Lezione 22 novembre 2024

venerdì 22 novembre 2024 11:11

Visto: $\varphi(t) = \left(\frac{f_1(t)}{g_1(t)}, \frac{f_2(t)}{g_2(t)} \right)$

$\varphi : \mathbb{A}^1 \setminus \Sigma' \longrightarrow Z(f) \subseteq \mathbb{A}^2$

allora si può estendere

$$\rho : \mathbb{P}^1 \setminus \Sigma \longrightarrow Z_p(hf) \subseteq \mathbb{P}^2$$

$$\rho(t_0 : t_1) = \left(t_0^{k_0} h g_1 \cdot h g_2 : t_0^{k_1} h g_2 \cdot h f_1 : t_0^{k_2} h g_1 \cdot h f_2 \right)$$

con $k_0, k_1, k_2 \geq 0$ opportuni in modo da ottenere 3 polin. omog. dello stesso grado

$$\rho|_{U_0 \setminus \Sigma} : \rho(1, t) = \left(1 \cdot (hg_1)(1,t)\,(hg_2)(1,t) : \dots \right)$$

$$= \left(a(hg_1) \cdot a(hg_2) : \dots \right)$$

$$= \left(g_1(t) \cdot g_2(t) : g_2(t) \cdot f_1(t) : g_1(t) \cdot f_2(t) \right)$$

$$\cap V_0 , \quad V_0 = \mathbb{P}^2 \setminus \{x_0 = 0\}$$

$$\updownarrow j_0$$

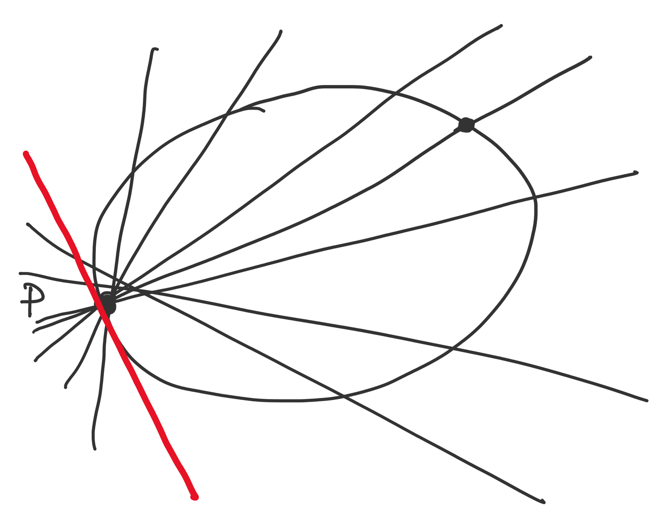
$$\mathbb{A}^2 \quad \Rightarrow \left(\frac{\cancel{g_2(t)} \cdot f_1(t)}{g_1(t) \cancel{g_2(t)}}, \frac{\cancel{g_1(t)} \cdot f_2(t)}{\cancel{g_1(t)} \cdot g_2(t)} \right)$$

Esempi: $d = 2$, $\quad g = \frac{(d-1)(d-2)}{2} - \sum \frac{m_i(m_i - 1)}{2}$

F irriducibile $\Rightarrow Z_p(F)$ è liscia

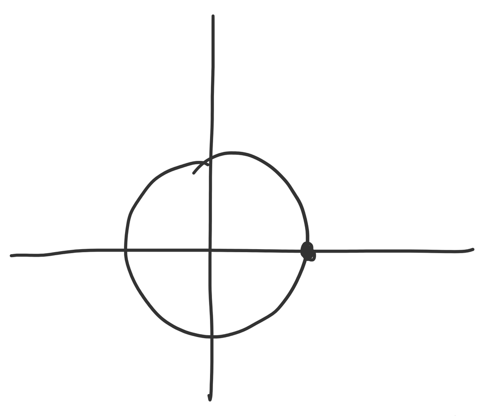
$g = \frac{(d-1)(d-2)}{2} = 0$: ogni conica liscia è razionale

Idea geometrica



es.: $x^2 + y^2 - 1 = 0$

fisso $P = (1, 0)$



generica retta per P: $t_0 \cdot (x - 1) + t_1 y = 0$

su U_0: $t_0 \neq 0$

$$(x - 1) + t \cdot y = 0$$

$$\boxed{x = ty + 1}$$

$$x = \frac{-2t^2}{t^2+1} + 1 = \frac{-t^2+1}{t^2+1}$$

$$(ty + 1)^2 + y^2 - 1 = 0$$

$$t^2 y^2 + 2ty + \cancel{1} + y^2 - \cancel{1} = 0$$

$$(t^2 + 1) y^2 + 2ty = 0$$

$$y \left((t^2+1) y + 2t \right) = 0 \quad \begin{cases} y = 0 \Rightarrow x = 1 \\ y = \frac{-2t}{t^2+1} \end{cases}$$

$\Rightarrow$ una parametr. raz. del cerchio è:

$$\varphi(t) = \left(\frac{-t^2+1}{t^2+1}, \frac{-2t}{t^2+1} \right)$$

Estensione a $\mathbb{P}^1 \setminus \Sigma$:

$$\rho(t_0 : t_1) = \left(h(t^2+1) : h(-t^2+1) : h(-2t) \cdot t_0 \right)$$

$$= \left(t_1^2 + t_0^2 : -t_1^2 + t_0^2 : -2 t_1 t_0 \right)$$

Esercizio: questa terna verifica l'eq.

$$x_1^2 + x_2^2 - x_0^2 = 0$$

e $\Sigma = \emptyset$