3 dicembr

$$R(x) = \frac{x^2+x+1}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$R(x)(x-1)\Big|_{x=1} = \frac{x^2+x+1}{(x-2)^2}\Big|_{x=1} = \frac{3}{1} = 3 = A$$

$$R(x)(x-2)^2\Big|_{x=2} = \frac{x^2+x+1}{x-1}\Big|_{x=2} = \frac{4+2+1}{1} = 7$$

$$\left(\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right)(x-2)^2\Big|_{x=2} = \frac{A}{x-1}(x-2) + B(x-2) + C\Big|_{x=2}$$

$$= C$$

$$R(x)x = \frac{(x^2+x+1)x}{(x-1)(x-2)^2} \xrightarrow{x\to+\infty} 1$$

$$\left(\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right)x = \frac{Ax}{x-1} + \frac{Bx}{x-2} + \frac{Cx}{(x-2)^2} \xrightarrow{x\to+\infty}$$

$$= A + B \qquad A+B=1 \qquad B = 1-A = -2$$

$$R(x)(x-2)^2 = \frac{x^2+x+1}{x-1} = \frac{A}{(x-1)}(x-2)^2 + B(x-2) + C$$

$$\left(\frac{x^2+x+1}{x-1}\right)'\Big|_{x=2} = B + \underbrace{A\left(\frac{(x-2)^2}{x-1}\right)'\Big|_{x=2}}_{0}$$

$$\left(\frac{(x-2)^2}{x-1}\right)' = \frac{2(x-2)(x-1)-(x-2)^2}{(x-1)^2}\Big|_{x=2} = 0$$

$$\left(\frac{x^2+x+1}{x-1}\right)' = \frac{(2x+1)(x-1)-(x^2+x+1)}{(x-1)^2}$$

$$= \frac{2x^2+x-2x-1-x^2-x-1}{(x-1)^2}$$

$$= \frac{x^2-2x-2}{(x-1)^2}\Big|_{x=2} = 4-4-2 = -2 = B$$

$$R(x)=\frac{x^2+x+1}{(x-1)(x-2)^3}=$$

$$=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{D}{(x-2)^3}$$

$$R(x)(x-1)\Big|_{x=1}=\frac{x^2+x+1}{(x-2)^3}\Big|_{x=1}=\frac{3}{-1}=-3$$

$$=A+(x-1)\left[\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{D}{(x-2)^3}\right]=A$$

$$R(x)(x-2)^3=\frac{A(x-2)^3}{x-1}+B(x-2)^2+C(x-2)+D$$

$$\frac{x^2+x+1}{x-1}\quad R(x)(x-2)^3=\frac{x^2+x+1}{x-1}=\frac{A(x-2)^3}{x-1}+B(x-2)^2+C(x-2)+D$$

$$R(x)(x-2)^3\Big|_{x=2}=\frac{x^2+x+1}{x-1}\Big|_{x=2}=D$$

$$\left(\frac{x^2+x+1}{x-1}\right)'\Big|_{x=2}=A\left(\frac{(x-2)^3}{x-1}\right)'+2B(x-2)+C\Big|_{x=2}$$

$$\left(\frac{x^2+x+1}{x-1}\right)'=\frac{(2x+1)(x-1)-x^2-x-1}{(x-1)^2}=$$

$$=\frac{x^2-2x-2}{(x-1)^2}\Big|_{x=2}=-2=C$$

$$\left(R(x)(x-2)^3\right)''\Big|_{x=2}=\left(\frac{x^2-2x-2}{(x-1)^2}\right)'\Big|_{x=2}=$$

$$=\frac{(2x-2)(x-1)^2-2(x^2-2x-2)(x-1)}{(x-1)^4}\Big|_{x=2}$$

$$=2\frac{(x-1)^2-(x^2-2x-2)}{(x-1)^3}\Big|_{x=2}$$

$$=2\frac{x^2-2x+1-x^2+2x+2}{(x-1)^3}\Big|_{x=2}$$

$$=\frac{6}{1^3}=6$$

$$A\left(\frac{(x-2)^3}{x-1}\right)''\Big|_{x=2}+\left(2B(x-2)\right)'\Big|_{x=2}=2B$$

$$6=2B\qquad B=3$$

$$2B=\frac{d^2}{dx^2}R(x)(x-2)^3\Big|_{x=2}$$

$$B=\frac{1}{2!}\frac{d^2}{dx^2}R(x)(x-2)^3\Big|_{x=2}=3$$

$$A=-3,\quad B=3\quad C=-2\quad D=7$$

$$R(x)=\frac{x^2+x+1}{(x-1)(x-2)^3}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{D}{(x-2)^3}$$

$$R(x)\,x=\frac{Ax}{x-1}+\frac{Bx}{x-2}+\frac{Cx}{(x-2)^2}+\frac{Dx}{(x-2)^3}$$

$R(x)\,x$, $x\to+\infty$, $\to 0$

$$=A+B$$

$$\boxed{A+B=0}$$

$$R(x) = \frac{x^2+x+1}{(x-2)^4} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{D}{(x-2)^4}$$

$A = 0$

$$R(x)(x-2)^4 = x^2+x+1 = A(x-2)^3 + B(x-2)^2 + C(x-2) + D$$

$$D = R(x)(x-2)^4\Big|_{x=2} = x^2+x+1\Big|_{x=2} = 7$$

$$C = \frac{d}{dx} R(x)(x-2)^4\Big|_{x=2} = \frac{d}{dx}(x^2+x+1)\Big|_{x=2} = 2x+1\Big|_{x=2} = 5$$

$$\frac{d}{dx}\left[A(x-2)^3 + B(x-2)^2 + C(x-2) + D\right]\Big|_{x=2} =$$

$$= 3A(x-2)^2 + 2B(x-2) + C\Big|_{x=2} = C$$

$$\frac{d}{dx} R(x)(x-2)^4 = 2x+1 = 3A(x-2)^2 + 2B(x-2) + C$$

$$\frac{d^2}{dx^2} R(x)(x-2)^4 = 2 = 6A(x-2) + 2B\Big|_{x=2}$$

$x=2$

$B = 1$ $\qquad B = \frac{1}{2!}\frac{d^2}{dx^2} R(x)(x-2)^4\Big|_{x=2}$

$$\frac{d^2}{dx^2} R(x)(x-2)^4 = 2 = 6A(x-2) + 2B$$

$$\frac{d^3}{dx^3} R(x)(x-2)^4\Big|_{x=2} = \frac{d}{dx} 2 = 0 = 6A$$

$$A = \frac{1}{3!}\frac{d^3}{dx^3} R(x)(x-2)^4\Big|_{x=2} = 0$$

$$R(x) = \frac{x^2+x+1}{(x-2)^4} = \frac{A_1}{x-2} + \frac{A_2}{(x-2)^2} + \frac{A_3}{(x-2)^3} + \frac{A_4}{(x-2)^4}$$

$i = 1,2,3,4$

$$A_i = \frac{1}{(4-i)!}\left(\frac{d}{dx}\right)^{4-i} R(x)(x-2)^4\Big|_{x=2}$$

$$R(x) = \frac{x+1}{x(x^2+1)} = \frac{x+1}{x(x-i)(x+i)} =$$

$$R(x) = \frac{A}{x} + \frac{B}{x-i} + \frac{C}{x+i}$$

$$A = R(x)\,x\Big|_{x=0} = \frac{x+1}{x^2+1}\Big|_{x=0} = 1$$

$$B = R(x)(x-i)\Big|_{x=i} = \frac{x+1}{x(x+i)}\Big|_{x=i} = \frac{i+1}{i\,2i} =$$

$$= -\frac{i+1}{2}$$

$$C = R(x)(x+i)\Big|_{x=-i} = \frac{x+1}{x(x-i)}\Big|_{x=-i} = \frac{-i+1}{-i\,.\,(-2i)} =$$

$$= -\frac{-i+1}{2}$$

$$\frac{x+1}{x(x^2+1)} = \frac{1}{x} - \frac{i+1}{2}\,\frac{1}{x-i} - \frac{1-i}{2}\,\frac{1}{x+i}$$

$$= \frac{1}{x} - \frac{1}{2}\,\frac{(1+i)(x+i)+(1-i)(x-i)}{x^2+1}$$

$$= \frac{1}{x} - \frac{1}{2}\,\frac{2x+(1+i)i-(1-i)i}{x^2+1}$$

$$= \frac{1}{x} - \frac{1}{2}\,\frac{2x\overbrace{-1-1}^{-2}}{x^2+1} =$$

$$= \frac{1}{x} - \frac{x-1}{x^2+1}$$