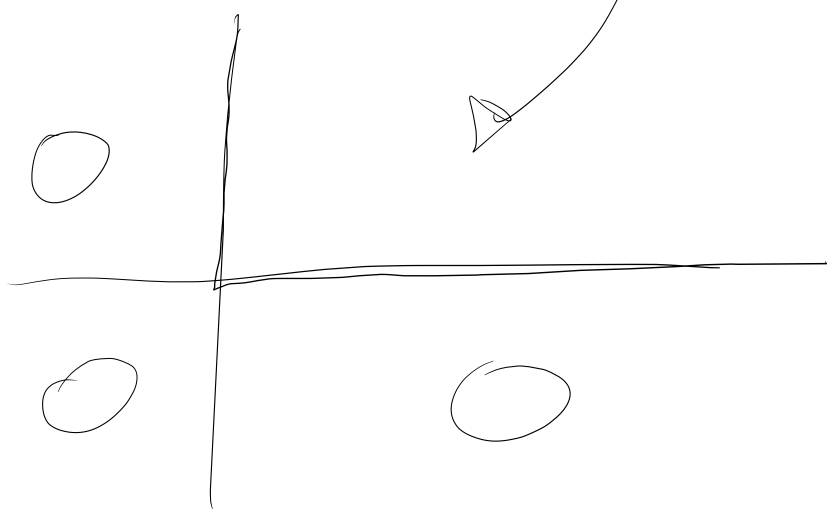
$$F_{X,Y}(x,y) = (1-e^{-x})(1-e^{-y})\left[1+\alpha e^{-x-y}\right] \qquad x,y \geq 0$$



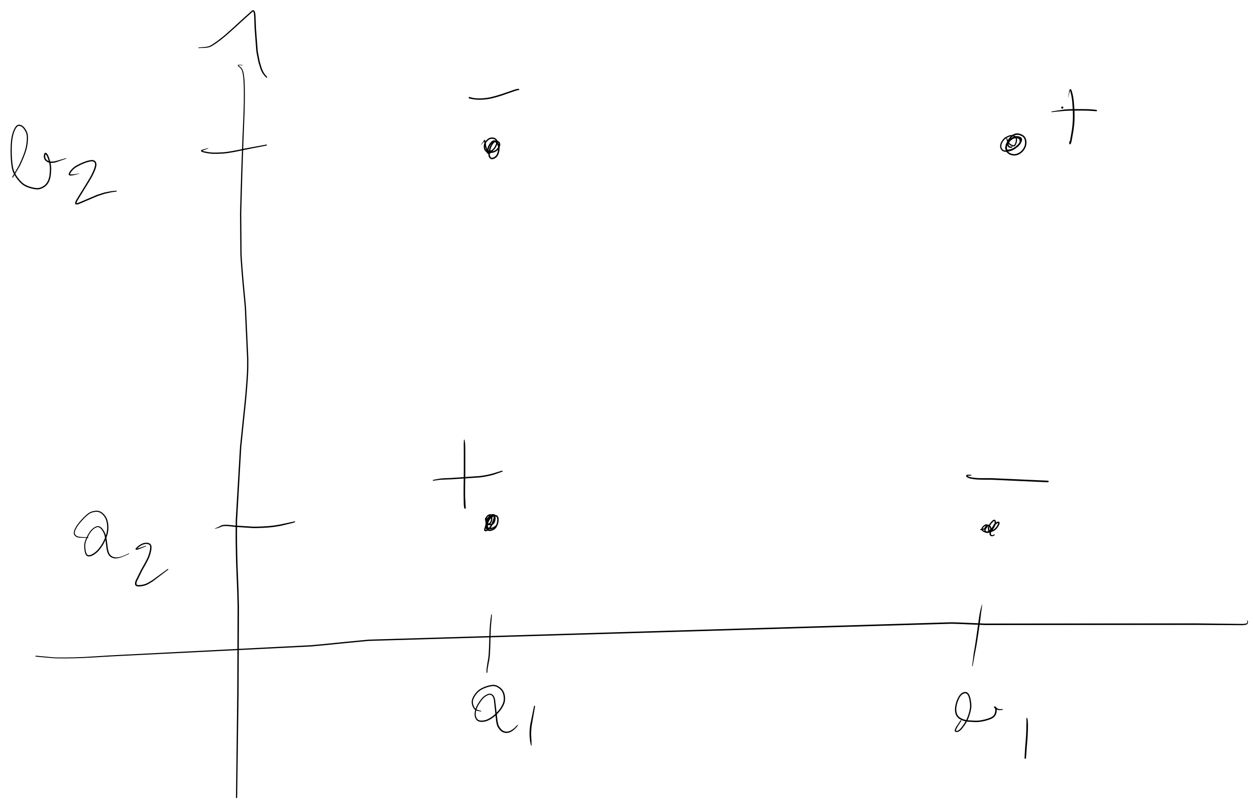
$$F_{X,Y}(x,0) = F_{X,Y}(0,y) = 0$$

CONTINUA

$$\lim_{\substack{x\to+\infty \\ y\to+\infty}} F_{X,Y}(x,y) = 1$$

$$\lim_{x\to-\infty} F_{X,Y}(x,y) = 0$$

$$\underbrace{\Delta^{(1)}_{a_1,b_1}\,\Delta^{(2)}_{a_2,b_2}\,F_{X,Y}}_{F(b_1,b_2)-F(a_1,b_2)-F(b_1,a_2)+F(a_1,a_2)} \geq 0$$



b2
a2
a1
b1
−
+
+
−

$$F_X(x) = \lim_{y \to +\infty} F_{X,Y}(x,y) = 1 - e^{-x}$$ ESP. STANDARD

$$F_{X,Y}(x,y) = (1-e^{-x})(1-e^{-y})\left[1+\alpha\, e^{-x} e^{-y}\right]$$

$$= C\left(F_X(x),\ F_Y(y)\right)$$

$$= F_X(x) \cdot F_Y(y) \left[1+\alpha\,(1-F_X(x))(1-F_Y(y))\right] \cancel{\alpha}$$

$$C(u,v) = u \cdot v \cdot \left[1+\alpha\,(1-u)(1-v)\right] \qquad 0 \le u,v \le 1$$

$$f_{X,Y}(x,y) = f_X(x)\, f_Y(y)\left[\underbrace{1+\alpha\,(2F_X(x)-1)(2F_Y(y)-1)}_{\geq 0}\right]$$

$$f_X(x) = e^{-x} \qquad f_Y(y) = e^{-y}$$

$$\alpha\,(2\overbrace{F_X(x)}^{U}-1)(2\overbrace{F_Y(y)}^{V}-1) = \qquad 0 \leq U,V \leq 1$$

$$\underbrace{\alpha}_{\in(-1,1)}\;\underbrace{\underbrace{(2U-1)}_{\in(-1,1)}\;\underbrace{(2V-1)}_{\in(-1,1)}}_{\in(-1,1)} \in (-1,1)$$

$$\Delta^{(1)}_{a_1,b_1}\Delta^{(2)}_{a_2,b_2}F_{X,Y}=\int_{a_1}^{b_1}\int_{a_2}^{b_2}\underbrace{f_{X,Y}(u,v)}_{\geq 0}\,du\,dv\geq 0$$

(X, Y, Z) NORMALE

$\alpha X + \beta Y + \gamma Z$ NORMALE UNIVARIATA

(X, Y) È NORMALE

$\alpha X + \beta Y = \alpha X + \beta Y + 0 \cdot Z$ NORMALE UNIVARIATA

$$U = c_1 X + c_2 Y + c_3 Z$$
$$V = c_4 X + c_5 Y + c_6 Z$$

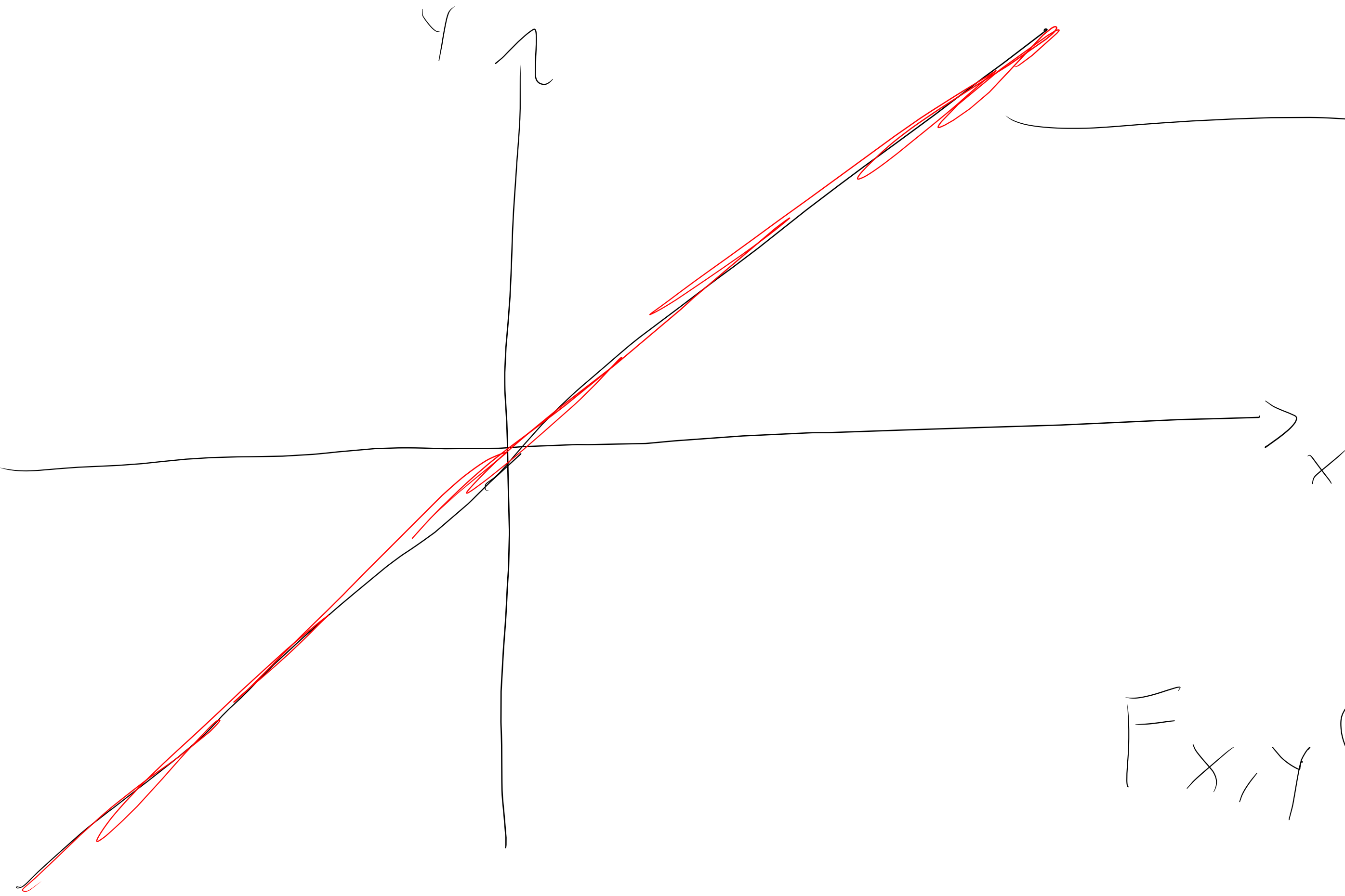
$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

OGNI C.L. DI U, V È UNA C.C. DI X, Y, Z

$X \sim N(0,1)$

$$\begin{bmatrix} X \\ X \end{bmatrix} \qquad z_1 X + z_2 X = (z_1 + z_2) X \sim N(0, \quad)$$

$$\underbrace{1}_{z_1} \cdot X + \underbrace{(-1)}_{z_2} X = X - X = 0$$



$r : y = x$

$$P\left(\begin{bmatrix} X \\ X \end{bmatrix} \in r\right) = 1$$

$$F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(u,v)\,du\,dv$$

NON È POSSIBILE

(RICAVARSI CHI È $F_{X,Y}$)

Σ È DIAGONALE $\quad \Sigma = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{bmatrix}$

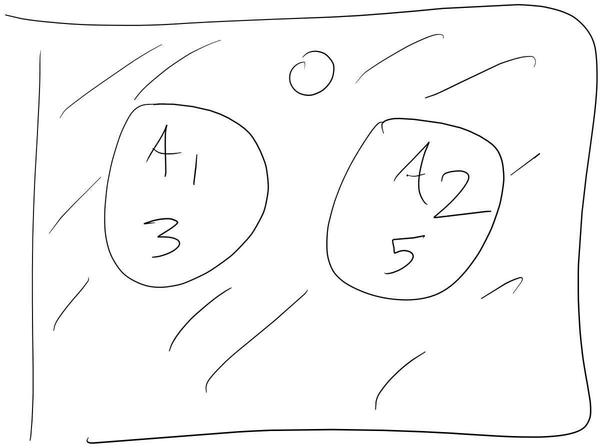
$$\Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\sigma_n^2} \end{bmatrix}$$

ALLORA SI VEDE CHE

$$f_{\underline{X}}(\underline{x}) = \prod_{i=1}^{n} f_{X_i}(x_i) \qquad f_{X_i}(x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{1}{2\sigma_i^2}(x_i-\mu_i)^2}$$

$$X = 3\,\mathbb{1}_{A_1} + 5\,\mathbb{1}_{A_2}$$

$$E[X] = 3P(A_1) + 5P(A_2)$$



$$A_1 \cap A_2 = \emptyset$$

$$X = 3\,\mathbb{1}_{A_1 \cup A_2} + 2\,\mathbb{1}_{A_2}$$

$$E[X] = 3\overbrace{P(A_1 \cup A_2)}^{P(A_1)+P(A_2)} + 2P(A_2)$$

$$= 3P(A_1) + 5P(A_2)$$

$$X = c \cdot \mathbb{1}_A \qquad P(A) = 1$$

$$E[X] = c\,P(A) = c$$

$x_1 \text{———} x_n$

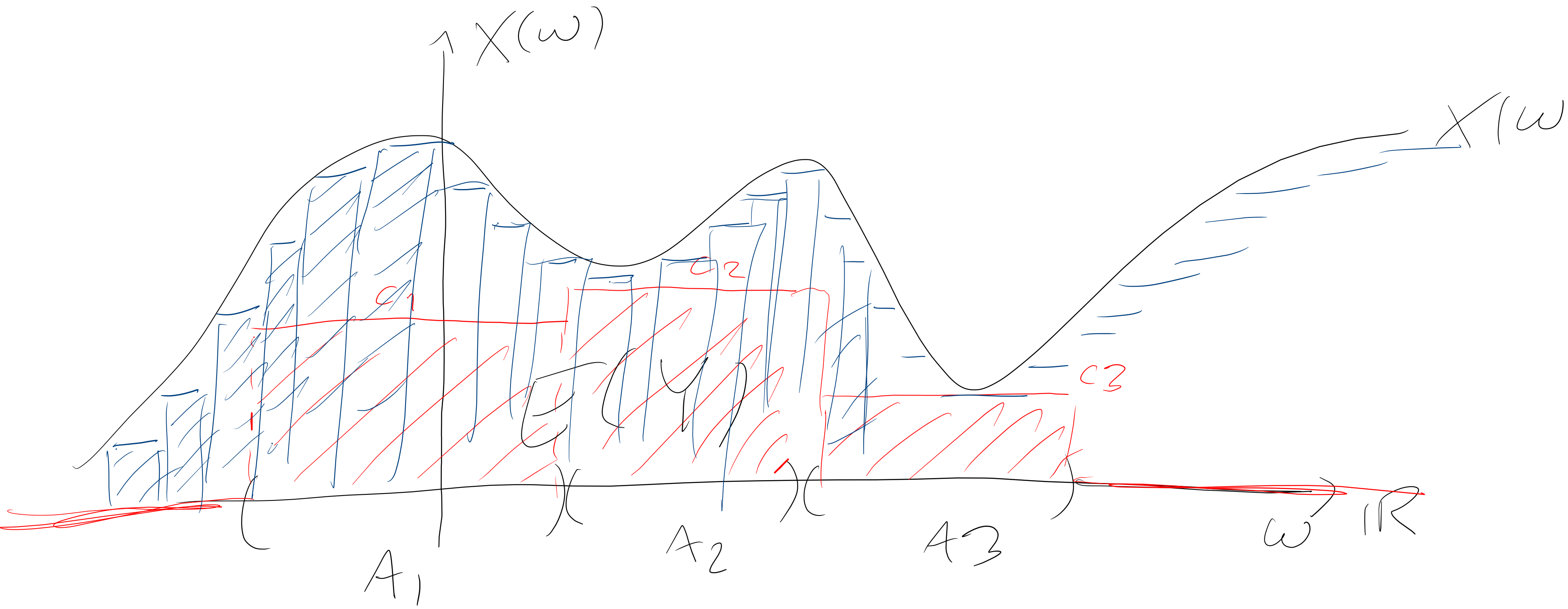
$p_1 \text{———} p_n$

$$X = \sum x_i \, 1_{\{X = x_i\}}$$

$$E[X] = \sum x_i p_i$$

MEDIA
ARITMETICA
DI x_1 —— x_n
PONDERATA CON
PESI p_1 —— p_n

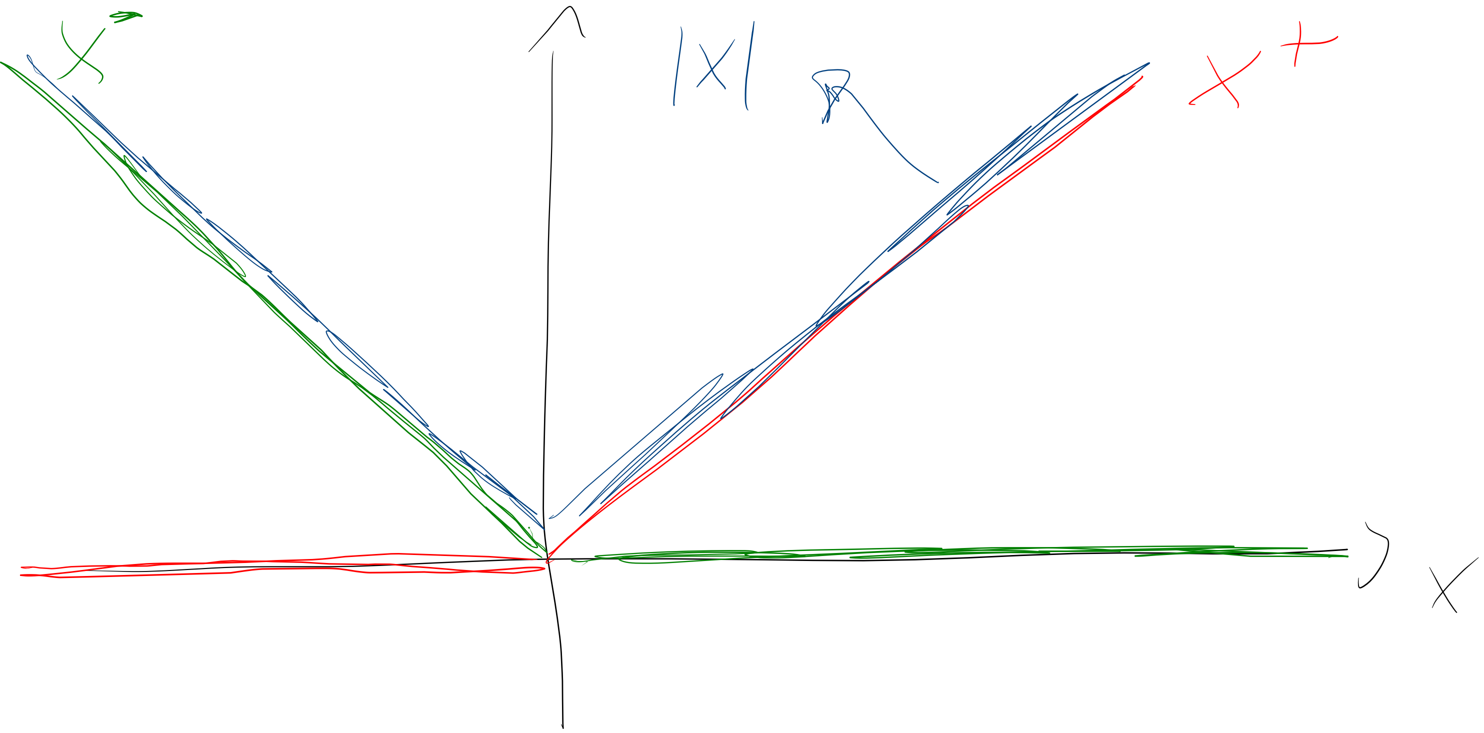
$\Omega = \mathbb{R}$



$0 \le Y \le X$

$$E[Y] = c_1 P(A_1) + c_2 P(A_2) + c_3 P(A_3)$$

$$Y = c_1 1_A + c_2 1_{A_2} + c_3 1_{A_3}$$



$$x^+ = \max(x, 0) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

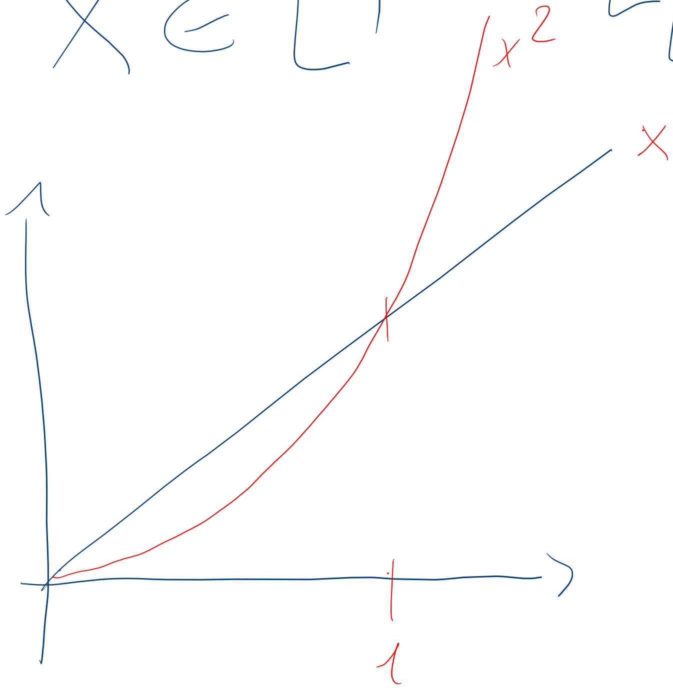
$$x^- = \max(-x, 0) = \begin{cases} 0 & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$x^+ - x^- = x$$

$$x^+ + x^- = |x|$$

$L^2 \subset L^1$ $\qquad$ $1 \le p < q \Rightarrow L^q \subset L^p$

$X \in L^q$ $\quad$ $E\left[|X|^q\right] < +\infty$ $\quad \Rightarrow X \in L^p$?



x^p

x^q $\quad x \ge 0$

$$|X|^p = \underbrace{\underbrace{|X|^p}_{\le 1} 1_{\{|X| \le 1\}}}_{\le 1} + \underbrace{\underbrace{|X|^p}_{\le |X|^q} 1_{\{|X| > 1\}}}_{\le |X|^q}$$

$\le 1 + |X|^q$ $\quad$ <u>INTEGRABILE</u>

$P(A) = 0$

$$X \mathbb{1}_A \begin{cases} X & \text{su } A \\ 0 & \text{su } A^c \end{cases} \qquad P(A^c) = 1$$

$$X \mathbb{1}_A = 0 \quad \text{q.c.} \implies X \mathbb{1}_A \in L^1, \; E[X \mathbb{1}_A] = 0$$

X 0 0.5

100 0.05

DIFFUSA SU $[0, 100]$ CON DENSITÀ $f(x) = \frac{0.45}{100}$

$0 < x < 100$

$$E[X] = 0 \times 0.5 + 100 \times 0.05 + \int_0^{100} x \, \frac{0.45}{100} \, dx$$