19 dicembre

$$f(x) = \int_{x^{-1}}^{\sqrt{x}} \underbrace{ch(lg(t+t^3))} \, dt$$

$$G(x) = \int_{x_0}^{x} g(t) \, dt \qquad g \in C^0([a,b])$$

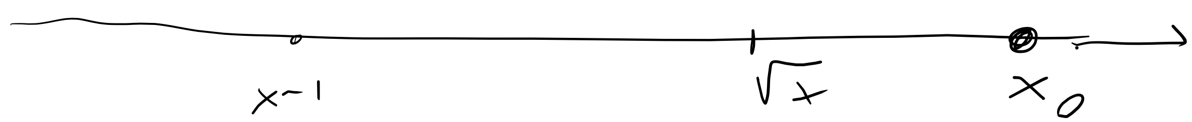
$$x_0 \in [a,b]$$

$$x \in [a,b]$$

$$G'(x) = g(x)$$

$$x_0 > 0$$

$$f(x) = \underbrace{\int_{x_0}^{\sqrt{x}} ch(lg(t+t^3)) \, dt}_{G(\sqrt{x})} - \underbrace{\int_{x_0}^{x^{-1}} ch(lg(t+t^3)) \, dt}_{G(x^{-1})}$$



$$G(x) = \int_{x_0}^{x} \underbrace{ch(lg(t+t^3))}_{g(t)} \, dt$$

$$f(x) = G(\sqrt{x}) - G(x^{-1})$$

$$f'(x) = G'(\sqrt{x}) \, (\sqrt{x})' - G'(x^{-1}) \, (x^{-1})'$$

$$= g(\sqrt{x}) \, \frac{1}{2} x^{-\frac{1}{2}} + g(x^{-1}) \, x^{-2}$$

$$\operatorname{ch}(\lg(\cancel{x}+\cancel{x}^3)) \overset{?}{\in} L((0,1]) \; ?$$

$$\operatorname{ch}(\lg(x+x^3)) \in L_{loc}((0,1])$$

$$\lim_{\varepsilon \to 0^+} \int_\varepsilon^1 \cosh(\lg(x+x^3))\,dx \overset{?}{\in} \mathbb{R}$$

$$g(x) = \operatorname{ch}(\lg(x+x^{-3})) = \frac{e^{\lg(x+x^3)} + e^{\lg(x+x^3)^{-1}}}{2} =$$

$$= \frac{x+x^3 + \frac{1}{x+x^3}}{2} =$$

$$= \frac{x+x^3}{2} + \frac{1}{2}\,\frac{1}{x+x^3} = \frac{x+x^3}{2} + \frac{1}{2}\,\frac{1}{x}\,\frac{1}{1+x^2}$$

$$\lim_{x\to 0} \frac{g(x)}{\frac{1}{x}} =$$

$$= \lim_{x\to 0} \frac{\frac{x+x^3}{2} + \frac{1}{2}\,\frac{1}{1+x^2}\,\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \left(\frac{x^2+x^4}{2} + \frac{1}{2}\,\frac{1}{(1+x^2)}\right) = \frac{1}{2} > 0$$

$$g(x) \notin L((0,1]) \Longleftrightarrow \frac{1}{x} \notin L((0,1]) \;\checkmark$$

$$\lim_{x \to \frac{\pi}{2}^+} \int_x^{2x} \tan(t^{-1})\, dt$$

Sappiamo che $\tan(y)$ è definita al di fuori di $\frac{\pi}{2} + n\pi \quad n \in \mathbb{Z}$

Il limite vorrebbe essere $\int_{\frac{\pi}{2}}^{\pi} \tan(t^{-1})\, dt$

ma solo se per $\frac{\pi}{2} \leq t \leq \pi$

$\frac{1}{t} \neq \frac{\pi}{2} + n\pi \quad \forall\, n \in \mathbb{Z}$

Ovviamente, se $\boxed{\frac{\pi}{2} \leq t \leq \pi}$ e se

$\frac{1}{t} = \frac{\pi}{2} + n\pi$ deve essere $n \geq 0$

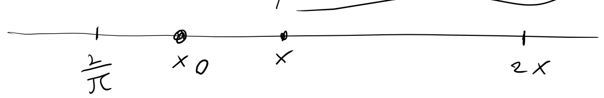
perché per $n < 0$ avrei $\frac{1}{t} < 0$ che è incompatibile con $t \geq \frac{\pi}{2}$.

$\frac{1}{t} = \frac{\pi}{2} + n\pi \qquad t = \dfrac{1}{\frac{\pi}{2} + n\pi} \leq \frac{2}{\pi} < \frac{\pi}{2}$

Pertanto $\underbrace{\tan(t^{-1})}_{g(t)} \in C^0\left(\left(\frac{2}{\pi}, +\infty\right)\right)$

$$\lim_{x \to \frac{\pi}{2}^+} \int_x^{2x} \tan(t^{-1})\, dt =$$

$$G(x) = \int_{x_0}^{x} \tan(t^{-1})\, dt$$



$$= \lim_{x \to \frac{\pi}{2}^+} \left[\int_{x_0}^{2x} \tan(t^{-1})\, dt - \int_{x_0}^{x} \tan(t^{-1})\, dt \right] =$$

$$= \lim_{x \to \frac{\pi}{2}^+} \left[G(2x) - G(x) \right] = G(\pi) - G\left(\tfrac{\pi}{2}\right)$$

$$= \int_{x_0}^{\pi} \tan(t^{-1})\, dt - \int_{x_0}^{\frac{\pi}{2}} \tan(t^{-1})\, dt = \int_{\frac{\pi}{2}}^{\pi} \tan(t^{-1})\, dt$$

e^{-x^2}

$$\lim_{x\to+\infty} \left(1+x-\sqrt{x^2+1}\right)^x =$$

$$= \lim_{x\to+\infty} e^{\lg\left(1+x-\sqrt{x^2+1}\right)^x} =$$

$$= \lim_{x\to+\infty} e^{\overset{\nearrow +\infty}{x}\,\lg\left(1+x-\sqrt{x^2+1}\right)}$$

$$1+x-\sqrt{x^2+1} = \left(1+x-\sqrt{x^2+1}\right)\frac{1+x+\sqrt{x^2+1}}{1+x+\sqrt{x^2+1}} =$$

$$= \frac{(1+x)^2-x^2-1}{1+x+\sqrt{x^2+1}} = \frac{2x}{1+x+\sqrt{x^2+1}} \xrightarrow[x\to+\infty]{} 1$$

$$x\lg\left(1+x-\sqrt{x^2+1}\right) = x\lg\frac{2x}{1+x+\sqrt{x^2+1}} =$$

$$= x\lg\frac{2}{1+\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}} =$$

$$= x\lg\frac{2}{2+\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}-1}$$

$$= x\ \overset{\nearrow 1}{\frac{\lg\left(1+\left(\frac{1}{1+\frac{1}{2}\frac{1}{x}+\frac{1}{2}\left(\sqrt{1+\frac{1}{x^2}}-1\right)}-1\right)\right)}{\frac{1}{2}\frac{1}{x}+\frac{1}{2}\left(\sqrt{1+\frac{1}{x^2}}-1\right)}}\ \frac{1}{2}\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}-1\right)$$

$$\lim_{y\to0}\frac{\lg(1+y)}{y}=1$$

$$= (1+o(1))\ \frac{1}{2}\left(1+x\left(\sqrt{1+\frac{1}{x^2}}-1\right)\right) =$$

$$\lim \frac{1}{2} + \frac{1}{2}\lim_{x\to+\infty} x\left(\sqrt{1+\frac{1}{x^2}}-1\right)$$

$$x\left(\sqrt{1+\frac{1}{x^2}}-1\right) = \frac{\sqrt{1+\frac{1}{x^2}}-1}{\frac{1}{x^2}\qquad x}$$

$$\lim_{y\to0}\frac{(1+y)^\alpha-1}{y}=\alpha \qquad = \underbrace{\frac{\left(1+\frac{1}{x^2}\right)^{\frac{1}{2}}-1}{\frac{1}{x^2}}}_{\to\frac{1}{2}}\quad \underbrace{\frac{1}{x}}_{\to 0}$$

$$\begin{cases} \overbrace{y''-2y'+y}^{L[y]}=e^x \\ y(0)=1,\ y'(0)=1 \end{cases}$$

$$r^2-2r+1=0$$

$$(r-1)^2=0 \qquad r=1 \quad \text{con molteplicità } 2.$$

$$y_h=C_1e^x+C_2xe^x$$

$$y_p=Ax^2e^x$$

$$L[Ax^2e^x]=A\,L[x^2e^x]=e^x$$

$$=A\left((x^2e^x)''-2(x^2e^x)'+x^2e^x\right)$$

$p(r)$

$$L[ue^{rx}]=u''e^{rx}+p'(r)u'e^{rx}+p(r)ue^{rx}$$

$$L[y]=y''+by'+cy \qquad p(r)=r^2+br+c$$

$$L[ue^{rx}]=(ue^{rx})''+b(ue^{rx})'+c\,ue^{rx}=$$

$$(ue^{rx})'=rue^{rx}+u'e^{rx}$$

$$(ue^{rx})''=r^2ue^{rx}+2ru'e^{rx}+u''e^{rx}$$

$$=(r^2ue^{rx}+\boxed{2ru'e^{rx}}+u''e^{rx})+b(rue^{rx}+\boxed{u'e^{rx}})$$

$$+c\,ue^{rx}=$$

$$=e^{rx}\left[u''+u'\underbrace{(2r+b)}_{p'(r)}+u\,p(r)\right]$$

$$A\,L[\overset{u}{x^2}e^x]=Ae^x\left[2+2x\overbrace{p'(1)}^{0}+x^2\overbrace{p(1)}^{0}\right]=2Ae^x$$

$$p(r)=(r-1)^2 \qquad p(1)=0=p'(1) \qquad =e^x$$

$$A=\frac{1}{2}$$

$$y_p=\frac{1}{2}x^2e^x$$

$$y=\left(C_1+C_2x+\frac{1}{2}x^2\right)e^x=C_1e^x+C_2xe^x+\frac{1}{2}x^2e^x$$

$$\begin{cases} y(0)=C_1 \quad =1 \\ y'(0)=C_1+C_2=1 \end{cases} \qquad (xe^x)'=e^x+xe^x\Big|_{x=0}=1$$

$$C_1=1 \qquad C_2=1-C_1=1-1=0$$

Es 11 da Limiti 2 Dove

$$\lim_{x\to 0^+} \frac{\sqrt{e^x+x} - ch(\sqrt{2x})}{e^{\sqrt{1+2x}} - e\sqrt{1+2x}}$$

denom $= e^{\sqrt{1+2x}} - e\sqrt{1+2x} =$

$$= e\left[e^{\sqrt{1+2x}-1} - \sqrt{1+2x} \right] =$$

$e^y = 1 + y + \frac{y^2}{2} + o(y^2)$ $\qquad y = \sqrt{1+2x} - 1$

$$= e\left[\cancel{1 - \sqrt{1+2x}} + \cancel{\sqrt{1+2x}-1} + \frac{(\sqrt{1+2x}-1)^2}{2} + o\left((\sqrt{1+2x}-1)^2\right)\right]$$

$$= \frac{e}{2}\left(\sqrt{1+2x}-1\right)^2 \left(1+o(1)\right)$$

$$\frac{\sqrt{1+2x}-1}{2x} = 1 + o(1)$$

$$\sqrt{1+2x}-1 = 2x\left(1+o(1)\right)$$

$$= 2e\,x^2\left(1+o(1)\right) = \text{denom}$$

num $\left(\sqrt{e^x+x} - ch(\sqrt{2x})\right) \cdot \frac{\sqrt{e^x+x} + ch(\sqrt{2x})}{\sqrt{e^x+x} + ch(\sqrt{2x})}$

$$= \frac{e^x + x - ch^2(\sqrt{2x})}{\sqrt{e^x+x} + ch(\sqrt{2x})} =$$

$$ch^2(y) = \frac{ch(2y)+1}{2}$$

$$= \frac{e^x + x - \frac{ch(2\sqrt{2x})+1}{2}}{2\left(1+o(1)\right)}$$

$$= \frac{2e^x + 2x - 1 - ch(2\sqrt{2x})}{4}$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$\frac{e^y + e^{-y}}{2} = \frac{1 + \cancel{y} + \frac{y^2}{2} + \cancel{\frac{y^3}{3!}} + \frac{y^4}{4!} + o(y^4)}{2} + \frac{1 - \cancel{y} + \frac{y^2}{2} - \cancel{\frac{y^3}{3!}} + \frac{y^4}{4!} + o(y^4)}{2}$$

$$= 1 + \frac{y^2}{2} + \frac{y^4}{4!} + o(y^4) \qquad y = 2\sqrt{2x}$$

$$= 1 + \frac{2^2 \cdot \cancel{2}x}{\cancel{2}} + \frac{2^4\, 2^2 x^2}{4!} + o(x^2)$$

$$= 1 + 4x + \frac{2^4}{6}x^2 + o(x^2) = 1 + 4x + \frac{8}{3}x^2 + o(x^2)$$

$$\frac{2e^x + 2x - 1 - ch(2\sqrt{2x})}{4}$$

$$= \frac{2\left(\cancel{1} + x + \frac{x^2}{2}\right) + 2x - \cancel{1} - 1 - 4x - \frac{8}{3}x^2 + o(x^2)}{4}$$

$$= \frac{1}{4}\left(1 - \frac{8}{3}\right)x^2 + o(x^2)$$

$$= \frac{1}{4}\left(-\frac{5}{3}\right)x^2 + o(x^2) = -\frac{5}{12}x^2 + o(x^2) = \text{num}$$

$$\lim_{x\to 0^+} \frac{-\frac{5}{12}\cancel{x^2}}{2e\,\cancel{x^2}} =$$

$$= -\frac{5}{24e}$$

Es n. 4 24/06/2024

$$f(x)=\int_0^x \frac{1}{1+t^2+t^4}\,dt$$

$P_6(x)$ polinomio di Mc Laurin.

$f(0)=0$ Siccome $f'(x)=\dfrac{1}{1+x^2+x^4}$

$\Rightarrow$ f è dispari. $\Rightarrow$ $P_6=P_5$

Posto $q_4(t)$ il polinomio di Mc Laurin di $\dfrac{1}{1+t^2+t^4}$ si avrà $P_5(x)=\int_0^x q_4(t)\,dt$

$$f(x)=\int_0^x \frac{1}{1+t^2+t^4}\,dt=\int_0^x \left(q_4(t)+o(t^4)\right)dt$$

$$f(x)=\underbrace{\int_0^x q_4(t)\,dt}_{\text{grado} \leq 5}+\underbrace{\int_0^x o(t^4)\,dt}_{o(x^5)}$$

$$\frac{1}{1+t^2+t^4}$$

$$\frac{1}{1+y}=1-y+y^2+o(y^2) \qquad y=t^2+t^4$$

$$\frac{1}{1+t^2+t^4}=1-(t^2+t^4)+(t^2+t^4)^2+\underbrace{o\left((t^2+t^4)^2\right)}_{o(t^4)}$$

$$o\left((t^2+t^4)^2\right)=o\left(\left(t^2(1+t^2)\right)^2\right)=o\left(t^4(1+t^2)^2\right)=$$
$$=o(t^4)$$

$$=1-(t^2+\cancel{t^4})+\cancel{t^4}+o(t^4)$$

$$\frac{1}{1+t^2+t^4}=1-t^2+o(t^4) \qquad q_4(t)=1-t^2$$

$$P_5(x)=x-\frac{x^3}{3}$$