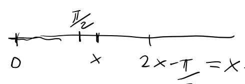
$$\lim_{x\to\frac{\pi}{2}^+}\int_x^{2x-\frac{\pi}{2}} \tan(t)\,dt =$$

$$= \lim_{x\to\frac{\pi}{2}^+}\int_x^{2x-\frac{\pi}{2}} \frac{\sin(t)}{\cos(t)}\,dt$$

$$\int_x^{2x-\frac{\pi}{2}} \frac{\sin(t)}{\cos(t)}\,dt =$$



$2x-\frac{\pi}{2} = x + \underbrace{x-\frac{\pi}{2}}_{>0}$

$t = s+\frac{\pi}{2}$ $\qquad s = t-\frac{\pi}{2}$

$dt = ds$

$$= \int_{x-\frac{\pi}{2}}^{2x-\pi}\frac{\sin(s+\frac{\pi}{2})}{\cos(s+\frac{\pi}{2})}\,ds =$$

$\sin(s+\frac{\pi}{2}) = \sin(s)\underset{0}{\cos(\tfrac{\pi}{2})} + \cos(s)\underset{1}{\sin(\tfrac{\pi}{2})}$

$\cos(s+\frac{\pi}{2}) = \cos(s)\underset{0}{\cos(\tfrac{\pi}{2})} - \sin(s)\underset{1}{\sin(\tfrac{\pi}{2})}$

$$= \int_{x-\frac{\pi}{2}}^{2x-\pi}\frac{\cos(s)}{-\sin s}\,ds$$

$$= -\int_{x-\frac{\pi}{2}}^{2x-\pi}\overbrace{\cos(s)\,\frac{s}{\sin s}}^{(1+o(1))}\,\frac{1}{s}\,ds$$

$$= -\int_{x-\frac{\pi}{2}}^{2x-\pi}(1+o(1))\,\frac{1}{s}\,ds =$$

$$= -\int_{x-\frac{\pi}{2}}^{2(x-\frac{\pi}{2})}\frac{1}{s}\,ds + \int_{x-\frac{\pi}{2}}^{2x-\pi}\frac{o(1)}{s}\,ds$$

$$= -\log s\Big]_{x-\frac{\pi}{2}}^{2(x-\frac{\pi}{2})} + \int_{x-\frac{\pi}{2}}^{2x-\pi}\frac{o(1)}{s}\,ds$$

$$= \underbrace{\log\frac{\cancel{x-\frac{\pi}{2}}}{2(\cancel{x-\frac{\pi}{2}})}}_{\log\frac{1}{2}} + \underbrace{\int_{x-\frac{\pi}{2}}^{2(x-\frac{\pi}{2})}\frac{o(1)}{s}\,ds}_{\downarrow x\to\frac{\pi}{2}^+}$$

$$\left|\int_{x-\frac{\pi}{2}}^{2(x-\frac{\pi}{2})}\frac{o(1)}{s}\,ds\right| = \left(x-\frac{\pi}{2}\right)\left|\frac{\overset{0}{o(1)}}{s}\right|_{s=c_x}$$

$\overset{0}{\uparrow}$ $x-\frac{\pi}{2} \le c_x \le \overset{0}{\uparrow}\, 2\left(x-\frac{\pi}{2}\right)$

$\frac{1}{2(x-\frac{\pi}{2})} \le \frac{1}{c_x} \le \frac{1}{x-\frac{\pi}{2}}$

$$\le \cancel{\left(x-\frac{\pi}{2}\right)}\,\frac{1}{\cancel{x-\frac{\pi}{2}}}\;o(1)\Big|_{s=c_x}\xrightarrow{x\to\frac{\pi}{2}^+} 0$$

$$\lim_{x\to\frac{\pi}{2}^+} c_x = 0$$

$$\int_x^{2x-\frac{\pi}{2}}\tan t\,dt = -\int_x^{2x-\frac{\pi}{2}}\frac{-\sin t}{\cos t}\,dt = -\log\cos t\Big]_x^{2x-\pi}$$

$$= \log\cos x - \log\cos 2x = \log\frac{\cos x}{\cos(2x-\frac{\pi}{2})} =$$

$$= \log\frac{\cos x}{\sin(2x)} = \log\frac{\cancel{\cos x}}{2\sin x\,\cancel{\cos x}} = \log\frac{1}{2} + \underbrace{\log\overset{\overset{1}{\uparrow\, x\to\frac{\pi}{2}}}{\frac{1}{\sin x}}}_{\downarrow\, 0}$$

$$\int_x^{2x-\frac{\pi}{2}} \tan t \; dt =$$

$$= \int_x^{2x-\frac{\pi}{2}} \frac{\sin(t)}{\cos(t)} \, dt$$

$$\sin(t) = \sin\left(\left(t-\frac{\pi}{2}\right)+\frac{\pi}{2}\right) = \cos\left(t-\frac{\pi}{2}\right) =$$

$$\cos y = \sum_{j=0}^{n} (-1)^j \frac{y^{2j}}{(2j)!} + o(y^{2n})$$

$$= \sum_{j=0}^{n} (-1)^j \frac{\left(t-\frac{\pi}{2}\right)^{2j}}{(2j)!} + o\left(\left(t-\frac{\pi}{2}\right)^{2n}\right)$$

$$\sin t = 1 - \frac{\left(t-\frac{\pi}{2}\right)^2}{2} + o\left(\left(t-\frac{\pi}{2}\right)^2\right)$$

$$\cos(t) = \cos\left(\left(t-\frac{\pi}{2}\right)+\frac{\pi}{2}\right) = -\sin\left(t-\frac{\pi}{2}\right) =$$

$$\sin(y) = \sum_{j=0}^{n} (-1)^j \frac{y^{2j+1}}{(2j+1)!} + o(y^{2n+1})$$

$$\cos t = -\sum_{j=0}^{n} (-1)^j \frac{\left(t-\frac{\pi}{2}\right)^{2j+1}}{(2j+1)!} + o\left(\left(t-\frac{\pi}{2}\right)^{2n+1}\right)$$

$$= -\left(t-\frac{\pi}{2}\right) + \frac{1}{6}\left(t-\frac{\pi}{2}\right)^3 + o\left(\left(t-\frac{\pi}{2}\right)^3\right)$$

$$-\int_x^{2x-\frac{\pi}{2}} \frac{\sin t}{\cos t} \, dt = -\int_x^{2x-\frac{\pi}{2}} \frac{1 + o\left(t-\frac{\pi}{2}\right)}{t-\frac{\pi}{2} + o\left(\left(t-\frac{\pi}{2}\right)^2\right)} \, dt$$

$$= -\int_x^{2x-\frac{\pi}{2}} \frac{1}{t-\frac{\pi}{2}} \underbrace{\frac{1+o\left(t-\frac{\pi}{2}\right)}{1+o\left(t-\frac{\pi}{2}\right)}}_{\left(1+o\left(t-\frac{\pi}{2}\right)\right)} \, dt$$

$$= -\int_x^{2x-\frac{\pi}{2}} \frac{1}{t-\frac{\pi}{2}} \, dt + \int_x^{2x-\frac{\pi}{2}} \frac{1}{\cancel{t-\frac{\pi}{2}}} \overbrace{o\left(t-\frac{\pi}{2}\right)}^{\cancel{\left(t-\frac{\pi}{2}\right)} o(1)} \, dt$$

$$= -\lg\left(t-\frac{\pi}{2}\right)\Big]_x^{2x-\frac{\pi}{2}} + \int_x^{2x-\frac{\pi}{2}} \underbrace{o(1)}_{t\to\frac{\pi}{2}} \, dt$$

$$= -\lg \frac{2x-\frac{\pi}{2}-\frac{\pi}{2}}{x-\frac{\pi}{2}} + \underbrace{\int_x^{2x-\frac{\pi}{2}} o(1) \, dt}_{\downarrow x\to\frac{\pi}{2} \;\; 0}$$

$=2$

$t\to\frac{\pi}{2} \to 0$

$\downarrow$

$\lg \frac{1}{2}$

$\sin(x)$ $\qquad\qquad$ x_0

$$\sin(x - x_0 + x_0) = \sin(x-x_0)\cos x_0 + \cos(x-x_0)\sin x_0$$

$$\sin(x-x_0) = \sum_{j=0}^{n} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} + o\left((x-x_0)^{2n+1}\right)$$

$$\cos(x-x_0) = \sum_{j=0}^{n} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} + o\left((x-x_0)^{2n}\right)$$

$$\sin(x) = P_{10}(x) + o\left((x-x_0)^{10}\right)$$

$$= \cos(x-x_0)\sin x_0 + \sin(x-x_0)\cos x_0$$

$$= \sin x_0 \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} + \cos(x_0) \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} + o((x-x_0)^{10})$$

$$\sin x = \sin x_0 \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} + \cos(x_0) \sum_{j=0}^{4} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} + o((x-x_0)^{10})$$

$$P_{2n}(x) = \sin x_0 \sum_{j=0}^{n} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} + \cos(x_0) \sum_{j=0}^{n-1} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!}$$

P_{2n}

$\sin(x) + \cos(x)$ $\qquad$ P_{2n+1}

$$\sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{j=0}^{n} (-1)^j \frac{x^{2j}}{(2j)!}$$

$$\sin x = \cos x_0 \sin(x-x_0) + \sin x_0 \cos(x-x_0)$$

$$= \cos x_0 \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} +$$

$$+ \sin x_0 \sum_{j=0}^{6} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} + o\left((x-x_0)^{11}\right) + o\left((x-x_0)^{12}\right)$$

$$\sin x = \cos x_0 \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} + \sin x_0 \sum_{j=0}^{5} (-1)^j \frac{(x-x_0)^{2j}}{(2j)!} +$$

$$+ \underbrace{\frac{(x-x_0)^{12}}{(12)!} + o\left((x-x_0)^{11}\right) + o\left((x-x_0)^{12}\right)}_{o\left((x-x_0)^{11}\right)}$$

$$P_{2n+1}(x) = \cos x_0 \sum_{j=0}^{n} (-1)^j \frac{(x-x_0)^{2j+1}}{(2j+1)!} + \sin x_0 \sum_{j=0}^{n} (-1) \frac{(x-x_0)^{2j}}{(2j)!}$$

$$\int_1^2 \frac{1}{\sqrt{x}\,\sqrt{3-x}}\,dx =$$

$x := y^2$

$y = \sqrt{x}$

$dy = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$

$\frac{1}{\sqrt{x}}\,dx = 2\,dy$

$$= 2\int_1^{\sqrt{2}} \frac{1}{\sqrt{3-y^2}}\,dy =$$

$y = \sqrt{3}\,u \qquad u = \frac{1}{\sqrt{3}}\,y$

$dy = \sqrt{3}\,du$

$$= 2\int_{\frac{1}{\sqrt{3}}}^{\sqrt{\frac{2}{3}}} \frac{\cancel{\sqrt{3}}}{\sqrt{\cancel{3}-\cancel{3}u^2}}\,du$$

$$= 2\int_{\frac{1}{\sqrt{3}}}^{\sqrt{\frac{2}{3}}} \frac{1}{\sqrt{1-u^2}}\,du = 2\arcsin u \Big]_{\frac{1}{\sqrt{3}}}^{\sqrt{\frac{2}{3}}}$$