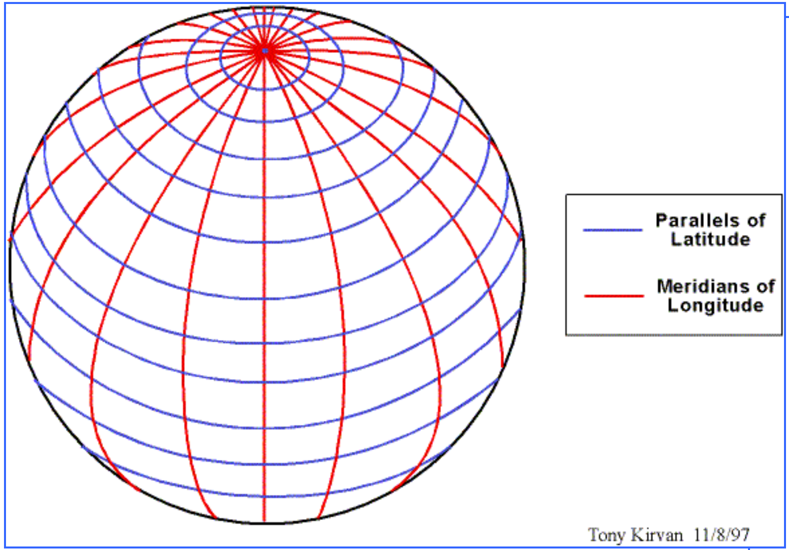
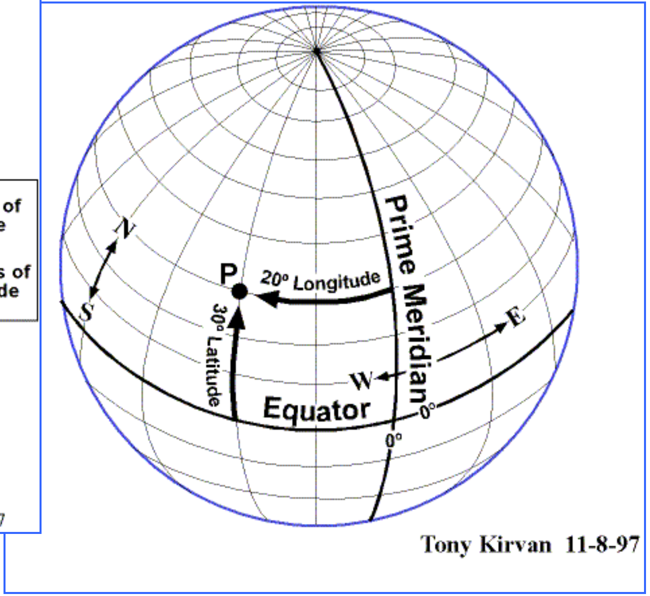
Geometria di un satellite

- Sfera Celeste
- Sistemi di Coordinate
- Studio Eclissi
- Geometria Terra / Satellite

SMAD Chapter 5 p. 95

Sfera Celeste 1/2





azimuth (longitudine ℓ)

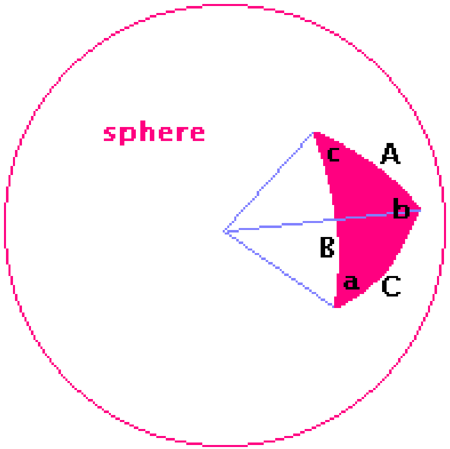
elevazione (latitudine λ)

$x = \cos \ell \cos \lambda$

$y = \sin \ell \cos \lambda$

$z = \sin \lambda$

Sfera Celeste 2/2



A spherical triangle consists of Great Circle Arcs, extending from the sphere's center, forming Great Circle Angles. Relations among arcs and angles are:

$$\cos(A) = \cos(B)\cos(C) + \sin(B)\sin(C)\cos(a)$$

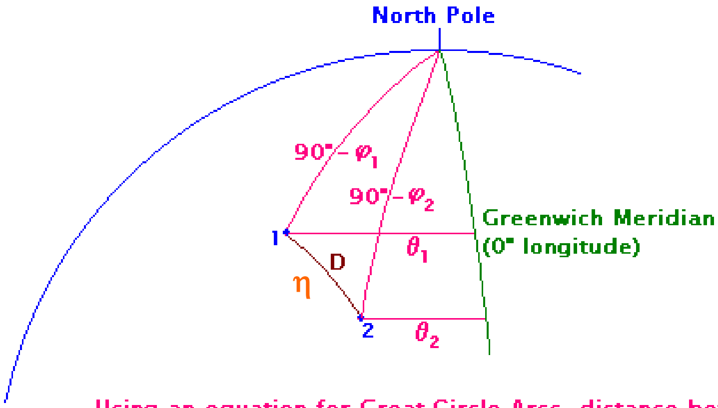
$$\cos(a) = -\cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

$$\sin(A)/\sin(a) = \sin(B)/\sin(b) = \sin(C)/\sin(c)$$

SMAD Appendix D
Table D-3 p. 907

φ_1, φ_2 elevazione

θ_1, θ_2 azimuth



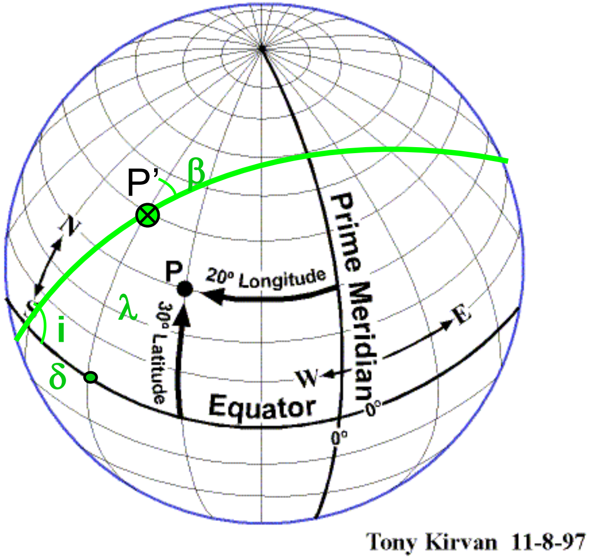
Using an equation for Great Circle Arcs, distance between 1 & 2 is estimated as:

$$\cos(\eta) = \cos(90^o - \varphi_1)\cos(90^o - \varphi_2) + \sin(90^o - \varphi_1)\sin(90^o - \varphi_2)\cos(\theta_1 - \theta_2)$$

$$D = 2\pi R_t/(2\pi)\ \mathrm{acos}(\sin(\varphi_1)\sin(\varphi_2) + \cos(\varphi_1)\cos(\varphi_2)\cos(\theta_1 - \theta_2))$$

Finestre di Lancio

P’ (30º W, 40º N)



$\lambda > i$?

$\lambda = i$?

$\lambda < i$?

SMAD Appendix D Table D-1 p. 905 riga 4 col. 3

SMAD Appendix D Table D-1 p. 905 riga 5 col. 3

$$\sin\beta = \cos i / \cos\lambda$$

$$\cos\delta = \cos\beta / \sin i$$

$$LST = \Omega + \delta$$

$$LST = \Omega + 180^o - \delta$$

SMAD chapter 6.4 p. 153-155

$$v_{sud} = - v_o \cos\gamma \cos\beta_L$$

$$v_{est} = v_o \cos\gamma \sin\beta_L - v_\lambda$$

$$v_r = v_o \sin\gamma \quad (v_z)$$

$$v_\lambda = 464.5 \cos\lambda \text{ m/s}$$

β azimuth di lancio γ angolo traiettoria volo al *burn-out*

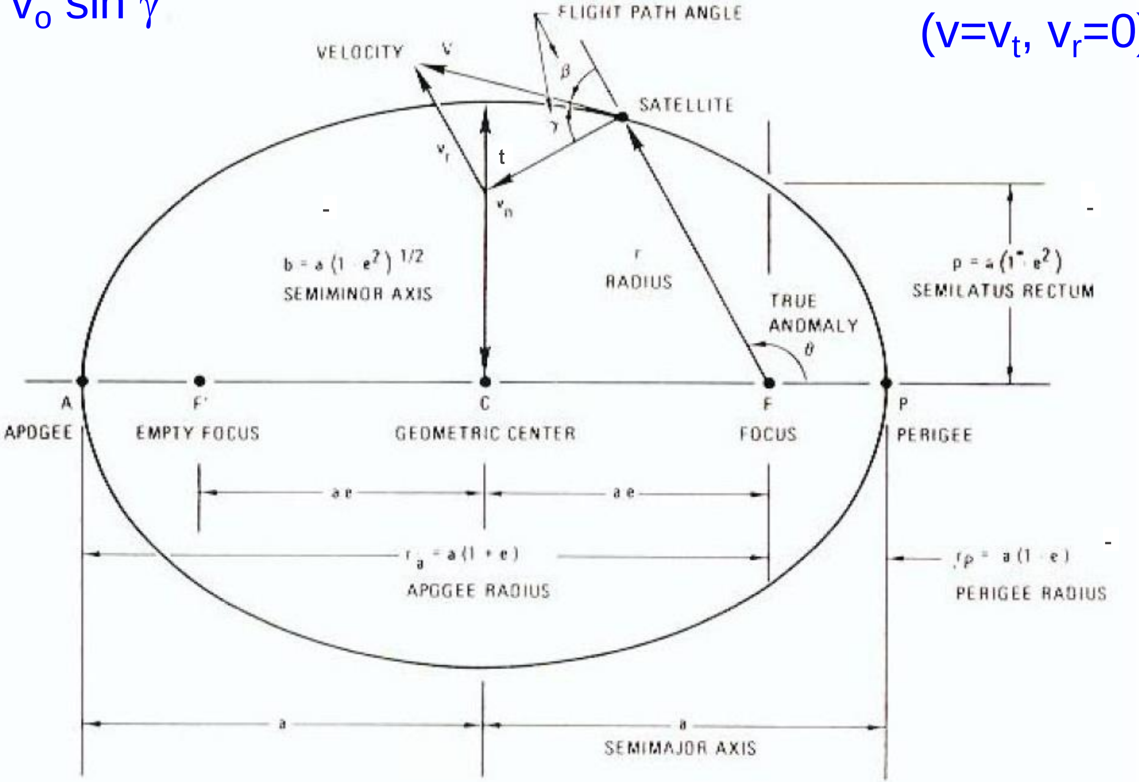
(vedi prossima slide)

Parametri Ellisse

$v_t = v_o \cos \gamma \ (*)$

$v_r = v_o \sin \gamma$

γ=0 per orbite circolari (v=v_t, v_r=0)



Sistemi di Coordinate 1/3

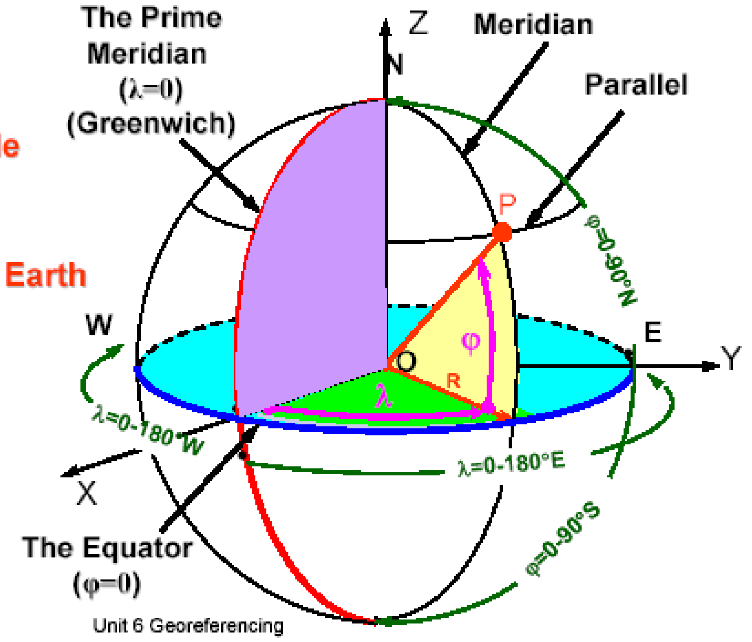
Sistema Geocentrico "Geografico"

λ - Geographic longitude

φ - Geographic latitude

R – Mean Radius of the Earth

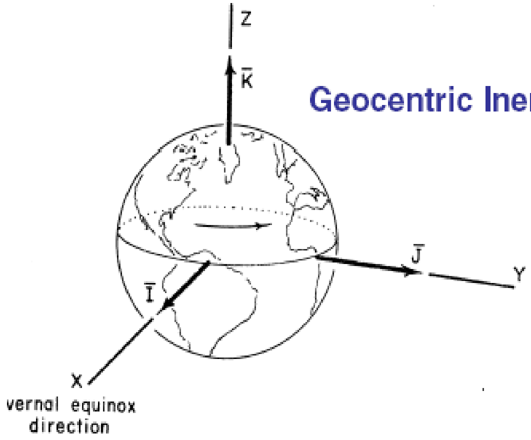
O - The Geo-Center



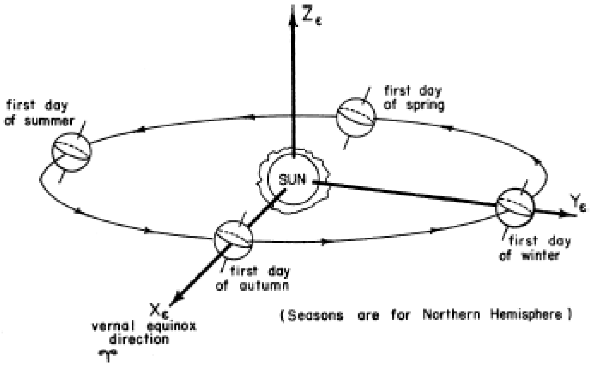
Attenzione all'indicazione lat/long !

Sistemi di Coordinate 2/3

Geocentric Inertial System

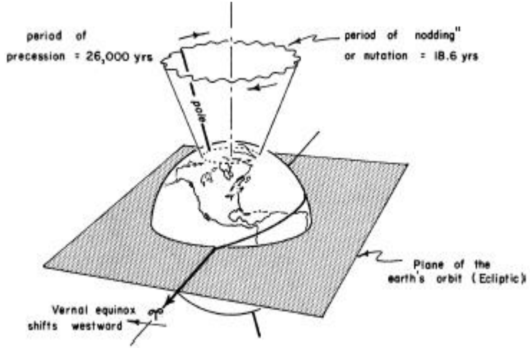


Heliocentric Inertial System



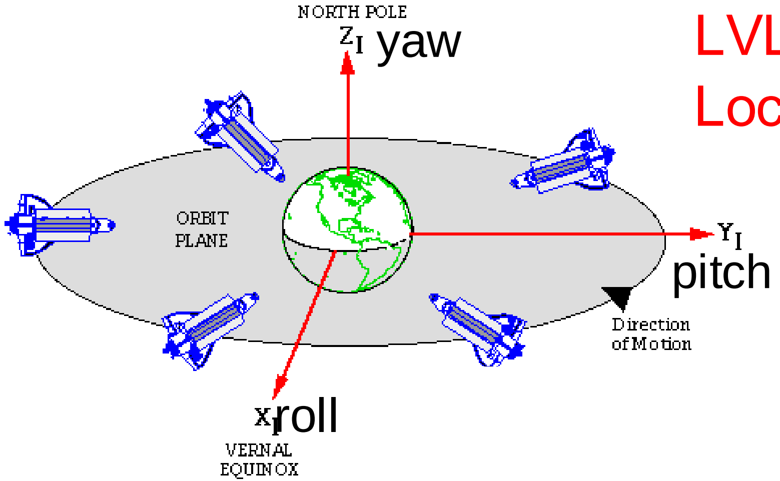
azimuth = ascensione retta α

elevazione = declinazione δ



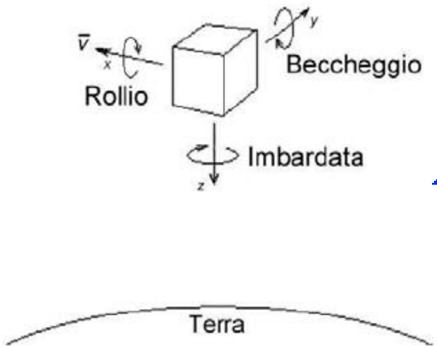
50.2786” westerly drift of the Vernal Equinox per year

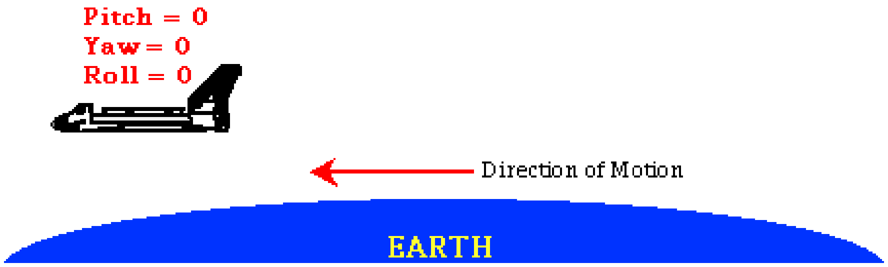
Sistemi di Coordinate 3/3



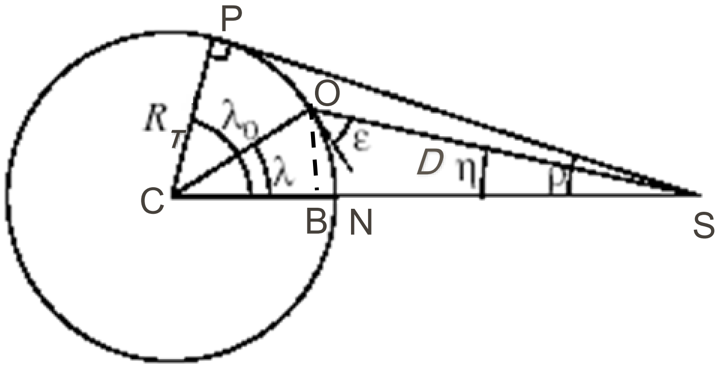
LVLH

Local Vertical – Local Horizontal





Geometria Terra / Satellite 1/3



ρ raggio angolare Terra

η angolo di nadir

ε elevazione

λ angolo centrale Terra
(swath width)

SMAD chapter 5.2
fig 5-13 p. 110-113

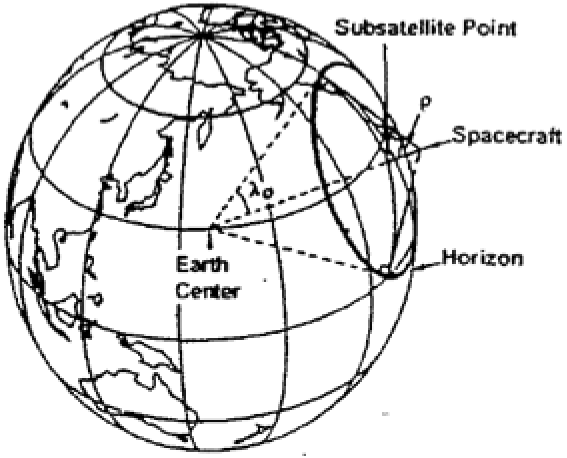
$\sin\rho = \cos\lambda_0 = R_T/R = R_T / (R_T+h)$

$\sin\eta = \cos\varepsilon \sin\rho$

$\lambda = \pi/2 - \eta - \varepsilon$

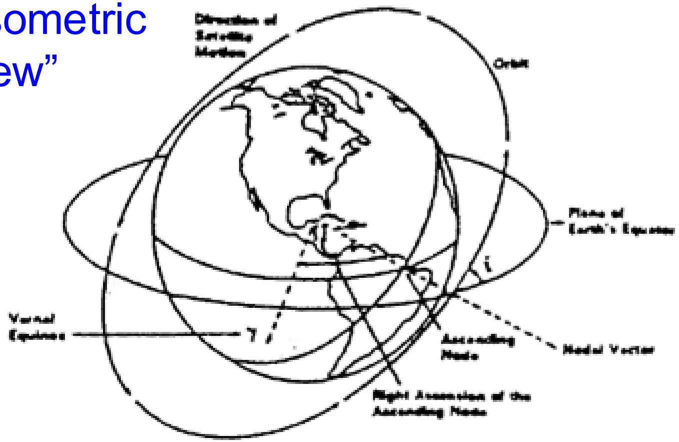
$\mathrm{tg}\,\eta = \sin\rho \sin\lambda / (1 - \sin\rho \cos\lambda)$

$D = R_T \sin\lambda / \sin\eta$

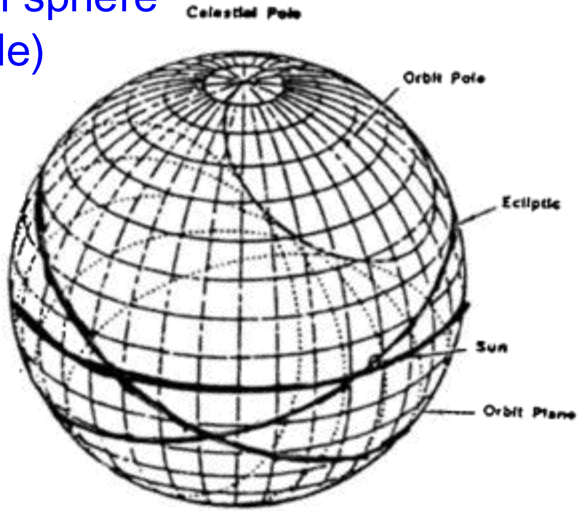


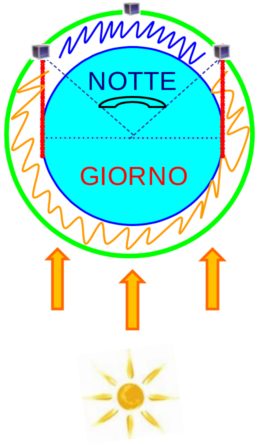
Geometria di un satellite 2/3

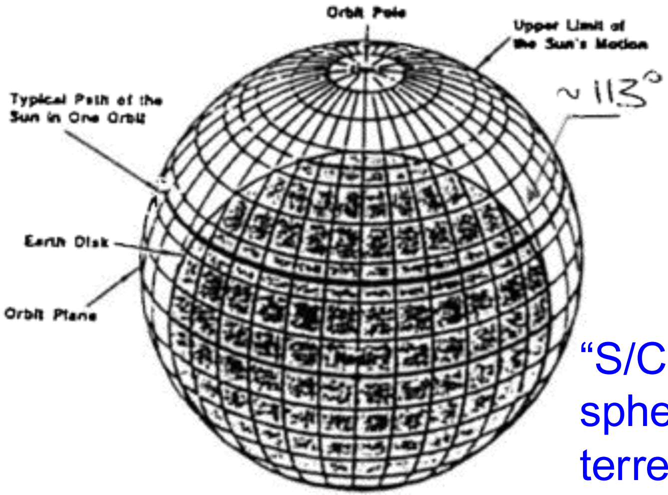
"isometric view"



"S/C centered celestial sphere" (inerziale)







"S/C centered celestial sphere" (riferimento terrestre)

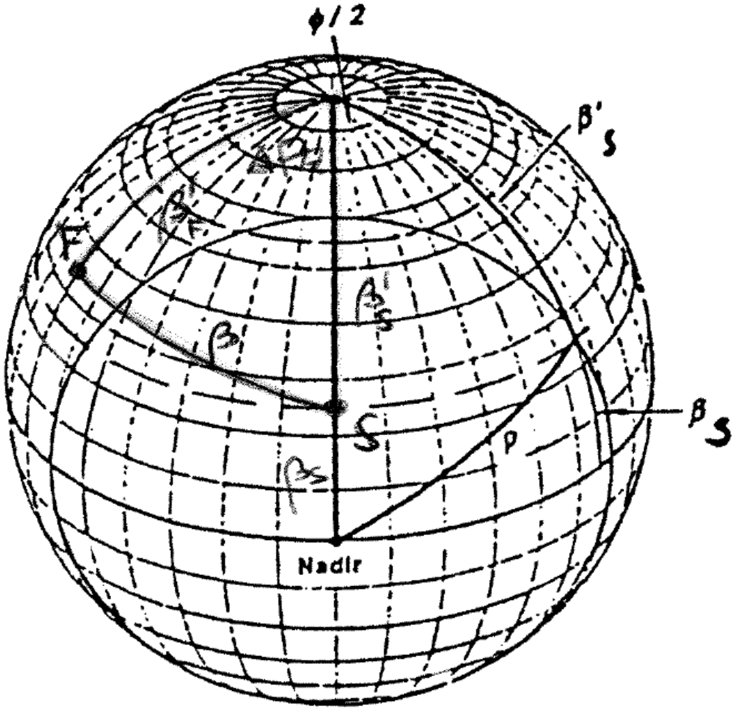
$h = 1000$ km, $i = 32°$ $\Rightarrow$

$\tau = 105$ min, $\rho = 60°$

Geometria di un satellite 3/3

"S/C centered celestial sphere"

$\cos \Phi/2 = \cos \rho / \cos \beta_s$



Analisi dell'eclissi da una LEO

$\beta_s = 25°$ $\Rightarrow \Phi/2 = 56.5°$

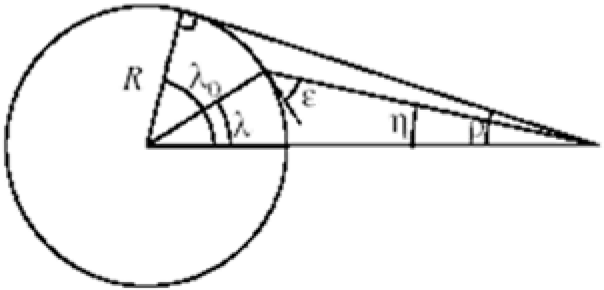
Durata fase notturna (eclisse): max e min

β_s : max e min β

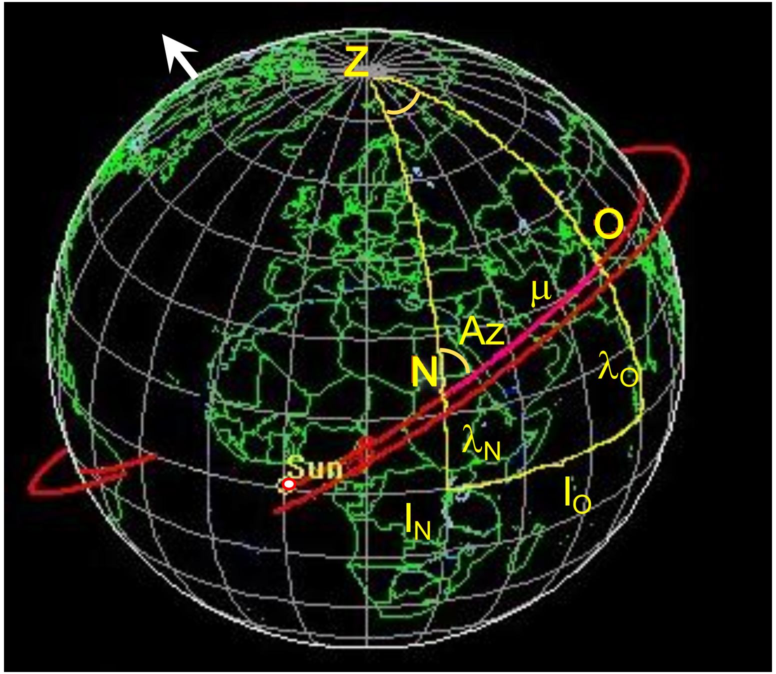
$\beta_F = 35°$, $Az_o = 70°$

$A = 0.5\ m^2$ $\Rightarrow$

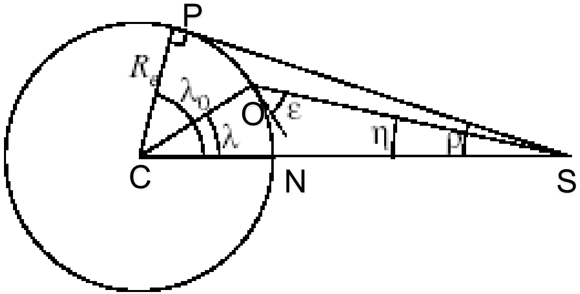
- "eclisse": $Az = Az_o \pm \Phi/2$
- "dietro": $\beta = \pm \pi/2$



Passaggio sopra la stazione 1/8



"Earth centered celestial sphere"



Meglio: geometria vista dal satellite!
Nota: quella segnata non è un' orbita del satellite ma un cerchio max che passa per O e N

N = Sub Satellite Point (l_N,λ_N)

O = Punto qls Terra (λ_O,λ_O) !!!

$\cos \mu = \sin \lambda_N \sin \lambda_O + \cos \lambda_N \cos \lambda_O \cos(l_O - l_N) \quad \Rightarrow \eta$

$\sin \lambda_O = \sin \lambda_N \cos \mu + \cos \lambda_N \sin \mu \cos Az \quad (\mu \equiv \lambda)$

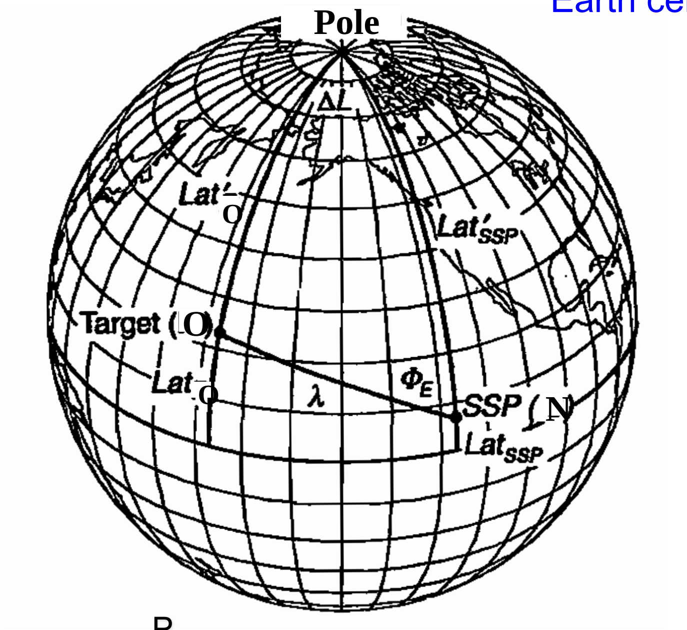
$\cos Az = (\sin \lambda_O - \sin \lambda_N \cos \mu) / \cos \lambda_N \sin \mu$

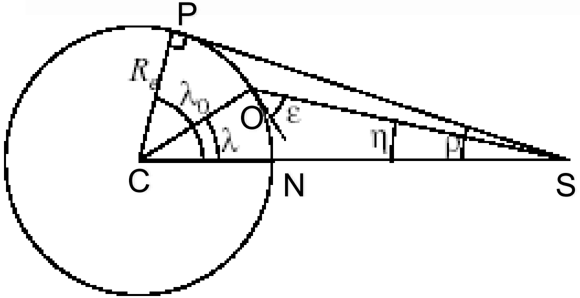
Risultato: angoli da satellite

SMAD chapter 5.2 fig 5-12 p. 112

Passaggio sopra la stazione 2/8

"Earth centered celestial sphere"



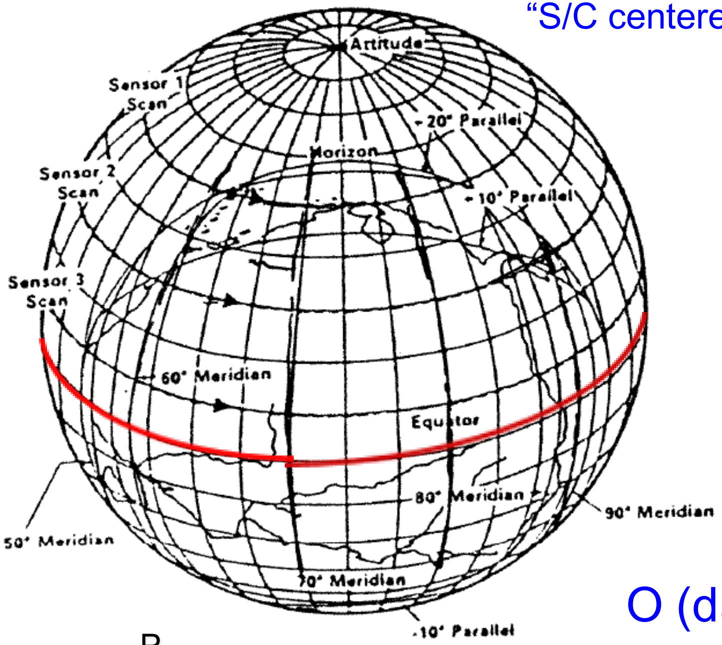


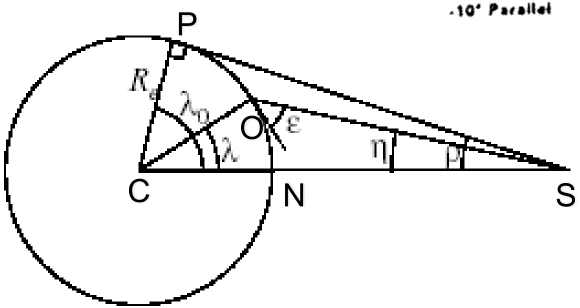
$l_O = 200^o$, $\lambda_O = 22^o$ (hawaii)

$l_N = 185^o$, $\lambda_N = 10^o$

Passaggio sopra la stazione 3/8

“S/C centered celestial sphere”





$l_O = 200^o$, $\lambda_O = 22^o$ (hawaii)

$l_N = 185^o$, $\lambda_N = 10^o$

$\Rightarrow$

$\rho = 59.8^o$, $\lambda_0 = 30.2^o$, $D_{max} = 3709$ km

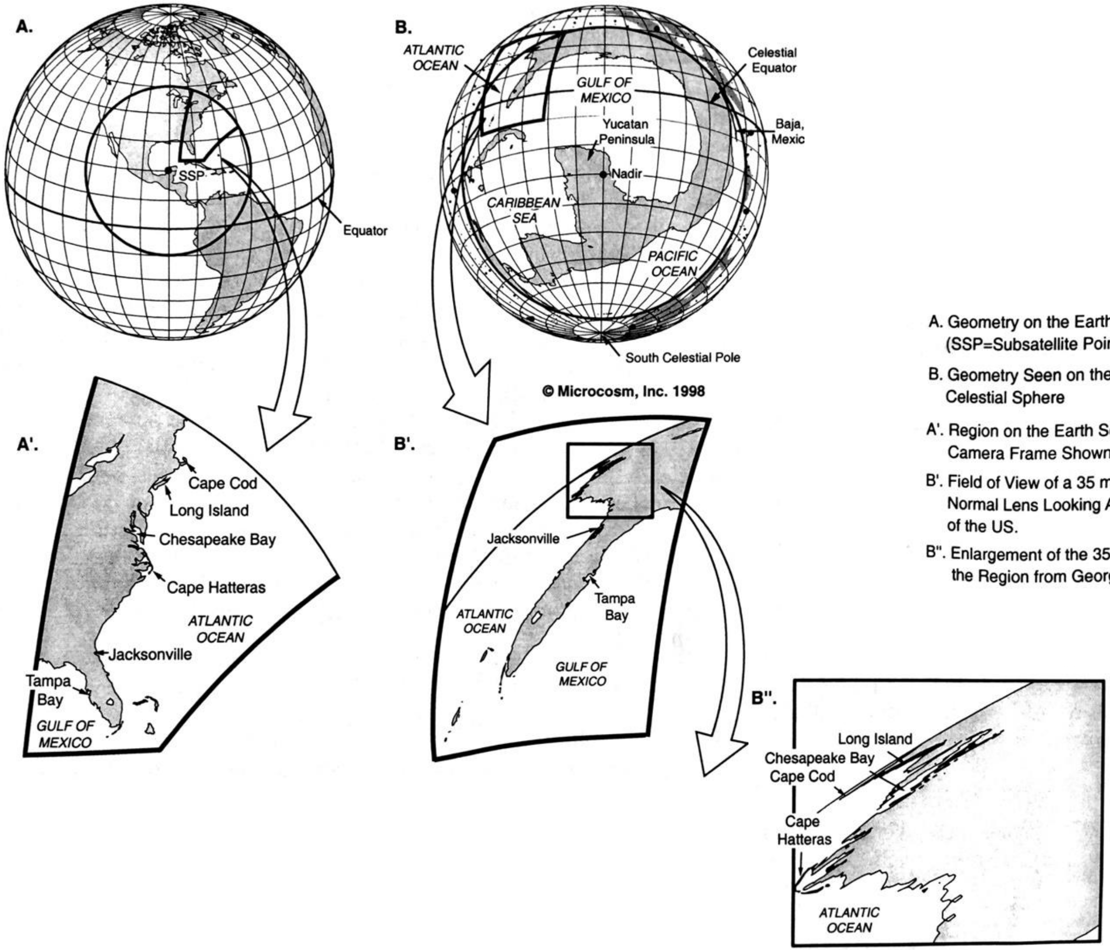
O (da N)

- $\lambda = 18.7^o$ (swath width)
- $Az = 48.3^o$

O (da satellite)

- $\eta = 56.8^o$ ($\varepsilon = 14.5^o$)
- $D = 2444$ km

Passaggio sopra la stazione 5/8



A. Geometry on the Earth's Surface (SSP=Subsatellite Point)

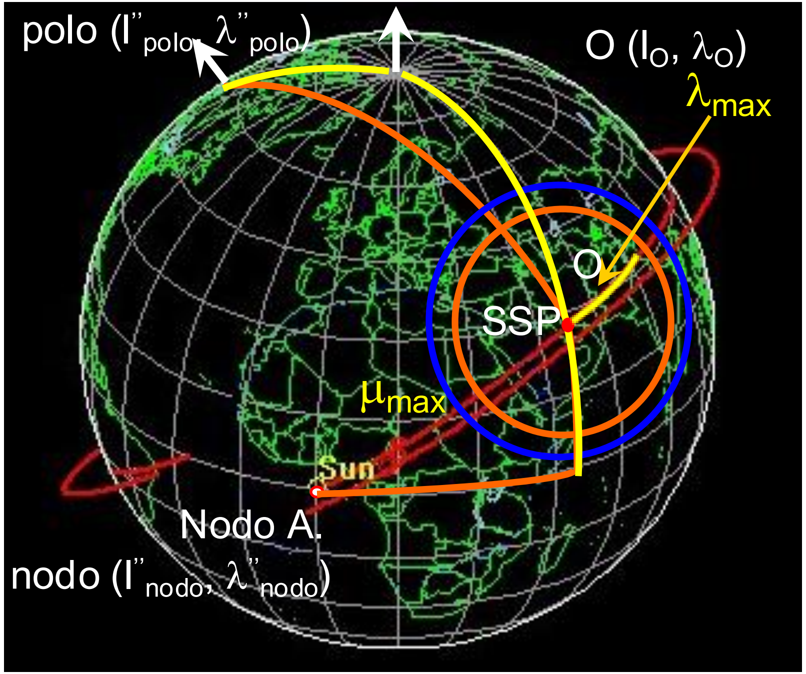
B. Geometry Seen on the Spacecraft Centered Celestial Sphere

A'. Region on the Earth Seen by the 35 mm Camera Frame Shown in (B')

B'. Field of View of a 35 mm Camera with a Normal Lens Looking Along the East Coast of the US.

B''. Enlargement of the 35 mm Frame Showing the Region from Georgia to Massachusetts.

Passaggio sopra la stazione 6/8



$\sin(l_O - l''_{nodo}) = \tan \lambda_O / \tan i$

$\sin \mu_{max} = \sin \lambda_O / \sin i$

O≡SSP

"Earth centered celestial sphere"

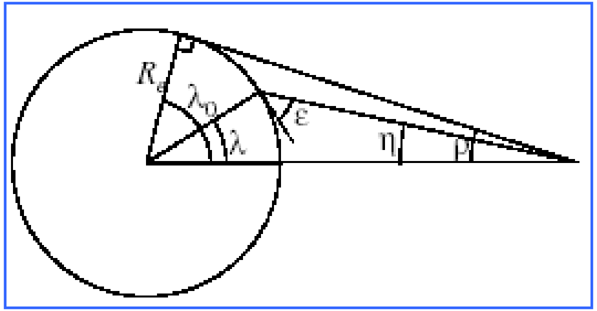
$\sin \eta_{max} = \cos \varepsilon_{min} \sin \rho$

$\lambda_{max} = \pi/2 - \eta_{max} - \varepsilon_{min}$

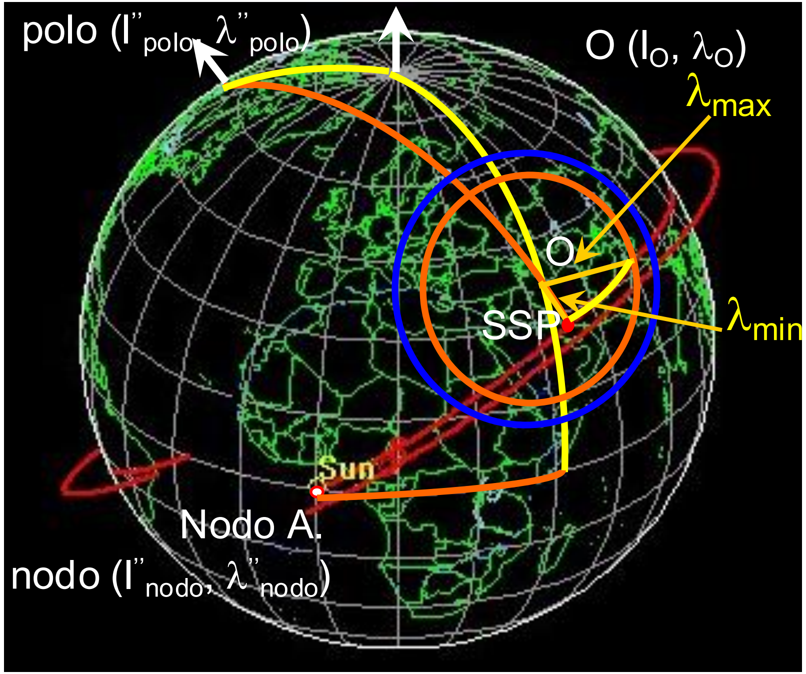
$D_{max} = R_T \sin \lambda_{max} / \sin \eta_{max}$

$\lambda''_{polo} = \pi/2 - i$ (lat)

$l''_{polo} = l''_{nodo} - \pi/2$ (long)



Passaggio sopra la stazione 7/8



SMAD chapter 5.3.1
fig 5-17 p. 118-121

"Earth centered celestial sphere"

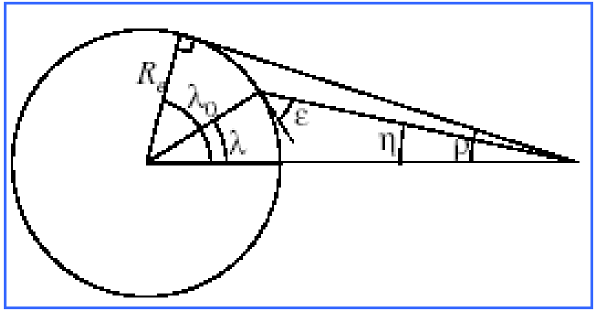
$\sin \eta_{max} = \cos \varepsilon_{min} \sin \rho$

$\lambda_{max} = \pi/2 - \eta_{max} - \varepsilon_{min}$

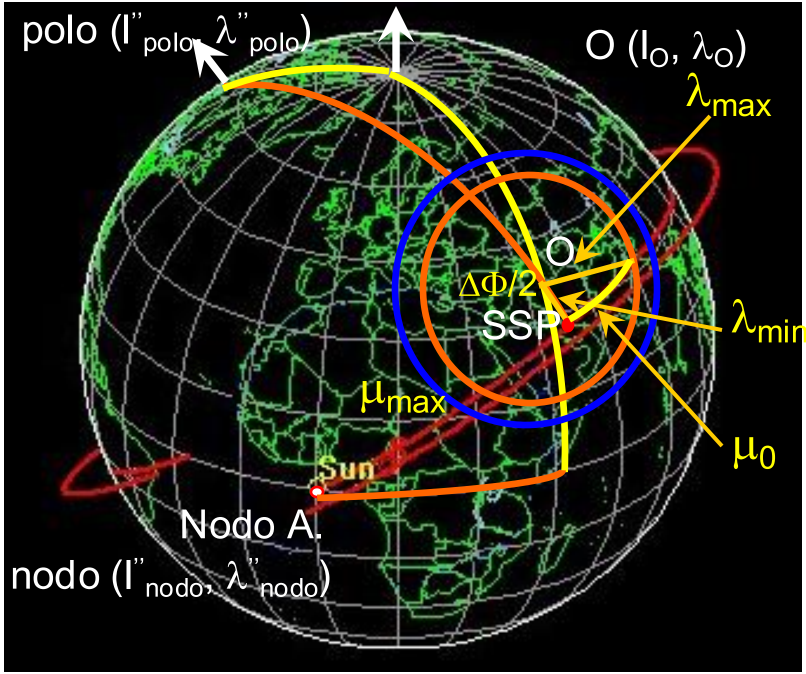
$D_{max} = R_T \sin \lambda_{max} / \sin \eta_{max}$

$\lambda''_{polo} = \pi/2 - i$ (lat)

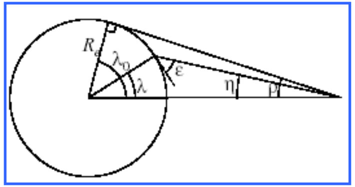
$l''_{polo} = l''_{nodo} - \pi/2$ (long)



Passaggio sopra la stazione 8/8



"Earth centered celestial sphere"



$\sin(l_O - l''_{nodo}) = \tan \lambda_O / \tan i$

$\sin \mu_{max} = \sin \lambda_O / \sin i$

O≡SSP

$\sin \lambda_{min} = \sin \lambda''_{polo} \sin \lambda_O +$

$+ \cos \lambda''_{polo} \cos \lambda_O \cos(l_O - l''_{polo})$

$\tan \eta_{min} = \sin \rho \sin \lambda_{min} / (1 - \sin \rho \cos \lambda_{min})$

$\varepsilon_{max} = \pi/2 - \eta_{min} - \lambda_{min}$

$R_T \sin \lambda_{min} = D_{min} \sin \eta_{min}$

$\omega_{max} = \dot{\theta}_{max} \cong v_{sat} / D_{min}$

$\cos \Delta\Phi/2 = \tan \lambda_{min} / \tan \lambda_{max}$

$\cos \mu_0 = \cos \lambda_{max} / \cos \lambda_{min}$

$T = (\tau / \pi) \operatorname{acos}(\cos \lambda_{max} / \cos \lambda_{min})$

Esercizio 1

Finestre di lancio:

Supponi che il punto Γ (corrispondentemente all'orbita in cui bisogna lanciare) si trovi a 30° W (rispetto al meridiano di Greenwich) e che la base di lancio sia a Trieste (lat = 45°, long = 14.5°) e l'inclinazione a cui vuoi mettere in orbita il satellite sia 60°, a=20000 km, e=0.67. Al momento del "burn-out" l'angolo di azimuth debba essere γ = 10° (θ=25°).

Valutare:

- il LST (lancio al nodo ascendente e discendente)
- la velocita' con cui bisogna lanciare il satellite (sud, est, z)

Ripetere lo stesso esercizio ponendo i =80°, 50°, 45° e 40°

Esercizio 1

				beta		delta		vlambda	vsud	vest	vz	vtot
phi	10	0,1745329		45,00	0,785398	35,26	0,61548	0,328885	-6,83396	6,505071	1,704142	9,587648
i	60	1,0471976				-20,76	339,236					0,22612
lambda	45	0,7853982				Omega						
l	14,5	0,2530727				9,24	0,161192		-20,76			
vlo	0,46511323	km/s				9,2	-100,24		-9,24			
hsat	400	6778				LST	asc		asc			
vsat	9,8138					44,50	deg	44,5	deg			
gamma	-30	-0,5235988	W									
	330		E			LST	disc		disc			
						44,50	deg	153,97	deg			
vr/vn		0,176327										

Esercizio 2

Finestre di lancio:

Supponi che Ω sia 9.2° (sistema inerziale, corrispondentemente all'orbita in cui bisogna lanciare) e che la base di lancio sia a Trieste (lat = 45°, long = 14.5°) e l'inclinazione a cui vuoi mettere in orbita il satellite sia 60°, a=20000 km, e=0.67. Al momento del "burn-out", il satellite si trovera' sulla sua orbita con un'anomalia vera θ=25°.

Valutare:

- il LST (lancio al nodo ascendente e discendente)
- la velocita' con cui bisogna lanciare il satellite (sud, est, z)

Esercizio 3

Elemento	Valore		Elemento	Valore	
h	400	km	Ω	90	gradi
e	0		ω	N/A	gradi
i	60	gradi	ν	0	gradi
				360	gradi

calcolare:

- la massima latitudine della traccia a Terra
- lo spostamento dei nodi per orbita

Esercizio 4

Nelle stesse condizioni, considerando un angolo di elevazione minimo (ε) pari a 25° (e successivamente per 10°), calcolare:

- la "swath width" ($2\lambda_{max}$)
- il campo di vista richiesto per coprire questo "swath" ($2\eta_{max}$)
- la distanza dal bordo della "swath" ($D(\lambda_{max})$)

Esercizio 4

Nelle stesse condizioni, considerando un angolo di elevazione minimo (ε) pari a 25° (e successivamente per 10°), calcolare:

- la “swath width” ($2\lambda_{max}$)
- il campo di vista richiesto per coprire questo “swath” ($2\eta_{max}$)
- la distanza dal bordo della “swath” ($D(\lambda_{max})$)

RESULTS (geometrical variables)		
Max Eclipse time	36,11	min
F	136,15	deg
Eclipse Time T_E =f(b_s)	35,00	min
	37,8%	
Day Time	57,56	min
	62,2%	
Swath Calculations		
Angular Earth Radius r	70,22	deg
	1,23	rad
Nadir angle h_{max}	58,52	deg
	1,02	rad
Earth central angle l_{max}	6,48	deg
	0,11	rad
Swath width 2 l_{max}	12,96	deg
	0,23	rad
Field of view for swath (FoV) 2 h_{max}	117,04	deg
	2,04	rad
Distance to the edge of swath D_{max}	844,02	km
Distance to the edge of swath on Earth S_{max}	722,11	km
Elevation minimum e_{min}	68,69	deg
	1,198806722	rad
Earth central angle l_{max}	1,31	deg
	0,022923754	rad
Swath Calculations	146,210832	km
	146,2076306	km
Area	67159,73432	km2
P/Area	2,97797E-10	W.m-2
	20	W

Esercizio 5

Nelle condizioni h=400 km, e=0, i=60°, se la stazione a Terra è posta ad una latitudine di 34° N e longitudine di 118° W e la longitudine del nodo ascendente durante il passaggio corrente è di 75° E, calcolare (usare ε_{min} = 10°):

- la distanza minima fra la S/C e la stazione a terra durante il passaggio corrente (874.6 km)
- la massima velocità angolare al passaggio corrente (30.14 deg/min)
- il tempo in vista della stazione a Terra durante il passaggio corrente (5.15 min)
- il massimo tempo in vista della stazione a Terra durante un passaggio sopra la stazione (6.21 min)
- longitudine della GS affinché la S/C passi sopra essa (232.1°, 97.9°)