CHIRAL ANOMALY IN 4d

$P = \bar\psi \gamma_5 \psi$

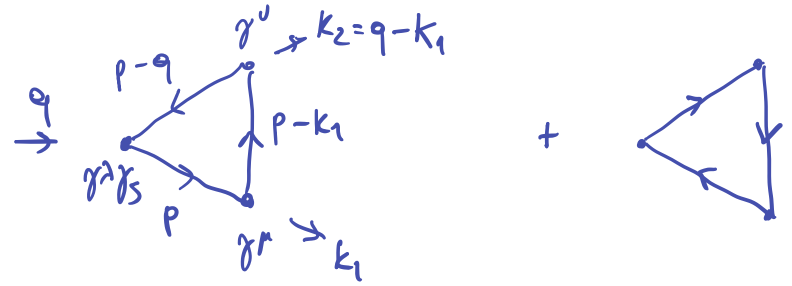
Ci calcoliamo i correlatori $\underbrace{\langle j^\mu j^\nu j_A^\lambda \rangle}_{T^{\mu\nu\lambda}}$ e $\underbrace{\langle P j^\mu j^\nu \rangle}_{T^{\mu\nu}}$

Perché anomalia solo in correlatori con 3 correnti? VEDI (*)

AWI : $q_\lambda T^{\mu\nu\lambda} = 2m\, T^{\mu\nu}$

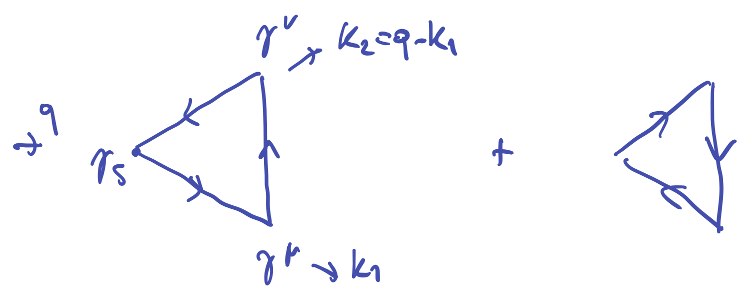
VWI : $k_{1\mu} T^{\mu\nu\lambda} = 0 = k_{2\nu} T^{\mu\nu\lambda}$

$\langle j^\mu j^\nu j_A^\lambda \rangle$



$$T^{\mu\nu\lambda} = i \int \frac{d^4p}{(2\pi)^4} (-)\, \mathrm{tr}\left[\frac{i}{\not p - m} \gamma^\lambda \gamma_5 \frac{i}{\not p - \not q - m} \gamma^\nu \frac{i}{\not p - \not k_1 - m} \gamma^\mu \right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$\langle j^\mu j^\nu P \rangle$



$$T^{\mu\nu} = i \int \frac{d^4p}{(2\pi)^4} (-)\, \mathrm{tr}\left[\frac{i}{\not p - m} \gamma_5 \frac{i}{\not p - \not q - m} \gamma^\nu \frac{i}{\not p - \not k_1 - m} \gamma^\mu \right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

Avremo bisogno della seguente identità:

$$\not q \gamma_5 = \gamma_5 (\not p - \not q - m) + (\not p - m)\gamma_5 + 2m\gamma_5$$

$$q_\lambda T^{\mu\nu\lambda} = 2m\, T^{\mu\nu} + R_1^{\mu\nu} + R_2^{\mu\nu}$$

$$R_1^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\overbrace{\frac{1}{\not p - \not k_2 - m}\gamma_5\gamma^\nu \frac{1}{\not p - \underbrace{\not q}_{=k_1+k_2} - m}\gamma^\mu}^{1°} - \overbrace{\frac{1}{\not p - m}\gamma_5\gamma^\nu \frac{1}{\not p - \not k_1 - m}\gamma^\mu}^{2°} \right)$$

$$R_2^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\frac{1}{\not p - \not k_1 - m}\gamma_5\gamma^\mu \frac{1}{\not p - \not q - m}\gamma^\nu - \frac{1}{\not p - m}\gamma_5\gamma^\mu \frac{1}{\not p - \not k_2 - m}\gamma^\nu \right)$$

Se $R_1 = R_2 = 0 \Rightarrow$ AWI è soddisfatta

→ Formalmente $R_1 \to 0$ se shiftiamo $p \to p + k_2$ nel 1° integrale e $p \to p - k_2$ nel 2° integrale

$$R_1^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\frac{1}{\not p - m}\gamma_5\gamma^\nu \frac{1}{\not p - \not k_1 - m}\gamma^\mu - \frac{1}{\not p - \not k_2 - m}\gamma_5\gamma^\nu \frac{1}{\not p - \not q - m}\gamma^\mu \right)$$

$$= -R_1^{\mu\nu} \Rightarrow R_1^{\mu\nu} = 0$$

Stesso conto per $R_2^{\mu\nu}$ $(p \to p \pm k_1)$

↳ siccome l'integrale diverge (div. lineare), le manipolazioni interne all'integrale non sono permesse $(\infty - \infty)$

→ dobbiamo regolarizzare la teoria.

Per questo conto useremo la regolarizzaz. di PAULI-VILLARS:

Introduciamo una specie ausiliaria di fermioni Ψ con STATISTICA OPPOSTA e massa M; alla fine del conto bisogna prendere il lim. $M \to \infty$ (statistica errata)

$\hookrightarrow$ • $T^{\mu\nu\lambda}_{reg} = T^{\mu\nu\lambda}(m) - T^{\mu\nu\lambda}(M)$

$$T^{\mu\nu\lambda}_{phys} \equiv \lim_{M\to\infty} T^{\mu\nu\lambda}_{reg}$$

• $T^{\mu\nu}_{phy} = \lim_{M\to\infty} T^{\mu\nu}_{reg} = \lim_{M\to\infty} \left(T^{\mu\nu}(m) - T^{\mu\nu}(M)\right) = T^{\mu\nu}(m)$

$T^{\mu\nu}$ è convergente e $T^{\mu\nu}(M) \sim \frac{1}{M} \xrightarrow[M\to\infty]{} 0$

Questa regolarizzazione fa una scelta specifica su di preservare tra AWI e VWI:

$$K_{1\mu} T^{\mu\nu\lambda}_{phy} = 0$$
$$K_{2\nu} T^{\mu\nu\lambda}_{phys} = 0 \quad \rightarrow \quad \text{VWI è PRESERVATA}$$

(Euristicam. $\mathcal{L}_{\Phi}$ non rompe $U(1)_V$)

AWI ?

Ora $T^{\mu\nu\lambda}_{reg}$ è finito ; in particolare $R_1{}^{\mu\nu}_{reg}$ e $R_2{}^{\mu\nu}_{reg}$ sono finiti $\longrightarrow$ quindi ora posso fare le manipolazioni all'interno dell'integrale $\Longrightarrow R_1{}^{\mu\nu}_{reg} = 0 = R_2{}^{\mu\nu}_{reg}$

$$\Rightarrow \quad q_\lambda T^{\mu\nu\lambda}_{reg} = 2m\, T^{\mu\nu}(m) - 2M\, T^{\mu\nu}(M)$$

$$\Rightarrow \quad q_\lambda T^{\mu\nu\lambda}_{phys} = 2m\, T^{\mu\nu}(m) - \underbrace{\lim_{M\to\infty} 2M\, T^{\mu\nu}(M)}_{\text{ora calcoleremo qta (violazione di AWI se} \neq 0)}$$

$$T^{\mu\nu}(M) = \int \frac{d^4p}{(2\pi)^4}\,(-)\,\mathrm{tr}\,\frac{1}{\not p - M}\,\gamma_5\,\frac{1}{\not p-\not q-M}\,\gamma^\nu\,\frac{1}{\not p-\not k_1-M}\,\gamma^\mu$$

$$+\begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$\frac{1}{a_1 a_2 a_3} = 2\int_0^1 dx_1 \int_0^{1-x_1} dx_2\, \frac{1}{\left[a_1 x_2 + a_2(1-x_1-x_2) + a_3 x_1\right]^3}$$

$$= -\int \frac{d^4p}{(2\pi)^4}\, 2\int_0^1 dx_1 \int_0^{1-x_1} dx_2\, \frac{\mathrm{tr}\left[(\not p+M)\gamma_5(\not p-\not q+M)\gamma^\nu(\not p-\not k_1+M)\gamma^\mu\right]}{\left[(p^2-M^2)x_2 + \left((p-q)^2-M^2\right)(1-x_1-x_2) + \left((p-k_1)^2-M^2\right)x_1\right]^3}$$

$$+\begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$\mathrm{Tr}\,\gamma_5\gamma^{\alpha_1}\cdots\gamma^{\alpha_n} = 0 \quad \text{für} \quad n \neq 4$$

$$= 4i\,\epsilon^{\alpha_1\cdots\alpha_4} \quad \text{für} \quad n = 4$$

$$\mathrm{tr}\left(\not p\,\gamma_5(\cancel{\not p}-\not q)\gamma^\nu\gamma^\mu\right) + \mathrm{tr}\left(\not p\,\gamma_5\gamma^\nu(\cancel{\not p}-\not k_1)\gamma^\mu\right) + \mathrm{tr}\left(\gamma_5(\not p-\not q)\gamma^\nu(\not p-\not k_1)\gamma^\mu\right)$$

$$= \cancel{\mathrm{tr}\left(\gamma_5\not p\not q\gamma^\nu\gamma^\mu\right)} + \cancel{\mathrm{tr}\left(\gamma_5\not p\gamma^\nu\not k_1\gamma^\mu\right)} - \cancel{\mathrm{tr}\left(\gamma_5\not p\gamma^\nu\not k_1\gamma^\mu\right)}$$

$$- \cancel{\mathrm{tr}\left(\gamma_5\not q\gamma^\nu\not p\gamma^\mu\right)} + \mathrm{tr}\left(\gamma_5\not q\gamma^\nu\not k_1\gamma^\mu\right) =$$

$$\not q = \cancel{\not k_1} + \not k_2$$

$$= 4i\,\epsilon^{\beta\nu\alpha\mu}k_{1\alpha}k_{2\beta} \qquad \epsilon^{\alpha\nu\beta\mu} = \epsilon^{\beta\alpha\nu\mu} = \epsilon^{\beta\mu\alpha\nu}$$

$$= 4i\,\epsilon^{\beta\mu\alpha\nu}k_{2\alpha}k_{1\beta}$$

$$= -\int\frac{d^4p}{(2\pi)^4}\,2\int_0^1 dx_1\int_0^{1-x_1}dx_2\,\frac{M\left(-4i\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\right)}{\left[x_2p^2 - x_2M^2 + (1-x_1-x_2)(p^2-2pq+q^2-M^2) + x_1(p^2-2pk_1+k_1^2-M^2)\right]^3} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$p^2 - M^2 - 2p\left(q(1-x_1-x_2) + k_1x_1\right) + q^2(1-x_1-x_2) + k_1^2x_1$$

$$= p^2 - 2pK - \bar M^2 \qquad K \equiv q(1-x_1-x_2)+k_1x \qquad \bar M \equiv M^2 - q^2(1-x_1-x_2) - k_1^2x_1$$

$$= 8iM\,\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2-2pk-\bar{M}^2)^3} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$p^0 = i l^0 \quad \Downarrow \quad p^2 = -l^2$$

$$-i \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+2pk+\bar{M}^2)^3} = \frac{-i\,\Gamma(3-2)}{(\bar{M}-k^2)^{3-2}(4\pi)^2\,\Gamma(3)}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2+2kp+b^2)^A} = \frac{\Gamma(A-d/2)}{(b^2-p^2)^{A-d/2}(4\pi)^{d/2}\,\Gamma(A)}$$

$$= \frac{-i}{(4\pi)^2} \frac{1}{2(\bar{M}^2-k^2)}$$

$$= \frac{1}{4\pi^2}\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{M}{M^2+\underbrace{f(x_1,x_2)}_{\text{non dip da } M}} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

stessa espressione del primo termine ma con $f(x_1,x_2)$ sostituita da una diversa $g(x_1,x_2)$

$$\lim_{M\to\infty} 2M\,T^{\mu\nu}(M) = \frac{1}{4\pi^2}\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \underbrace{\lim_{M\to\infty}\left(\frac{2M^2}{M^2+f(x_1,x_2)} + \frac{2M^2}{M^2+g(x_1,x_2)}\right)}_{=4}$$

$$= \frac{1}{\pi^2}\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \underbrace{\int_0^1 dx_1 \int_0^{1-x_1} dx_2}_{= \int_0^1 dx_1 (1-x_1) = 1-\frac{1}{2} = \frac{1}{2}}$$

$$q_\lambda T^{\mu\nu\lambda}_{phys} = 2m\,T^{\mu\nu}(m) \underbrace{- \frac{1}{2\pi^2}\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}}_{\text{ANOMALIA}}$$

$$\partial^z_\lambda \langle j^\mu(x)\, j^\nu(y)\, j^\lambda_A(z)\rangle = -i\int \frac{dq}{(2\pi)^4}\int\frac{dk_1}{(2\pi)^4}\int\frac{dk_2}{(2\pi)^4} e^{-ik_1 x - ik_2 y + iqz}\,\cdot$$

$$\cdot\, iq_\lambda T^{\mu\nu\lambda}(q,k_1,k_2)$$

$$= \int\frac{dq}{(2\pi)^4}\int\frac{dk_2}{(2\pi)^4} e^{-ik_2(y-x)+iq(z-x)}\left(-\frac{1}{2\pi^2}\epsilon^{\mu\nu\alpha\beta} iq_\alpha(-i)k_{2\beta}\right)$$

$$= -\frac{1}{2\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial^z_\alpha\,\delta(z-x)\,\partial^y_\beta\,\delta(y-x)$$

$$\partial^z_\lambda\langle j^\lambda_A(z)\rangle_A = \partial^z_\lambda\langle j^\lambda_A(z)\rangle_{free} + \dots - \frac{1}{2}\int dx_1\, dx_2\, A_{\rho_1}(x_1)A_{\rho_2}(x_2)\,\cdot$$

$$\cdot\,\partial_\lambda\langle j^{\rho_1}(x_1)\, j^{\rho_2}(x_2)\, j^\lambda_A(z)\rangle + \dots$$

A ↑ campo di gauge esterno accoppiato alla corrente vettoriale j^μ

$$= -\int dx_1\, dx_2\, A_{\rho_1}(x_1)A_{\rho_2}(x_2)\left(-\frac{1}{4\pi^2}\right)\epsilon^{\rho_1\rho_2\alpha\beta}\partial^z_\alpha\,\delta(z-x_1)\,\partial^{x_2}_\beta\,\delta(x_2-\underset{=z}{x_1})$$

$$= \frac{1}{16\pi^2}\epsilon^{\rho_1\rho_2\alpha\beta}F_{\alpha\rho_1}F_{\beta\rho_2}$$

$$= -\frac{1}{16\pi^2}\epsilon^{\alpha\rho_1\beta\rho_2}F_{\alpha\rho_1}F_{\beta\rho_2}$$

$$= -\frac{1}{16\pi^2}\epsilon^{\mu_1\nu_1\mu_2\nu_2}F_{\mu_1\nu_1}F_{\mu_2\nu_2}$$

NON-ABELIAN FIELDS

$\mathcal{L} = \bar{\Psi}\, i \not{\partial} \Psi = \bar{\Psi}_i\, i\gamma^\mu (\partial_\mu \delta^{ij}) \Psi_j$ $\quad U(1)_{AV}$, $SU(N)_{AV}$, $N = \dim R$

Campi Ψ stanno in una rap R di G

$\Rightarrow \quad j^{\mu a} = \bar{\Psi} \gamma^\mu t^a_R \Psi$ $\qquad a \in$ Adj rap.

$j^{\mu a}_A = \bar{\Psi} \gamma^\mu \gamma_5 t^a_R \Psi \qquad P = \bar{\Psi} \gamma_5 t^a_R \Psi$

In questa situazione ci sono correlatori a più punti che contribuiscono all'anomalia (cioè $\langle \underbrace{j \cdots j}_{n} \rangle$ con $n \geq 3$)

$$T^{abc\,\mu\nu\lambda}(k_1, k_2) = i \int dx\, dy\, e^{ik_1 x + ik_2 y} \langle j^{a\mu}(x)\, j^{b\nu}(y)\, j_A^{c\lambda}(0) \rangle$$

$$T^{abc\,\mu\nu} = \quad '' \quad P^c(0)$$

$\downarrow$

AWI: $$q_\lambda T^{abc\,\mu\nu\lambda} = 2m\, T^{abc\,\mu\nu}_{(m)} - \underbrace{\frac{C^{abc}}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}}_{\text{ABJ anomaly per il caso non-abeliano}}$$

$$C^{abc} = \frac{1}{2} \mathrm{tr}_R \{ t^a_R, t^b_R \} t^c_R$$

Se manteniamo la CORRENTE ASSIALE ABELIANA $j^\lambda_A = \bar{\Psi} \gamma^\mu \gamma_5 \Psi$ (j^λ_A è un "color singlet", cioè in rap. triviale di G),

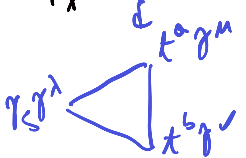
$\updownarrow$ legata a simm. $\Psi \mapsto e^{i\gamma_5 \beta} \Psi$

(mentre $j^{\lambda a}_A = \bar{\Psi} \gamma^\mu \gamma_5 t^a_R \Psi$ è legata a $\Psi \mapsto e^{i\gamma_5 \beta^a t^a_R} \Psi$)

e quella vett. non-ab. $j^{\mu a}_V = \bar{\Psi} \gamma^\mu t^a_R \Psi$

allora :

$$q_\lambda T^{ab\,\mu\nu\lambda} = 2m\, T^{ab\,\mu\nu} - \frac{c(R)\, \delta^{ab}}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \qquad (*)$$



$\mathrm{tr}\,(t^a t^b \mathbb{1})$

Accoppiamo la corrente vettoriale $j^{\mu a}$ a un campo di gauge non-abeliano A^a_μ. L'anomalia (*) produrrà ancora termini del tipo $(\partial_\xi A^a_\beta)^2$. Come contribuiscono ~~le altre~~ gli altri termini dell' WI anomale?

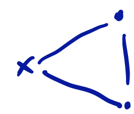
Devono contribuire in maniera consistente col fatto che $\partial_\lambda j^\lambda_A$ è un SINGOLETTO sotto G

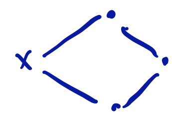
$$\langle \partial_\lambda j^\lambda_A \rangle_A = - \frac{c(R)\,\delta^{ab}}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^b_{\alpha\beta} \qquad (m=0)$$

SINGLET ANOMALY

$$= - \frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} \, \mathrm{tr}\, F_{\mu\nu} F_{\alpha\beta}$$

$\epsilon^{\mu\nu\alpha\beta}\delta^{ab} F^a_{\mu\nu} F^b_{\alpha\beta}$

$\epsilon^{\mu\nu\alpha\beta}\partial_\mu A^a_\nu \partial_\alpha A^a_\beta \longrightarrow$  triangolo

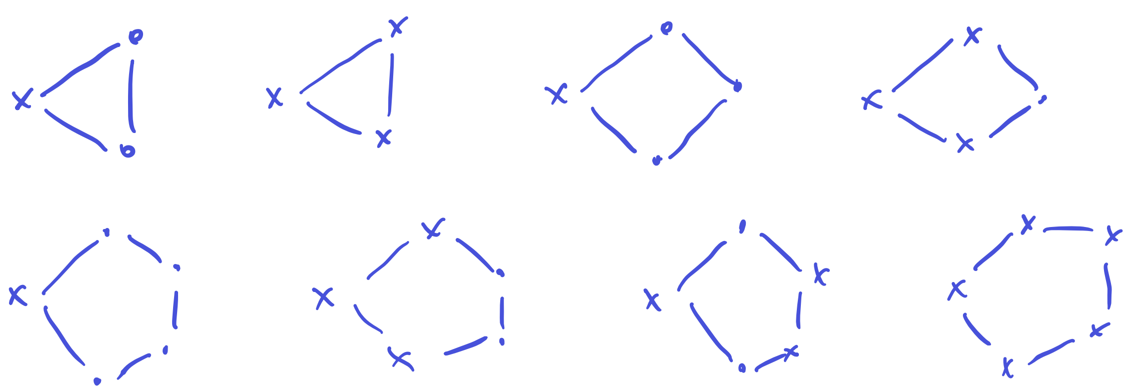
$\epsilon^{\mu\nu\alpha\beta}\partial_\mu A^a_\nu A^c_\alpha A^d_\beta f^{bcd} \longrightarrow$  quadrato

$$\epsilon^{\mu\nu\alpha\beta} A^c_\mu A^d_\nu A^e_\alpha A^f_\beta \underbrace{f^{ecd} f^{aef}}$$

$$= \# \left(f^{acd} f^{aef} + f^{aed} f^{efc} + f^{afd} f^{ace} \right) = 0 \quad \text{Jacobi id.}$$

Se anche la corrente assiale viene presa non-ab. $j^{\lambda a}_A = \bar\psi \gamma^\mu \gamma_5 t^a_R \psi$

$\Rightarrow$ molti diagrammi contribuiscono $\rightarrow$ molte WI sono anomale nella teoria libera



Se accoppiamo $j^{\mu a}$ a A^a_μ :

$$\langle (D_\mu j^\mu_A)^a \rangle = -\frac{1}{16\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left(t^a F_{\mu\nu}F_{\rho\sigma}\right) \qquad (*)$$

COVARIANT ANOMALY

Finora abbiamo sempre accoppiato A^a_μ alla CORRENTE VETT., cioè abbiamo GAUGIATO la simmetria preservata. Infatti se la simm. di gauge è ANOMALA la teoria quantistica risulta INCONSISTENTE: 1) perdiamo la relazione di equivalenza necessaria per tener conto della ridondanza della descrizione 2) alcune proprietà come la rinormalizzabilità e l'unitarietà vengono violate.

Campi chirali

Pensiamo di accoppiare correnti R-handed e L-handed a diversi campi vett. A^R_μ e A^L_μ.

Otteniamo:

$$\langle (D^H_\mu j^{H\mu})^a \rangle = \eta_H \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left(t^a \partial_\mu\left(A^H_\nu \partial_\rho A^H_\sigma + \frac{i}{2} A^H_\nu A^H_\rho A^H_\sigma\right)\right)$$

$H = L, R$

CONSISTENT ANOMALY

$$\eta_H = \begin{cases} -1 & H = L \\ +1 & H = R \end{cases}$$

Notiamo che otteniamo lo stesso risultato formale (a meno del segno) per L e R.

Esiste anche un altro tipo di anomalia COVARIANT ANOMALY

per correnti L, R (diversa regolarizzazione rispetto a CONSIST.)

$$\langle (D_\mu j^{H\mu})^a \rangle = \eta_H \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left(t^a F^H_{\mu\nu} F^H_{\rho\sigma}\right) \qquad H = L, R$$

$$j^{L,R\,\mu\,a} = \bar{\psi} \gamma^\mu t^a \left(\frac{1 \pm \gamma_5}{2}\right) \psi = \frac{1}{2}\left(j^{\mu a} \pm j_A^{\mu a}\right)$$

$$A_\mu^{L,R} = A_\mu^V \pm A_\mu^A$$

In $\mathcal{L}$

$\sim j^{L\mu a} A_\mu^{aL} + j^{R\mu a} A_\mu^{aR}$

$\Leftrightarrow j^\mu A_\mu^{aV} + j_A^\mu A_\mu^{aA}$

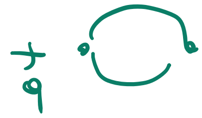
$\hookrightarrow$ se $A_\mu^A \equiv 0 \Rightarrow A_\mu^{L,R} = A_\mu$

e $\langle (D_\mu j_A^\mu)^a \rangle = \langle (D_\mu j^{L\mu})^a \rangle - \langle (D_\mu j^{R\mu})^a \rangle =$

$$= 2\left[-\frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left(t^a F_{\mu\nu} F_{\rho\sigma}\right)\right] = -\frac{1}{16\pi^2} \epsilon\, \mathrm{tr}(tFF)$$

cioè (*).

(A) – Quando ho $\langle j^{\mu_1}(x_1)\cdots j^{\mu_n}(x_n)\rangle$ $n>3$ gli integrali sono convergenti e quindi manipolazioni formali sono lecite e portano a $\partial\langle j\cdots j\rangle = 0$.

– Per due correnti:  conto simile a funz. a 2 pt. $\langle AA\rangle = \langle jj\rangle$ a 1-loop in QED

$$\langle j_A^\mu j_A^\lambda\rangle = (i^2)\, i\int \frac{d^4p}{(2\pi)^4}(-)\,\underbrace{\mathrm{tr}\left[(\not p+m)\gamma^\lambda\gamma_5(\not p-\not q+m)\gamma^\mu\gamma_5\right]}_{\mathrm{tr}\,(\not p+m)\gamma^\lambda(\not p-\not q-m)\gamma^\mu}\frac{1}{(p^2-m^2)((p-q)^2-m^2)}$$

$$\Rightarrow \langle j_A j_A\rangle = \langle jj\rangle\Big|_{m^2\mapsto -m^2}$$

ma $\langle jj\rangle$ è proporzionale a $q^2 g^{\mu\nu} - q^\mu q^\nu$

$\Rightarrow \partial\langle j_A j_A\rangle = 0$

$$\langle j^\mu j_A^\lambda\rangle = (i^2)\, i\int \frac{d^4p}{(2\pi)^4}(-)\,\underbrace{\mathrm{tr}\left[(\not p+m)\gamma^\lambda\gamma_5(\not p-\not q+m)\gamma^\mu\right]}_{=-\mathrm{tr}\left[\not p\gamma^\lambda\gamma_5\not q\gamma^\mu\right]\propto q_\alpha p_\beta\epsilon^{\alpha\beta\mu\lambda}}\frac{1}{(p^2-m^2)((p-q)^2-m^2)}$$

$$\Rightarrow \partial_\lambda\langle j^\mu j_A^\lambda\rangle = 0$$

– $\langle j^\mu\rangle = \langle j_A^\mu\rangle = 0$ per invarianza di Lorentz.