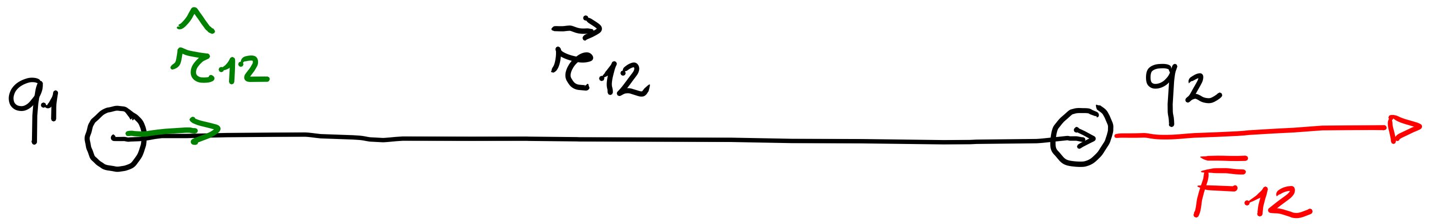
ELETTROSTATICA

$\oplus$ protone

$\ominus$ elettrone

$e = 1{,}602 \cdot 10^{-19}\ C$

SI Coulomb C



$$\vec{F}_{12} = K \cdot \frac{q_1 \cdot q_2}{|\vec{r}_{12}|^2}\, \hat{r}_{12}$$

$$[K] = \frac{N}{C^2} \cdot m^2$$

$$K = 9 \cdot 10^9\ \frac{Nm^2}{C^2}$$

$$K = \frac{1}{4\pi\varepsilon_0}$$

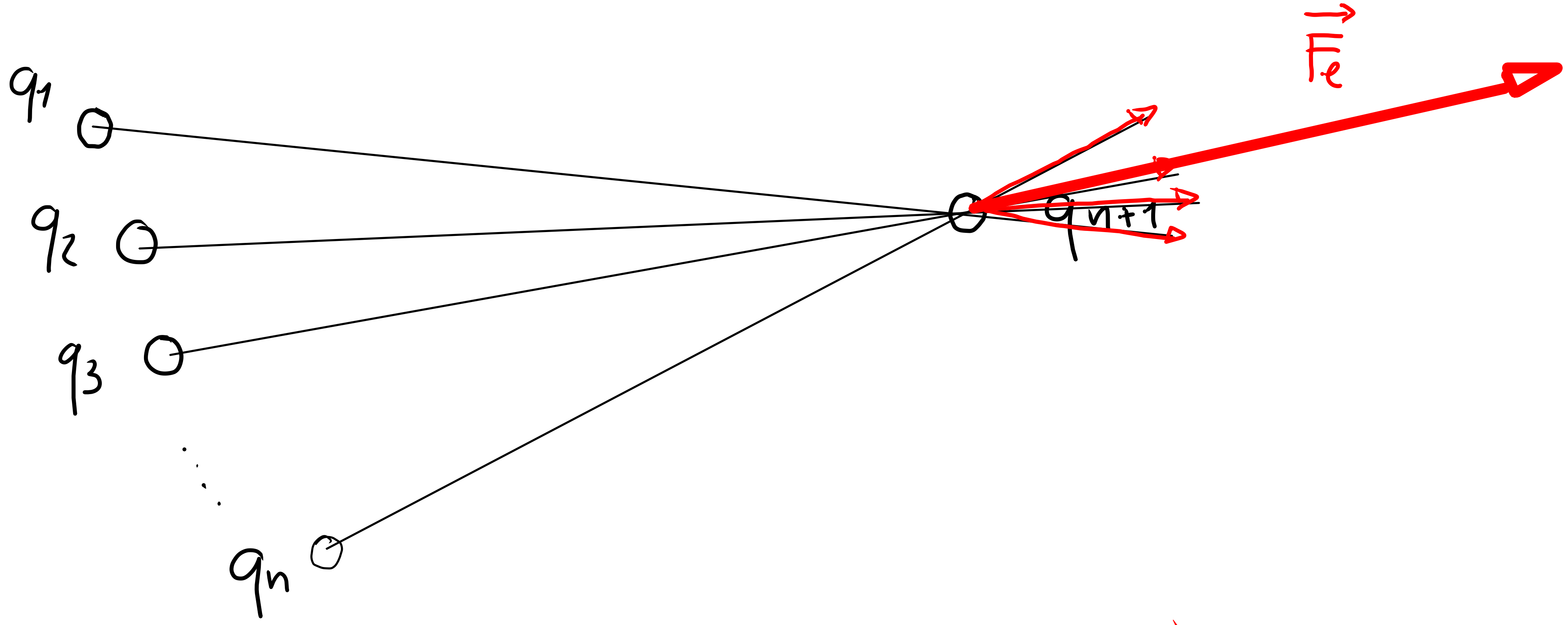
costante dielettrica del vuoto

$$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ \frac{C^2}{Nm^2}$$

$\varepsilon_0 \rightarrow \varepsilon_0 \cdot \varepsilon_r \leftarrow$ costante dielettrica relativa

$\varepsilon_r > 1$

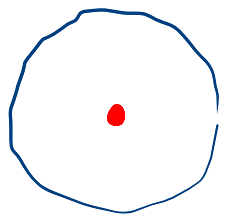
PRINCIPIO DI SOVRAPPOSIZIONE

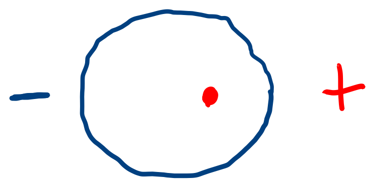


CONDUTTORI e ISOLANTI (o DIELETTRICI)

conduttori e^- "liberi" ⇒ elettroni di conduzione

isolanti e^- "legati" ⇒ gli atomi (o le molecole) si polarizzano

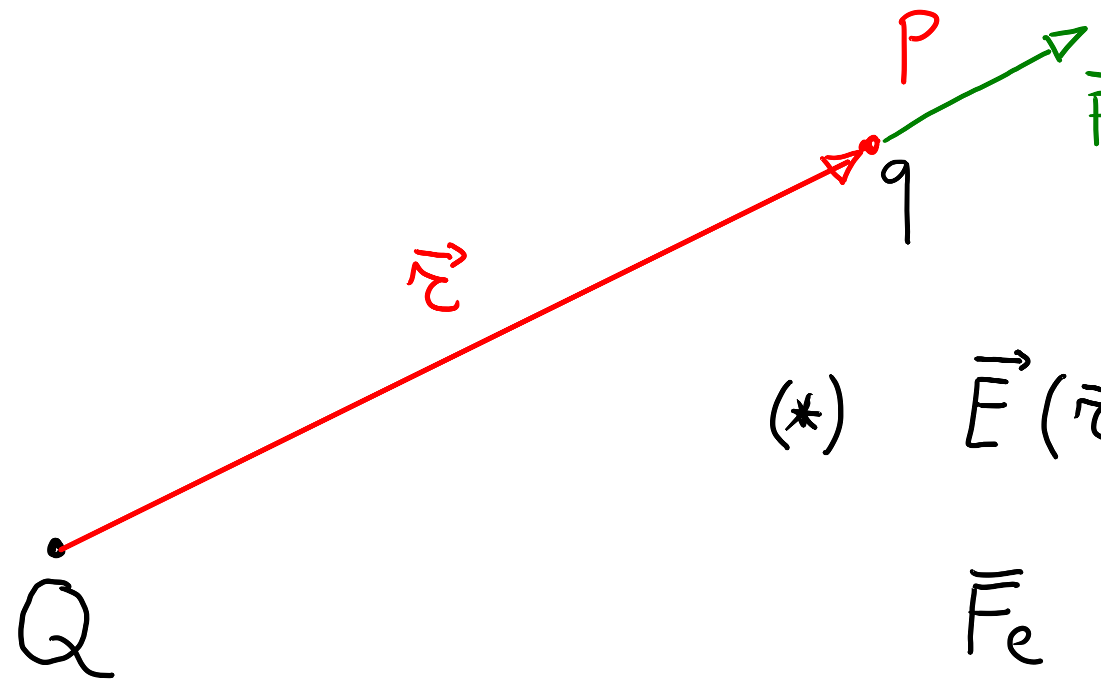




CAMPO ELETTRICO

Q carica sorgente

q carica di prova



$$\vec{F}_e = K \frac{Q \cdot q}{r^2} \hat{r}$$

$$(*) \quad \vec{E}(\vec{r}) = \frac{\vec{F}_e}{q} = K \frac{Q}{r^2} \hat{r}$$

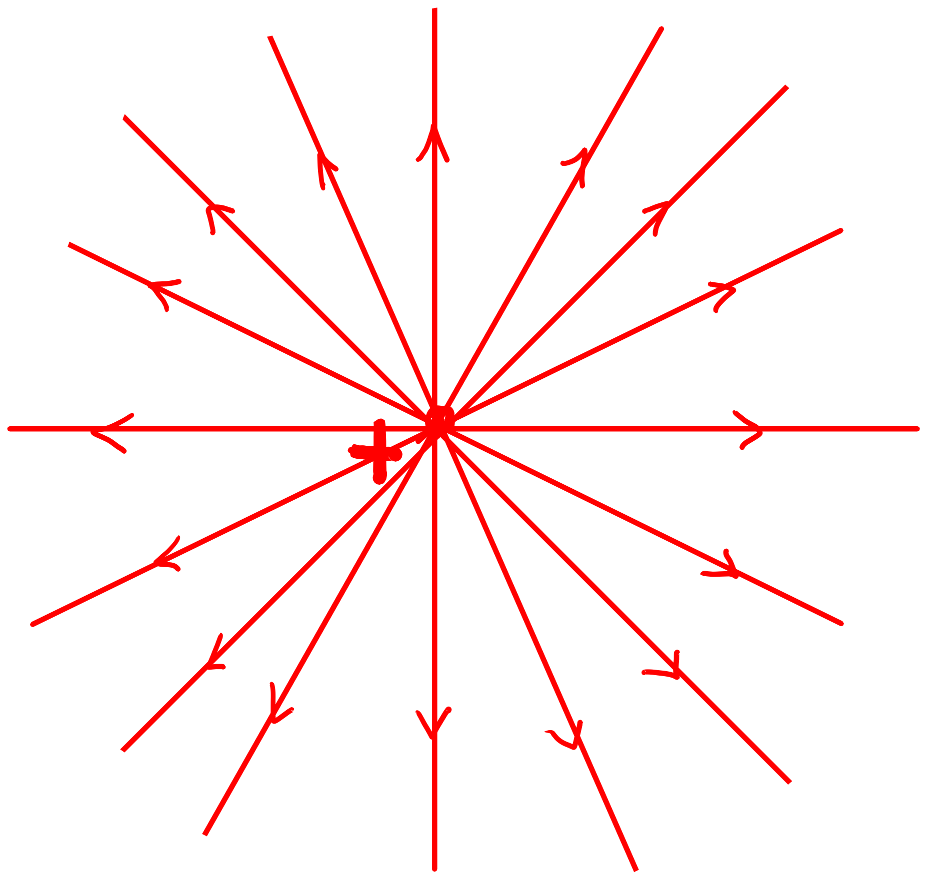
$$\vec{F}_e = q \cdot \vec{E}(\vec{r})$$

$$(*) \quad [E] = \frac{N}{C}$$

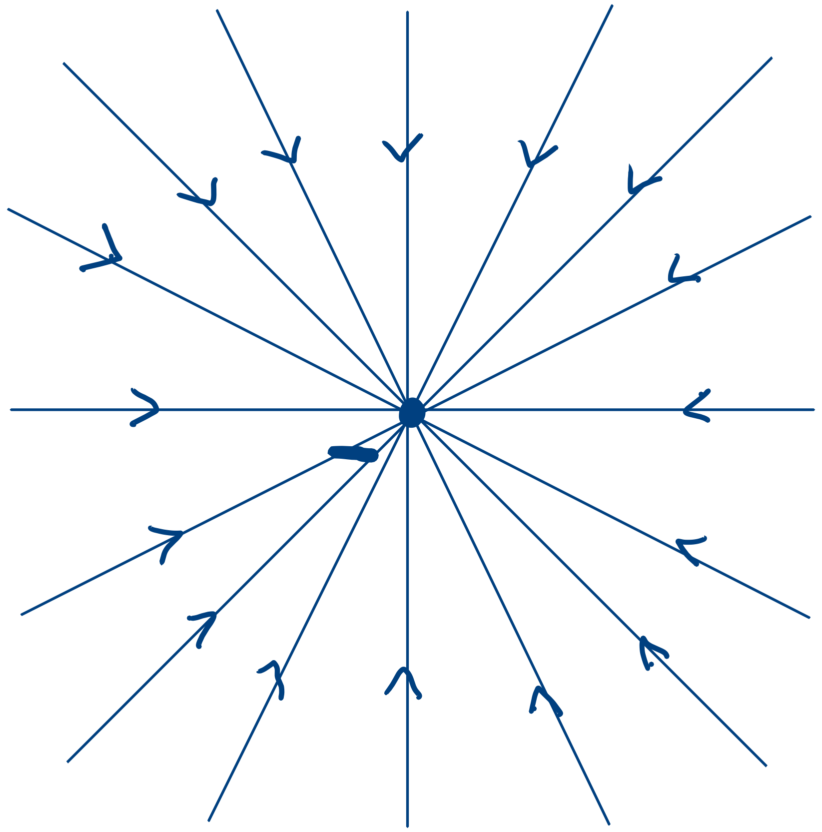
$$\boxed{\vec{E}(\vec{r}) = \frac{K Q \hat{r}}{r^2}}$$

ESEMPI

carica puntiforme +

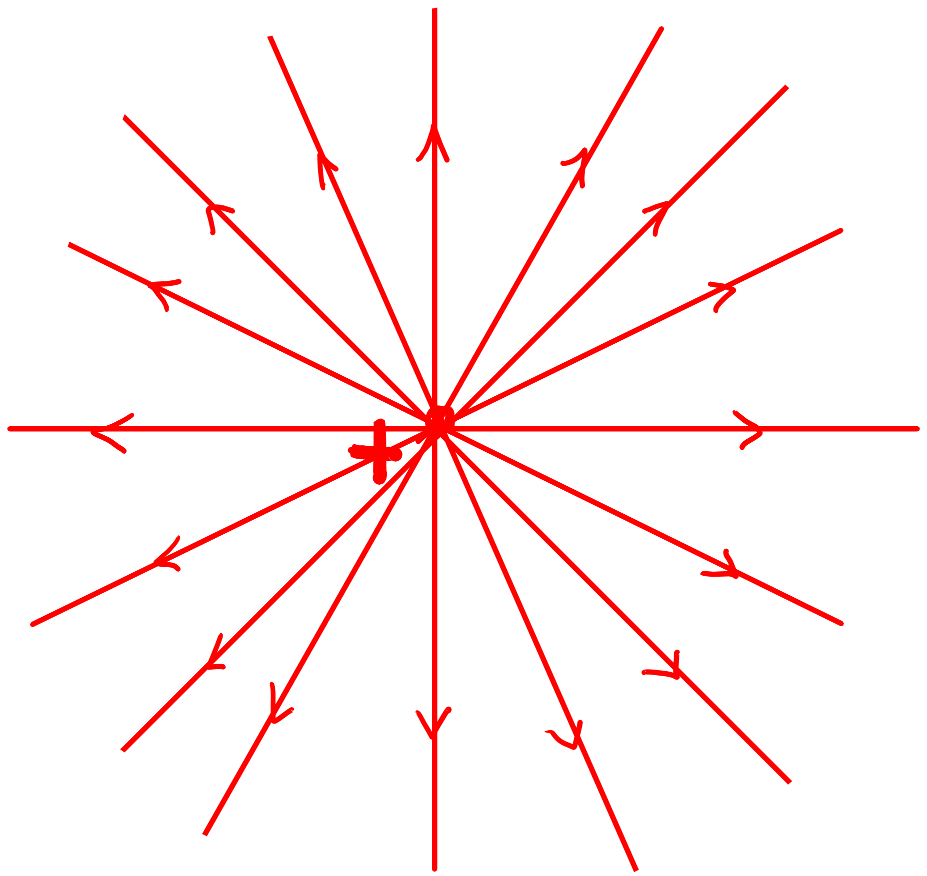


carica puntiforme -

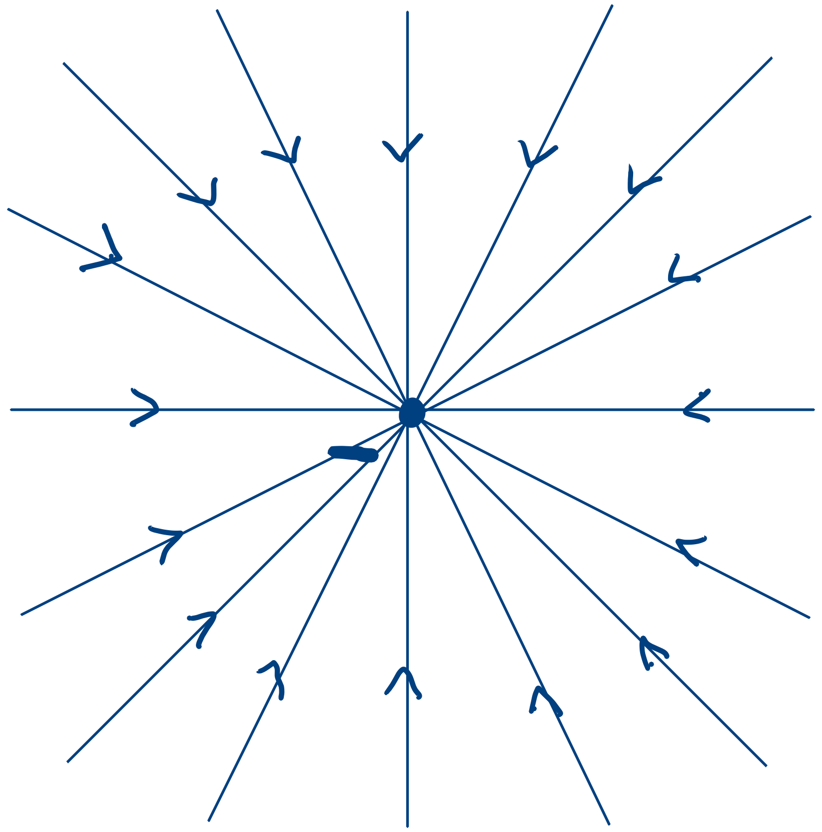


ESEMPI

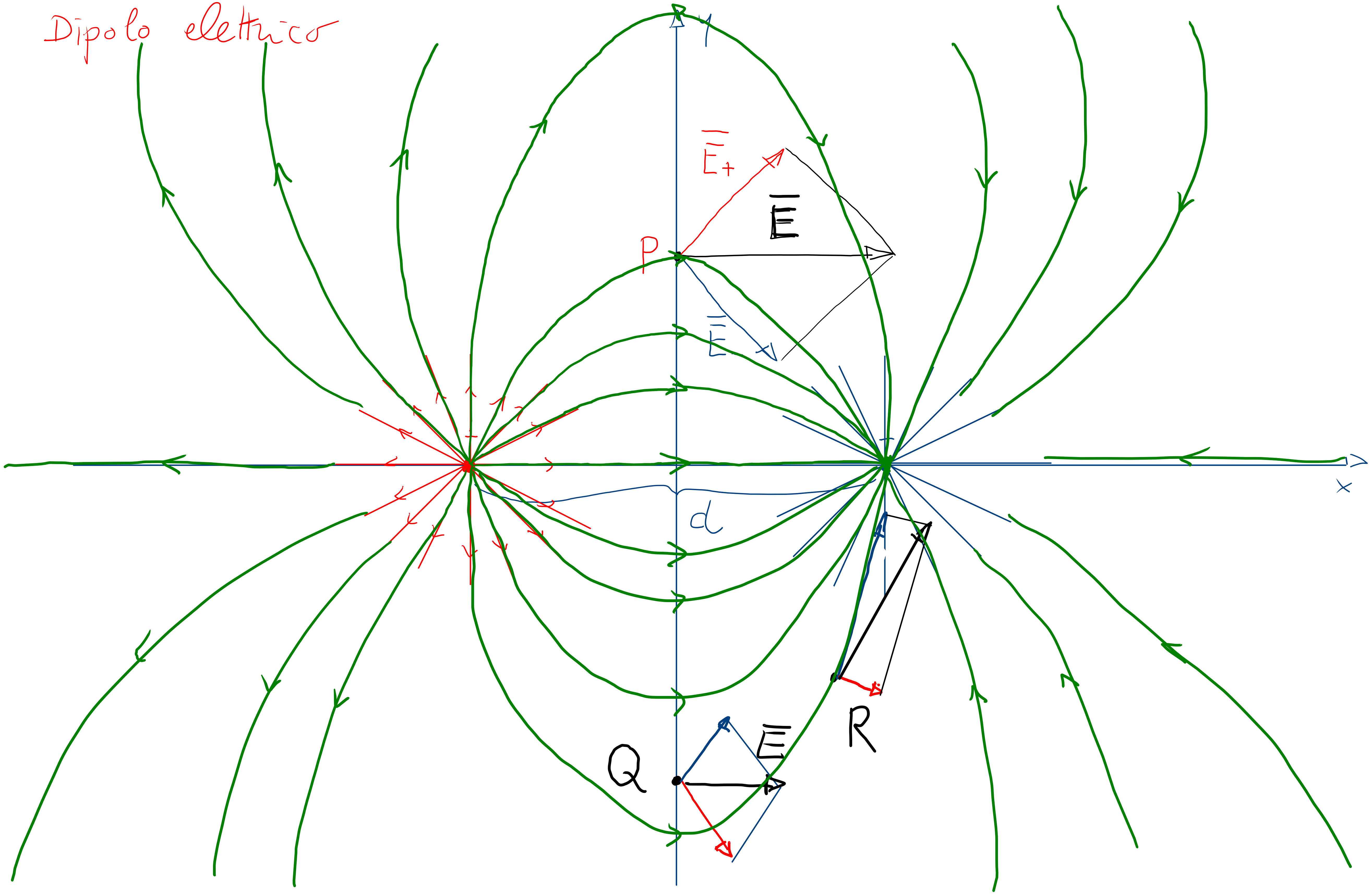
carica puntiforme +

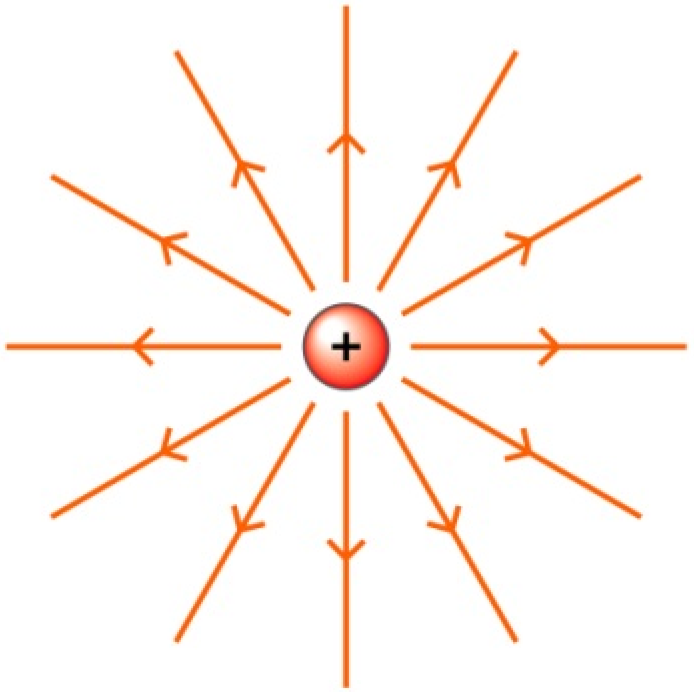


carica puntiforme −

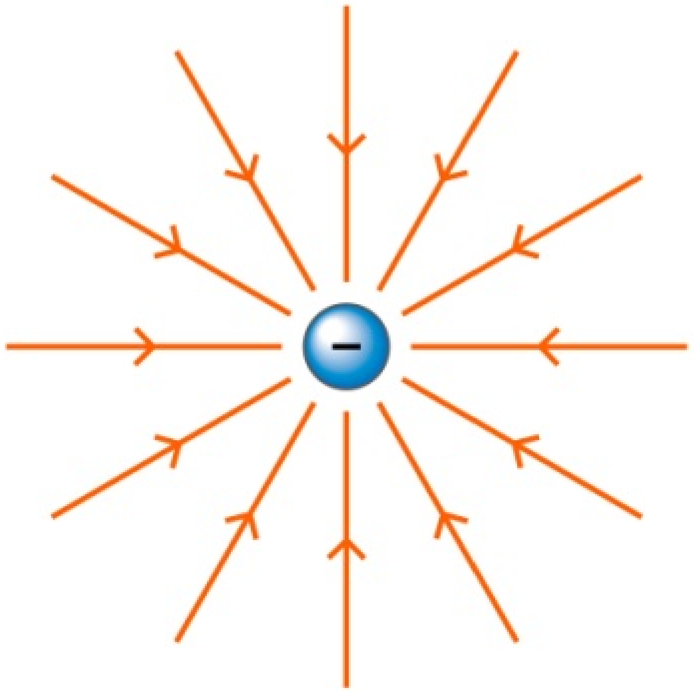


Dipolo elettrico

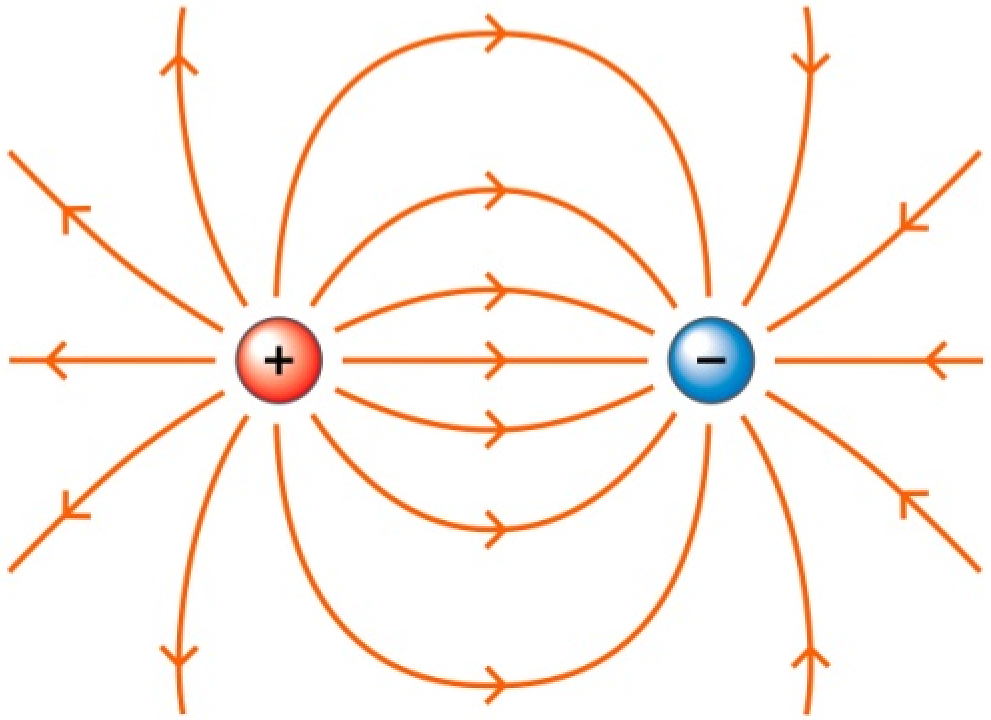




a Linee di forza del campo elettrico creato da una carica positiva.

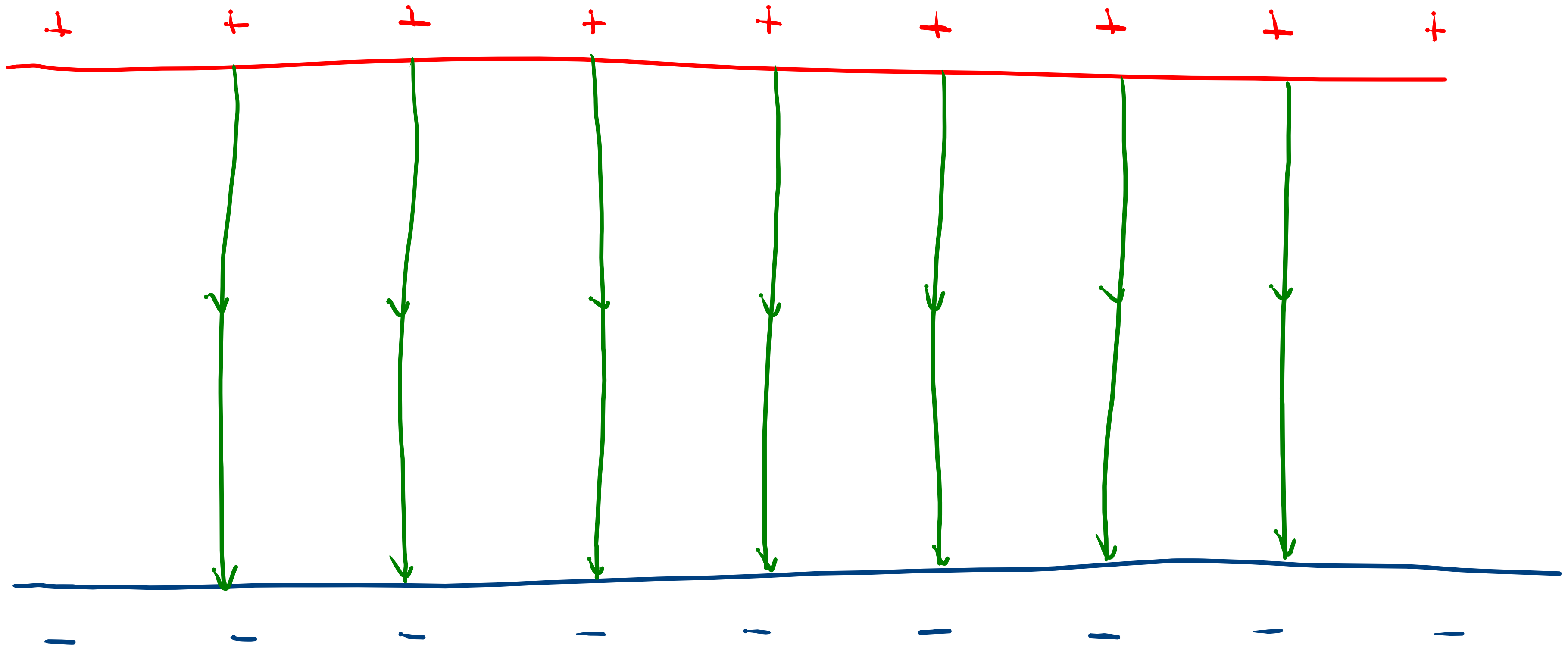


b Linee di forza del campo creato da una carica negativa.



c Linee di forza del campo creato da due cariche uguali e opposte (dipolo elettrico).

Due piani infiniti di cariche, + e −



POTENZIALE ELETTRICO

$$\mathcal{L} = U_A - U_B = -\Delta U$$

$$\mathcal{L}_{\infty \bar{r}} = \underset{\substack{\parallel \\ 0}}{U_\infty} - U(\bar{r})$$

U energia potenziale elettrostatica

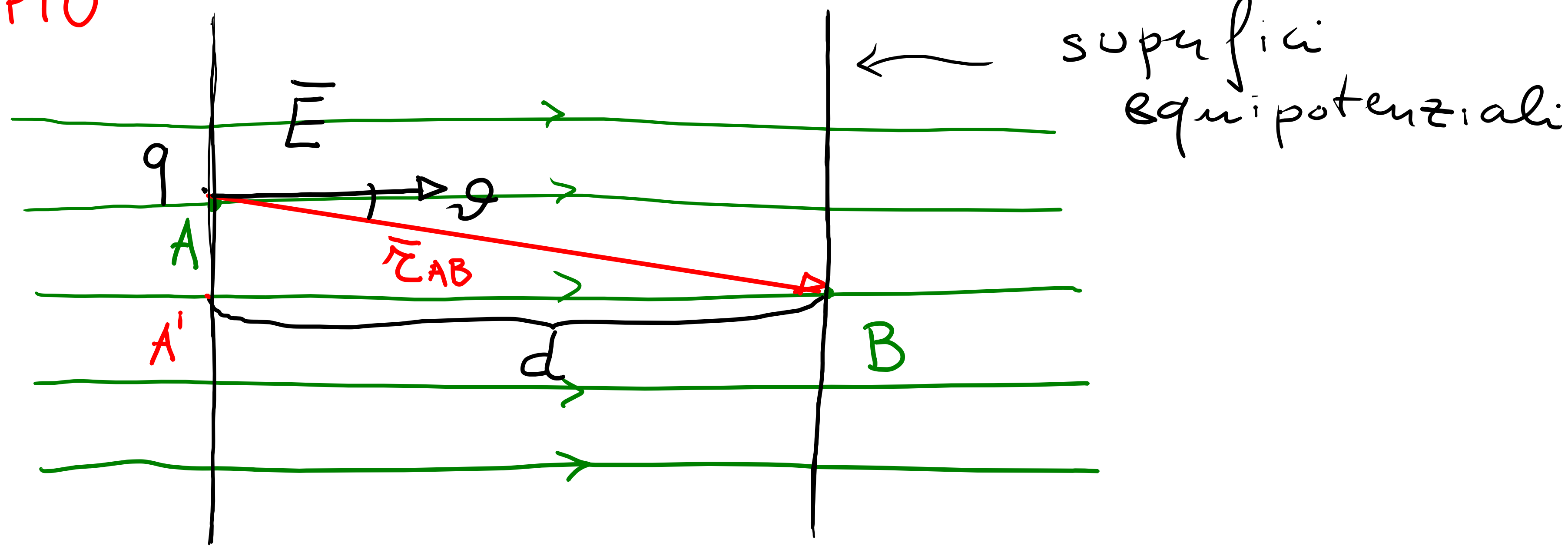
$$U(\bar{r}) = -\mathcal{L}_{\infty \bar{r}}$$

= lavoro fatto contro le forze elettrostatiche per portare q dall'∞ in $\vec{r}$.

Potenziale elettrico $\quad V(\bar{r}) = \dfrac{U(\bar{r})}{q}$

$$[V] = \frac{J}{C} = \text{Volt} = V$$

ESEMPIO



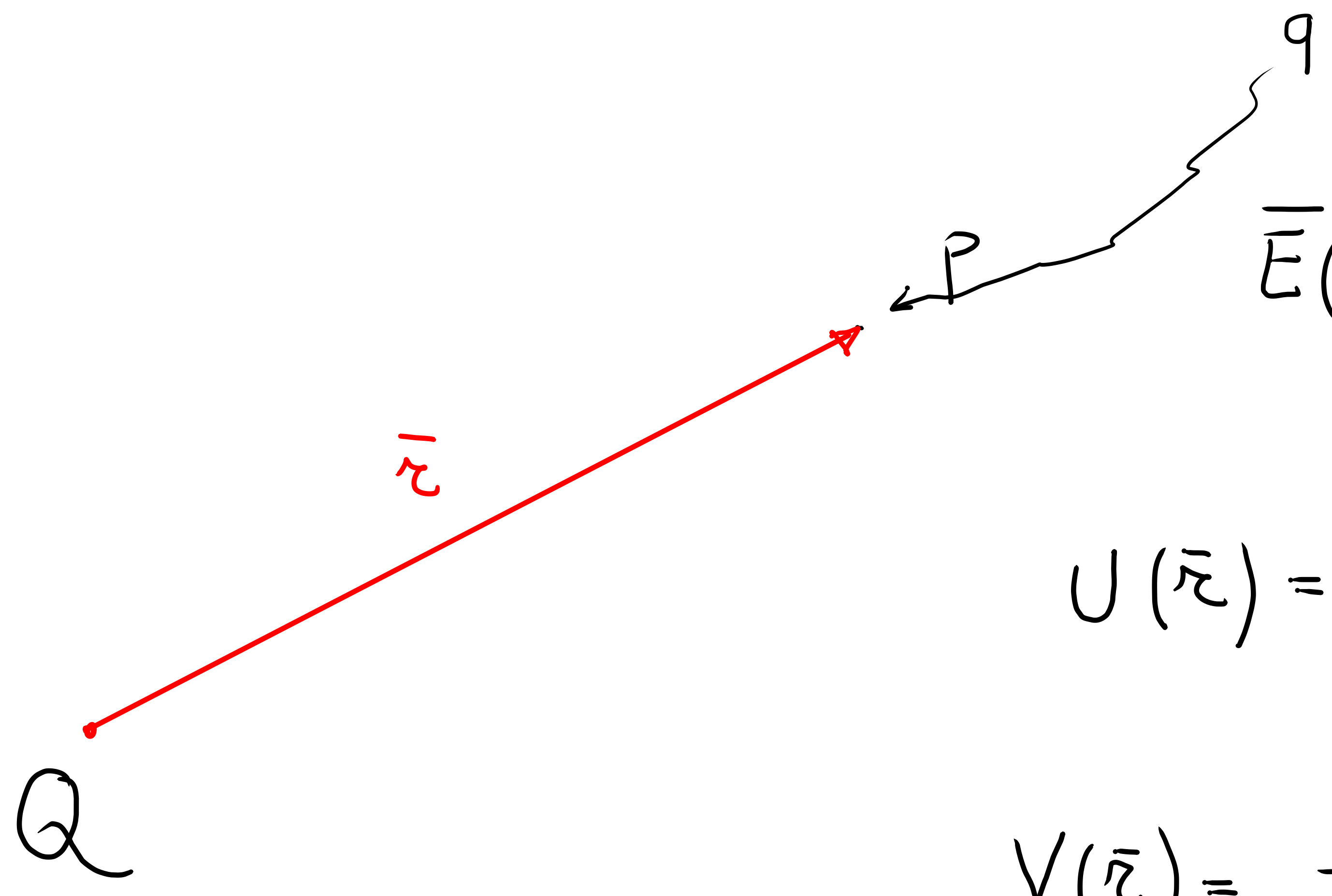
$$\mathcal{L}_{AB} = U_A - U_B$$
$$= \vec{F} \cdot \vec{r}_{AB} = q\vec{E} \cdot \vec{r}_{AB} = q|\vec{E}| \cdot |\vec{r}_{AB}| \cos\vartheta = qEd$$

$$\frac{U_A - U_B}{q} = V_A - V_B = \vec{E} \cdot \vec{r}_{AB} = E \cdot d \quad (*)$$

$$V_A = V_B + \underbrace{\vec{E} \cdot \vec{r}_{AB}}_{>0} \qquad V_A > V_B \quad \text{nel nostro caso}$$
$$V_A = V_{A'}$$

$$(*) = E \cdot d = V_A - V_B \quad \Rightarrow \quad E = -\frac{\Delta V}{d} \qquad [E] = \frac{V}{m} = \frac{N}{C}$$

POTENZIALE DI UNA CARICA PUNTIFORME



$$\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

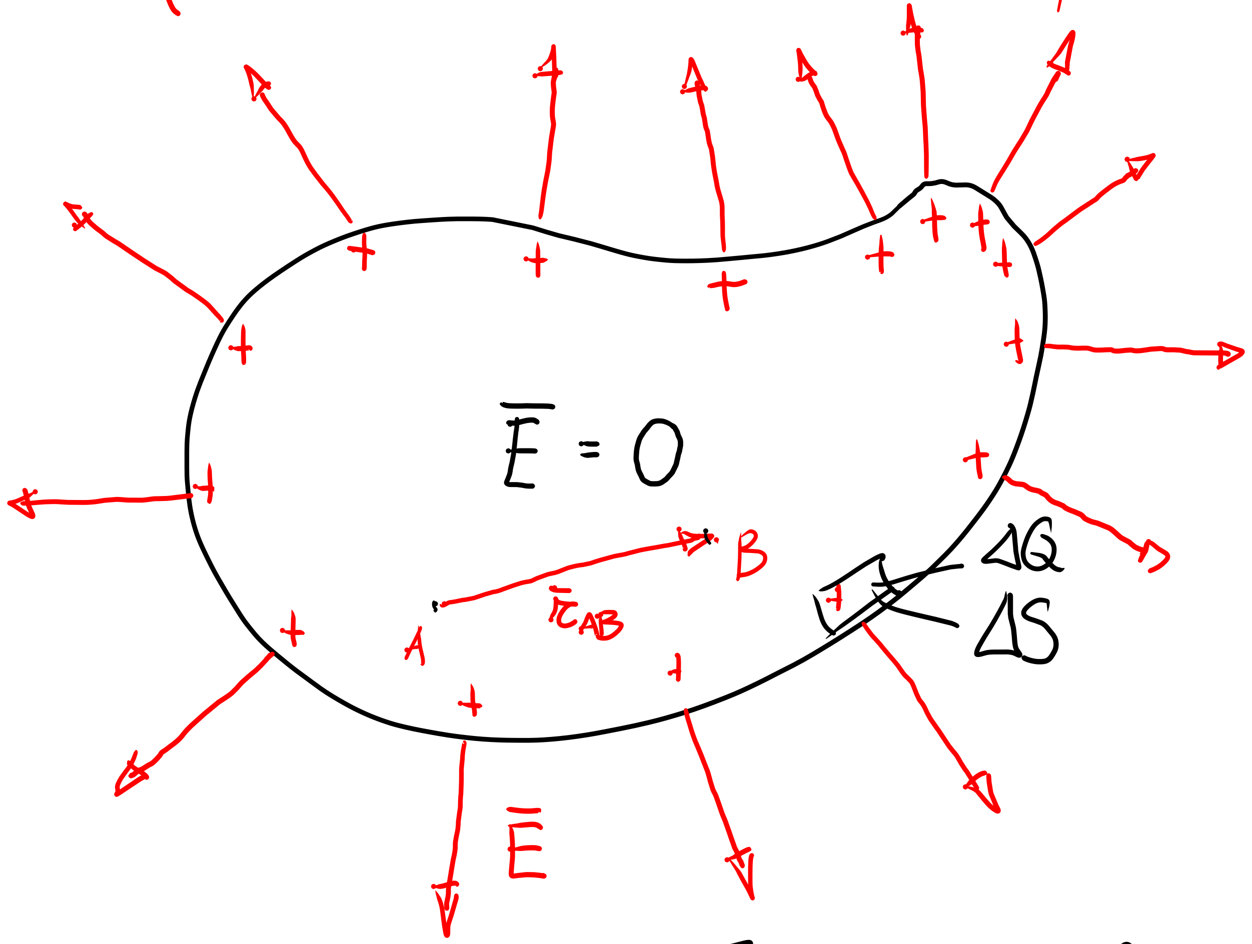
$$U(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$$

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}}$$

il potenziale dipende solo dalla distanza r da Q

CAMPO E POTENZIALE ELETTRICI DI UN CONDUTTORE
(in condizioni statiche, di equilibrio)



- Q carica depositata si distribuisce sulla superficie
- all'interno $\bar{E} = 0$ altrimenti no equilibrio
- V è lo stesso ovunque

$$\Delta V = -\underbrace{\bar{E}}_{=0} \cdot \bar{\tau}_{AB} = 0$$

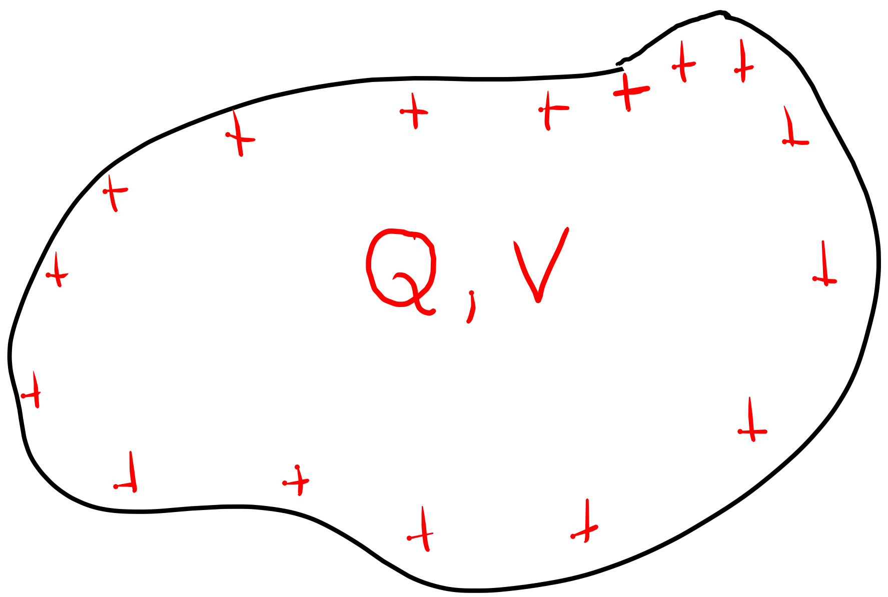
- all'esterno, $\bar{E} \perp$ superficie altrimenti no equilibrio
- anche sulla superficie si ha $\Delta V = -\bar{E} \cdot \bar{\tau}_{AB} = 0$ $(\bar{E} \perp \bar{\tau}_{AB})$

  $\Delta V = 0$ superficie equipotenziale

- $E = \dfrac{\sigma}{\varepsilon_0}$ $\qquad \sigma$ densità superficiale di carica

$$\sigma = \frac{\Delta Q}{\Delta S} \qquad [\sigma] = \frac{C}{m^2}$$

CAPACITÀ DI UN CONDUTTORE



$Q_1 \quad V_1$

$Q_2 \quad V_2$

...

$Q \propto V$

è proporzionale a ...

costante di proporzionalità

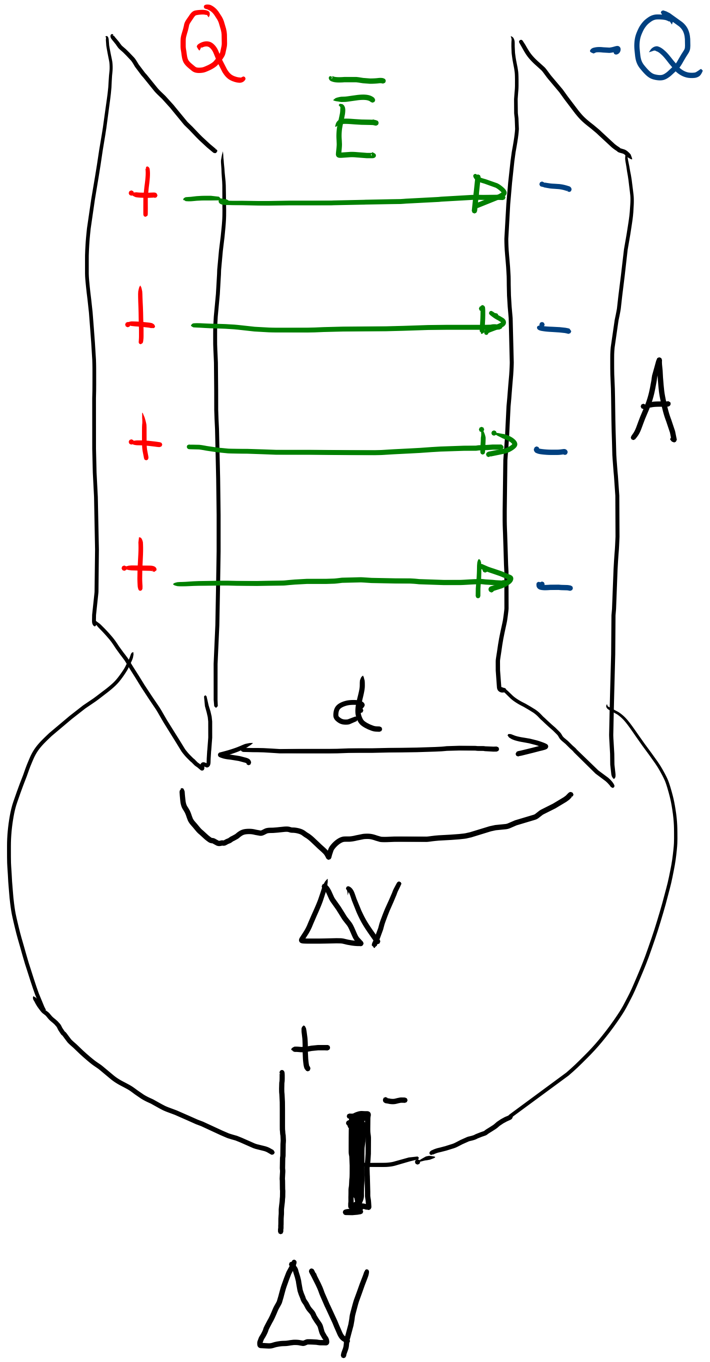
$Q = CV$

capacità del conduttore

$$C = \frac{Q}{V}$$

$$[C] = \frac{\text{Coulomb}}{\text{Volt}} = \text{Farad} = F = \frac{C}{V}$$

CONDENSATORE (a facce piane parallele)



$$E = \frac{\Delta V}{d} \qquad \Delta V = E \cdot d$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \sigma = \frac{Q}{A} \qquad Q = \sigma A = \varepsilon_0 E \cdot A$$

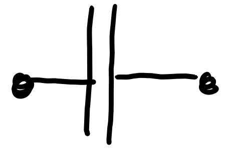
$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 \cancel{E} \cdot A}{\cancel{E} \cdot d} = \varepsilon_0 \frac{A}{d}$$

$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$

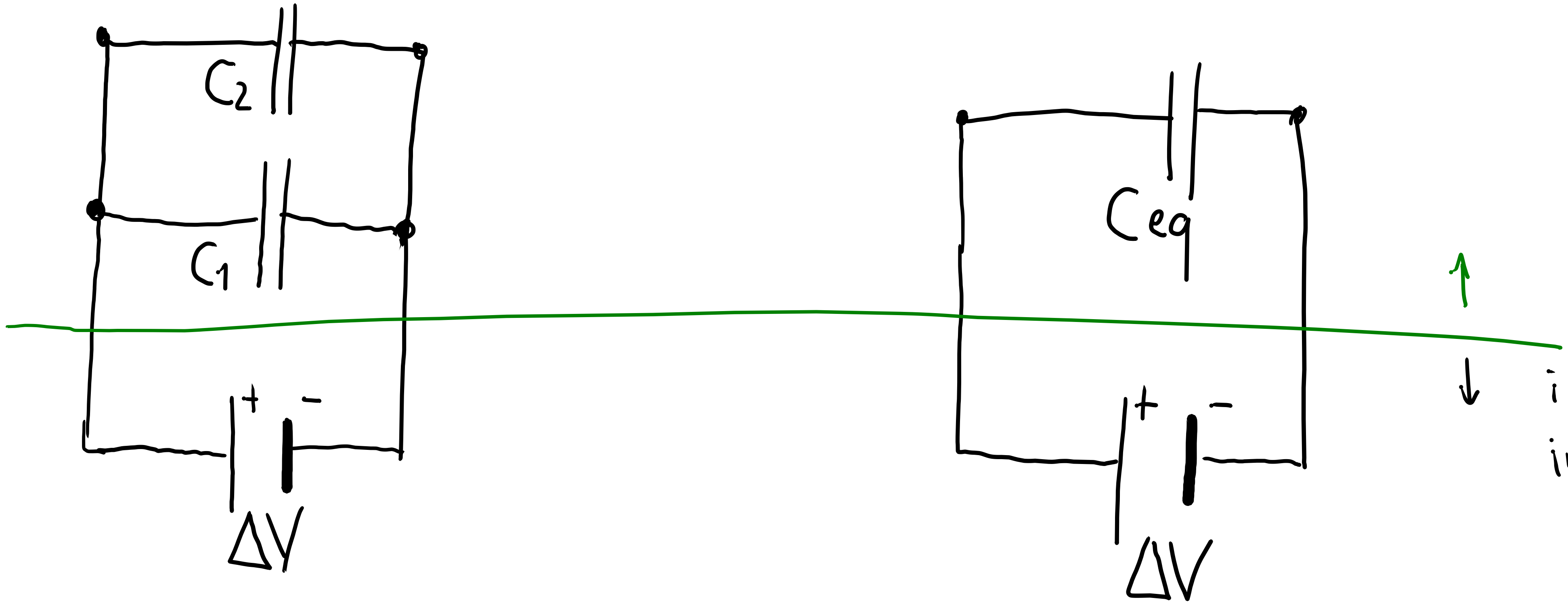
area

distanza

con dielettrico tra le armature $\varepsilon_0 \to \varepsilon_0 \cdot \varepsilon_r$

Il simbolo circuitale del condensatore è 

CONDENSATORI IN PARALLELO



i due circuiti sono indistinguibili

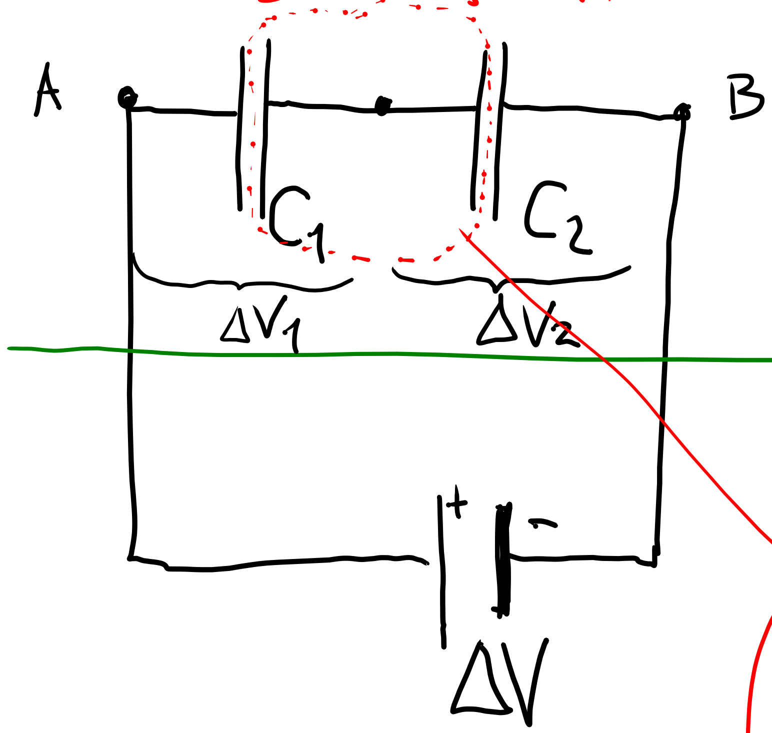
$$C_1 = \frac{Q_1}{\Delta V} \qquad C_{eq} = \frac{Q}{\Delta V} \qquad Q = Q_1 + Q_2$$

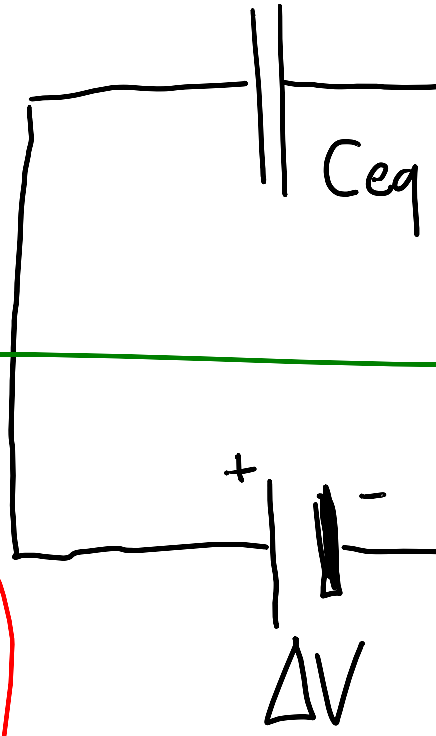
$$C_2 = \frac{Q_2}{\Delta V} \qquad C_{eq} = \frac{Q_1 + Q_2}{\Delta V} = \frac{Q_1}{\Delta V} + \frac{Q_2}{\Delta V} = C_1 + C_2$$

$$\boxed{C_{eq} = C_1 + C_2}$$

per n condensatori in parallelo $C_{eq} = C_1 + C_2 + \dots + C_n$

CONDENSATORI IN SERIE





indistinguibili

$$C_1 = \frac{Q_1}{\Delta V_1} \qquad C_{eq} = \frac{Q}{\Delta V}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

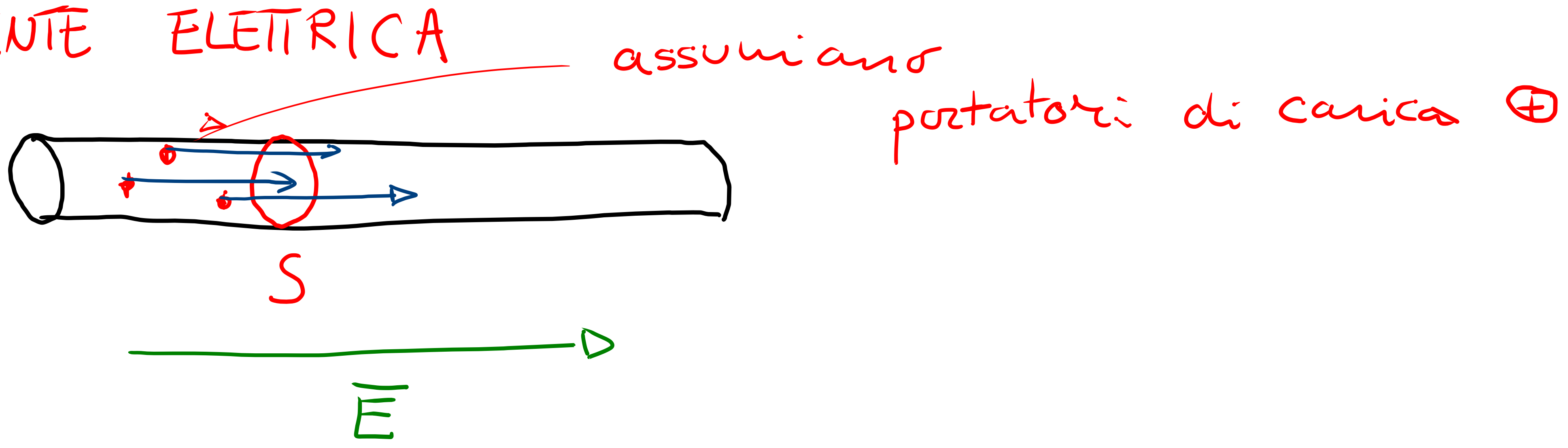
$$\Downarrow$$

$$C_2 = \frac{Q_2}{\Delta V_2} \qquad Q_1 = Q_2 = Q \qquad C_{eq} = \frac{Q}{\Delta V_1 + \Delta V_2}$$

$$\frac{1}{C_{eq}} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{\Delta V_1}{Q_1} + \frac{\Delta V_2}{Q_2} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

CORRENTE ELETTRICA



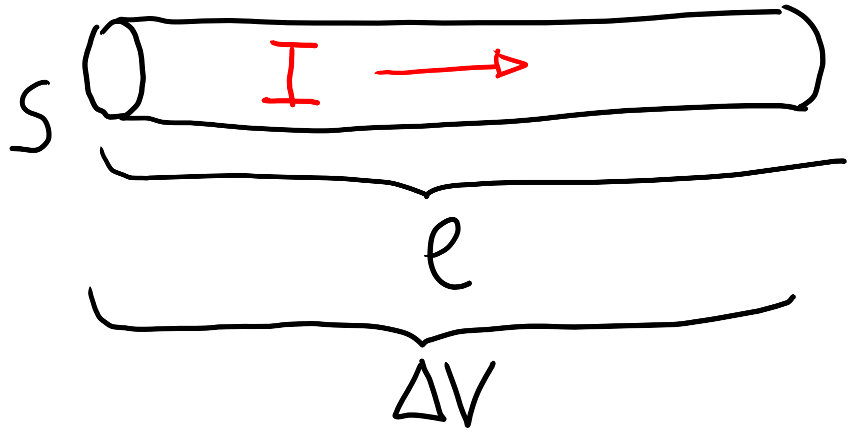
Δt $\quad$ ΔQ attraversa S in Δt

$$i_m = \frac{\Delta Q}{\Delta t}$$

$$[i_m] = \frac{C}{s} = \text{Ampere} = A \qquad 1C = A \cdot s$$

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

LEGGI DI OHM



$$\Delta V \propto I$$

$$\boxed{\Delta V = R I}$$

resistenza del conduttore

$$[R] = \frac{[\Delta V]}{[I]} = \frac{V}{A} = \Omega$$

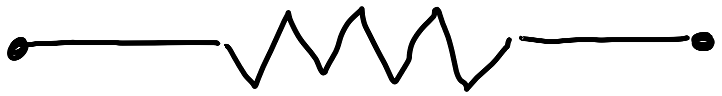
Ohm

$$\boxed{R = \rho \frac{\ell}{S}}$$

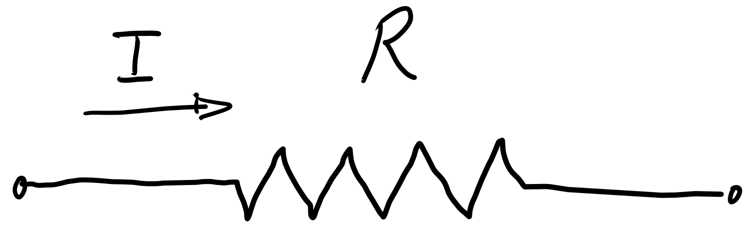
resistività

Simbolo circuitale

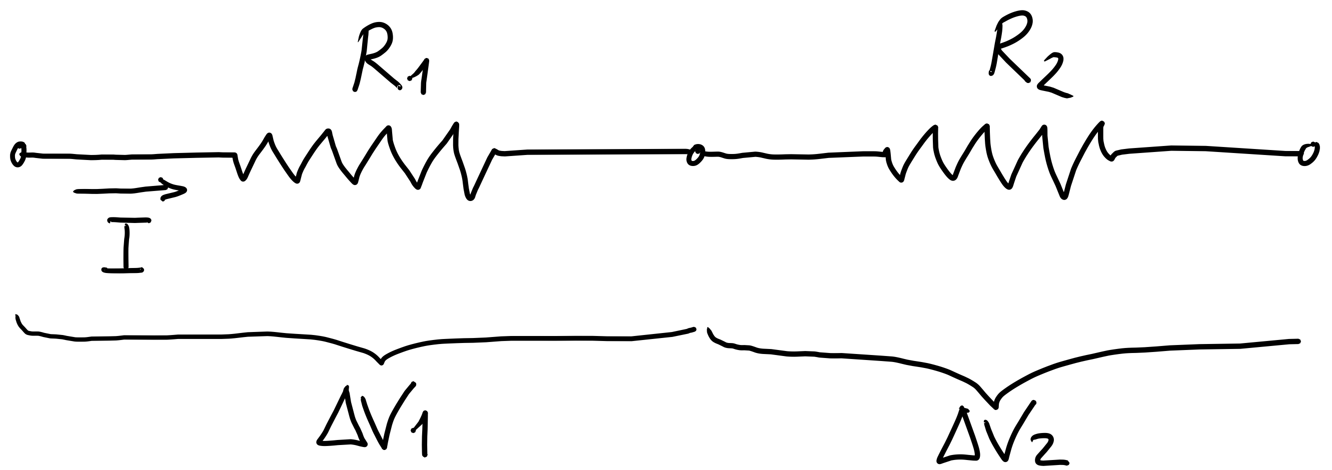
R

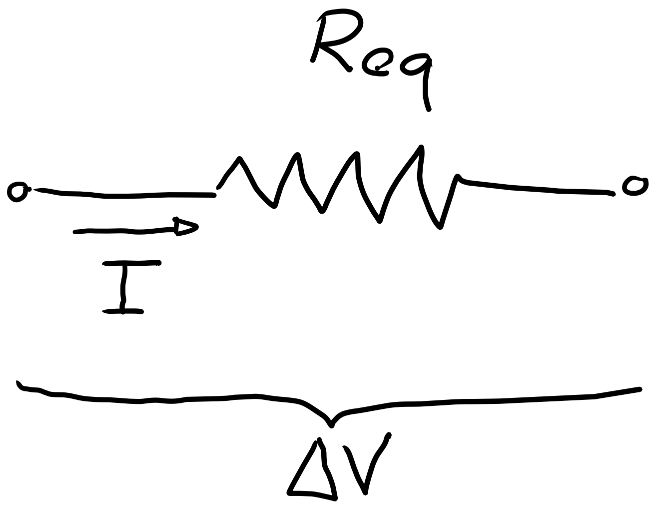


RESISTORI IN SERIE (attraversati dalla stessa I)



$$\Delta V = R I$$

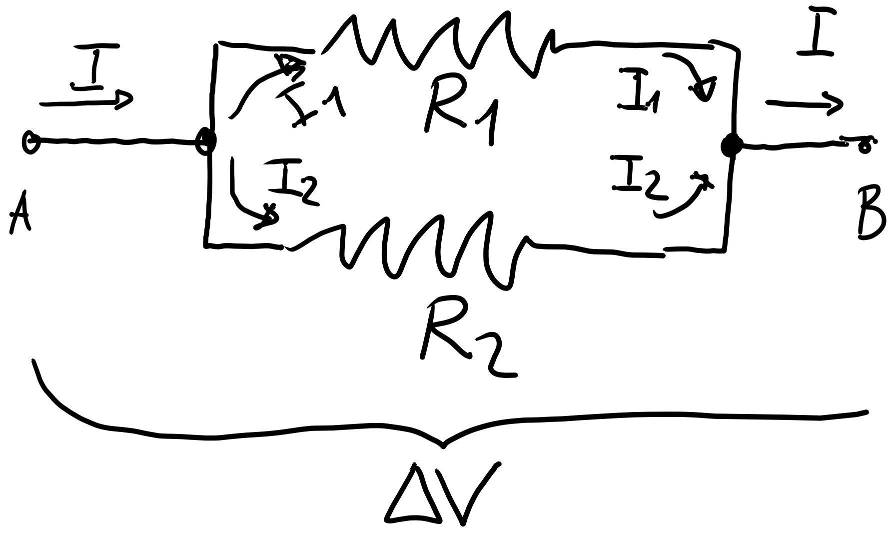


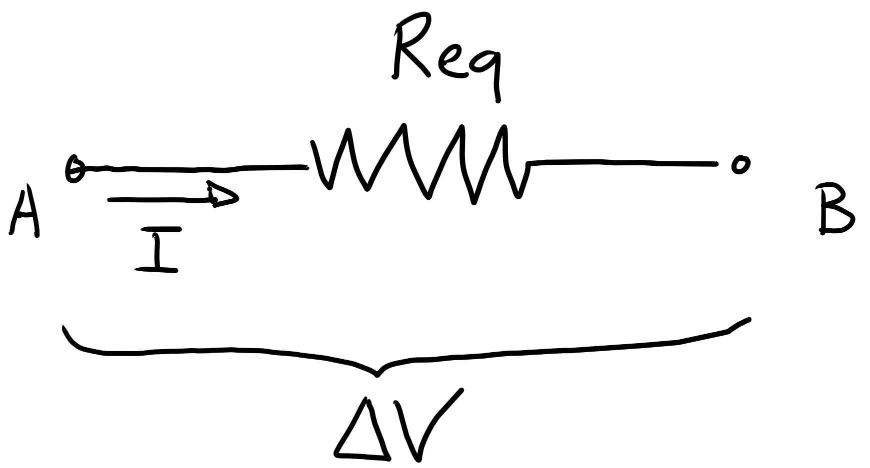


$$\left.\begin{array}{l} \Delta V = \Delta V_1 + \Delta V_2 = R_1 I + R_2 I \\ \Delta V = R_{eq}\, I \end{array}\right\} \quad \boxed{R_{eq} = R_1 + R_2}$$

per n resistori in serie $\quad R_{eq} = R_1 + R_2 + \dots + R_n$

RESISTORI IN PARALLELO (con ai capi la stessa ΔV)





$$I = I_1 + I_2$$

$$\Delta V = R_{eq}\, I$$

$$\Delta V = R_1 I_1$$
$$\Delta V = R_2 I_2$$

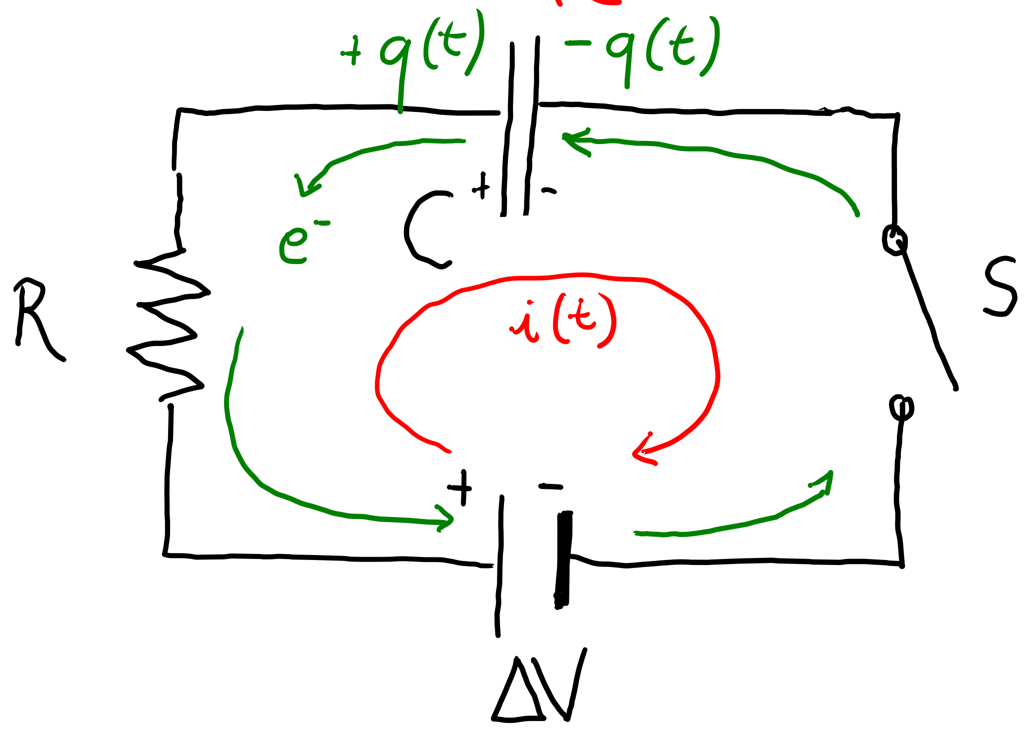
$$\frac{\cancel{\Delta V}}{R_{eq}} = \frac{\cancel{\Delta V}}{R_1} + \frac{\cancel{\Delta V}}{R_2}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Per n resistori in parallelo

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

CIRCUITO RC - Carica del condensatore



S aperto: non circola corrente

C scarico: $q(t=0) = 0$

A $t=0$ chiudiamo S

$\Rightarrow$ circola $i(t)$

$\Rightarrow$ sul condensatore si deposita $q(t)$

La carica del condensatore si ferma quando sulle armature troviamo:

$$Q = C \cdot \Delta V$$

$\tau = RC$ costante di tempo del circuito RC

$$q(t) = \overbrace{C \cdot \Delta V}^{Q} \left(1 - e^{-\frac{t}{\tau}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$[\tau] = s \qquad [\tau] = [RC] = \Omega F = \frac{\cancel{V}}{A} \cdot \frac{C}{\cancel{V}} = \frac{C}{\frac{C}{s}} = s$$

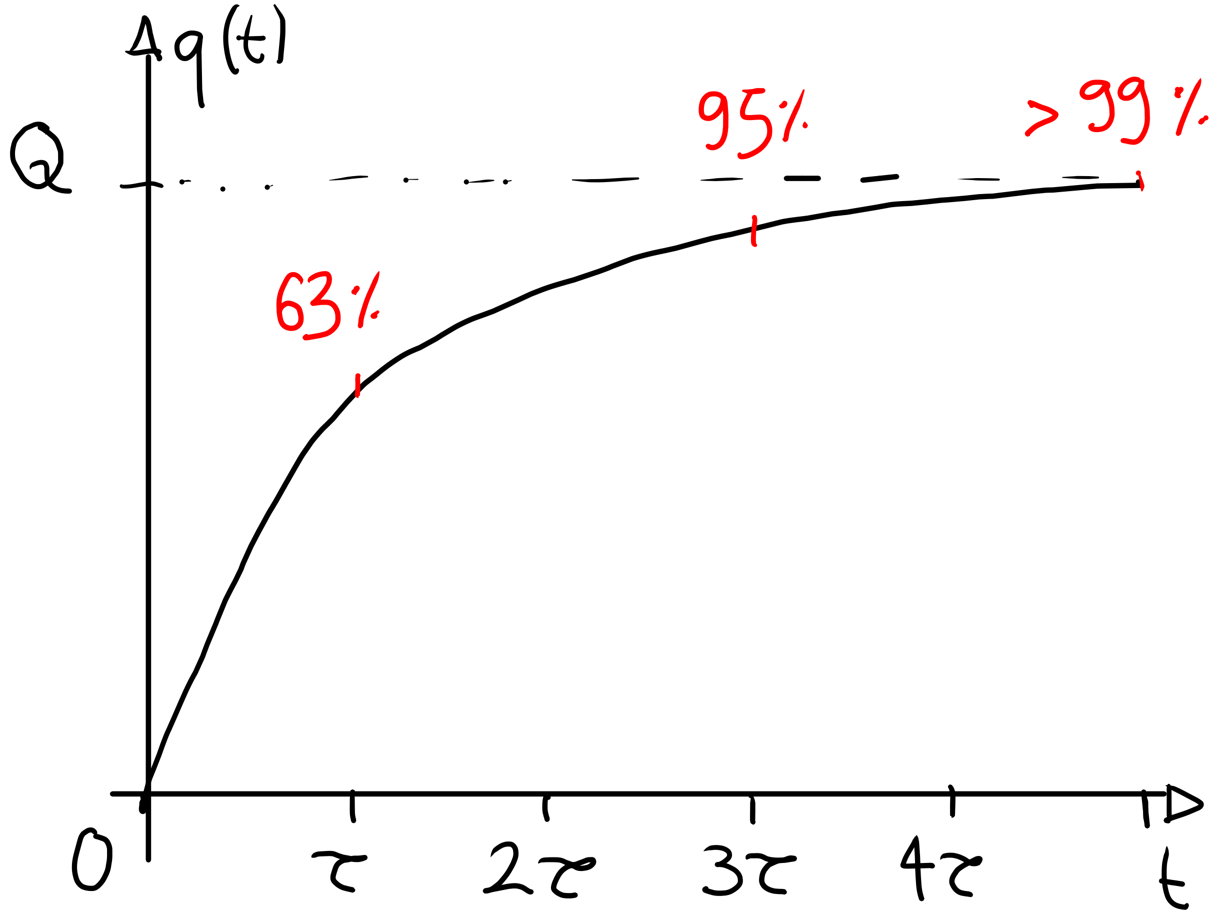
- $q(t) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$

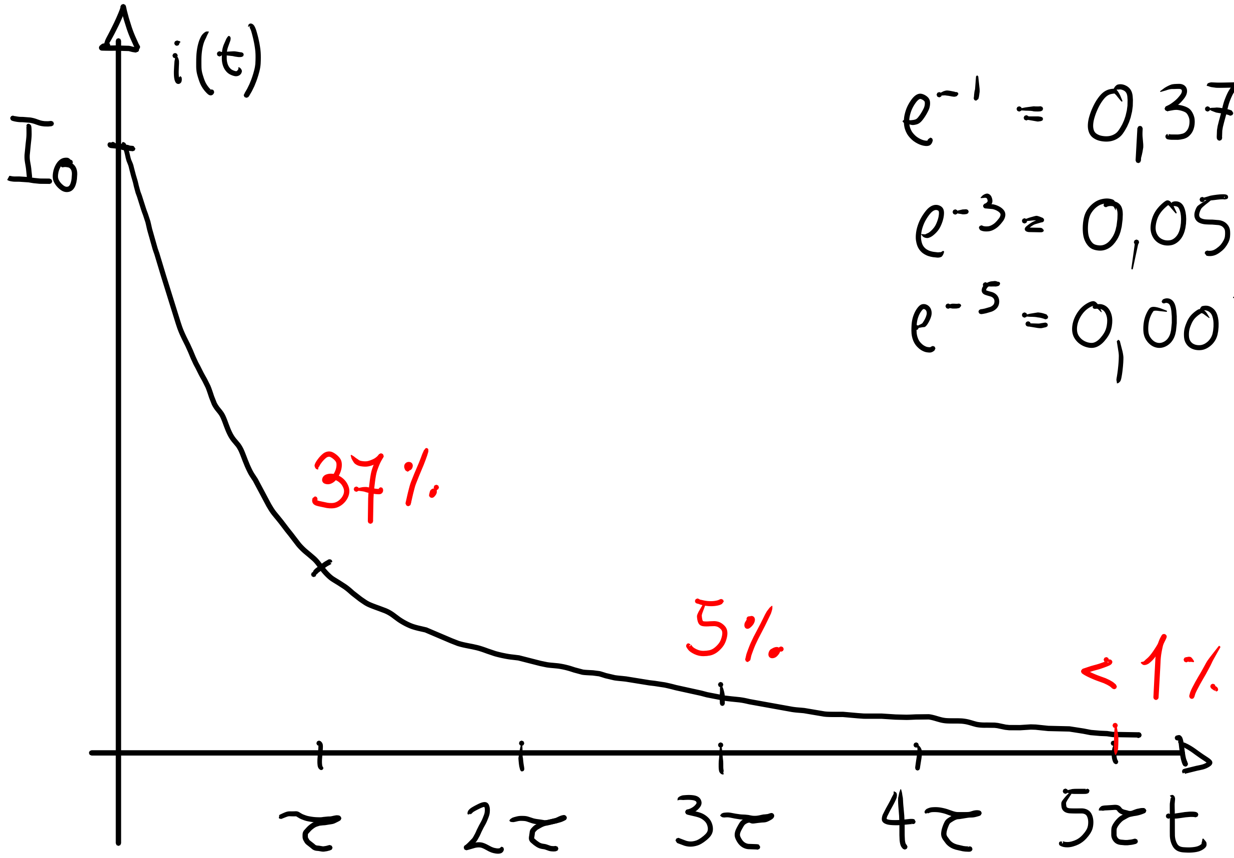
  con $Q = C \cdot \Delta V$

- $i(t) = \frac{dq(t)}{dt} = Q\left[-e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right)\right]$

  $= \frac{Q}{RC}\, e^{-\frac{t}{\tau}} = \underbrace{\frac{\cancel{C}\Delta V}{R\cancel{C}}}_{I_0 = \frac{\Delta V}{R}}\, e^{-\frac{t}{\tau}}$

  $= I_0\, e^{-\frac{t}{\tau}}$



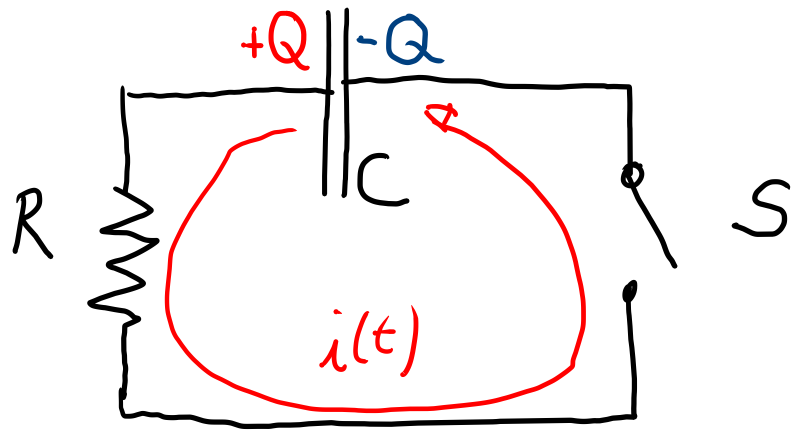


$e^{-1} = 0{,}37$

$e^{-3} = 0{,}05$

$e^{-5} = 0{,}007$

CIRCUITO RC - Scarica del condensatore



$$q(t=0) = Q$$

al tempo $t=0$ chiudiamo S

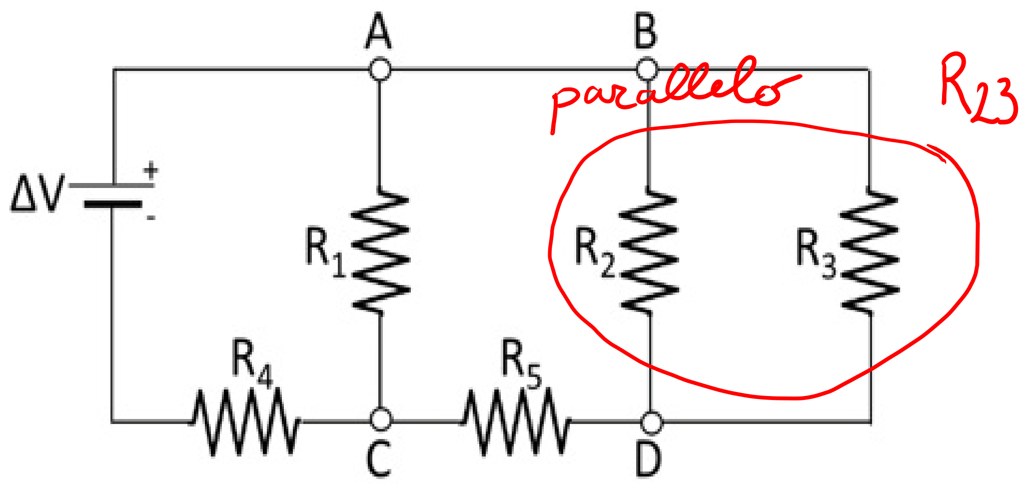
$$q(t) = Q\, e^{-\frac{t}{\tau}} \qquad \tau = RC$$

circola in verso opposto (*)

$$i(t) = \frac{dq(t)}{dt} = Q\, e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right) = -\underbrace{\frac{Q}{RC}}_{I_0}\, e^{-\frac{t}{\tau}} = -I_0\, e^{-\frac{t}{\tau}}$$

(*) rispetto alla carica

Nel circuito in figura il generatore di tensione fornisce una differenza di potenziale $\Delta V = 24$ V. Le resistenze R_1, R_2, ed R_3 sono uguali tra di loro, $R_1 = R_2 = R_3 = 10\ \Omega$, come pure sono uguali tra di loro le resistenze R_4 ed R_5, che invece valgono $R_4 = R_5 = 25\ \Omega$. Calcolare:



a) La resistenza R_{23} equivalente alle resitenze R_2 ed R_3, collocate tra i nodi B e D:

i) $R_{23} = \dfrac{R_2 \cdot R_3}{R_2 + R_3}$ ii) $R_{23} =$ 5,0 Ω

b) La resistenza R_{eq} equivalente all'intero sistema di cinque resistenze:

i) $R_{eq} =$ vedi calcolo ii) $R_{eq} =$ 32,5 Ω

c) La corrente I_4, che attraversa la resistenza R_4:

i) $I_4 = \Delta V / R_{eq}$ ii) $I_4 =$ 0,74 A

d) La corrente I_1, che attraversa la resistenza R_1:

i) $I_1 = 3/4\ I_4$ ii) $I_1 =$ 0,55 A

e) Le correnti ed I_5, I_2, ed I_3, che attraversano rispettivamente le resistenze R_5, R_2 ed R_3:

i) $I_5 = 1/4\ I_4$ ii) $I_5 =$ 0,19 A

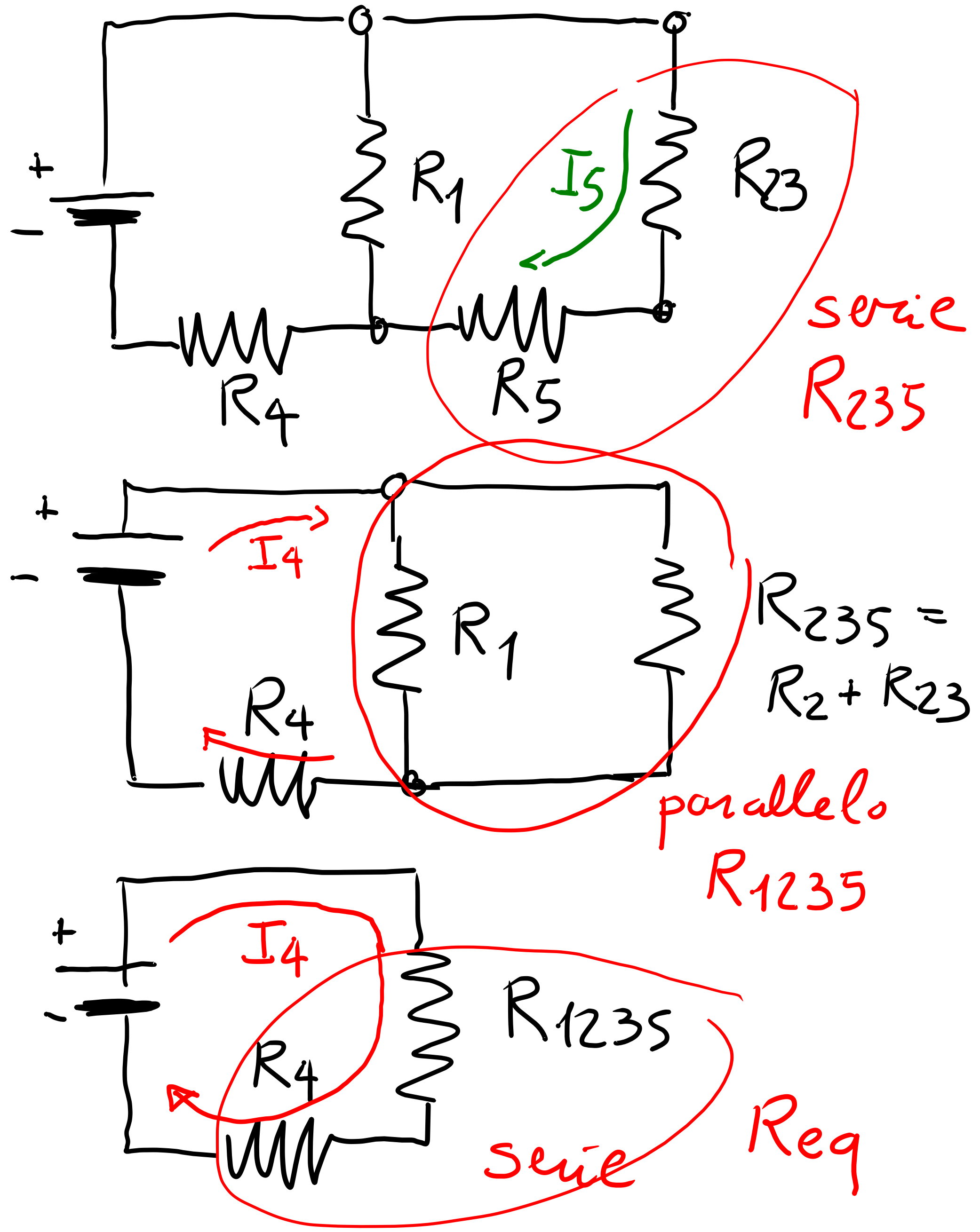
i) $I_2 = 1/2\ I_5 = 1/8\ I_4$ ii) $I_2 =$

i) $I_3 = 1/2\ I_5 = 1/8\ I_4$ ii) $I_3 =$

a) $$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_3 + R_2}{R_2 R_3}$$

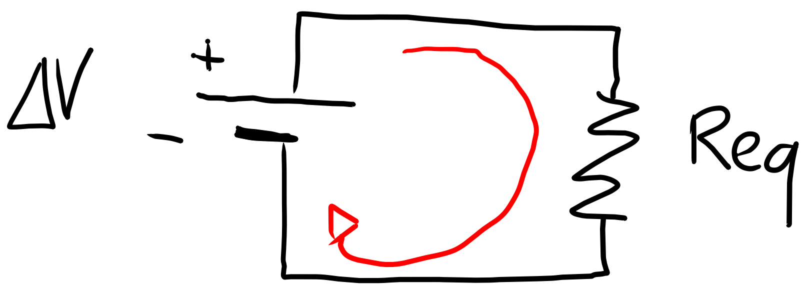
$$R_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100\ \Omega^2}{20\ \Omega} = 5\ \Omega$$

b)

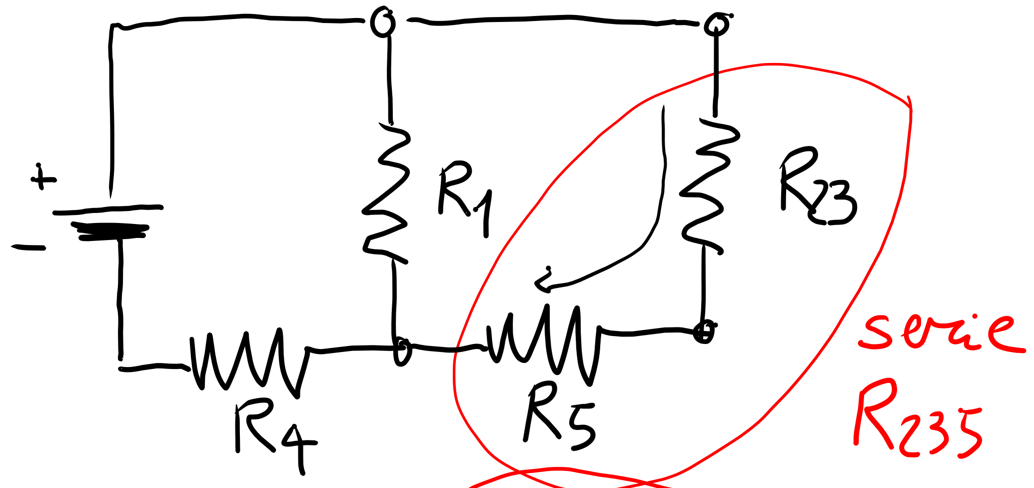


a) $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_3 + R_2}{R_2 R_3}$

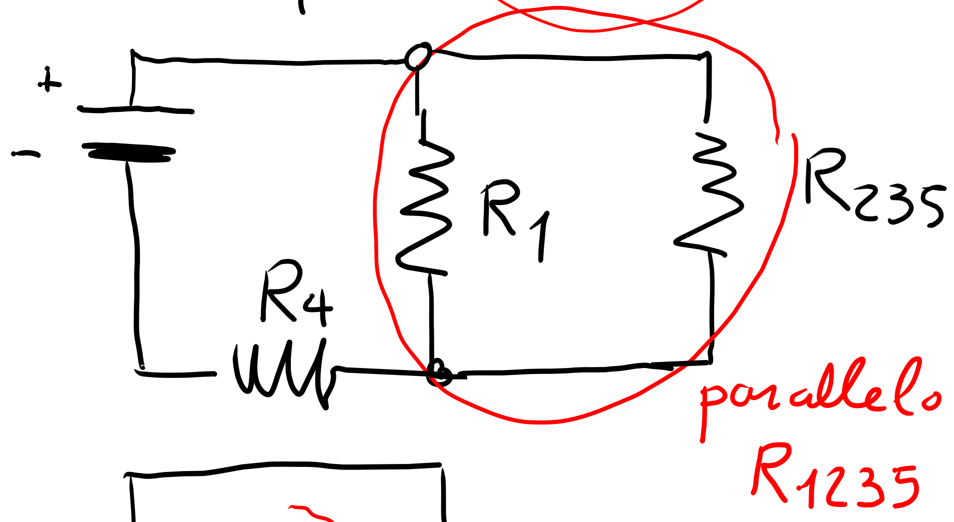
$$R_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100\ \Omega^2}{20\ \Omega} = 5\ \Omega$$



b)

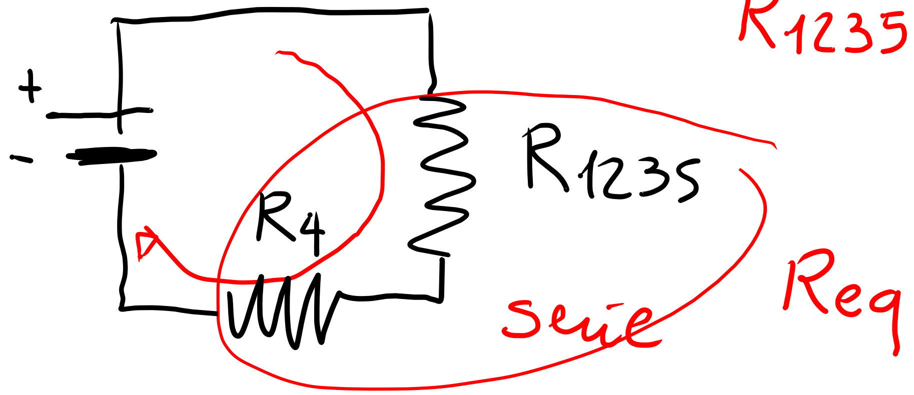


$$R_{235} = R_5 + R_{23} = 25\ \Omega + 5\ \Omega = 30\ \Omega$$



$$(R_{1235})^{-1} = \frac{1}{R_1} + \frac{1}{R_{235}} = \frac{R_{235} + R_1}{R_1 \cdot R_{235}}$$

$$R_{1235} = \frac{R_1 \cdot R_{235}}{R_1 + R_{235}} = \frac{10\ \Omega \cdot 30\ \Omega}{10\ \Omega + 30\ \Omega} = \frac{15}{2}\ \Omega = 7,5\ \Omega$$

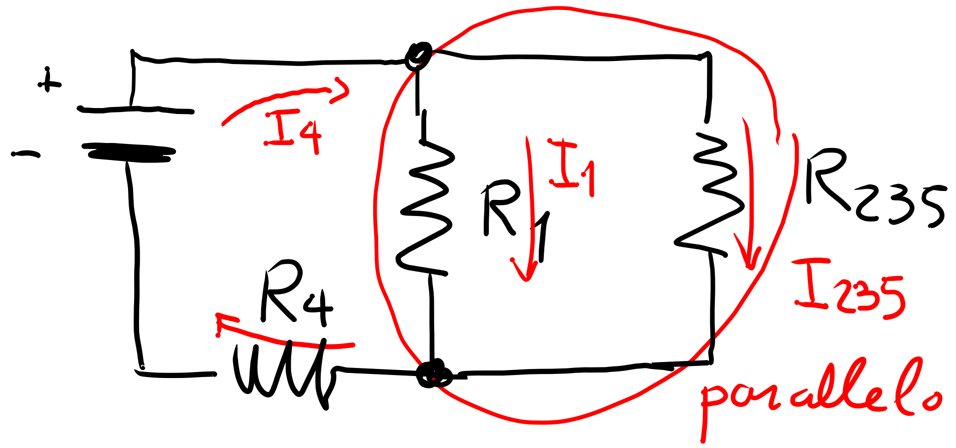


$$R_{eq} = R_{1235} + R_4 = 7,5\ \Omega + 25\ \Omega = 32,5\ \Omega$$

c) I_4 è la stessa corrente che attraversa R_{eq}, ovvero

$$I_4 = \frac{\Delta V}{R_{eq}} = \frac{24\ V}{32,5\ \Omega} = 0,74\ A$$

d)



$$\begin{cases} I_4 = I_1 + I_{235} \\ I_1 R_1 = I_{235} \cdot R_{235} \end{cases}$$

$$\begin{cases} I_4 = I_1 + \frac{R_1}{R_{235}} I_1 = I_1 \left(1 + \frac{R_1}{R_{235}} \right) \\ I_{235} = I_1 \frac{R_1}{R_{235}} \end{cases}$$

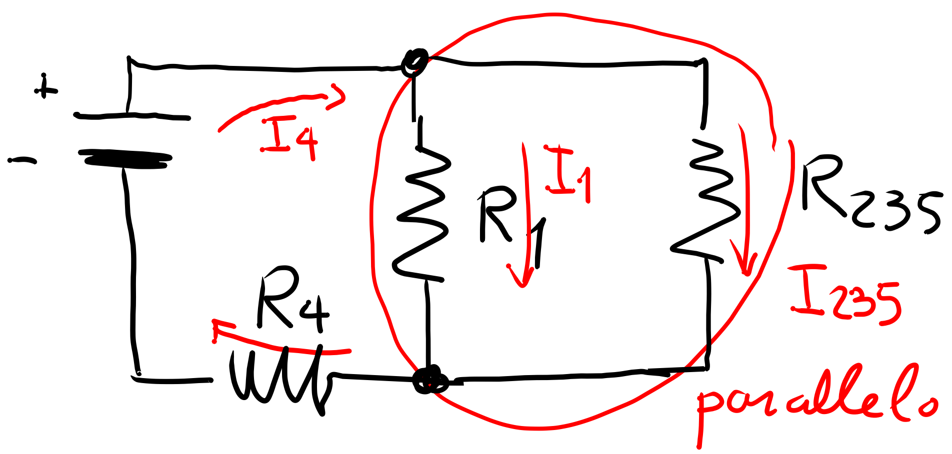
$$\begin{cases} I_1 = \frac{I_4}{1 + \frac{R_1}{R_{235}}} = \frac{1}{1 + \frac{10}{30}} I_4 = \frac{1}{1+\frac{1}{3}} I_4 = \frac{1}{\frac{4}{3}} I_4 = \frac{3}{4} I_4 \\ \dots \end{cases}$$

$$\begin{cases} \dots \\ I_{235} = \frac{1}{4} I_4 = \end{cases}$$

c) I_4 è la stessa corrente che attraversa R_{eq}, ovvero

$$I_4 = \frac{\Delta V}{R_{eq}} = \frac{24\ V}{32{,}5\ \Omega} = 0{,}74\ A$$

d)



$$\begin{cases} I_4 = I_1 + I_{235} \\ I_1 R_1 = I_{235} \cdot R_{235} \end{cases}$$

$$\begin{cases} I_4 = I_1 + \frac{R_1}{R_{235}} I_1 = I_1\left(1 + \frac{R_1}{R_{235}}\right) \\ I_{235} = I_1 \frac{R_1}{R_{235}} \end{cases}$$

$$\begin{cases} I_1 = \dfrac{I_4}{1 + \frac{R_1}{R_{235}}} = \dfrac{1}{1 + \frac{10}{30}} I_4 = \dfrac{1}{1+\frac{1}{3}} I_4 = \dfrac{1}{\frac{4}{3}} I_4 = \dfrac{3}{4} I_4 \\ \dots \end{cases}$$

$$\begin{cases} \dots \\ I_{235} = \frac{1}{4} I_4 = \end{cases}$$

con $I_4 = \frac{\Delta V}{Req}$