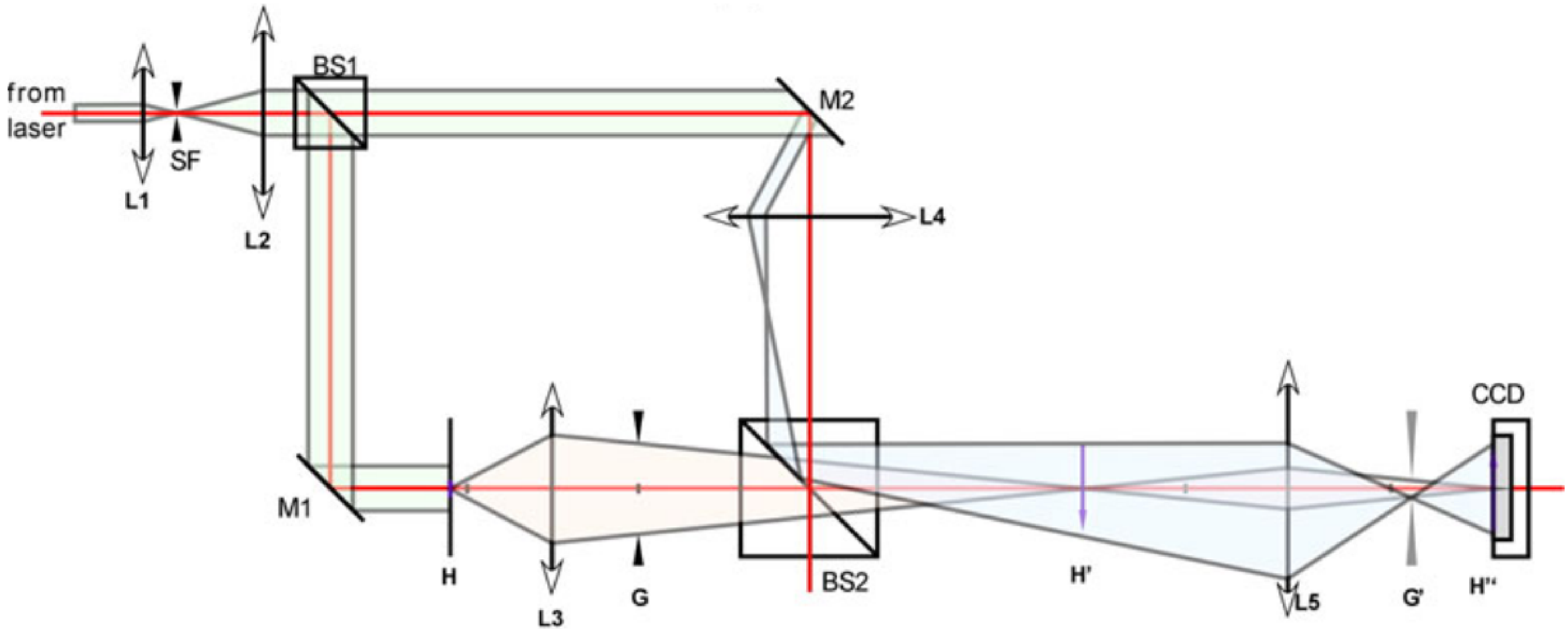


Fringe interferometry



Twyman-Green interferometer

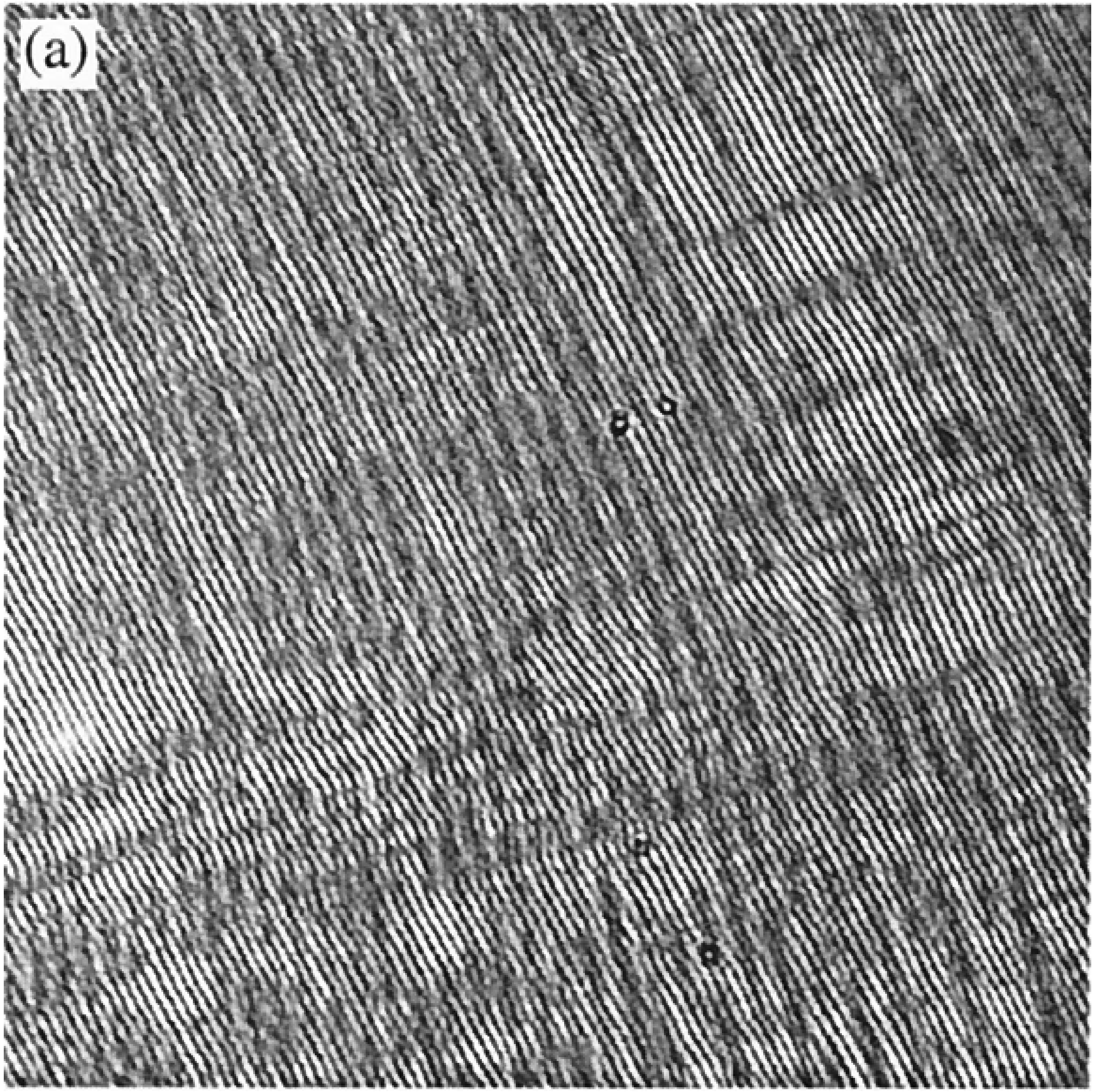
Visible light interferometer



Mach-Zehnder interferometer

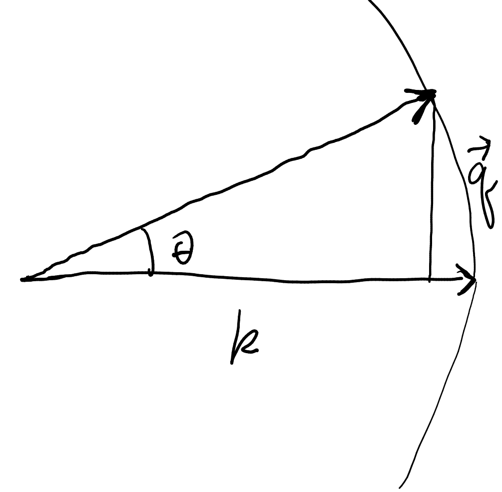
Source: M. K. Kim, SPIE Rev. **1**, 018005 (2010).

Fringe interferometry



Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

Two tilted plane waves:



$$1 + e^{i\vec{q}\cdot\vec{r}}$$

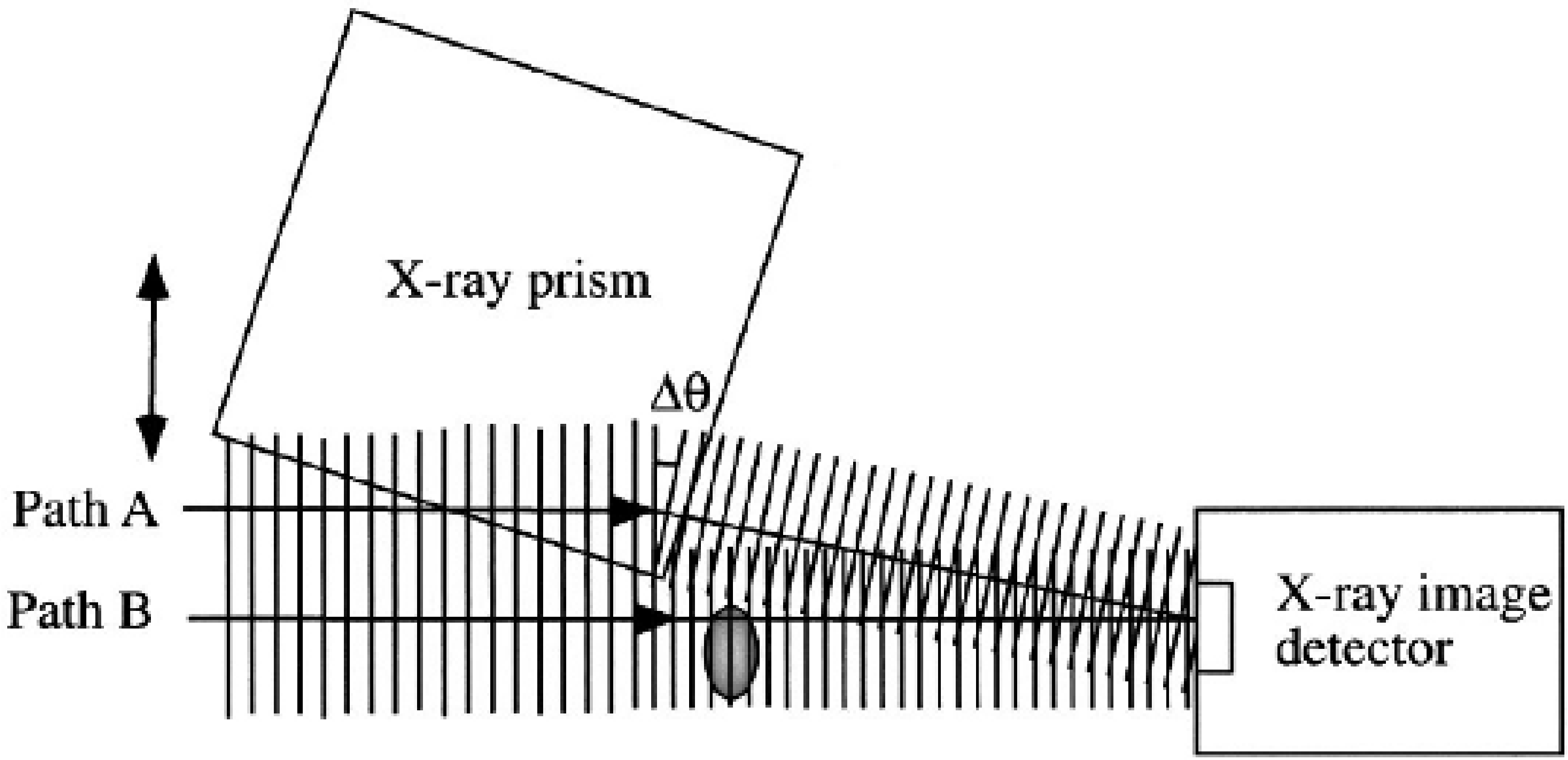
along z — at an angle θ $\left(\sin\theta = \frac{|q|}{k}\right)$

$$I = \left|1 + e^{i\vec{q}\cdot\vec{r}}\right|^2 = 1 + e^{i\vec{q}\cdot\vec{r}} + e^{-i\vec{q}\cdot\vec{r}} + 1 = 2 + 2\cos(\vec{q}\cdot\vec{r})$$

$\hookrightarrow$ oscillation with spatial frequency $\vec{n} = \vec{q}/2\pi$

With a sample in: $\left|a(\vec{r}) + e^{i\vec{q}\cdot\vec{r}}\right|^2$

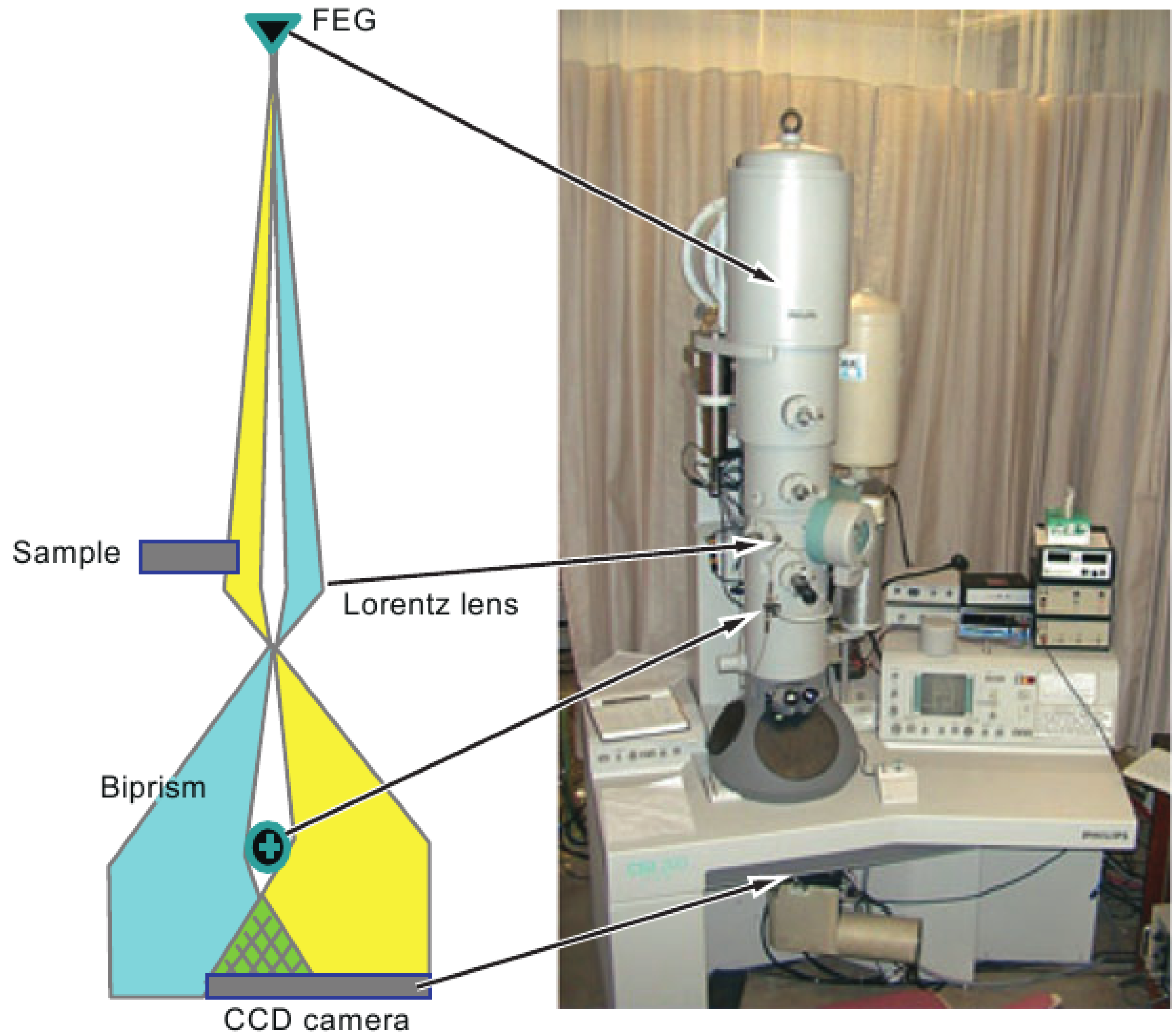
Off-axis X-ray holography



Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

Off-axis electron holography

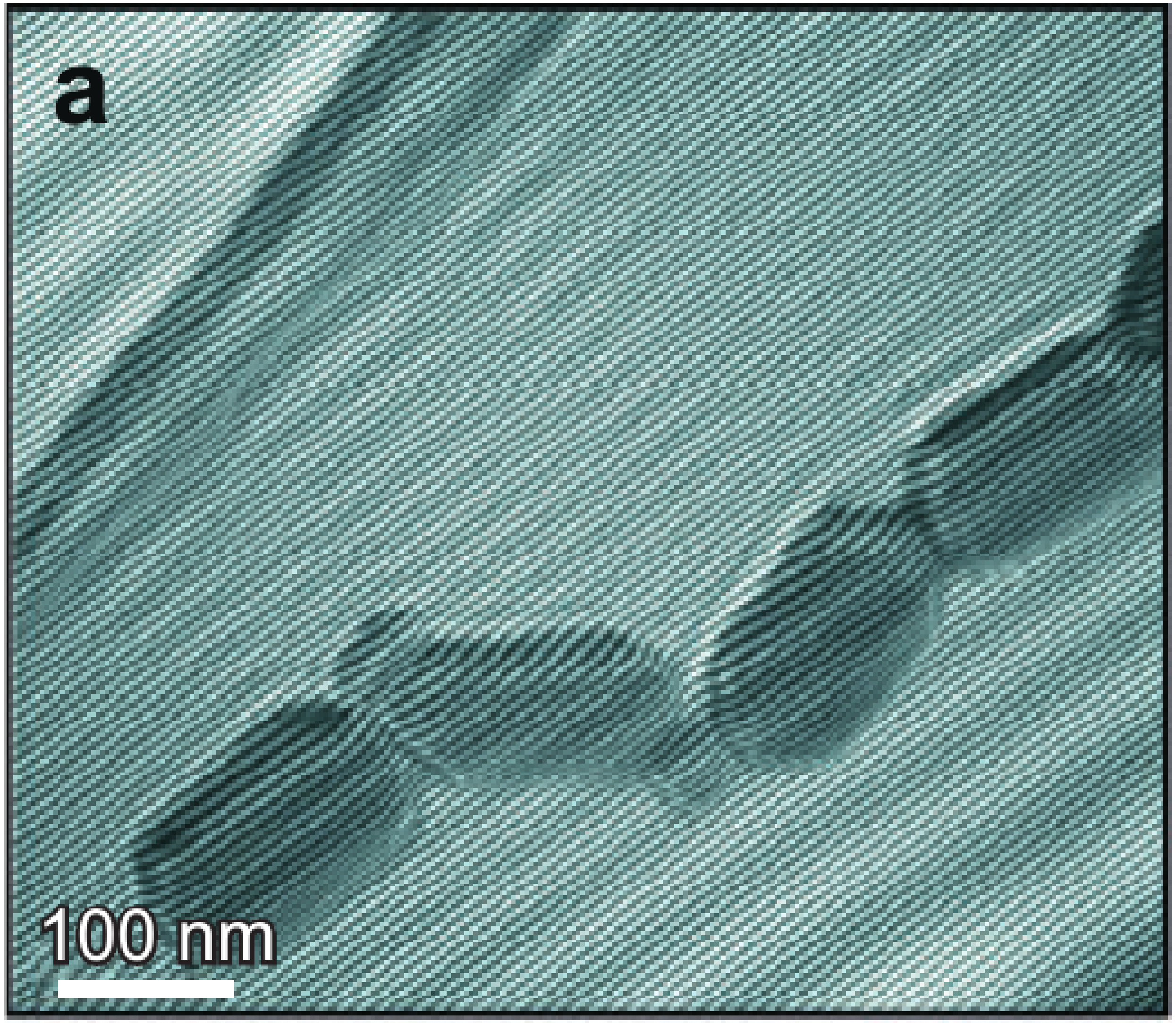
Electron microscopy



Hologram **Philips CM200 FEG**

Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

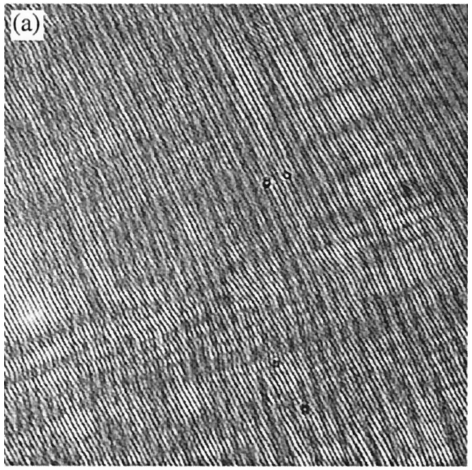
Off-axis electron holography

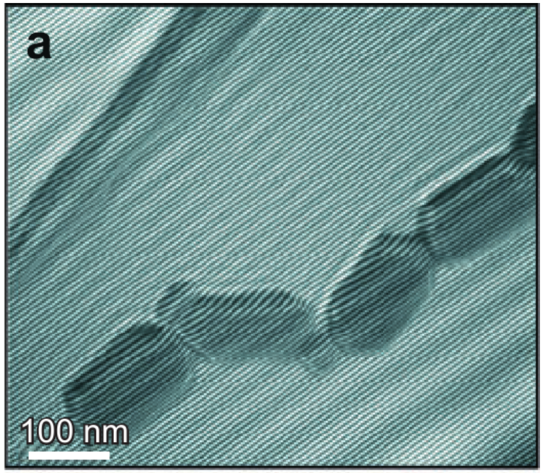


$$\mathcal{F}\{f(\vec{r})\} = F(\vec{u}) \qquad \mathcal{F}\{f(\vec{r})e^{i\vec{u}_0\cdot\vec{r}}\} = F(\vec{u}-\vec{u}_0)$$

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Fringe interferometry





$$\psi = \underset{\text{object}}{\psi_o} + \underset{\text{reference}}{\psi_r}$$

tilt

$$\psi_r(\vec r) = A e^{i\vec q\cdot\vec r}$$

$$\psi_o(\vec r) = A\underbrace{a(\vec r)\, e^{i\varphi(\vec r)}}_{\text{complex-valued transmission function}}$$

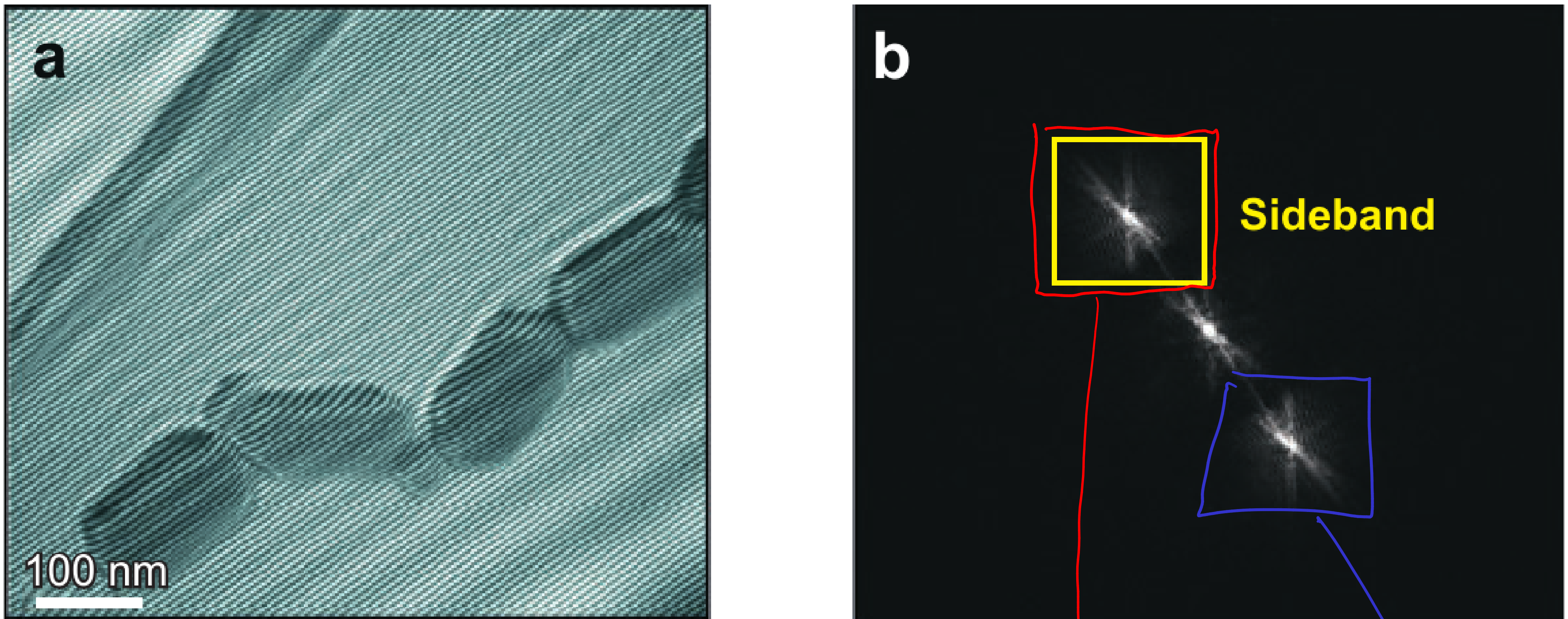
a, φ are unknown to be retrieved from measurement

Measurement:

$$|\psi|^2 = (\psi_o + \psi_r)(\psi_o + \psi_r)^*$$

$$I = |A|^2\left(1 + |a(r)|^2 + \underbrace{\overbrace{a(\vec r)\, e^{i(\vec q\cdot\vec r - \varphi)}}^{\psi_o(\vec r)e^{i\vec q\cdot\vec r}} + \overbrace{a(\vec r)\, e^{-i(\vec q\cdot\vec r - \varphi)}}^{\psi_o^*(\vec r)e^{-i\vec q\cdot\vec r}}}_{2a(\vec r)\cos(\vec q\cdot\vec r - \varphi(\vec r) + \varphi_o)}\right)$$

Off-axis holography



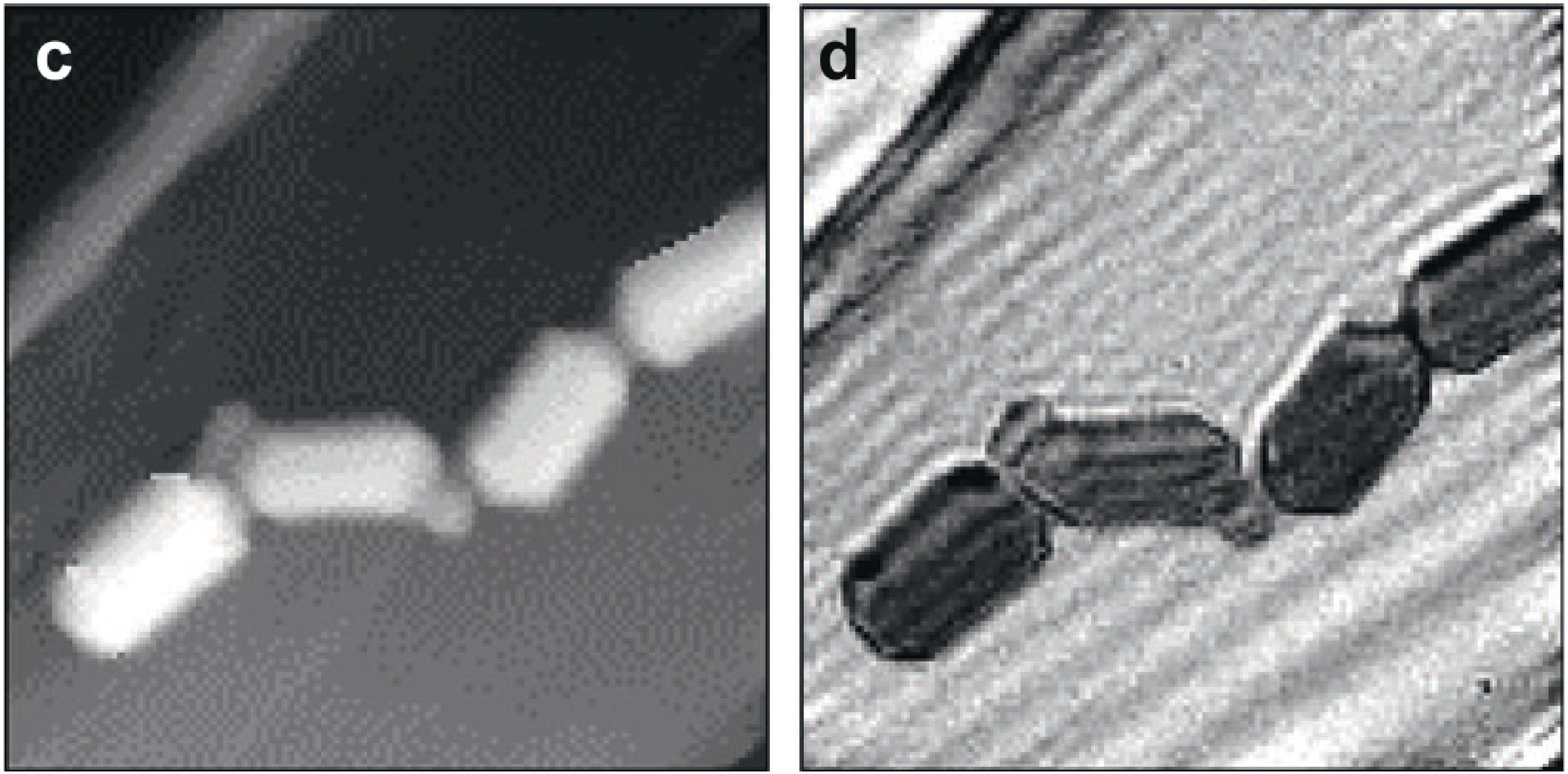
$$\mathcal{F}\{|\psi|^2\} = |A|^2\left[\mathcal{F}\{a^2(r)+1\} + \boxed{\mathcal{F}\{\psi_0\}\left(\vec{u}+\frac{\vec{q}}{2\pi}\right)} + \boxed{\mathcal{F}\{\psi_0^*\}\left(\vec{u}-\frac{\vec{q}}{2\pi}\right)}\right]$$

solution: crop, inverse F.T. !

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis holography

Price paid: loss of resolution



phase (φ)

attenution (a)

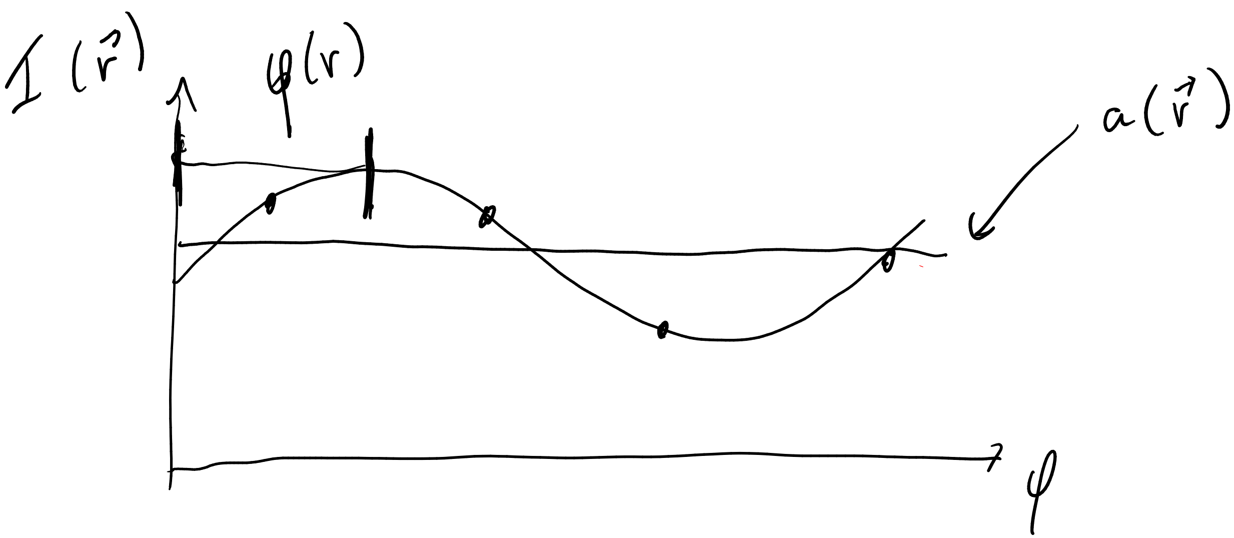
Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Phase stepping

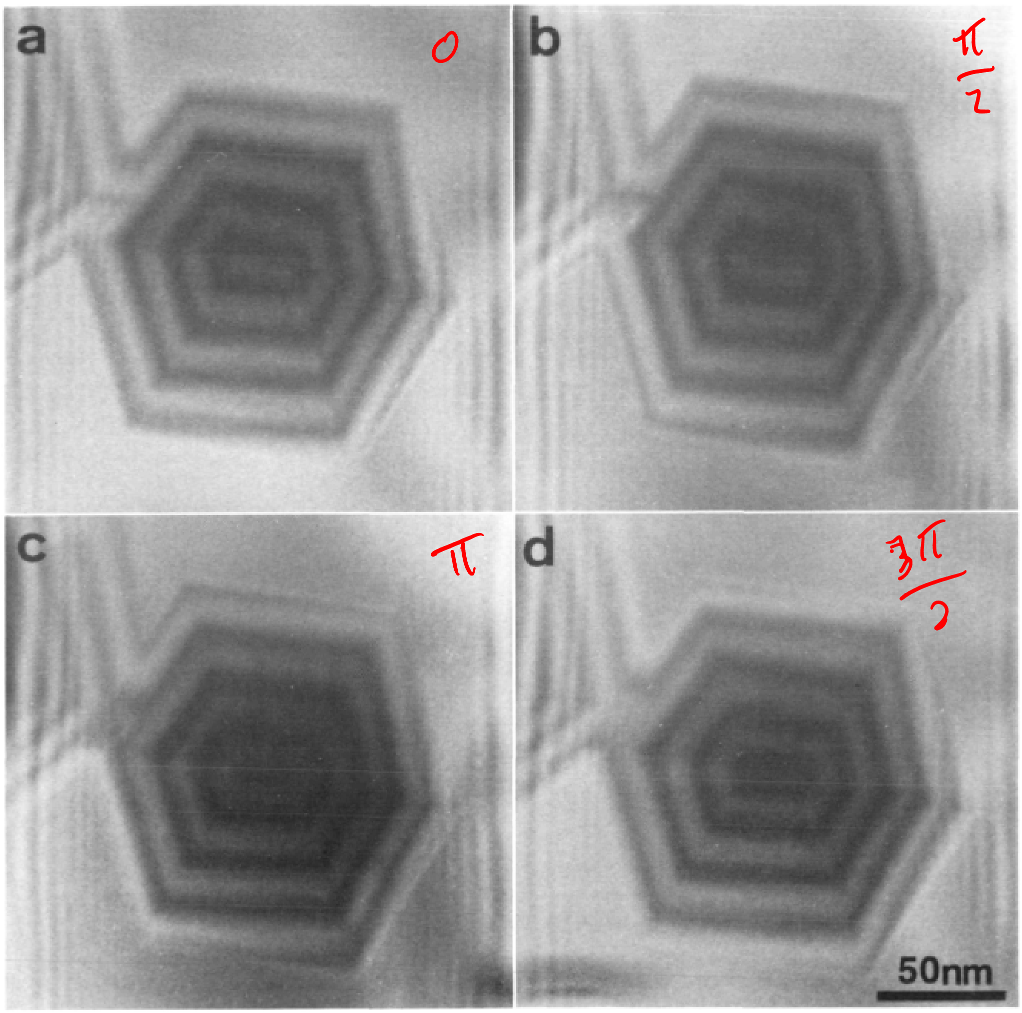
- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.

$$e^{i(\vec{q}\cdot\vec{r})} \qquad e^{i(\vec{q}\cdot\vec{r}+\frac{\pi}{2})} \qquad e^{i(\vec{q}\cdot\vec{r}+\pi)} \qquad e^{i(\vec{q}\cdot\vec{r}+\frac{3\pi}{2})}$$



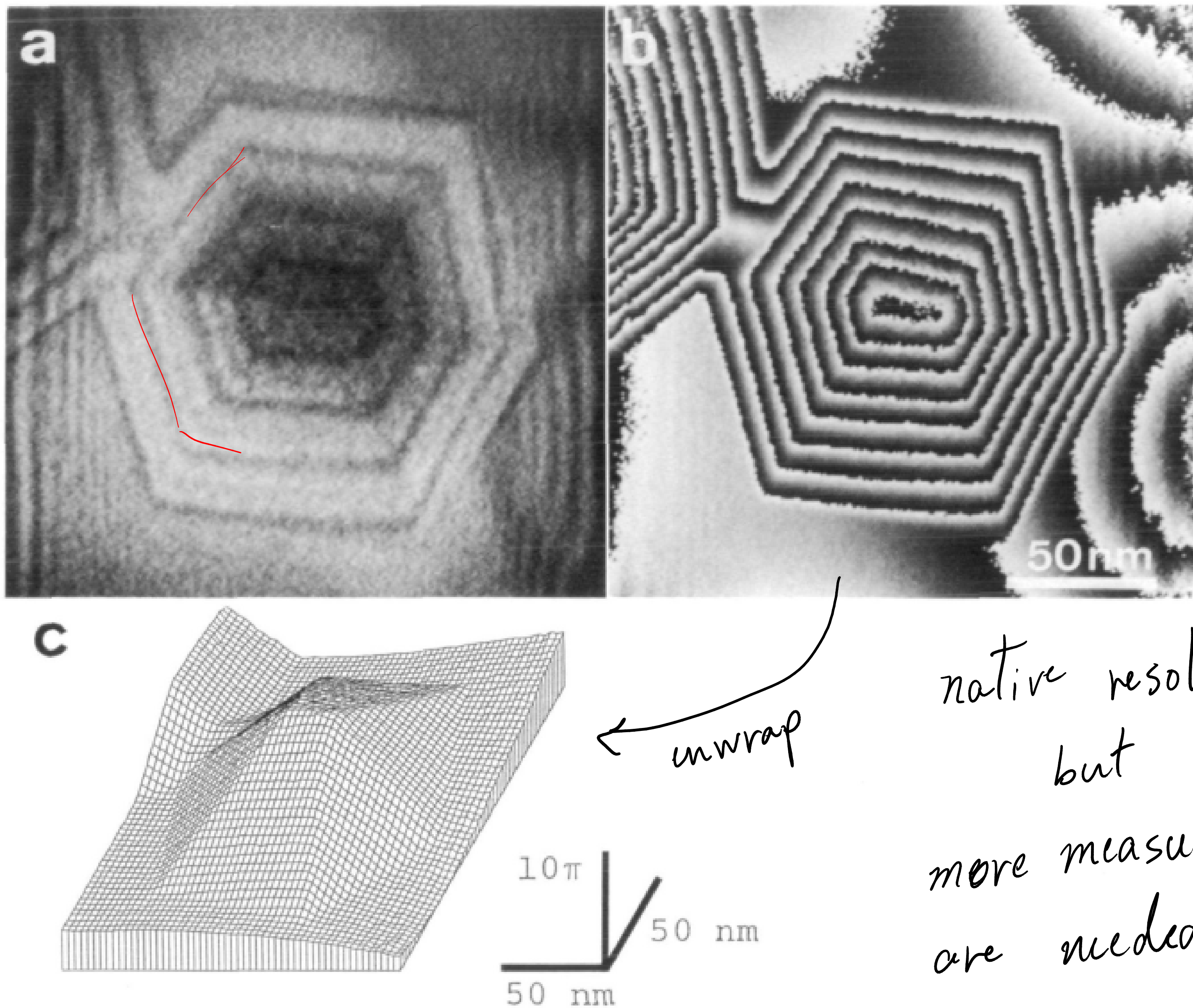
Fringe scanning



Electron microscopy

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Fringe scanning



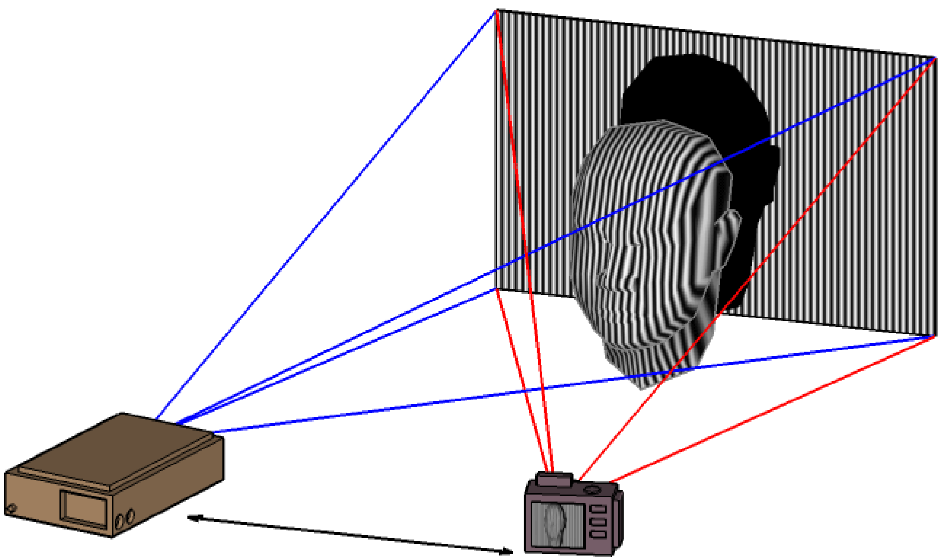
unwrap

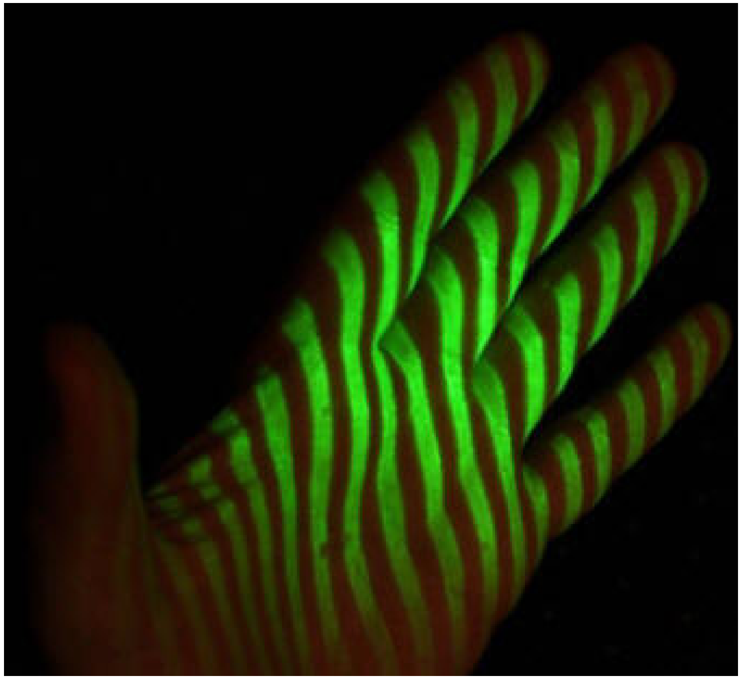
native resolution
but
more measurements
are needed!

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





Phase unwrapping

- Phase is measured only in the interval [0, 2π)
- Physical phase shifts (which can be larger) are wrapped on this interval

  → Any multiple of 2π is possible

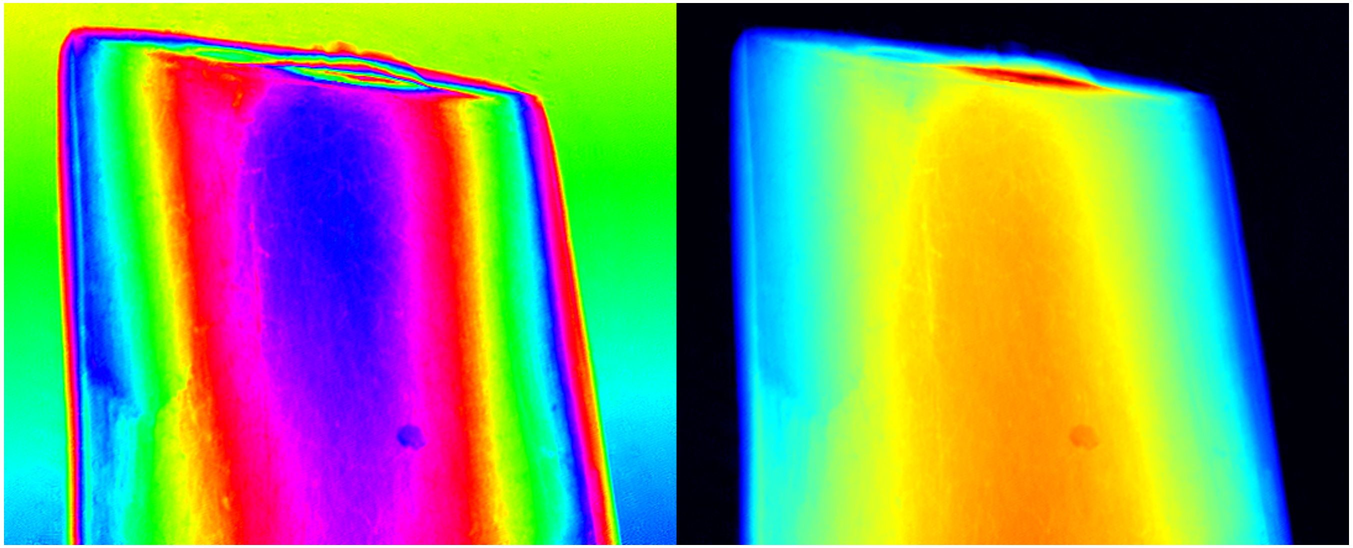
- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

path following methods

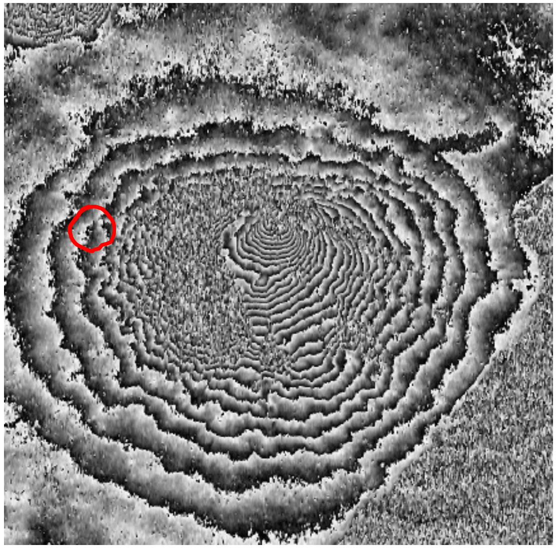
phase vortex connection

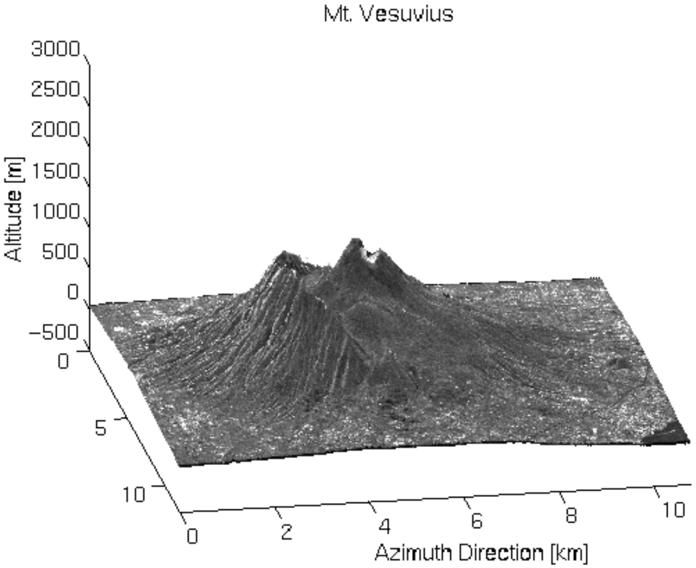
Complex-valued images

Phase unwrapping



phase vortex



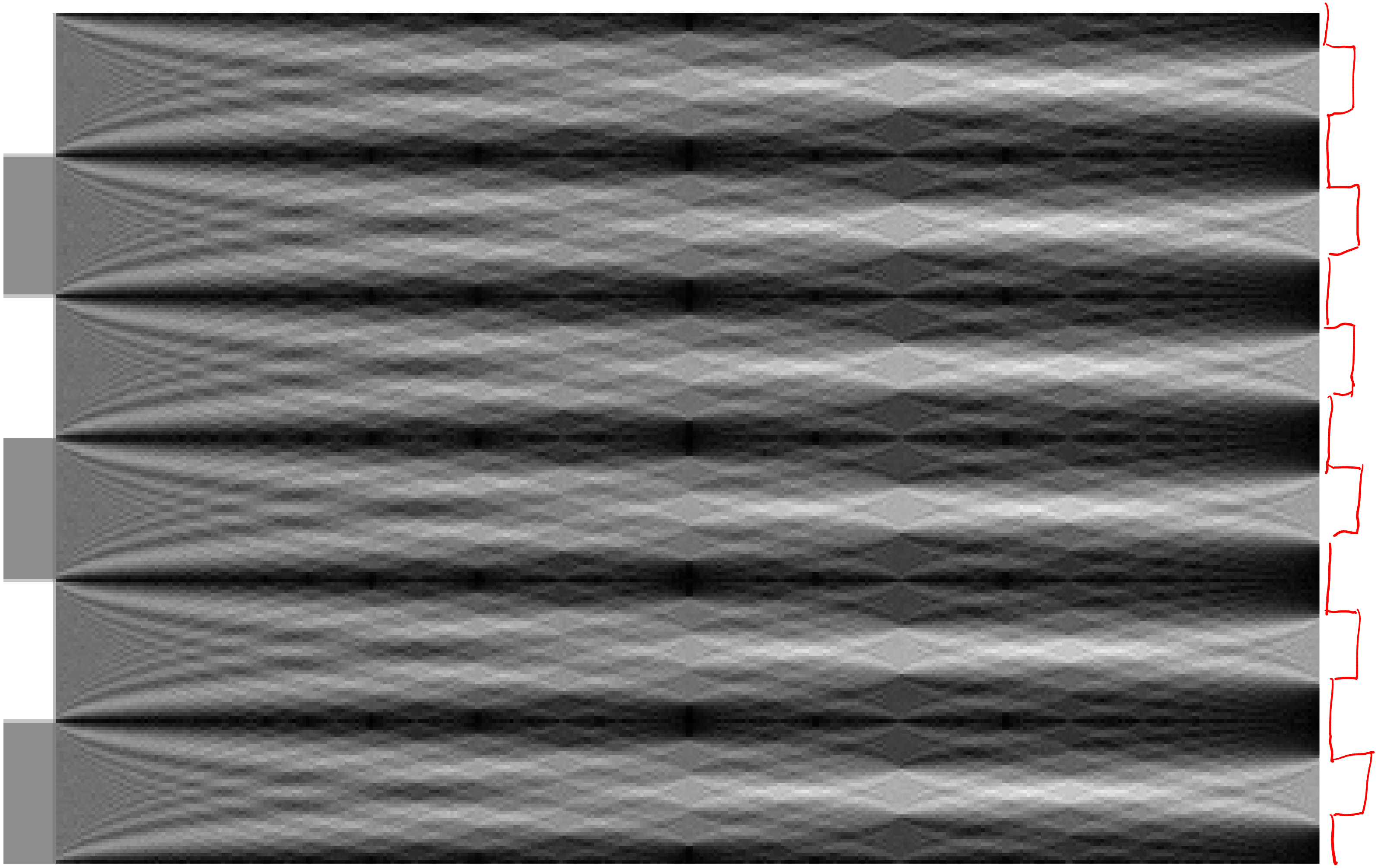


Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

Grating interferometry

Diffraction from a grating

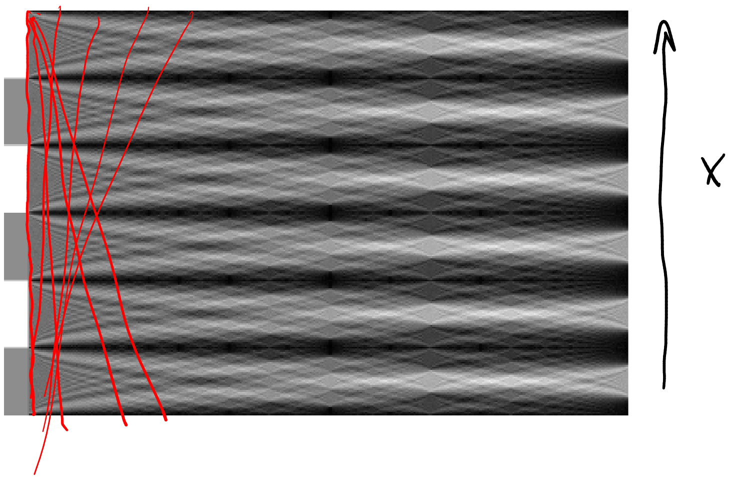
"Talbot carpet"

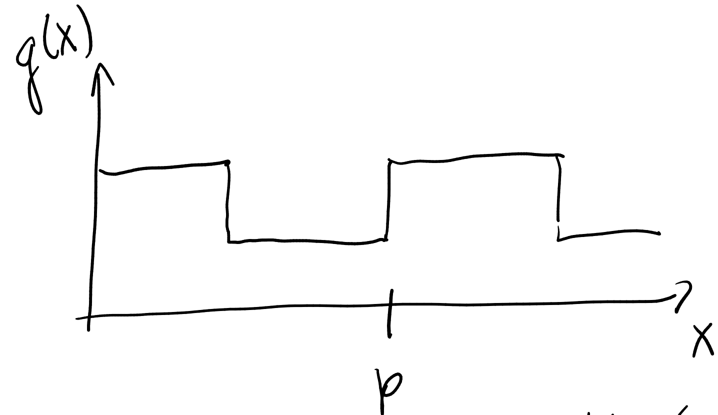


periodicity along the propagation axis : Talbot effect

Grating interferometry

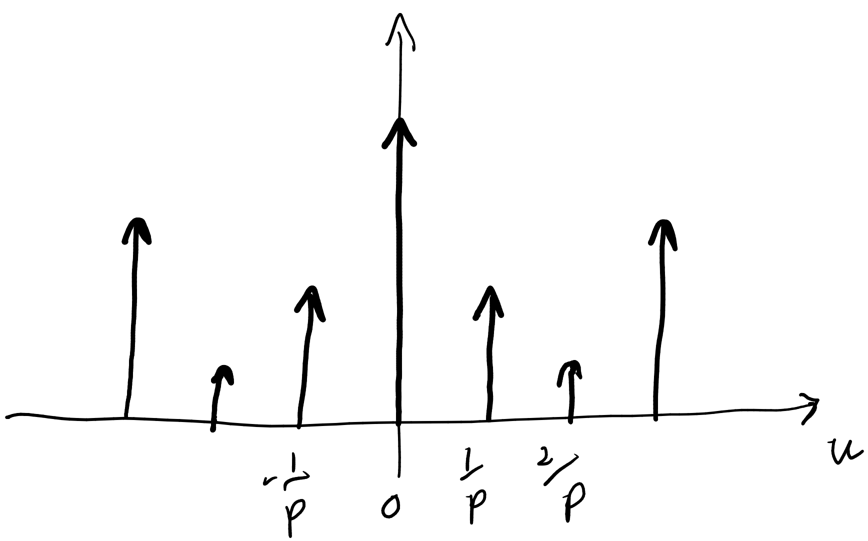
Diffraction from a grating

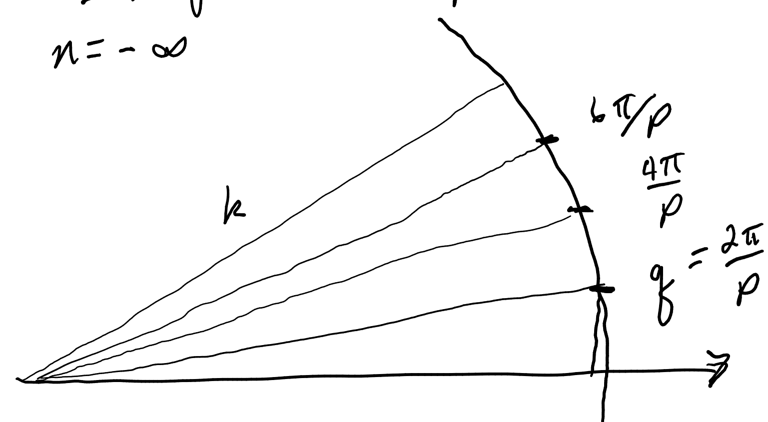




$$g(x) = \sum_{-\infty}^{\infty} g_n \, e^{2\pi i x n/p}$$

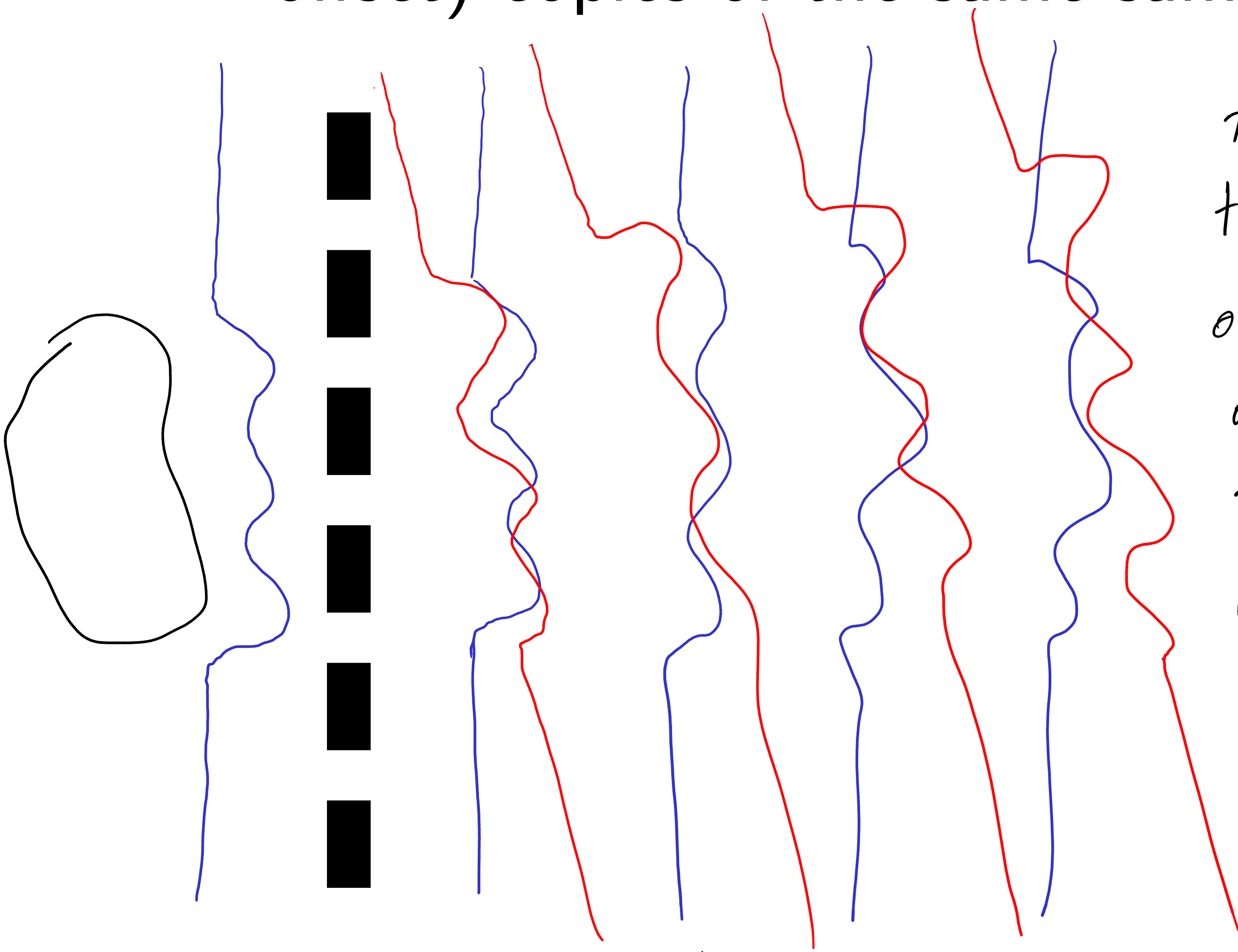
$$G(u) = \sum_{n=-\infty}^{\infty} g_n \, \delta\left(u - \frac{n}{p}\right)$$





Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.



the wavefield passed the grating is made of multiple copies of the incident wavefield travelling in different directions

$$\sin\theta = \frac{q}{k} = \frac{\frac{2\pi n}{p}}{\frac{2\pi}{\lambda}} = n\frac{\lambda}{p}$$

for $n=1$, the lateral shift between images $= z\sin\theta = \frac{\lambda z}{p}$

Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

e.g. if only orders ± 1 are relevant: $\longleftarrow$ $g_0 = 0$

exit of grating

$$\Psi(\vec{r}; z=0) = \psi_0(\vec{r}) \cdot g(\vec{r})$$

$$\psi(\vec{r}; z) = \mathcal{F}^{-1}\left\{ \mathcal{F}\left\{ \psi_0(\vec{r})\, g(\vec{r}) \right\} e^{-i\pi u^2 \lambda z} \right\}$$

$$= \mathcal{F}^{-1}\left\{ \underline{\Psi}_0(\vec{u}) * G(\vec{u})\, e^{-i\pi u^2 \lambda z} \right\}$$

$$= \mathcal{F}^{-1}\left\{ \Psi_0(\vec{u}) * \left(g_1 \delta\left(u - \frac{1}{p}\right) + g_{-1} \delta\left(u + \frac{1}{p}\right)\right) e^{\cdots} \right\}$$

$$= \mathcal{F}^{-1}\left\{ g_1 \Psi_0\left(\vec{u} - \frac{\hat{x}}{p}\right) + g_{-1} \Psi_0\left(\vec{u} + \frac{\hat{x}}{p}\right) e^{-\pi i u^2 \lambda z} \right\}$$

$$\mathcal{F}^{-1}\left\{\psi_0\left(\vec{u}-\frac{\hat{x}}{p}\right)e^{-\pi i u^2 \lambda z}\right\} \qquad \vec{u}' = \vec{u} - \frac{\hat{x}}{p}$$

$$u^2 = \left(\vec{u}' + \frac{\hat{x}}{p}\right)^2 = u'^2 + \frac{1}{p^2} + 2\frac{u'_x}{p}$$

$$\downarrow$$

$$= \mathcal{F}^{-1}\left\{\psi_0(\vec{u}')\, e^{-i\pi\lambda z\left(u'^2 + \frac{1}{p^2} + 2\frac{u'_x}{p}\right)}\right\}$$

$$e^{-i\pi\frac{\lambda z}{p^2}} \int d^2u'\, \underbrace{\psi_0(u')\, e^{-i\pi\lambda z u'^2}}\; \underbrace{e^{-2\pi i \frac{\lambda z}{p} u'_x}\; e^{2\pi i\left(\vec{u}' + \frac{\hat{x}}{p}\right)\cdot\vec{r}}}_{e^{2\pi i\, \vec{u}'\cdot\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)} \cdot e^{\frac{2\pi i x}{p}}}$$

$$= e^{-i\pi\frac{\lambda z}{p^2}}\, g_1\, e^{\frac{2\pi i x}{p}}\, \psi_0\left(\vec{r} - \frac{\lambda z}{p}\hat{x}; z\right)$$

$$\psi(\vec{r}; z) = e^{-i\pi\lambda z/p^2}\left(g_1 e^{2\pi i x/p}\,\psi_0\left(\vec{r} - \frac{\lambda z}{p}; z\right) + g_{-1} e^{-\frac{2\pi i x}{p}}\,\psi_0\left(\vec{r} + \frac{\lambda z}{p}; z\right)\right)$$

$g_1 = g_{-1}^* \in \mathbb{R}$ for simplicity

$$I = |\psi(\vec{r};z)|^2 \simeq |g_1|^2 \left(\left|\psi_0\left(\vec{r} - \frac{\lambda z}{p}\hat{x}; z\right)\right|^2 + \left|\psi_0\left(\vec{r} + \frac{\lambda z}{p}\hat{x}; z\right)\right|^2 \right)$$

$$+ 2\,\mathrm{Re}\left\{ g_1^2\, e^{4\pi i x/p}\, \psi_0\left(\vec{r} - \frac{\lambda z}{p}\hat{x}; z\right) \psi_0^*\left(\vec{r} + \frac{\lambda z}{p}\hat{x}; z\right) \right\}$$

$$\psi_0(\vec{r};z) = a(\vec{r})\, e^{i\varphi(\vec{r})}$$

$$I \simeq |g_1|^2 \underbrace{\left(a^2\left(\vec{r} - \frac{\lambda z}{p}\right) + a^2\left(\vec{r} + \frac{\lambda z}{p}\right)\right)}_{\simeq 2a^2(\vec{r})} + \overbrace{2a\left(\vec{r} + \frac{z\lambda}{p}\right) a\left(\vec{r} - \frac{z\lambda}{p}\right)}^{\simeq 2a^2(\vec{r})} |g_1|^2$$

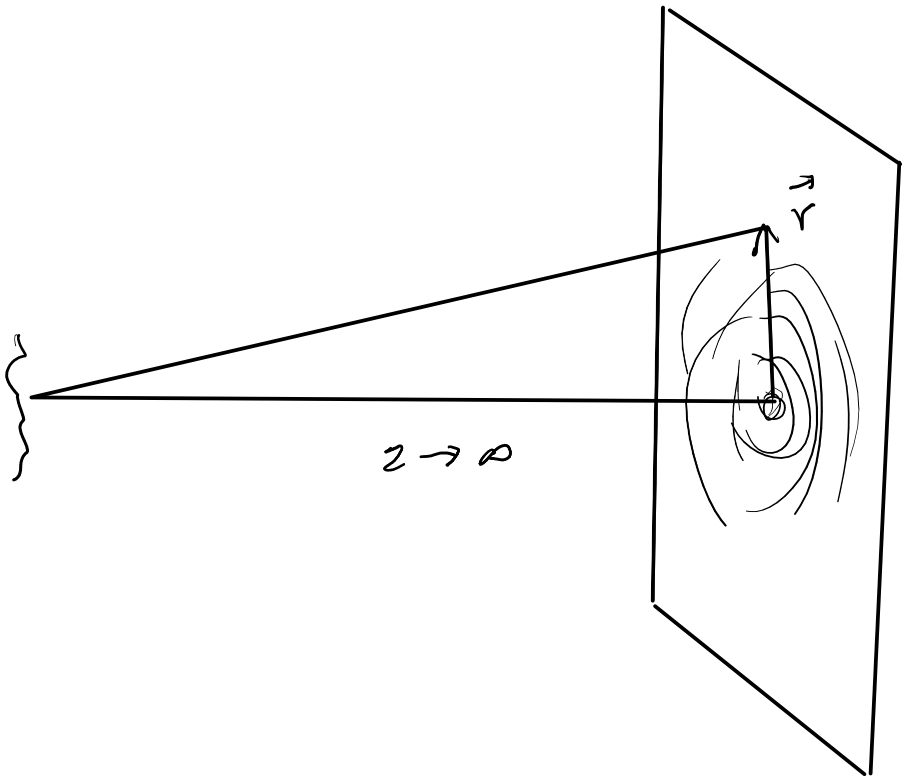
$$\cos\left(\underbrace{\varphi\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) - \varphi\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}_{\simeq \frac{2\lambda z}{p}\frac{\partial \varphi}{\partial x}} + \frac{4\pi x}{p} \right)$$

$$I(\vec{r}) \simeq 2|g_1|^2 a^2(\vec{r}) \left(1 + \cos\left(\frac{2\lambda z}{p}\frac{\partial \varphi}{\partial x} + 4\pi x/p \right)\right)$$

analysis yields $a^2(\vec{r})$ and $\frac{\partial \varphi}{\partial x}$ $\rightarrow$ "Differential phase contrast"

Far-field diffraction

The Fraunhofer regime



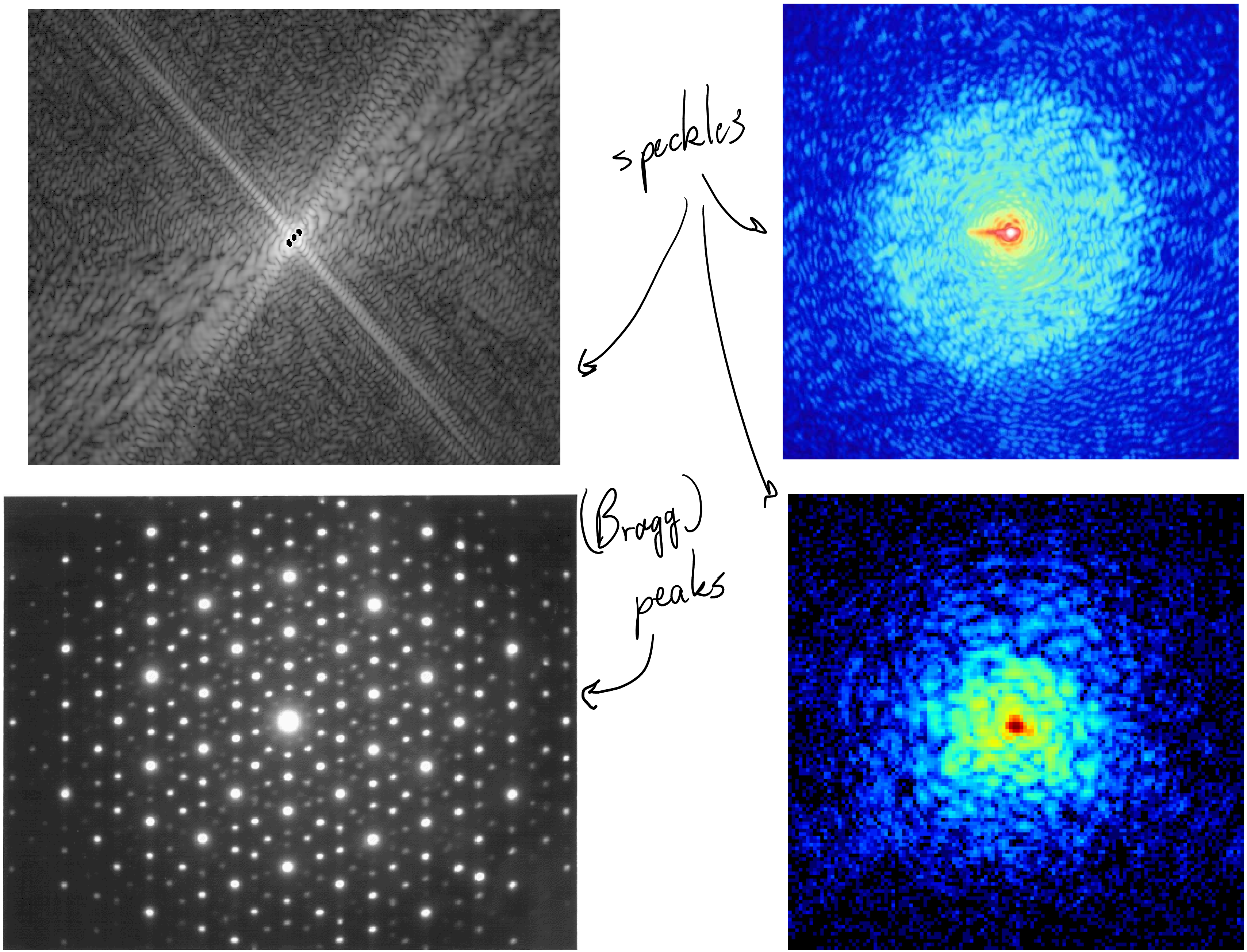
$$\frac{\vec{r}}{z} = \frac{\vec{q}}{k} = \lambda \vec{u}$$

$$\left|\psi(\vec{r}; z \to \infty)\right|^2 \propto \left|\mathcal{F}\psi\right|^2 \left(\vec{u} = \frac{\vec{r}}{\lambda z}\right)$$

$$\downarrow$$

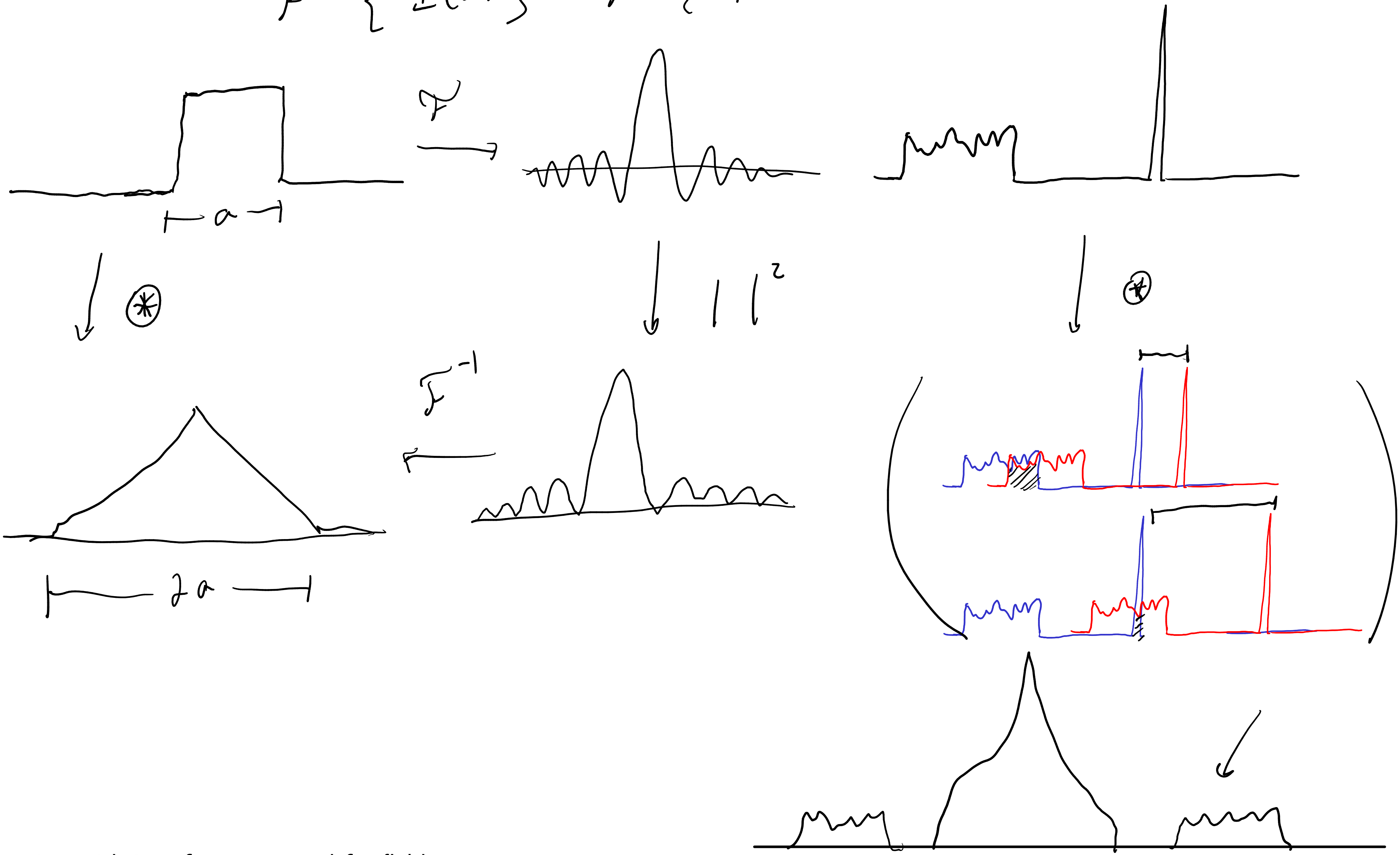
$$I(\vec{u})$$

Diffraction patterns



Diffraction and autocorrelation

$$\mathcal{F}^{-1}\{I(u)\} = \mathcal{F}^{-1}\{\psi(\vec{u})\cdot\psi^*(\vec{u})\} = \psi(\vec{r}) \circledast \psi(\vec{r})$$



Fourier transform holography

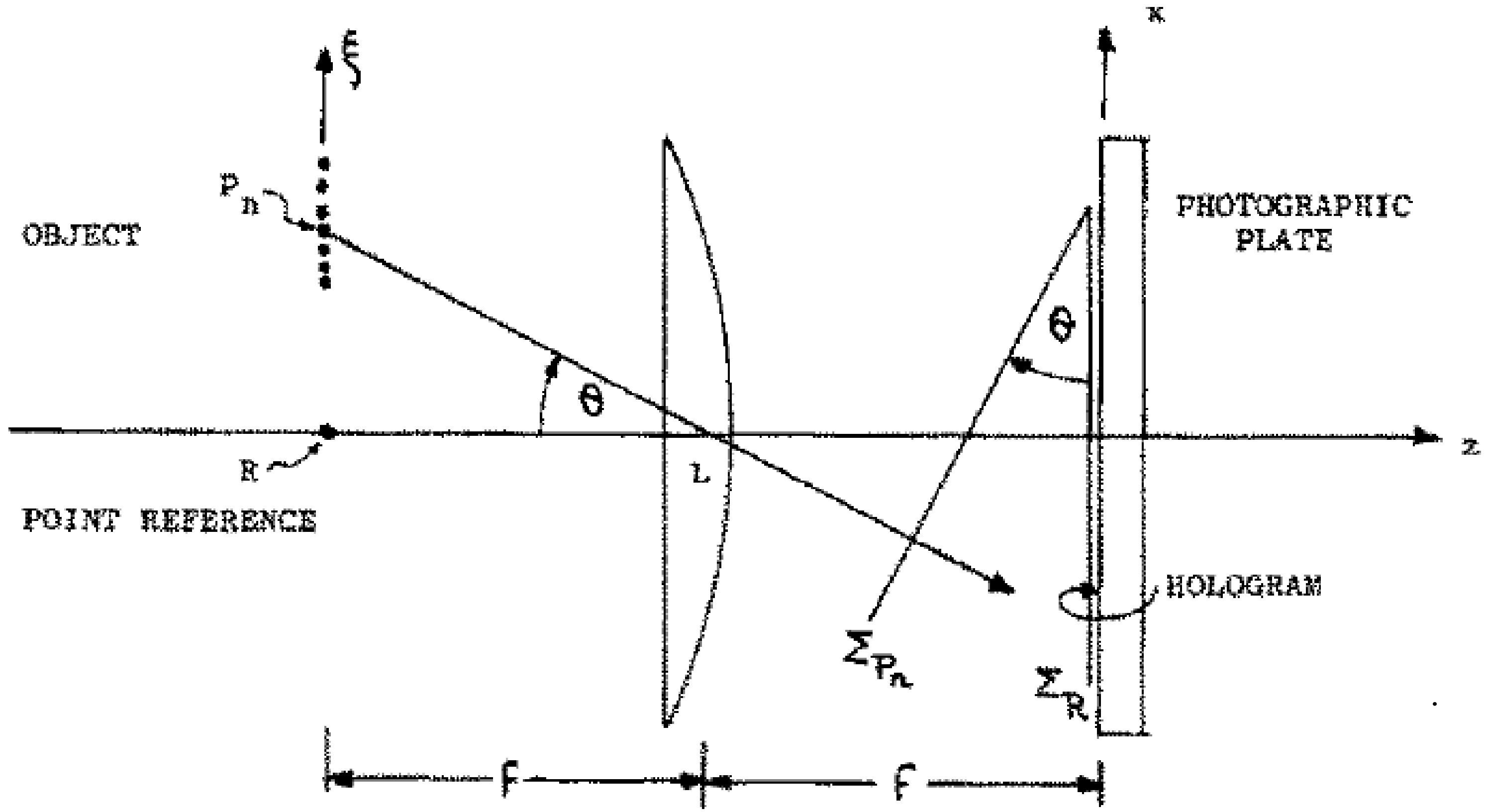
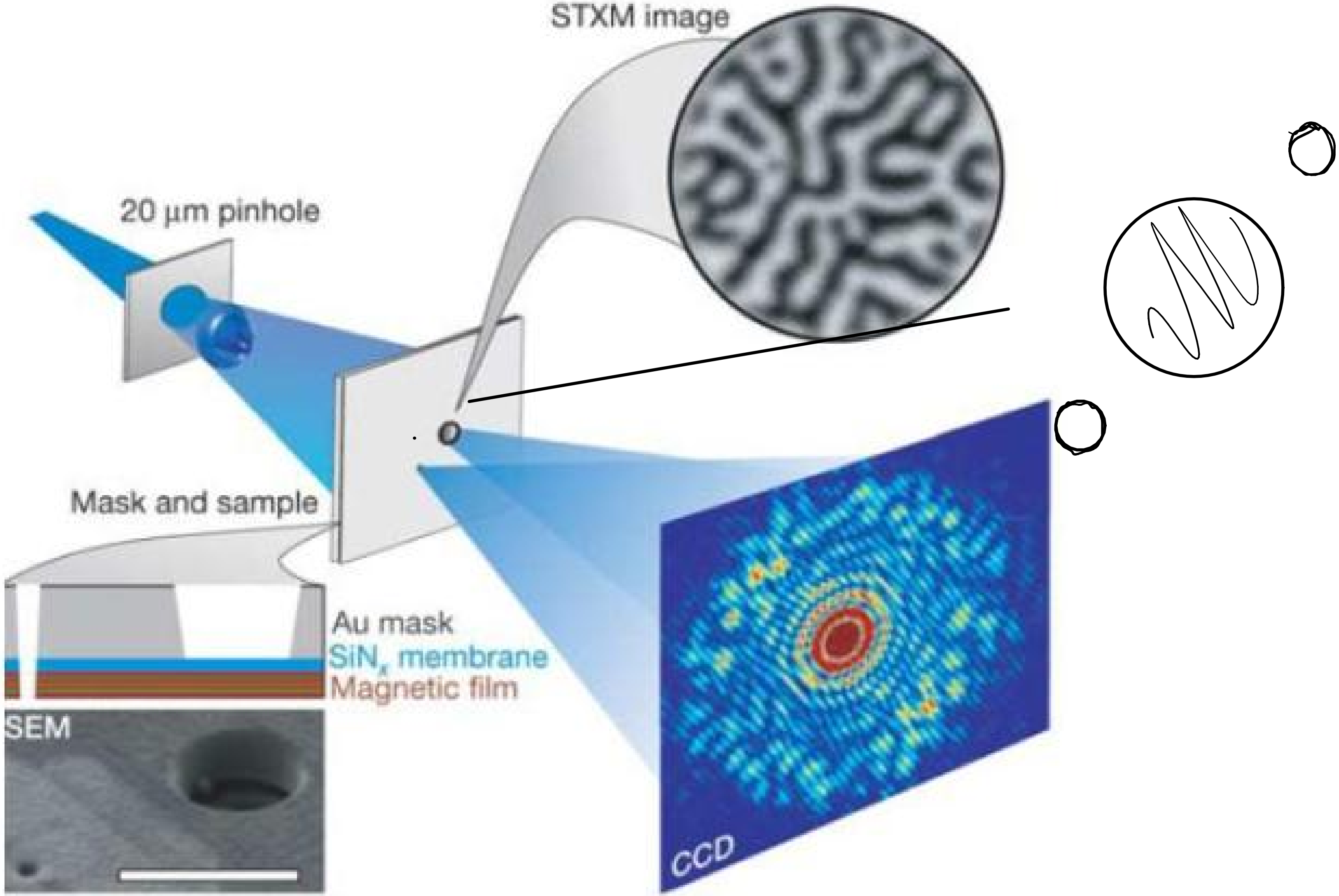


Fig. 1. Recording of a Fourier-transform hologram with a lens L. Σ_R = reference wavefront.

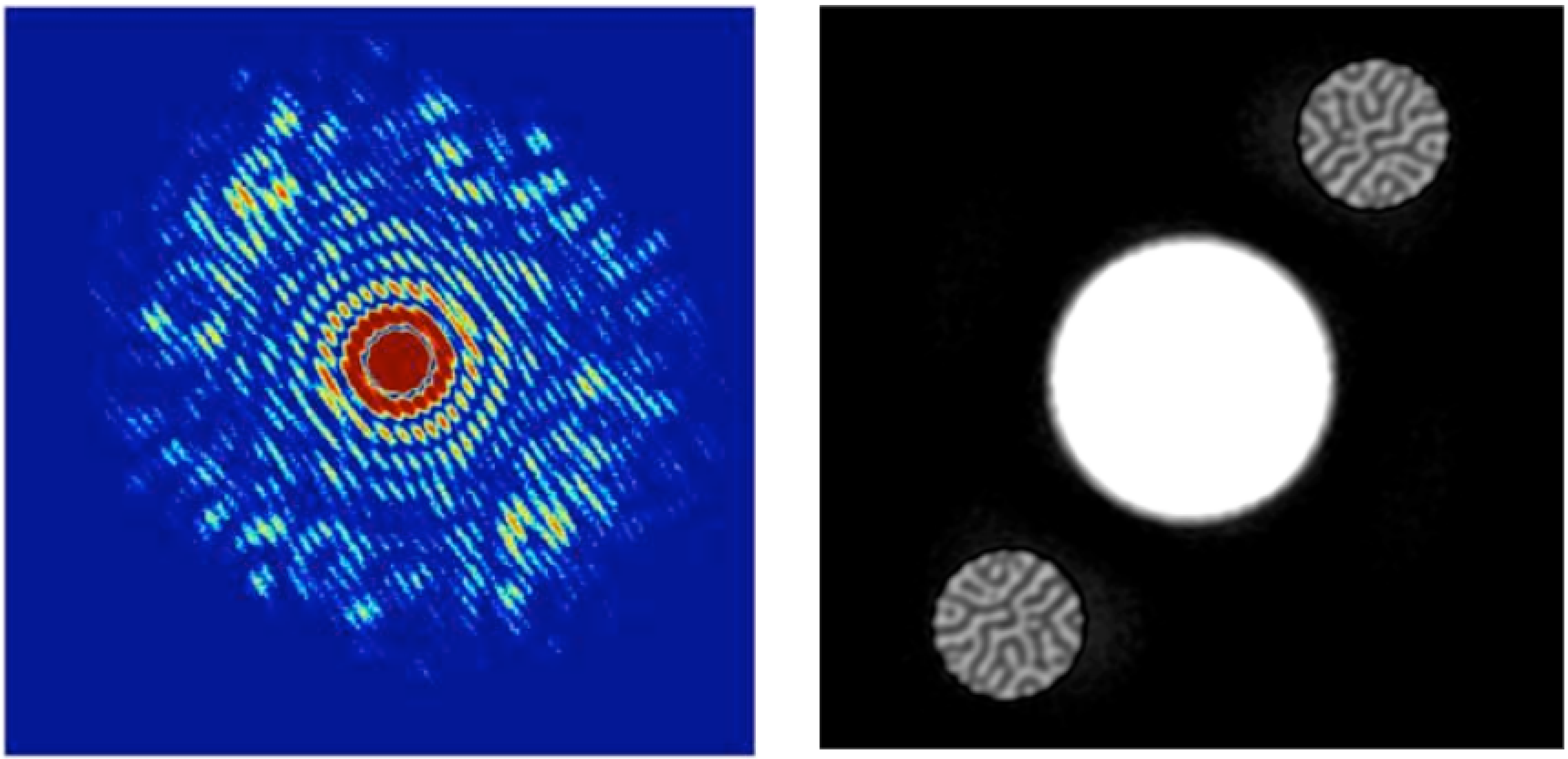
Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

Fourier transform holography



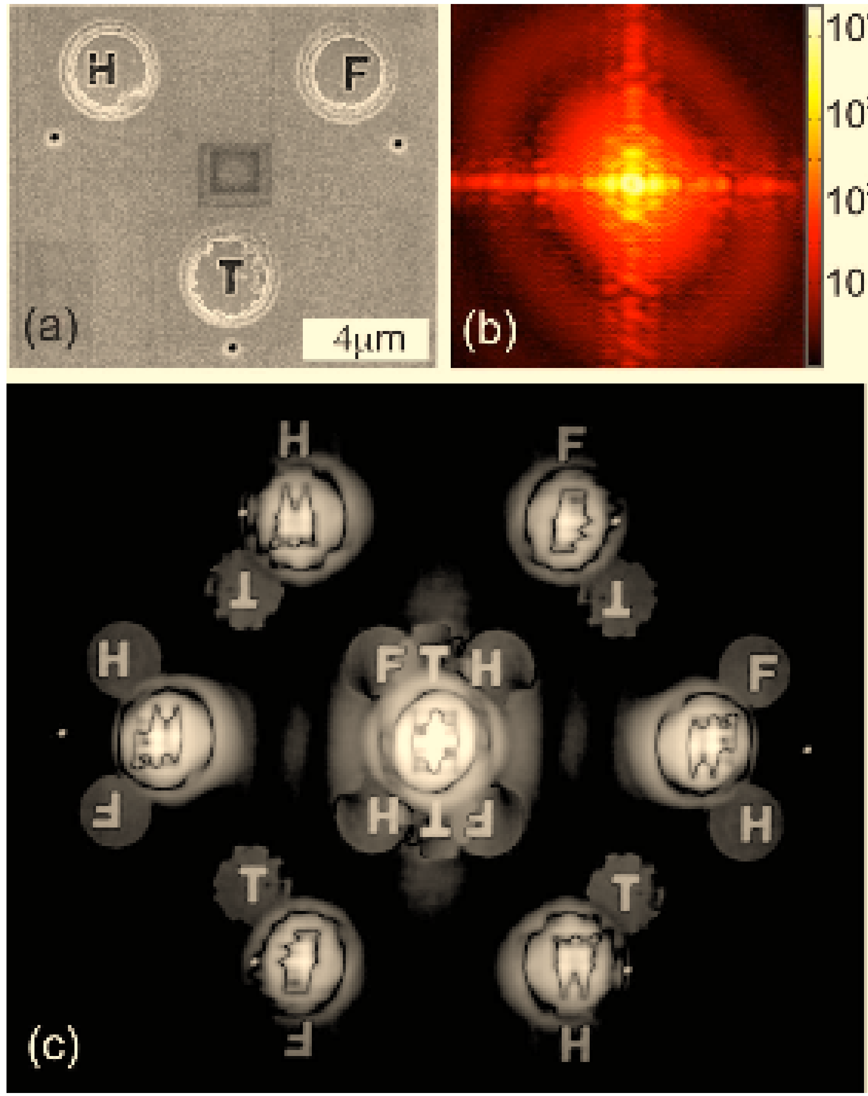
Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

Fourier transform holography

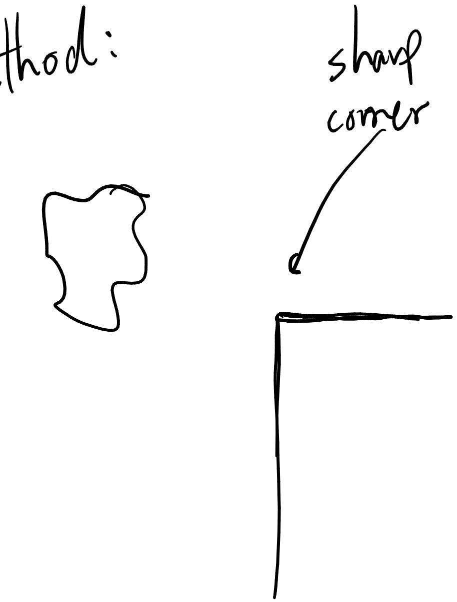


Fourier transform holography

Multiple references



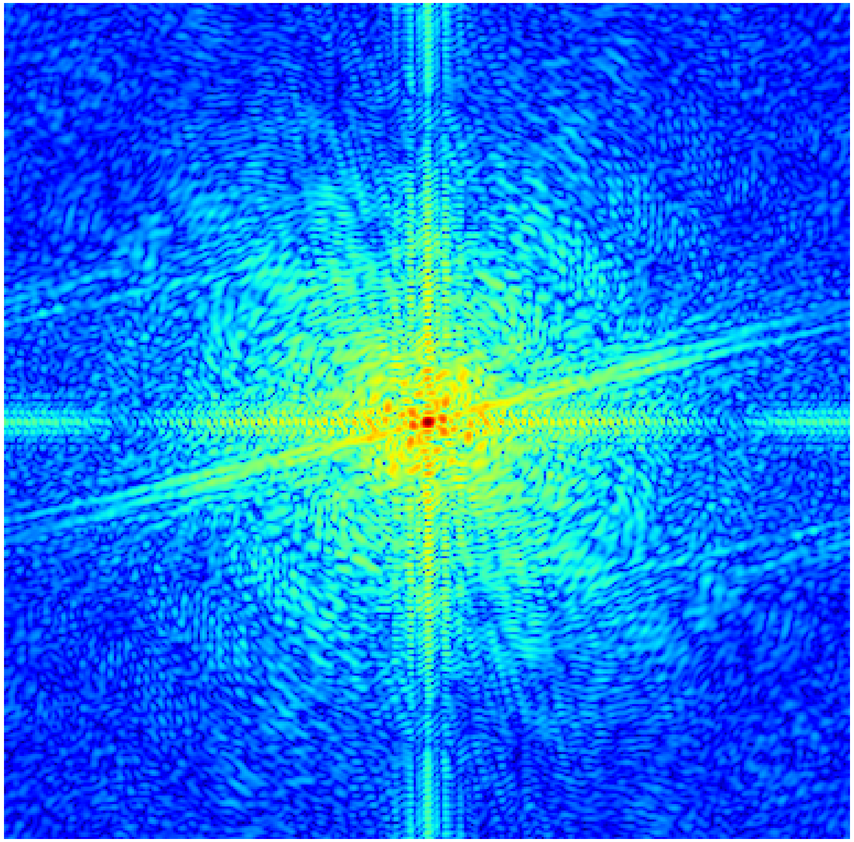
Another similar method:



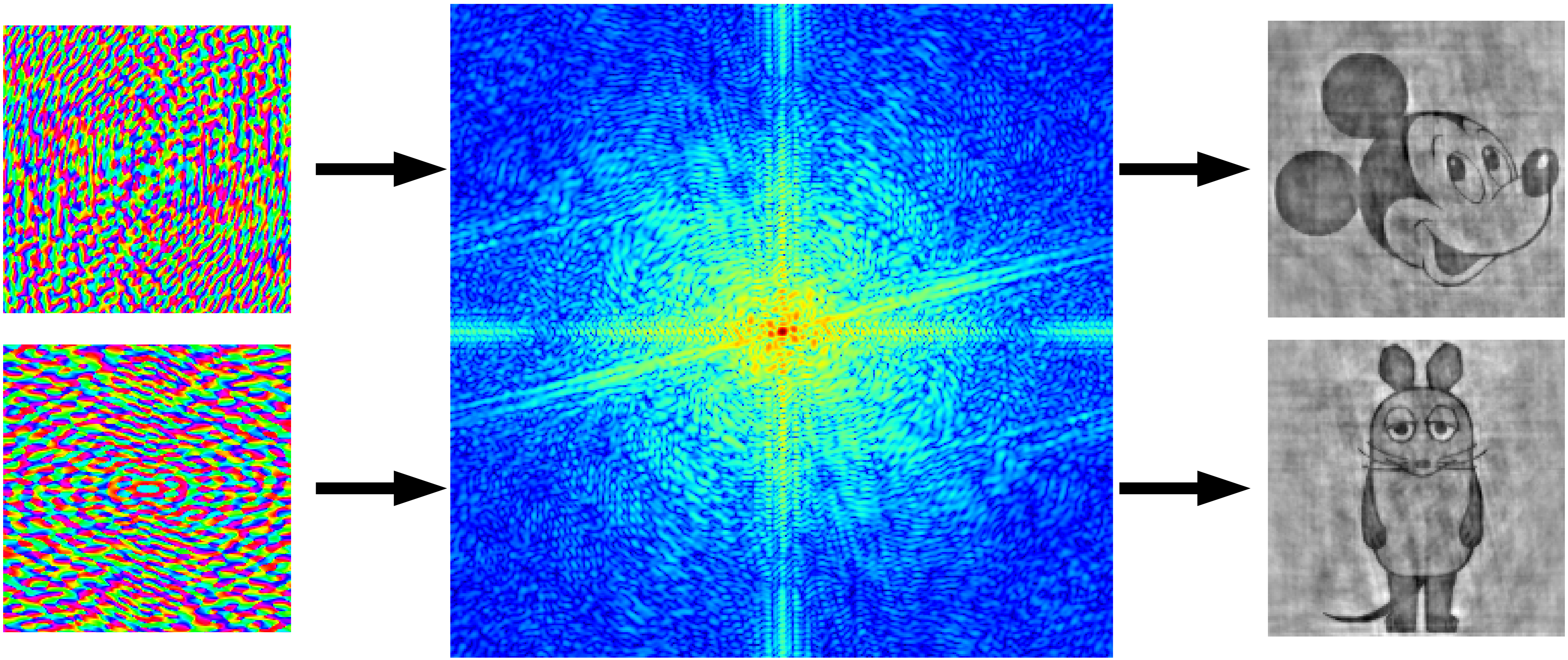
"HERALDO"

Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

Coherent diffractive imaging

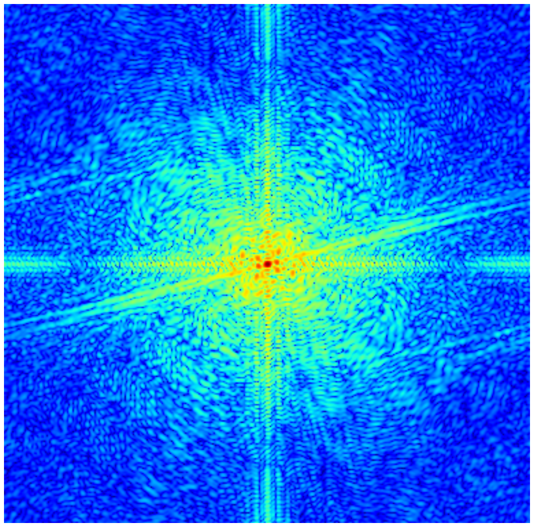


The phase problem

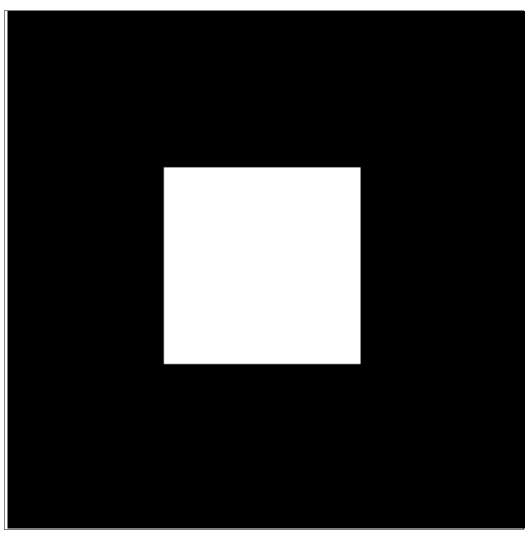


Coherent diffractive imaging

Two constraints

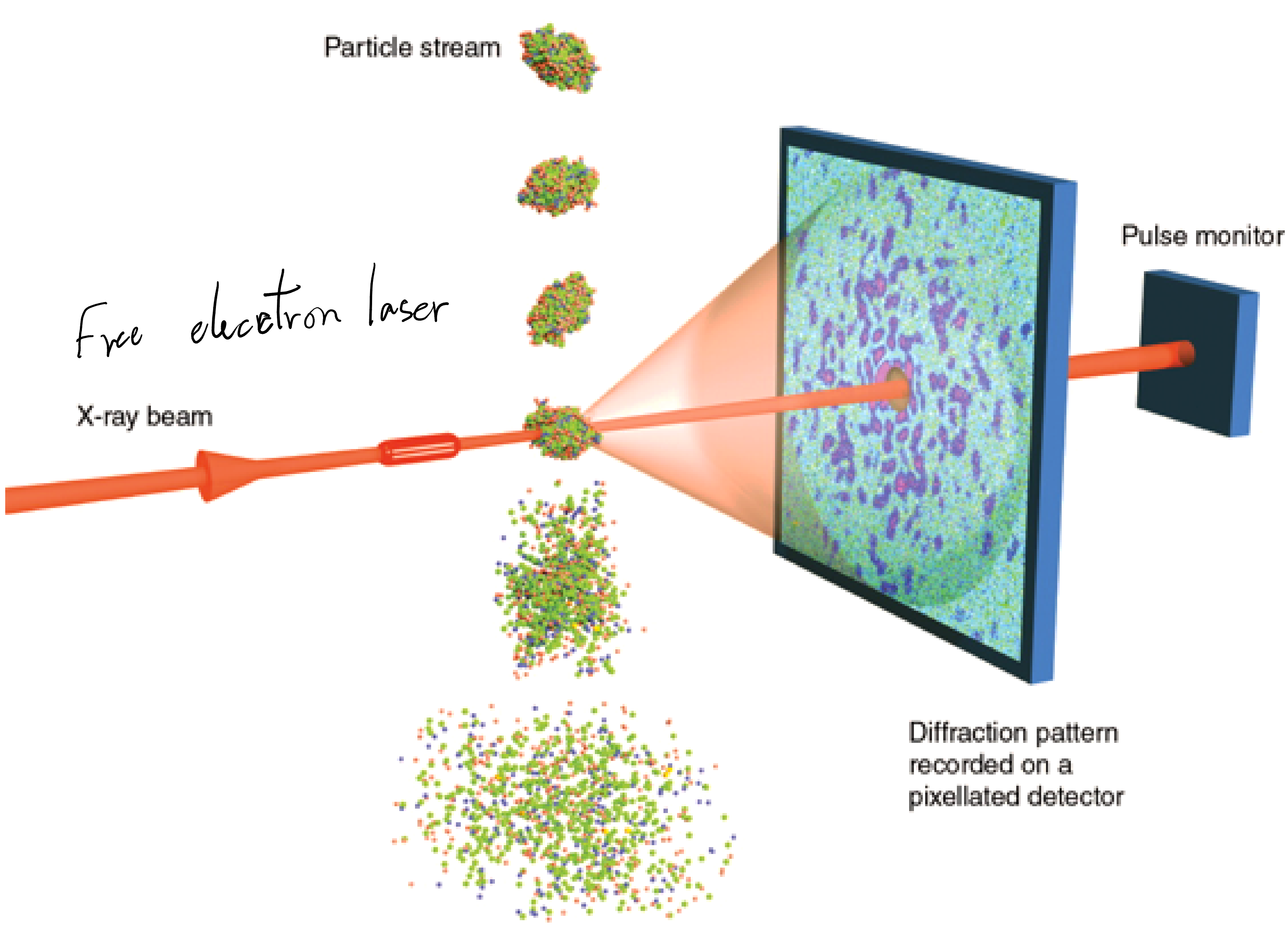


1. Solution is consistent with measured Fourier amplitudes



2. Solution is isolated

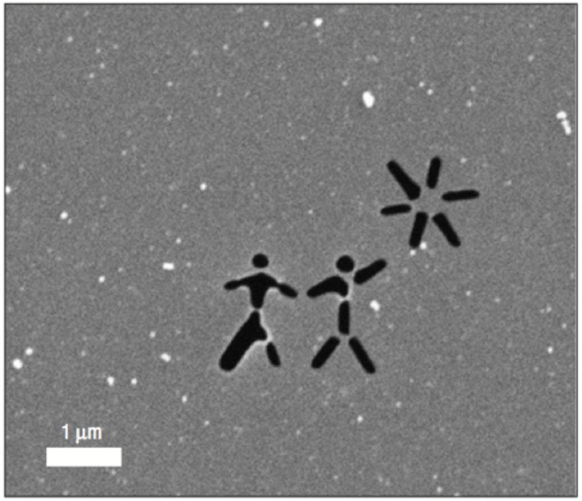
Radiation damage limits on radiation

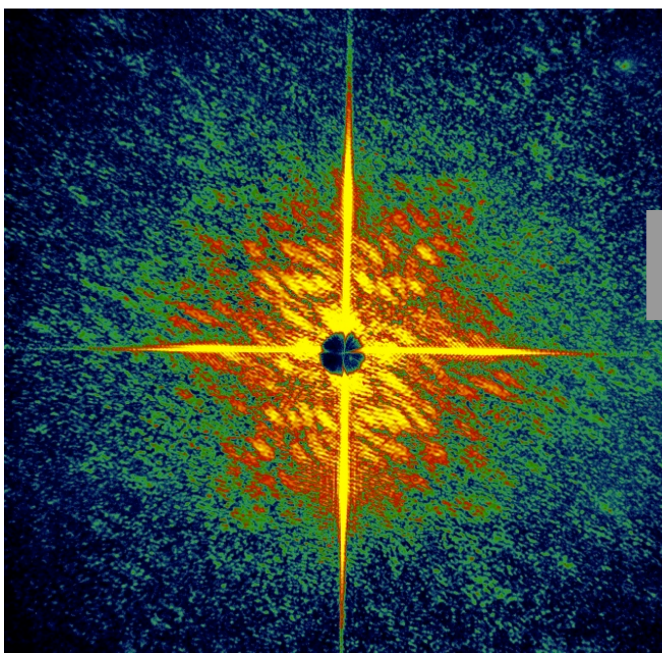


R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

"Diffraction before destruction"

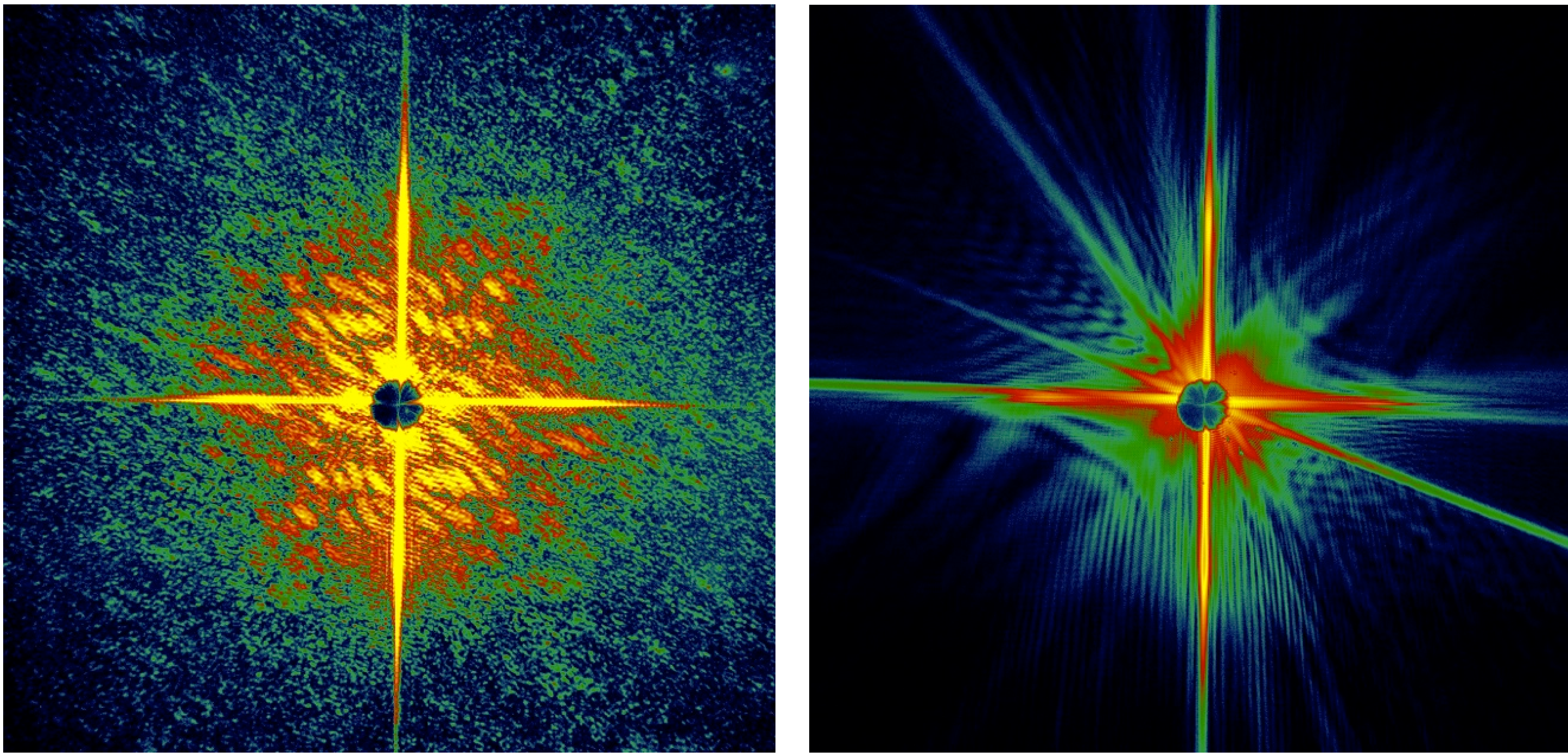




H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

“Diffraction before destruction”

The imaging pulse vaporized the sample



H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction

Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld

Von R. Hegerl und W. Hoppe

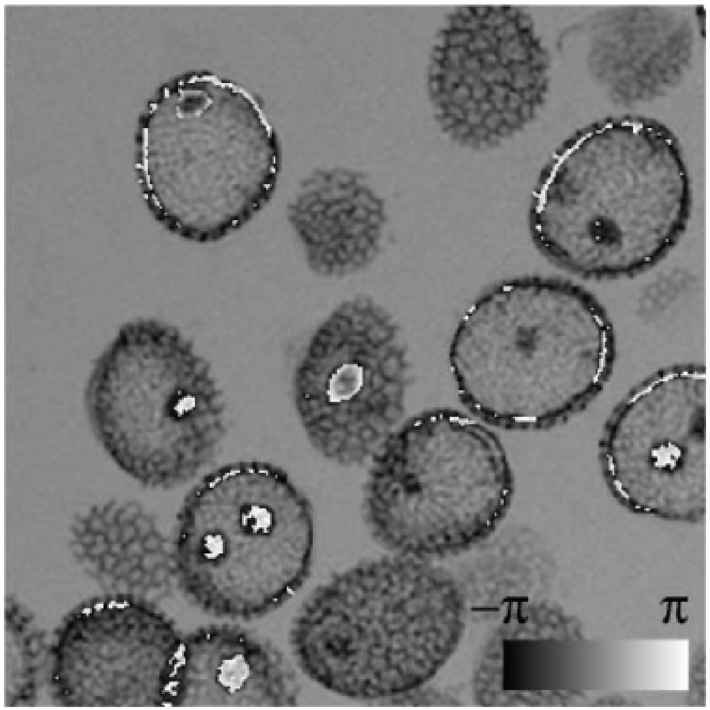
Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" (πτυξ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

1969 - 1970

Ptychography

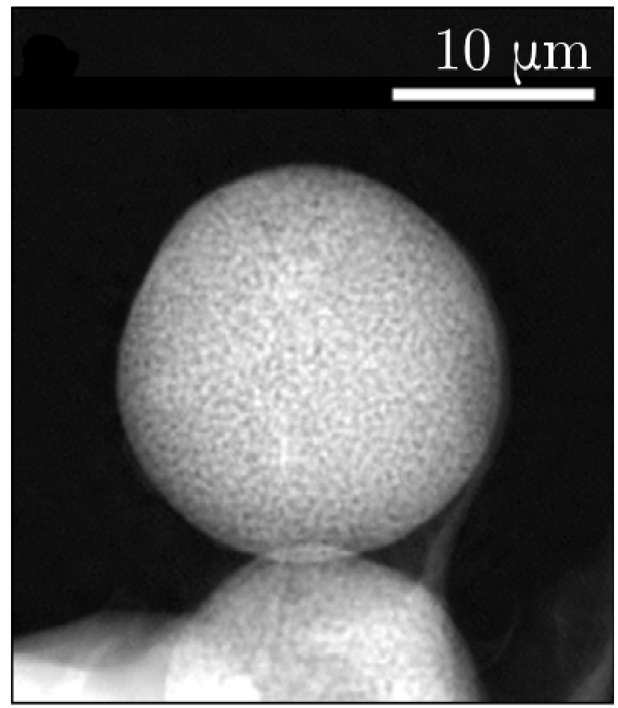
A few examples

Visible light



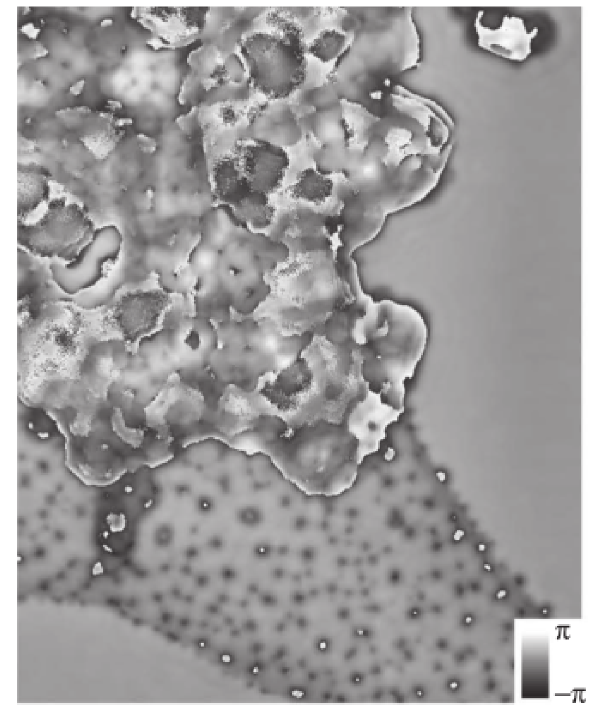
A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

X-rays



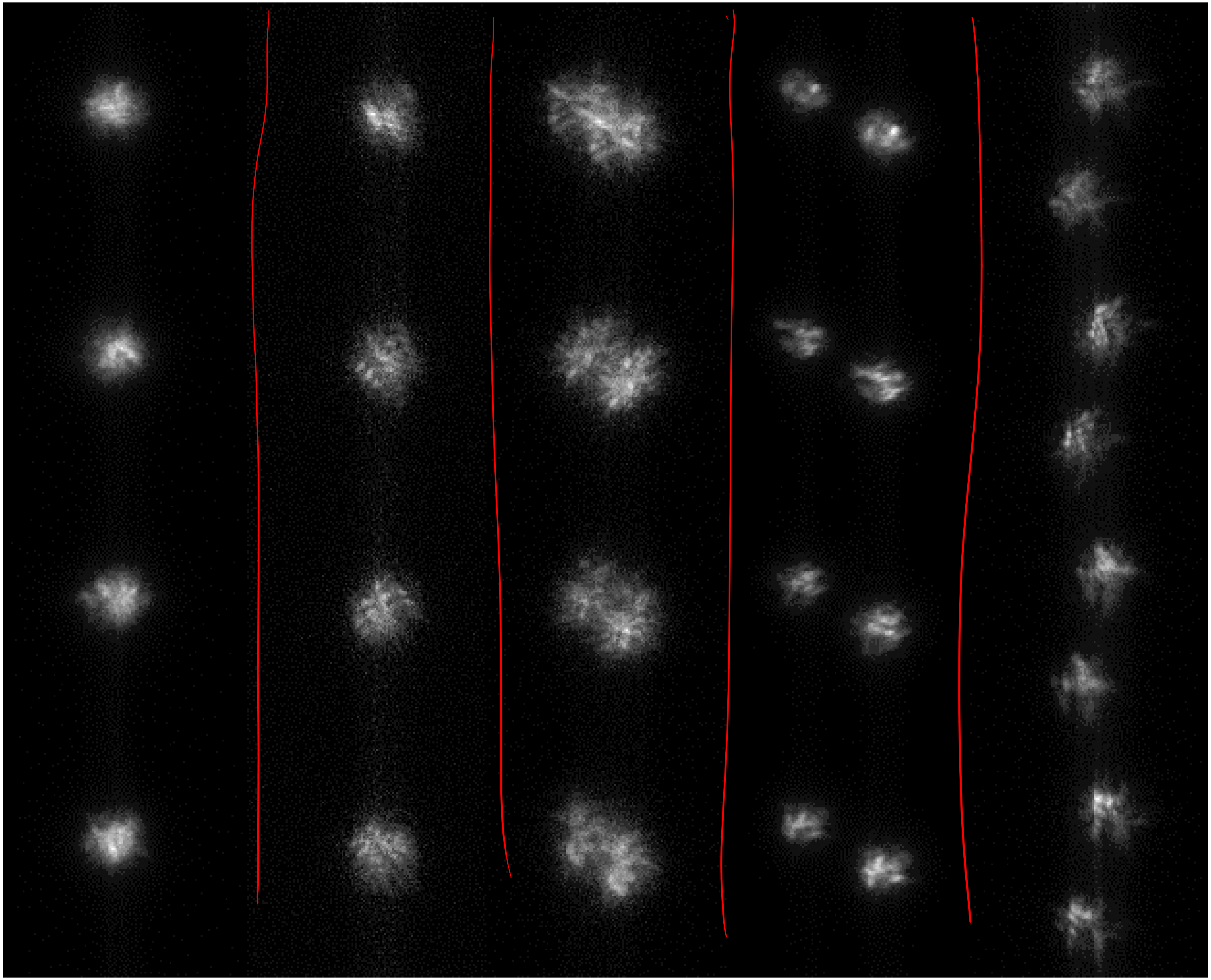
P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).

electrons



M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

Speckle imaging in astronomy



result of air turbulence

Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

Speckle imaging in astronomy

Model

one measurement:

$$I(\vec{r}) = O * P$$

instantaneous point-spread function

$$\tilde{I}(\vec{u}) = \tilde{O} \cdot M \leftarrow \text{MTF}$$

$$|\tilde{I}(\vec{u})|^2 = |\tilde{O}|^2 |M|^2$$

can be modeled from fluid dynamics

average over multiple independent measurements

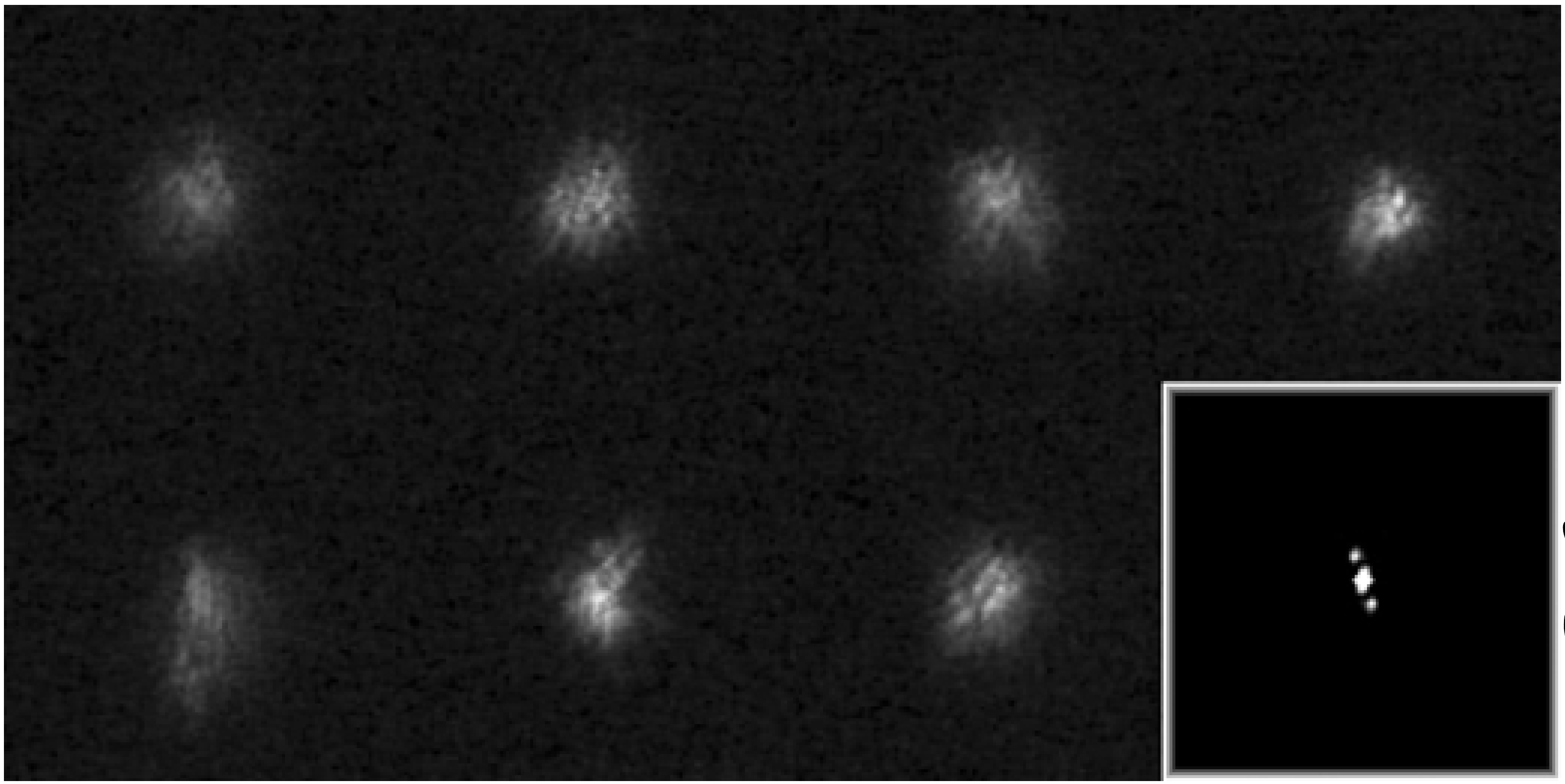
$$\langle |\tilde{I}(\vec{u})| \rangle = |\tilde{O}|^2 \langle |M|^2 \rangle$$

$$|\tilde{O}|^2 \simeq \frac{\langle |\tilde{I}|^2 \rangle}{\langle |M_{model}|^2 \rangle}$$

recovering O from $|\tilde{O}|^2$ same coherent diffractive imaging

Speckle imaging in astronomy

Retrieval of the autocorrelation



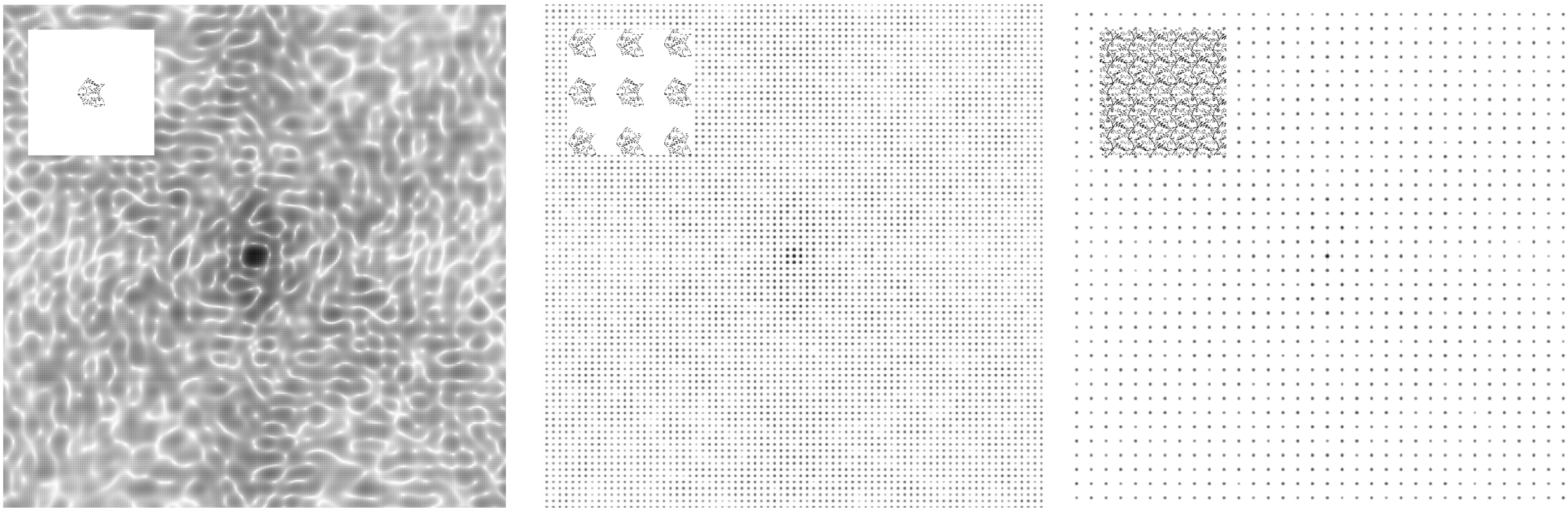
autocorrelation of O

$\mathcal{F}^{-1}\{|\tilde{O}|^2\}$

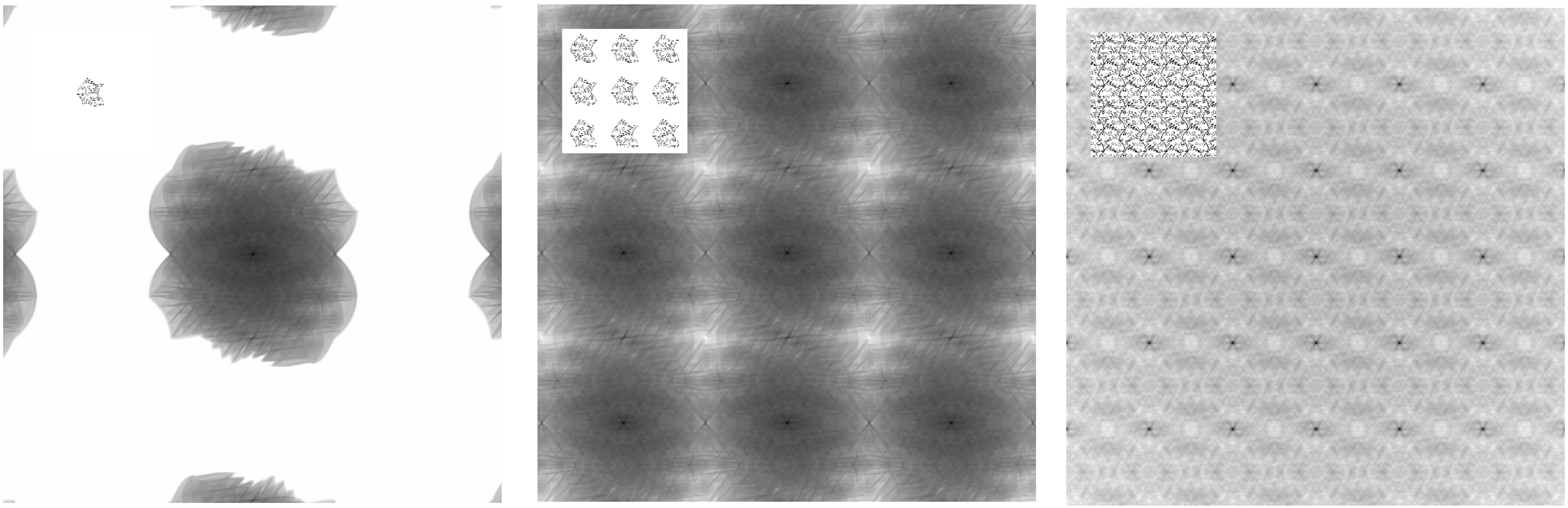
Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

Crystallography

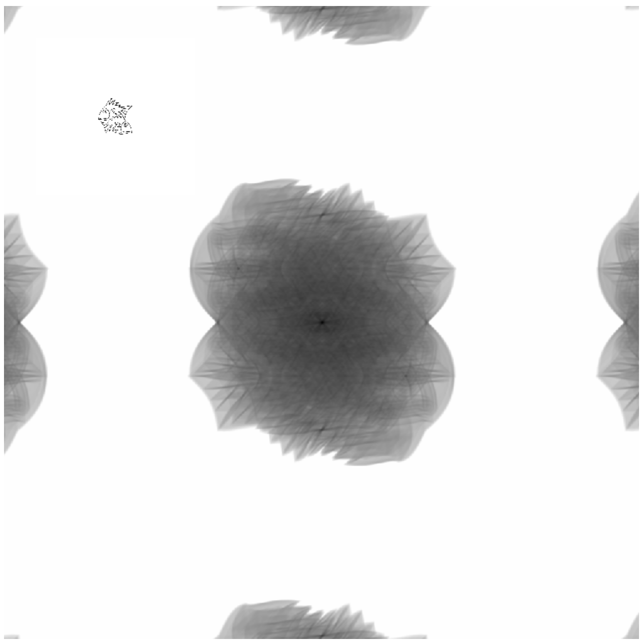
Diffraction by a crystal: Bragg peaks

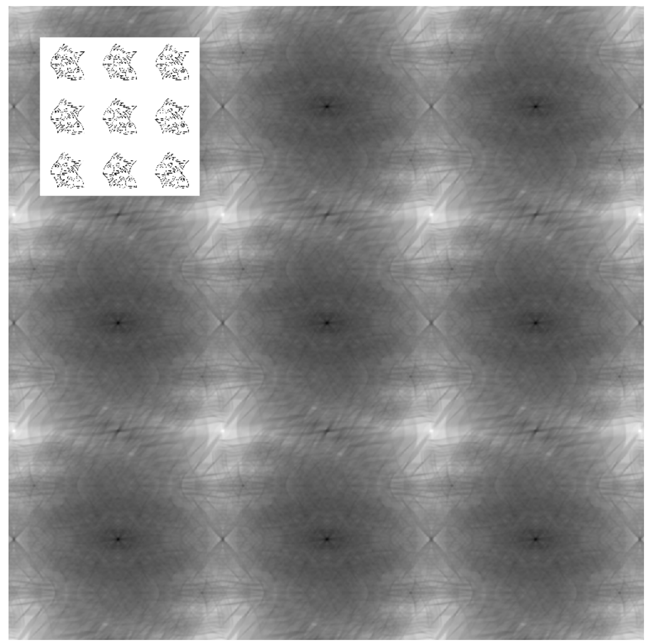


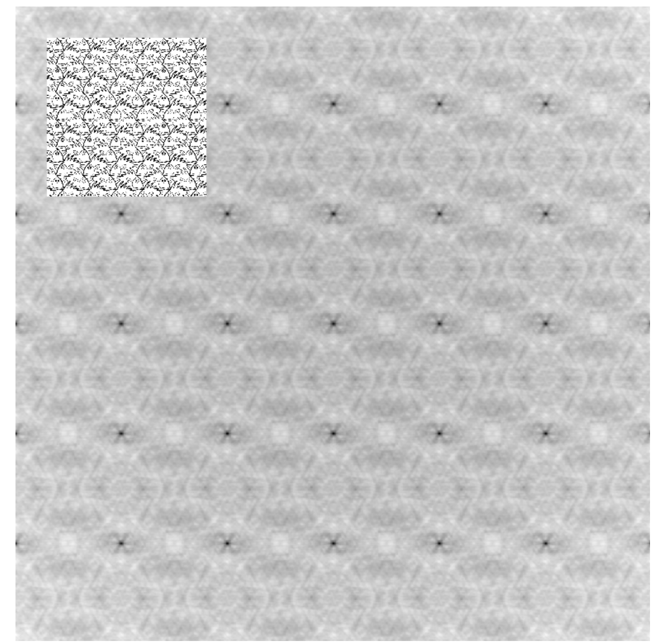
Crystallography
Fourier transform of intensity: autocorrelation



Crystallography

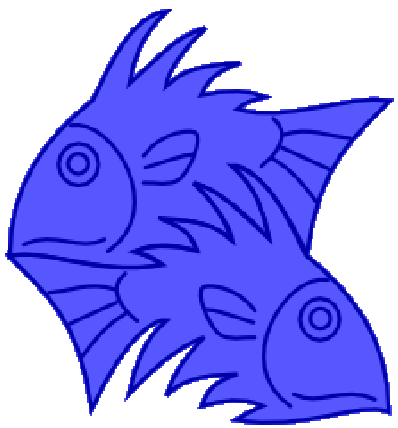




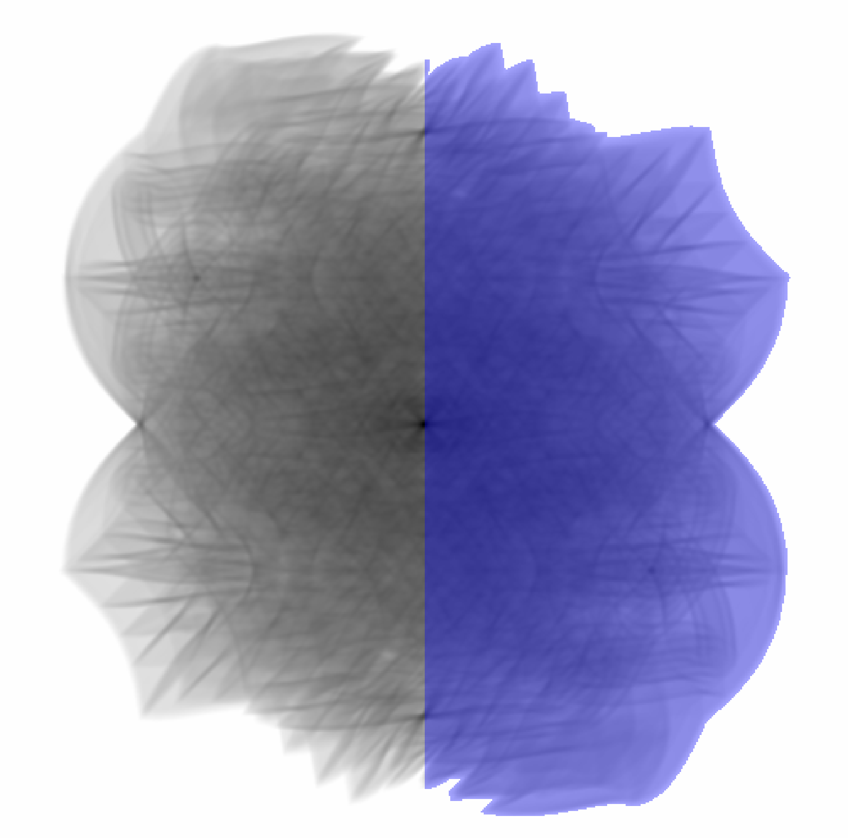


Crystallography

Problem is overconstrained with an isolated sample



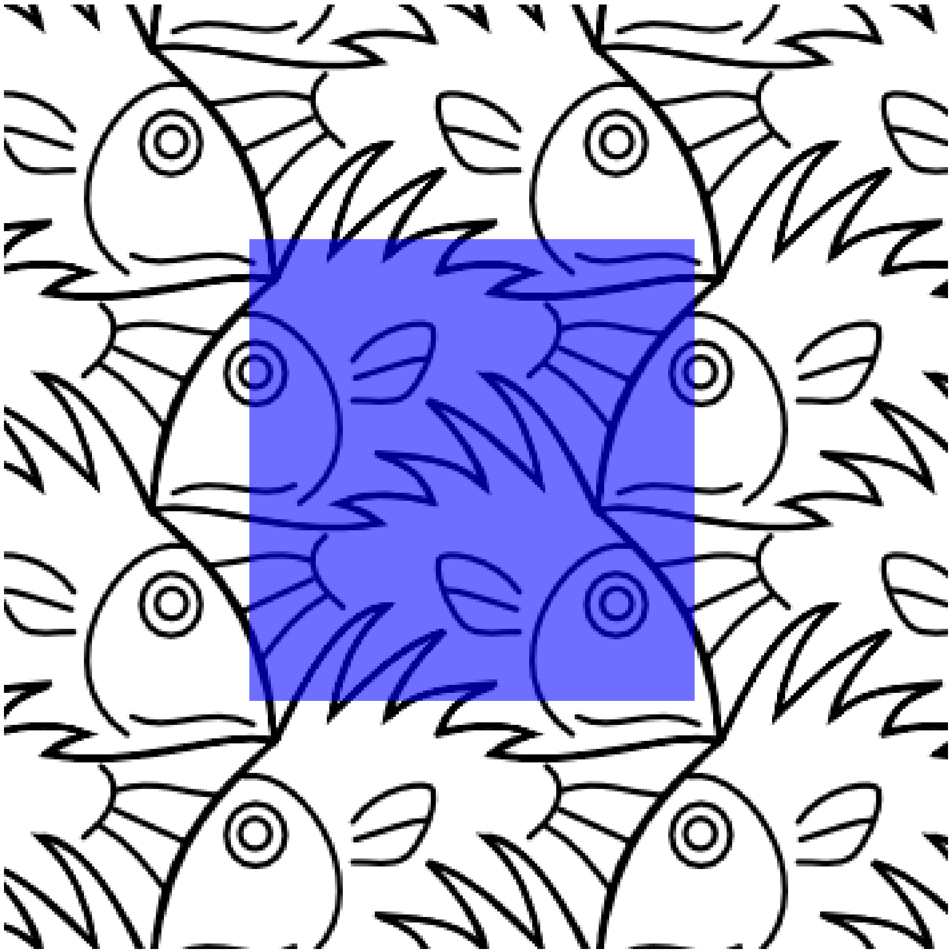
unknowns = N

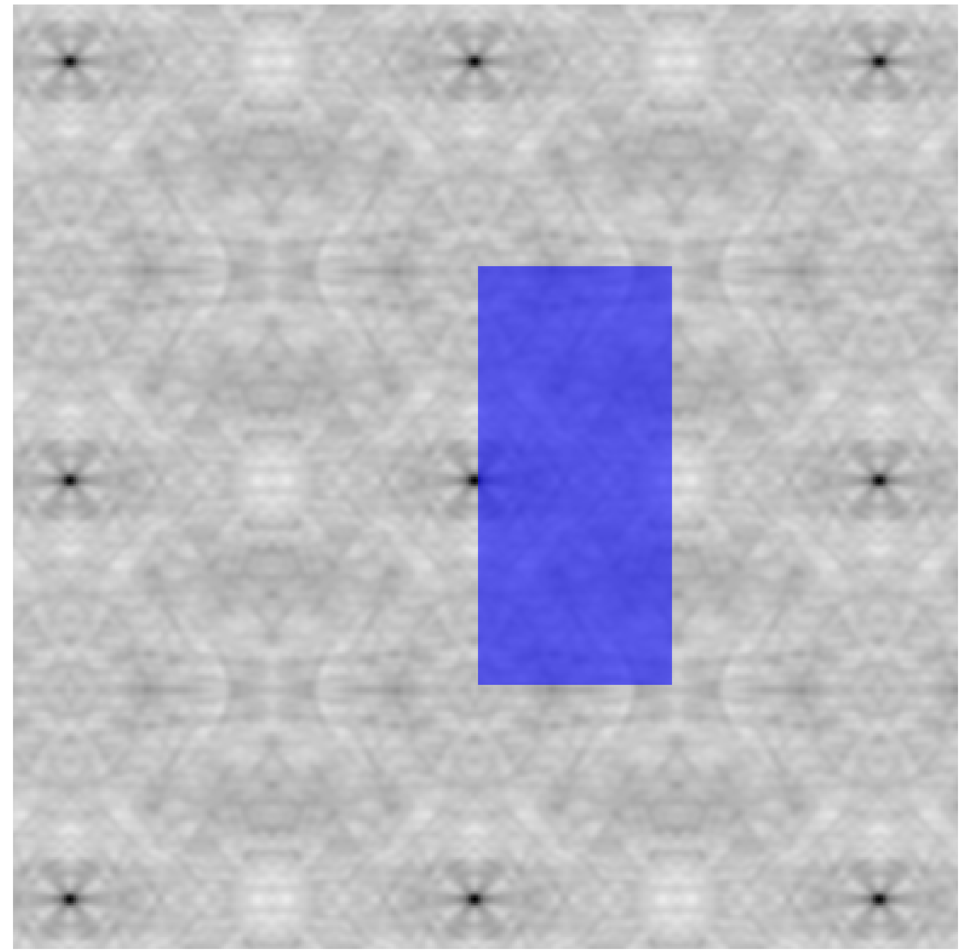


constraints ≥ 2N

Crystallography

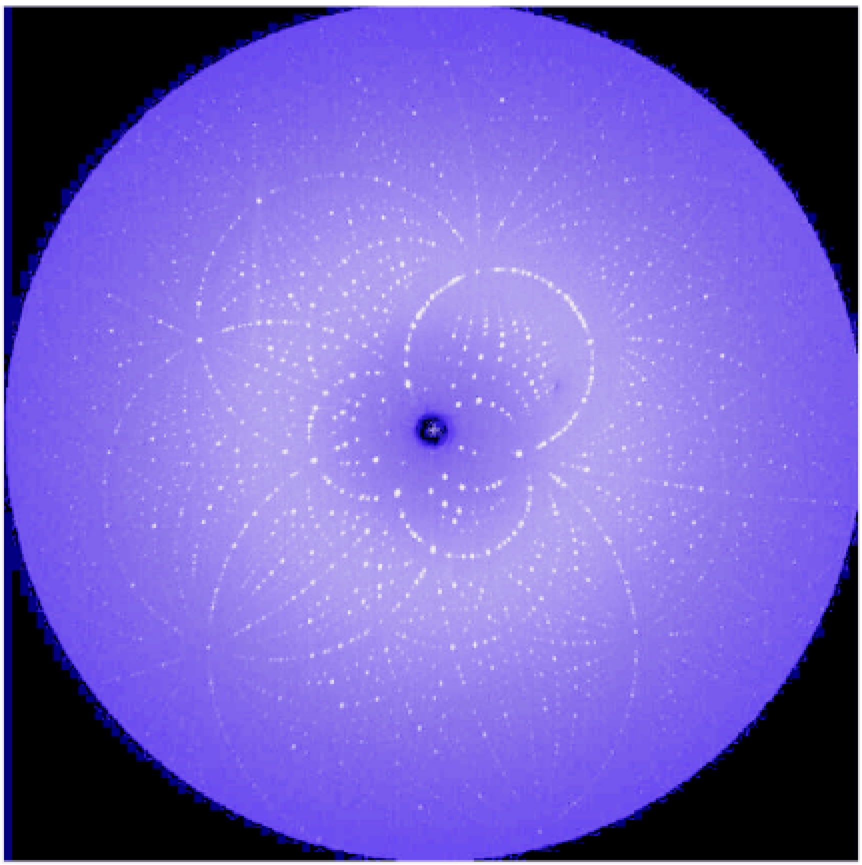
Problem is **under**constrained with a crystal





unknowns = N **constraints = N/2**

Crystallography
Structure determination



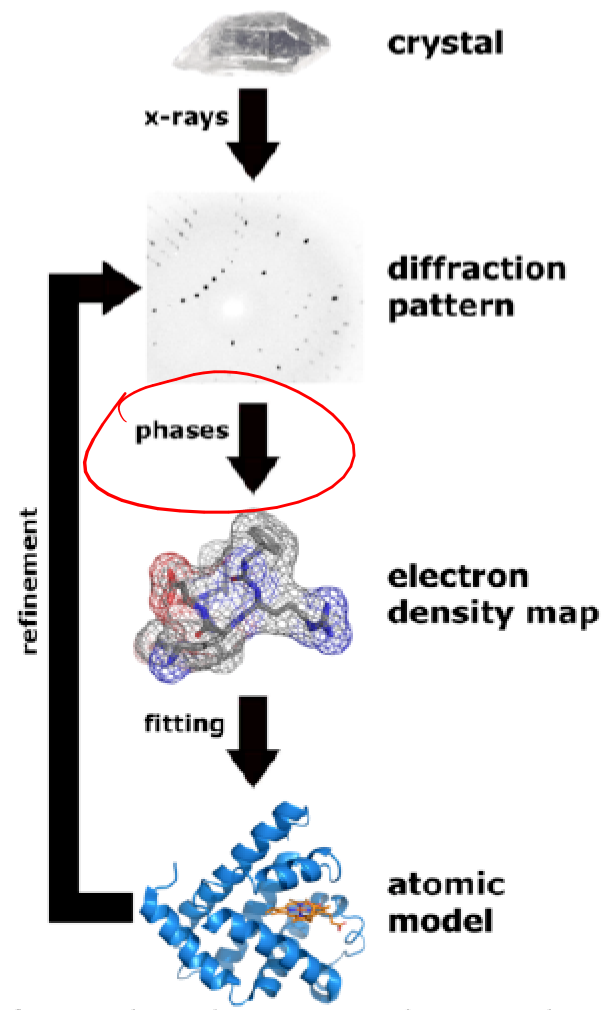


Image from Wikimedia courtesy Thomas Splettstoesser

Crystallography

Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

Summary

Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)