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Corso di Termofluidodinamica Computazionale

Homework No. 3
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Problem 3

Problem description

Consider the two simple 2D grids constituted by triangular cells illustrated in figures 1 and 2. The first, *regular* one constituted by equilateral triangles, has been taken directly from Chapt. 9 in [1]. The second, *irregular* one, is a *deformed* version of the former. The coordinates of the mesh nodes (vertices) are provided in the associated files `grid1.nod` and `grid2.nod`, and are also given, for convenience, in tables 1 and 2, respectively.

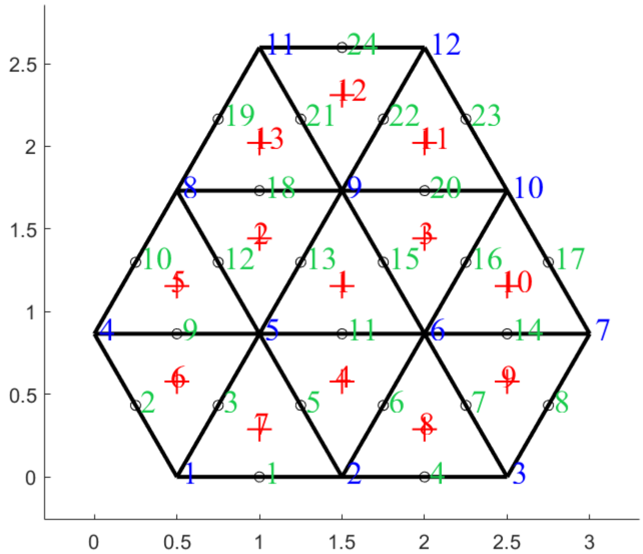

Figure 1: Grid 1 for problem 3.

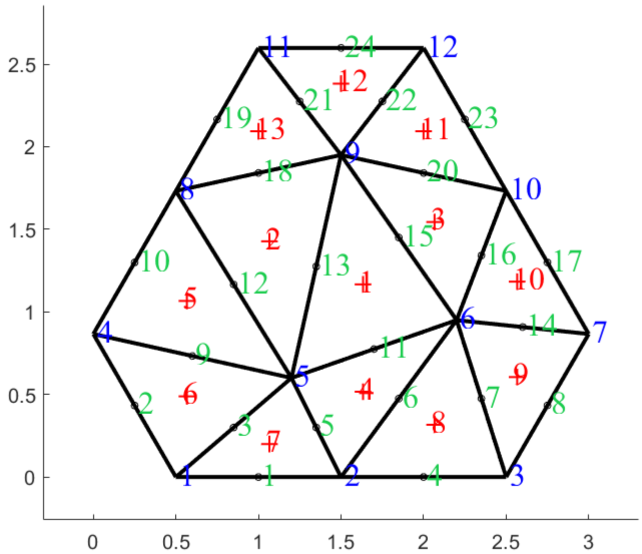

Figure 2: Grid 2 for problem 3.

Node	x	y
1	0.5	0.0
2	1.5	0.0
3	2.5	0.0
4	0.0	0.8660254
5	1.0	0.8660254
6	2.0	0.8660254
7	3.0	0.8660254
8	0.5	1.7320508
9	1.5	1.7320508
10	2.5	1.7320508
11	1.0	2.5980762
12	2.0	2.5980762

Table 1: Coordinates of the nodes (vertices) of the grid of figure 1.

Node	x	y
1	0.5	0.0
2	1.5	0.0
3	2.5	0.0
4	0.0	0.8660254
5	1.2	0.6000000
6	2.2	0.9500000
7	3.0	0.8660254
8	0.5	1.7320508
9	1.5	1.9500000
10	2.5	1.7320508
11	1.0	2.5980762
12	2.0	2.5980762

Table 2: Coordinates of the nodes (vertices) of the grid of figure 2.

The cell definitions are stored in the cell connectivity matrix which, in this problem, is the same for both grids. This is a matrix of node (vertex) numbers where each row of the matrix contains the connectivity - the nodes - of the corresponding cell. It is given in the file `grid.ele` and also in table 3.

Question

Consider the three following cases of temperature distribution, and corresponding (exact) analytical expression of the gradient:

Linear

$$T(x, y) = 100x + 30y + 5$$
$$\nabla T(x, y) = 100\,\mathbf{i} + 30\,\mathbf{j}$$

Cell	Nodes		
1	5	6	9
2	5	9	8
3	6	10	9
4	2	6	5
5	4	5	8
6	1	5	4
7	1	2	5
8	2	3	6
9	3	7	6
10	6	7	10
11	9	10	12
12	9	12	11
13	8	9	11

Table 3: Cell connectivity for the grids of figures 1 and 2.

Quadratic

$$\begin{aligned} T(x,y) &= 100\left(x^2+y^2+1\right) \\ \nabla T(x,y) &= 200x\,\mathbf{i}+200y\,\mathbf{j} \end{aligned}$$

Cubic

$$\begin{aligned} T(x,y) &= 100\left(x^3+y^2+xy+5\right) \\ \nabla T(x,y) &= 100\left(3x^2+y\right)\mathbf{i}+100\left(2y+x\right)\mathbf{j} \end{aligned}$$

Compute numerically, for the three temperature distributions, the temperature gradient at the centroid of cell *1* for both grids, using the cell-based Gauss-Green method with *and* without interpolation[1] and compare the results with the exact gradient.

For the calculations develop a code/script (MATLAB, OCTAVE, Scilab or Python recommended) accordingly to the requirements described in problem 4 of Homework 2015-16.

[1]For grid 1, since the cell faces are midway between the centroids of the two attached cells, using weighted interpolation has no effect.

References

[1] F. Moukalled, L. Mangani, M. Darwish, *The Finite Volume Method in Computational Fluid Dynamics: An Advanced Introduction With OpenFOAM and Matlab*, Springer-Verlag, 2015.