1

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \Rightarrow \boxed{\overleftrightarrow{\rho}^{-1} \vec{E} = \left(\overleftrightarrow{\rho}_1^{-1} + \overleftrightarrow{\rho}_2^{-1}\right)\vec{E}}$$ ADDITIVITY of CURRENTS

$$\rho^{-1}_{xx} = \frac{\rho_1}{\rho_1^2 + R_1^2 H^2} + \frac{\rho_2}{\rho_2^2 + R_2^2 H^2} \stackrel{def}{=} \frac{\rho(H)}{\rho^2(H) + R^2 H^2}$$

def di ρ_{xx}

$$\rho^{-1}_{xy} = \frac{R_1 \cancel{H}}{\rho_1^2 + R_1^2 H^2} + \frac{R_2 \cancel{H}}{\rho_2^2 + R_2^2 H^2} = \frac{R\cancel{H}}{\rho^2(H) + R^2 H^2}$$

1: +
2: −

$$\boxed{\begin{array}{ll} R_1 = \dfrac{1}{nec} & R_2 = -R_1 = \dfrac{-1}{nec} \\ \rho_1 = \dfrac{1}{ne^2\mu_1} & \rho_2 = \dfrac{1}{ne^2\mu_2} \end{array}}$$

$$\boxed{\begin{array}{l} \rho(H) = ? \\ R = ? \end{array}}$$

Remember: mobility: μ=τ/m

1)
$$\frac{\frac{1}{\cancel{n}\cancel{e^2}\mu_1}}{\frac{1}{n^{\cancel{2}}e^{\cancel{4}2}\mu_1^2} + \frac{H^2}{n^{\cancel{2}}\cancel{e^2}c^2}} + \frac{\frac{1}{\cancel{n}\cancel{e^2}\mu_2}}{\frac{1}{n^{\cancel{2}}e^{\cancel{4}2}\mu_2^2} + \frac{H^2}{n^{\cancel{2}}\cancel{e^2}c^2}} = \frac{\rho}{\rho^2 + R^2 H^2}$$

2)
$$\frac{\frac{1}{\cancel{nec}}}{\frac{1}{n^{\cancel{2}}e^{\cancel{4}3}\mu_1^2} + \frac{H^2}{n^{\cancel{2}}e^{\cancel{2}}c^2}} + \frac{-\frac{1}{\cancel{nec}}}{\frac{1}{n^{\cancel{2}}e^{\cancel{4}3}\mu_2^2} + \frac{H^2}{n^{\cancel{2}}e^{\cancel{2}}c^2}} = \frac{R}{\rho^2 + R^2 H^2}$$

1)
$$\frac{\cancel{1/\mu_1}}{\frac{c^2 + H^2 e^2 \mu_1^2}{n e^2 c^2 \cancel{\mu_1^2}}} + \frac{\cancel{1/\mu_2}}{\frac{c^2 + H^2 e^2 \mu_2^2}{n e^2 c^2 \cancel{\mu_2^2}}} = \frac{\rho}{\rho^2 + R^2 H^2}$$

$$\frac{n e^2 c^2 \mu_1}{c^2 + H^2 e^2 \mu_1^2} + \frac{n e^2 c^2 \mu_2}{c^2 + H^2 e^2 \mu_2^2} = \frac{\rho}{\rho^2 + R^2 H^2}$$

Poups $\boxed{\rho = \tilde{\rho} \cdot \dfrac{1}{ne^2} \quad ; \quad R = \tilde{R} \cdot \dfrac{1}{nec}}$ ($\tilde{R}$ is adimensional)

$$\frac{\rho}{\rho^2 + R^2 H^2} = \frac{\tilde{\rho}/\cancel{ne^2}}{\frac{\tilde{\rho}^2}{n^{\cancel{2}}e^{\cancel{4}2}} + \frac{\tilde{R}^2 H^2}{n^{\cancel{2}}e^{\cancel{2}}c^2}} = \frac{\tilde{\rho} \cdot n e^2 c^2}{\tilde{\rho}^2 c^2 + \tilde{R}^2 e^2 H^2}$$

$$\frac{R}{\rho^2 + R^2 H^2} = \frac{\tilde{R}/\cancel{nec}}{\frac{\tilde{\rho}^2}{n^{\cancel{2}}e^{\cancel{4}3}} + \frac{\tilde{R}^2 H^2}{n^{\cancel{2}}e^{2}c^2}} = \frac{\tilde{R} \cdot n e^3 c}{\tilde{\rho}^2 c^2 + \tilde{R}^2 e^2 H^2}$$

Ex. 1

1) $$\frac{\mu_1}{c^2+A^2\mu_1^2}+\frac{\mu_2}{c^2+A^2\mu_2^2}=\frac{\tilde{\rho}}{\tilde{\rho}^2c^2+A^2\tilde{R}^2} \rightarrow \frac{1}{\tilde{\rho}^2c^2+A^2\tilde{R}^2}=\frac{1}{\tilde{\rho}}(\;)$$

with $A^2 \equiv e^2H^2$

2) $$\frac{\mu_1^2}{c^2+A^2\mu_1^2}-\frac{\mu_2^2}{c^2+A^2\mu_2^2}=\frac{\tilde{R}}{\tilde{\rho}^2c^2+A^2\tilde{R}^2}$$

1) in 2):
ds ds

$$\frac{\mu_1^2}{c^2+A^2\mu_1^2}-\frac{\mu_2^2}{c^2+A^2\mu_2^2}=\frac{\tilde{R}}{\tilde{\rho}}\left(\frac{\mu_1}{c^2+A^2\mu_1^2}+\frac{\mu_2}{c^2+A^2\mu_2^2}\right)$$

$$(\mu_1^2-\mu_2^2)c^2=\frac{\tilde{R}}{\tilde{\rho}}\left(\mu_1c^2+A^2\mu_1\mu_2^2+\mu_2c^2+A^2\mu_1^2\mu_2\right)$$

$$=\frac{\tilde{R}}{\tilde{\rho}}\left[(\mu_1+\mu_2)c^2+(\mu_1+\mu_2)A^2\mu_1\mu_2\right]$$

$$\boxed{\frac{\tilde{\rho}}{\tilde{R}}=\frac{c^2+A^2\mu_1\mu_2}{c^2(\mu_1-\mu_2)}} \qquad (\mu_1\neq\mu_2)$$

Rewrite 1) as:

$$\frac{\mu_1c^2+A^2\mu_1\mu_2^2+\mu_2c^2+A^2\mu_1^2\mu_2}{(c^2+A^2\mu_1^2)(c^2+A^2\mu_2^2)}=\frac{1}{\tilde{R}}\cdot\frac{\tilde{\rho}/\tilde{R}}{\left(\tilde{\rho}/\tilde{R}\right)^2c^2+A^2}$$

$$\frac{(\mu_1+\mu_2)\cancel{(c^2+A^2\mu_1\mu_2)}}{(c^2+A^2\mu_1^2)(c^2+A^2\mu_2^2)}=\frac{1}{\tilde{R}}\cdot\frac{\cancel{c^2+A^2\mu_1\mu_2}}{c^2(\mu_1-\mu_2)}\cdot\frac{1}{\left[\frac{c^2+A^2\mu_1\mu_2}{c^2(\mu_1-\mu_2)}\right]^2c^2+A^2}$$

$$\boxed{\tilde{R}}=\frac{(c^2+A^2\mu_1^2)(c^2+A^2\mu_2^2)\cdot c^{\cancel{4}\,2}(\mu_1-\mu_2)^{\cancel{2}}\;\big/\;(\mu_1+\mu_2)}{\cancel{c^2(\mu_1-\mu_2)}\left[(c^4+A^4\mu_1^2\mu_2^2+\cancel{2c^2A^2\mu_1\mu_2})c^2+A^2c^{\cancel{4}}(\mu_1^2+\mu_2^2\cancel{-2\mu_1\mu_2})\right]}$$

$$=\frac{\cancel{(c^2+A^2\mu_1^2)}\cancel{(c^2+A^2\mu_2^2)}c^2(\mu_1-\mu_2)}{\cancel{(c^2+A^2\mu_1^2)}\cancel{(c^2+A^2\mu_2^2)}c^2(\mu_2+\mu_1)}=\boxed{\frac{\mu_1-\mu_2}{\mu_1+\mu_2}}$$

Since $\tilde{R} = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$

$\Rightarrow$ ~~subst~~ subst. in $\frac{\tilde{\rho}}{\tilde{R}} = \frac{c^2 + A^2\mu_1\mu_2}{c^2(\mu_1 - \mu_2)}$:

$$\boxed{\tilde{\rho} = \frac{\cancel{\mu_1 - \mu_2}}{\mu_1 + \mu_2} \cdot \frac{c^2 + A^2\mu_1\mu_2}{c^2\cancel{(\mu_1 - \mu_2)}} = \frac{1}{\mu_1 + \mu_2} + \frac{e^2H^2}{c^2}\frac{\mu_1\mu_2}{\mu_1 + \mu_2}}$$

$\Delta\tilde{\rho}(H)$ always > 0

$$\Delta\rho(H) = \frac{1}{ne^2}\frac{e^2H^2}{c^2}\frac{\mu_1\mu_2}{\mu_1+\mu_2}$$

1.2 $\boxed{R = \frac{1}{nec} \cdot \tilde{R} = \frac{1}{nec}\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}}$

if the mobilities are = the total Hall effects is 0. (system with perfect e-h symmetry)

1.3 $\mu_1 = \mu_2$

Sheet 2), eq. 2) $\Rightarrow$ $\tilde{R} = 0$ $\Rightarrow$ $\boxed{R = 0}$

and $\tilde{\rho}$ from 1): $\frac{1}{\tilde{\rho}c^2} = 2\frac{\mu_1}{c^2 + A^2\mu_1^2}$

$$\tilde{\rho} = \frac{c^2 + A^2\mu_1^2}{2\mu_1 c^2} = \frac{1}{2\mu_1} + \frac{A^2\mu_1}{2c^2}$$

as it can be deduced also from last eqs of (1.1)

$A^2 = e^2H^2$

$$\rho = \tilde{\rho}\cdot\frac{1}{ne^2} = \frac{\cancel{6}}{2ne^2m} + \frac{e^2H^2m}{2c^2\cancel{6}}$$