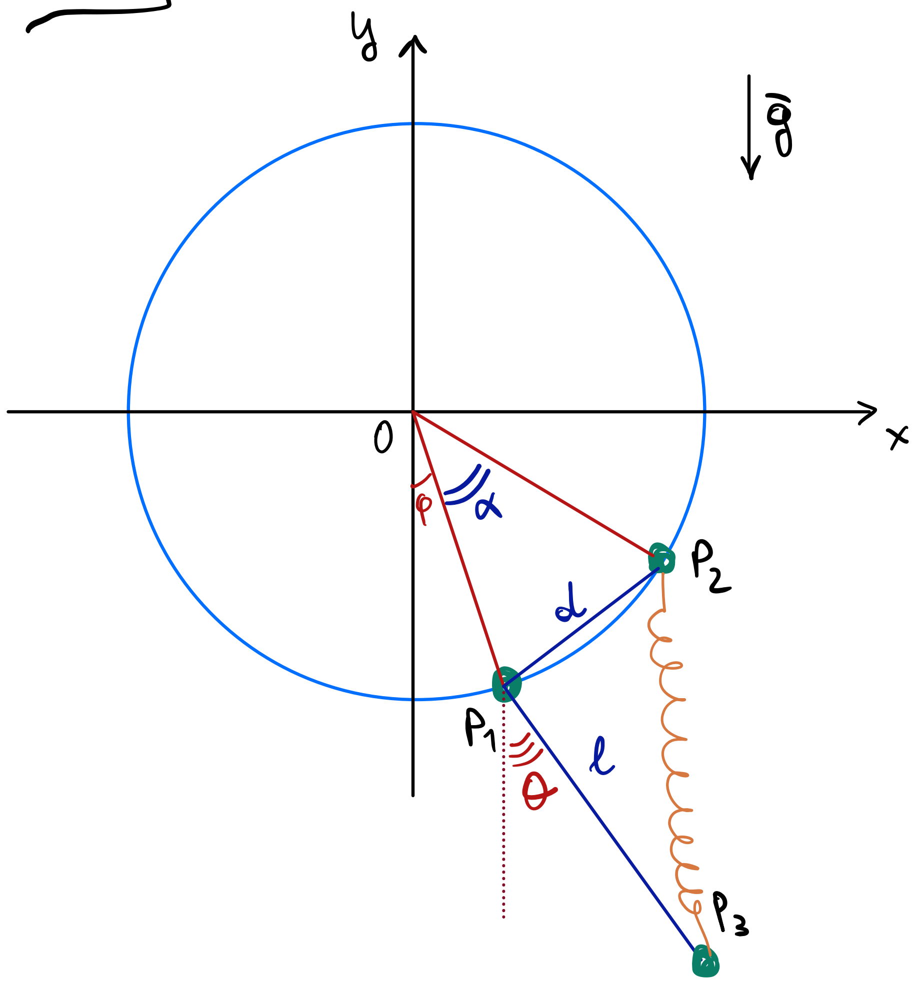
ES 2



$$d = 2R\,\text{sen}\,\frac{\alpha}{2}$$

$$x_1 = R\,\text{sen}\,\varphi$$

$$y_1 = -R\cos\varphi$$

$$x_2 = R\,\text{sen}\,(\varphi+\alpha) =$$

$$= R\,\text{sen}\,\alpha\cos\varphi + R\cos\alpha\,\text{sen}\,\beta$$

$$y_2 = -R\cos(\varphi+\alpha) =$$

$$= -R\cos\alpha\cos\varphi + R\,\text{sen}\,\alpha\,\text{sen}\,\varphi$$

$$x_3 = R\,\text{sen}\,\varphi + \ell\,\text{sen}\,\theta$$

$$y_3 = -R\cos\varphi - \ell\cos\theta$$

1)

$$T = \frac{1}{2} m R^2 \dot{\varphi}^2 + \frac{2 m R^2 \dot{\varphi}^2}{2} + \frac{m}{2}\left[\left(R\dot{\varphi}\cos\varphi + l\dot{\theta}\cos\theta\right)^2 + \left(R\dot{\varphi}\operatorname{sen}\varphi + l\dot{\theta}\operatorname{sen}\theta\right)^2 \right]$$

$$2lR\dot{\varphi}\dot{\theta}\left(\cos\varphi\cos\theta + \operatorname{sen}\varphi\operatorname{sen}\theta\right)$$

$$= \frac{1}{2}\left(m + 2m + m\right) R^2 \dot{\varphi}^2 + \frac{m}{2} l^2 \dot{\theta}^2 + \frac{m}{2}\, 2lR\dot{\varphi}\dot{\theta}\cos(\varphi - \theta)$$

$$= 2mR^2\dot{\varphi}^2 + \frac{m}{2} l^2 \dot{\theta}^2 + mlR\dot{\theta}\dot{\varphi}\cos(\varphi - \theta)$$

$$A = \begin{pmatrix} 4mR^2 & mlR\cos(\varphi-\theta) \\ mlR\cos(\varphi-\theta) & ml^2 \end{pmatrix}$$

$$V = -2mgR\cos\varphi - 2mgR\cos(\varphi + \alpha) - mgl\cos\theta$$

$$L = 2mR^2\dot{\varphi}^2 + \frac{ml^2}{2}\dot{\theta}^2 + mlR\dot{\theta}\dot{\varphi}\cos(\varphi - \theta)$$

$$+ 2mgR\cos\varphi + 2mgR\cos(\varphi + \alpha) + mgl\cos\theta$$

2)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = \frac{d}{dt}\left(4mR^2\dot{\varphi} + mlR\dot{\theta}\cos(\varphi - \theta)\right) =$$

$$= 4mR^2\ddot{\varphi} + mlR\ddot{\theta}\cos(\varphi-\theta) - mlR\dot{\theta}\dot{\varphi}\operatorname{sen}(\varphi - \theta)$$

$$\frac{\partial L}{\partial \varphi} = -mlR\dot{\theta}\dot{\varphi}\operatorname{sen}(\varphi-\theta) - 2mgR\operatorname{sen}\varphi - 2mgR\operatorname{sen}(\varphi+\alpha)$$

$$\ddot{\varphi} + \frac{l}{R}\ddot{\theta}\cos(\varphi-\theta) + \frac{2g}{R}\operatorname{sen}\varphi + \frac{2g}{R}\operatorname{sen}(\varphi+\alpha) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}\left(ml^2\dot{\theta} + mlR\dot{\varphi}\cos(\varphi-\theta)\right) = ml^2\ddot{\theta} + mlR\ddot{\varphi}\cos(\varphi-\theta) + mlR\dot{\varphi}\dot{\theta}\operatorname{sen}(\varphi-\theta$$

$$\frac{\partial L}{\partial \theta} = mlR\dot{\theta}\dot{\varphi}\operatorname{sen}(\varphi-\theta) - mgl\operatorname{sen}\theta$$

$$\ddot{\theta} + \frac{R}{l}\ddot{\varphi}\cos(\varphi-\theta) + \frac{g}{l}\operatorname{sen}\theta = 0$$

3) $V = -2mgR\cos\varphi - 2mgR\cos(\varphi+\alpha) - mgl\cos\theta$ $\qquad \operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen}\frac{p+q}{2}\cos\frac{p-q}{2}$

$\partial_\varphi V = 2mgR\operatorname{sen}\varphi + 2mgR\operatorname{sen}(\varphi+\alpha) = 4mgR\cdot\cos\frac{\alpha}{2}\cdot\operatorname{sen}\left(\varphi+\frac{\alpha}{2}\right)$

$\partial_\theta V = mgl\operatorname{sen}\theta$

Pti equil: $(\varphi,\theta) = \left(-\frac{\alpha}{2}, 0\right) \quad \left(-\frac{\alpha}{2}, \pi\right) \quad \left(\pi-\frac{\alpha}{2}, 0\right) \quad \left(\pi-\frac{\alpha}{2}, \pi\right)$

$\partial^2_\varphi V = 4mgR\cos\frac{\alpha}{2}\cos\left(\varphi+\frac{\alpha}{2}\right)$

$\partial^2_\theta V = mgl\cos\theta$

$\partial_\varphi\partial_\theta V = 0$

$$\partial^2 V\Big|_{-\frac{\alpha}{2},0} = \begin{pmatrix} 4mgR\cos\frac{\alpha}{2} & \\ & mgl \end{pmatrix} \quad \text{stab}$$

$$\partial^2 V\Big|_{-\frac{\alpha}{2},\pi} = \begin{pmatrix} >0 & \\ & -mgl \end{pmatrix} \quad \text{instab.}$$

$$\partial^2 V\Big|_{\pi-\frac{\alpha}{2},0} = \begin{pmatrix} <0 & \\ & >0 \end{pmatrix} \quad \text{instab.} \qquad \partial^2 V\Big|_{\pi-\frac{\alpha}{2},\pi} = \begin{pmatrix} <0 & \\ & <0 \end{pmatrix} \quad \text{instab.}$$

4)
$$A = \begin{pmatrix} 4mR^2 & ml R\cos(\varphi-\theta) \\ ml R\cos(\varphi-\theta) & ml^2 \end{pmatrix}\Bigg|_{-\frac{\alpha}{2},0} = \begin{pmatrix} 4mR^2 & ml R\cos\alpha/2 \\ ml R\cos\alpha/2 & ml^2 \end{pmatrix}$$

$$B = \begin{pmatrix} 4mgR\cos\frac{\alpha}{2} & \\ & mgl \end{pmatrix}$$

$$\det(B-\lambda A) = \det\begin{pmatrix} 4\, g/R \cos\frac{\alpha}{2} - 4\lambda & -\lambda\, l/R \cos\alpha/2 \\ -\lambda\, l/R\cos\alpha/2 & gl/R^2 - \lambda\, l^2/R^2 \end{pmatrix} \cdot m^2 R^4$$

$$= m^2 R^4 \det\begin{pmatrix} 4\left(\frac{g}{R}\cos\frac{\alpha}{2} - \lambda\right) & -\lambda \frac{l}{R}\cos\alpha/2 \\ -\lambda l/R \cos\alpha/2 & \frac{l^2}{R^2}\left(g/l - \lambda\right) \end{pmatrix}$$

$$= l^2 m^2 R^2 \left[4\left(\lambda - \frac{g}{R}\cos\frac{\alpha}{2}\right)\left(\lambda - g/l\right) - \lambda^2 \cos^2\frac{\alpha}{2} \right]$$

$$[\ldots] = \left(4 - \cos^2\frac{\alpha}{2}\right)\lambda^2 - 4g\left(\frac{1}{R}\cos\frac{\alpha}{2} + \frac{1}{l}\right)\lambda + 4\frac{g^2}{Rl}\cos\frac{\alpha}{2}$$

$$\frac{1}{R}\cos\frac{\alpha}{2} = 1/l \rightarrow \cos\alpha/2 = R/l \qquad \underline{R<l} \qquad \boxed{\cos\frac{\alpha}{2} = \frac{R}{l}}$$

$$\left(4 - R^2/l^2\right)\lambda^2 - 8\frac{g}{l}\lambda + 4\left(\frac{g}{l}\right)^2$$

$$4\left(\lambda - \frac{g}{l}\right)^2 - \left(\frac{R\lambda}{l}\right)^2 =$$

$$= \left(2\lambda - 2\frac{g}{l} - \frac{R\lambda}{l}\right)\left(2\lambda - 2\frac{g}{l} + \frac{R\lambda}{l}\right)$$

$$\lambda_1 = \frac{2g/l}{2 - R/l} \qquad \lambda_2 = \frac{2g/l}{2 + R/l}$$

Lagrangiana Linearizzata:

$$A = \begin{pmatrix} 4mR^2 & mR^2 \\ mR^2 & ml^2 \end{pmatrix} \qquad B = \begin{pmatrix} 4mgR^2/l & \\ & mgl \end{pmatrix}$$

$$L = \frac{1}{2}\delta\dot{\bar{q}} \cdot A\delta\dot{\bar{q}} - \frac{1}{2}\delta\bar{q} \cdot B\delta\bar{q} = \qquad \delta\varphi = \varphi + \alpha/2$$

$$\delta\theta = \theta - 0$$

$$= 2mR^2\,\delta\dot{\varphi}^2 + \frac{1}{2}ml^2\,\delta\dot{\theta}^2 + mR^2\,\delta\dot{\varphi}\,\delta\dot{\theta}$$

$$- 2mg\,R^2/l\,\delta\varphi^2 - \frac{1}{2}mgl\,\delta\theta^2$$

5) $B - \lambda_1 A \propto \begin{pmatrix} (2R/\ell)^2 & 2R/\ell \\ 2R/\ell & 1 \end{pmatrix} \rightarrow \vec{u}_1 = \begin{pmatrix} 1 \\ -2R/\ell \end{pmatrix}$

$B - \lambda_2 A \propto \begin{pmatrix} (2R/\ell)^2 & -2R/\ell \\ -2R/\ell & 1 \end{pmatrix} \rightarrow \vec{u}_2 = \begin{pmatrix} 1 \\ 2R/\ell \end{pmatrix}$

$$\begin{pmatrix} \varphi(t) \\ \theta(t) \end{pmatrix} = \begin{pmatrix} -\alpha/2 \\ 0 \end{pmatrix} + A_1 \vec{u}_1 \cos(\sqrt{\lambda_1}\, t + \phi_1) + A_2 \vec{u}_2 \cos(\sqrt{\lambda_2}\, t + \phi_2)$$

6)

Forze cost. $F\vec{i}_x$ su P_2

$\delta V = -F x_2 = -FR \operatorname{sen}(\varphi + \alpha)$

$$\left.\frac{\partial (V + \delta V)}{\partial \varphi}\right|_{\varphi=0} = R(-F\cos\alpha + 2mg \operatorname{sen}\alpha)$$

$= 0$ se $F = 2mg \operatorname{tg}\alpha$

ES 1)

6) $H = p_1 q_2 - p_2 q_1$ $\leftarrow$ componente del mom. angolare lungo asse z

genera rotazioni attorno tale asse

7) ES: $p_1^2 + p_2^2$, $q_1^2 + q_2^2$, $p_1 q_1 + p_2 q_2$

(cioè quantità invarianti sotto rotazioni)

8) $\{\tilde{p}\, e^{\alpha \tilde{q}} - \frac{\tilde{p}}{\alpha}, \alpha \tilde{q} + \ln \tilde{p}\} =$

$= \alpha \{\tilde{p}, \tilde{q}\} e^{\alpha \tilde{q}} - \{\tilde{p}, \tilde{q}\} =$

$+ \tilde{p}\, \{e^{\alpha \tilde{q}}, \ln \tilde{p}\}$

$= - \cancel{\alpha e^{\alpha \tilde{q}}} + 1 + \cancel{\tilde{p}} \left(\cancel{\alpha e^{\alpha \tilde{q}}} \frac{1}{\cancel{\tilde{p}}} - 0 \right) = 1$ //