Quantum Field Theory 1

Test Exam

Questions

1. Write down the Klein-Gordon equation for a free relativistic particle (with all units explicitly shown). Show that plane waves are solutions of the equation and derive the energy spectrum. What is the problem with it?

2. Show that the Klein-Gordon equation satisfies a continuity equation and write down the explicit expression of the 4-current $j^\mu(x)$. Be careful in making sure that it has the correct dimensions. What is the problem here? Show it with a specific example.

3. Write down the free-particle Dirac equation (with all units explicitly shown) in the form
$$i\hbar\frac{\partial}{\partial t}\psi(x) = H_D\psi(x),$$
where H_D is the Dirac Hamiltonian, and starting from the requirement that the relativistic relation $E^2 = p^2c^2 + m^2c^4$ is satisfied, derive all relevant constraints on α_i and β. Write an explicit representation of α_i and β.

4. Show that the Dirac equation satisfies a continuity equation and write down the explicit expression of the 4-current $j^\mu(x)$. Be careful in making sure that it has the correct dimensions. Show that this current does not have the problem, which shows up for the current associated to the Klein-Gordon equation.