9 gennaio

$$\lim_{x \to 0} \frac{x\left(e^{3x-x^2}-1\right)\cos(3x)}{\cos(x+x^2)-1} \quad \cos(3x) \to 1$$

$$\lim_{y\to 0} \frac{\cos(y)-1}{y^2} = \frac{1}{2}$$

$$\lim_{y\to 0} \frac{e^{y}-1}{y} = 1$$

$$= \lim_{x\to 0} x \, \frac{e^{3x-x^2}-1}{3x-x^2} \, (3x-x^2) \, \frac{(x+x^2)^2}{\cos(x^2+x)-1} \, \frac{1}{(x+x^2)^2}$$

$\frac{e^{3x-x^2}-1}{3x-x^2} \to 1$, $\frac{(x+x^2)^2}{\cos(x^2+x)-1} \to -2$

$$= -2 \lim_{x\to 0} \frac{x\,(3x-x^2)}{(x+x^2)^2}$$

$$= -2 \lim_{x\to 0} \frac{\cancel{x}\;3\cancel{x}\left(1-\frac{x}{3}\right)}{\cancel{x^2}\,(1+x)^2} = -6$$

$\left(1-\frac{x}{3}\right) \to 1$, $(1+x)^2 \to 1$

$$\lim_{x\to+\infty}\left(x^2 e^{\frac{x+3}{x^2-4}} - x^2 - x\right) \qquad \frac{x+3}{x^2-4} \to 0$$

$$e^y = 1 + y + \frac{y^2}{2} + o(y^2) \qquad y \to 0$$

$$x^2\left(\cancel{1} + \frac{x+3}{x^2-4} + \frac{1}{2}\left(\frac{x+3}{x^2-4}\right)^2 + \underbrace{o\left(\left(\frac{x+3}{x^2-4}\right)^2\right)}_{o\left(\frac{1}{x^2}\right)}\right) - \cancel{x^2} - x$$

$$= x^2\frac{x+3}{x^2-4} + \underbrace{\frac{1}{2}x^2\left(\frac{x+3}{x^2-4}\right)^2}_{\to \frac{1}{2}} - x + \cancel{o(1)}$$

$$\lim_{x\to+\infty}\left(x^2\frac{x+3}{x^2-4} - x + \frac{1}{2}\right)$$

$$x^2\frac{x+3}{x^2-4} = x^2(x+3)\frac{1}{x^2-4} =$$

$$= \cancel{x^2}(x+3)\frac{1}{\cancel{x^2}}\,\frac{1}{1-\frac{4}{x^2}}$$

$$\boxed{\frac{1}{1-y} = 1 + y + y^2 + o(y^2) = 1 + y + o(y)}$$

$$= (x+3)\left(1 - \frac{4}{x^2} + o\left(\frac{1}{x^2}\right)\right)$$

$$= x + 3 - \frac{4}{x} + o\left(\frac{1}{x}\right) = x + 3 + o(1)$$

$$\lim_{x\to+\infty}\left(\cancel{x} + 3 + o(1) - \cancel{x} + \frac{1}{2}\right) = \frac{7}{2}$$

13

$$\lim_{x\to+\infty} \underbrace{x^4\left(e^{\frac{2x^2}{2x^2+1}} - e^{\cos\frac{1}{x}}\right)}_{f(x)} \qquad \lim_{y\to 0}\frac{e^y-1}{y} = 1$$

$$f(x) = x^4\, e^{\cos\frac{1}{x}} \left(e^{\frac{2x^2}{2x^2+1} - \cos\left(\frac{1}{x}\right)} - 1\right) \qquad e^{\cos\frac{1}{x}} \to e(1+o(1))$$

$$= x^4\, e\,(1+o(1)) \left(e^{\underbrace{\frac{2x^2}{2x^2+1} - \cos\left(\frac{1}{x}\right)}_{g(x)}} - 1\right)$$

$$g(x) = \frac{2x^2}{2x^2+1} - \cos\left(\frac{1}{x}\right) = \frac{\cancel{2x^2}}{\cancel{2x^2}\left(1+\frac{1}{2x^2}\right)} - \cos\left(\frac{1}{x}\right)$$

$$\cos(y) = 1 - \frac{y^2}{2} + o(y^2) = \boxed{1 - \frac{y^2}{2} + \frac{y^4}{4!} + o(y^4)}$$

$$\frac{1}{1+y} = 1 - y + o(y) = \boxed{1 - y + y^2 + o(y^2)}$$

$$= \cancel{1} - \cancel{\frac{1}{2x^2}} + o\left(\frac{1}{x^2}\right) - \left(\cancel{1} - \cancel{\frac{1}{2x^2}} + o\left(\frac{1}{x^2}\right)\right)$$

$$f(x) = x^4\, o\left(\frac{1}{x^2}\right)$$

$$g(x) = \frac{1}{1+\frac{1}{2x^2}} - \cos\left(\frac{1}{x}\right) = \cancel{1} - \cancel{\frac{1}{2x^2}} + \left(\frac{1}{2x^2}\right)^2 + o\left(\frac{1}{x^4}\right)$$

$$- \left(\cancel{1} - \cancel{\frac{1}{2x^2}} + \frac{1}{4!}\,\frac{1}{x^4} + o\left(\frac{1}{x^4}\right)\right)$$

$$= \left(\frac{1}{4} - \frac{1}{4!}\right)\frac{1}{x^4} + o\left(\frac{1}{x^4}\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{6}\right)\frac{1}{x^4} + o\left(\frac{1}{x^4}\right) = \frac{5}{24}\,\frac{1}{x^4} + o\left(\frac{1}{x^4}\right)$$

$$f(x) = x^4\, e\,(1+o(1)) \left(\frac{e^{g(x)}-1}{g(x)}\right) g(x) \qquad \frac{e^{g(x)}-1}{g(x)} \to 1$$

$$\lim_{x\to+\infty} f(x) = e \lim_{x\to+\infty} x^4\, g(x) =$$

$$= e \lim_{x\to+\infty} \cancel{x^4}\,\frac{5}{24}\,\frac{1}{\cancel{x^4}}\,\cancel{(1+o(1))} = e\,\frac{5}{24}$$

$$\lim_{x \to +\infty} \frac{x^2 \lg\left(\frac{5+x^2}{3+x^2}\right) - 2}{e^{\sin\left(\frac{5}{x}\right)} - 1 - \frac{5}{x}}$$

$$\sin(y) = y - \frac{y^3}{3!} + o(y^3)$$

$$\sin\left(\frac{5}{x}\right) = \frac{5}{x} - \frac{1}{3!}\left(\frac{5}{x}\right)^3 + o\left(\frac{1}{x^3}\right) = \frac{5}{x} + o\left(\frac{1}{x^2}\right)$$

$$e^z = 1 + z + \frac{z^2}{2} + o(z^2)$$

$$e^{\sin\left(\frac{5}{x}\right)} = 1 + \sin\frac{5}{x} + \frac{\sin^2\frac{5}{x}}{2} + o\left(\frac{1}{x^2}\right)$$

$$= 1 + \left(\frac{5}{x} + o\left(\frac{1}{x^2}\right)\right) + \frac{1}{2}\left(\frac{5}{x} + o\left(\frac{1}{x^2}\right)\right)^2 + o\left(\frac{1}{x^2}\right)$$

$$= 1 + \frac{5}{x} + \frac{25}{2x^2} + o\left(\frac{1}{x^2}\right)$$

$$\text{denom} = e^{\sin\left(\frac{5}{x}\right)} - 1 - \frac{5}{x} = \cancel{1 + \frac{5}{x}} + \frac{25}{2x^2} + o\left(\frac{1}{x^2}\right) \cancel{- 1 - \frac{5}{x}}$$

$$\text{denom} = \frac{25}{2x^2}\left(1 + o(1)\right)$$

$$\text{num} = x^2 \lg\left(\frac{5+x^2}{3+x^2}\right) - 2 = x^2 \lg\left(\frac{\cancel{x^2}\left(1 + \frac{5}{x^2}\right)}{\cancel{x^2}\left(1 + \frac{3}{x^2}\right)}\right) - 2$$

$$= x^2 \lg\left(\frac{1 + \frac{5}{x^2}}{1 + \frac{3}{x^2}}\right) - 2 =$$

$$= x^2 \lg\left(1 + \frac{5}{x^2}\right) - x^2 \lg\left(1 + \frac{3}{x^2}\right) - 2$$

$$\lg(1+y) = y - \frac{y^2}{2} + o(y^2)$$

$$= x^2\left(\cancel{\frac{5}{x^2}} - \frac{25}{2}\frac{1}{x^4} + o\left(\frac{1}{x^4}\right)\right) - x^2\left(\cancel{\frac{3}{x^2}} - \frac{9}{2}\frac{1}{x^4} + o\left(\frac{1}{x^4}\right)\right) \cancel{-2}$$

$$= \frac{1}{2x^2}(-25 + 9) + o\left(\frac{1}{x^2}\right) = \frac{-16}{2}\frac{1}{x^2} + o\left(\frac{1}{x^2}\right)$$

$$\lim_{x \to +\infty} \frac{\frac{-16}{2}\cancel{\frac{1}{x^2}}}{\frac{25}{2}\cancel{\frac{1}{x^2}}} = -\frac{16}{25}$$