Financial Econometrics

June 17th 2016

Exercise 1 (3/10) Consider the following APT-style model:

$$r_A = \beta_0 + \beta_1 r_M + \beta_2 y + \beta_3 \pi + u$$

where r_A and r_M are excess returns on, respectively, an asset A and a market index; y is the growth rate of industrial production; π is the inflation rate.

You have been given a sample containing 70 data points and you have estimated the model by OLS. You are now interested in reducing the given model - if appropriate - and for this purpose you want to assess the joint significance of y and π. In other words, you want to compare the above model with the reduced CAPM-like one:

$$r_A = \beta_0 + \beta_1 r_M + v$$

which you have in turn estimated. Call the first model *unrestricted* (U) and the second one *restricted* (R). From estimation you have obtained the following quantities of interest:

- URSS the sum of squared residuals from model U
- RRSS the sum of squared residuals from model R

Which statistical test would you use?

1. specify the null and the alternative hypotheses
2. describe the testing procedure

Exercise 2 (3/10) Consider a stochastic process Y_t. Consider another process Z_t and the regression

$$Y_t = \delta_0 + \delta_1 Z_t + \varepsilon_t \tag{1}$$

With respect to the regression (1),

1. define *weak* (or *covariance*) stationarity for (any process) Y_t or Z_t
2. how would you test whether (1) is a spurious or a cointegrating regression?
3. briefly discuss the consequences of the results in the previous point 2. on the properties of $\hat{\delta}_{OLS}$

Exercise 3 (4/10) Consider the simple bivariate linear model

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

1. Answer the following question: if you write the model in matrix form, as $y = X\beta + \varepsilon$, who is X?
2. Write the formula for the ordinary least squares estimator $\hat{\beta}_{OLS}$ using both summations and matrix notation
3. Derive the ordinary least squares estimator $\hat{\beta}_{OLS}$ from the first-order conditions *(You have the choice between using either summations or linear algebra)*