Enrico Nobile
Dipartimento di Ingegneria e Architettura
Università degli Studi di Trieste

Corso di Termofluidodinamica Computazionale

Homework No. 1
AA 2019/2020

Marzo 2020

Proposed problem

A plane fin of uniform cross-section, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 50$ W/(m K); it has a thickness t, a length L and its width w is large in comparison with its thickness. In this case the fin is cooled by *both* convection and radiation:

- The convective heat transfer coefficient is h =50 W/(m^2 K) and the surrounding fluid temperature is T_∞.
- The radiation heat transfer can be accounted for assuming that the fin surface is gray-diffuse, with a global emissivity ϵ = 0.7, that the fluid is not participating nor emitting, and that the temperature of the surroundings is T_s.

The base of the fin is maintained at a temperature $T_b = 400$ °C, while the tip of the fin can be assumed perfectly insulated.

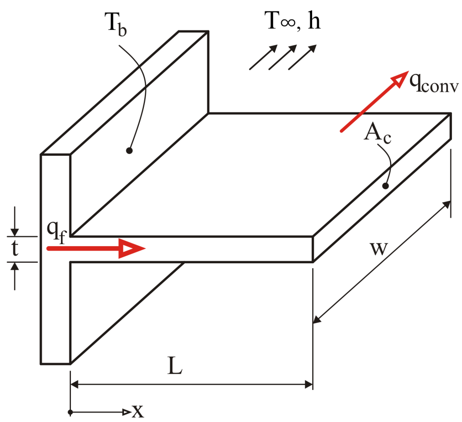

Figure 1: Straight fin with uniform cross-section.

Assuming a *1D* temperature distribution, i.e. $T \approx T(x)$, compute, with the Finite Volume method (FV), the heat flux per unit width of the fin q'_{num} [W/m], using a number N of FVs equal to $N = 10$, 20, 40, and 80.

Consider the three following cases:

1. In this first case the fin has a length $L = 20$ mm and a thickness $t = 1$ mm, the fluid temperature and the surroundings temperature are the same, i.e. $T_\infty \equiv T_s$ = -273.15 °C = 0 K. In this way, it is possible to verify the result by comparison with the simplified analytical expression given in [1], not reported here for brevity, which gives q'_{appr} = 1186.0 [W/m].
2. For the second case, repeat the calculation but assume T_∞ = 25 °C and T_s = 15 °C.
3. For the third case, consider a fin with a different geometry (lower aspect-ratio): $L = 20$ mm and thickness $t = 10$ mm. All other data are the same as in the second case.

TIP # 1

Due to radiation heat transfer the problem is *non-linear*, and therefore it requires an iterative approach with a proper linearization. In this case it can be done by observing that the heat transfer by radiation can be expressed in a form similar to that of convection heat transfer:

$$\begin{aligned} q'_{rad} &= \epsilon\sigma A_f \left(T_f^4 - T_s^4\right) \\ &= A_f h_r \left(T_f - T_s\right) \end{aligned}$$

where T_f is the (local) temperature of the fin, σ = 5.670367×10^{-8} W/(m^2 K^4) is the Stefan-Boltzmann constant and

$$h_r = \epsilon\sigma \left(T_f^2 + T_s^2\right)\left(T_f + T_s\right)$$

is the *radiation heat transfer coefficient*. Since h_r *depends* on T_f, an iterative procedure, with a check of convergence, should be applied.
In this way, the total heat transfer can be expressed as

$$q'_{tot} = q'_{conv} + q'_{rad} = A_f \left[h\left(T_f - T_\infty\right) + h_r\left(T_f - T_s\right)\right]$$

Needless to say, proper attention must be paid, in this case, to the unit of measure of temperature.

TIP # 2

In order to verify, at least partially, the correctness of the script, a possible solution is to switch off the radiation heat transfer contribution, and compare the numerical result with the analytical solution [2] valid for longitudinal fins of uniform cross-section with insulated tip:

$$q'_f = \sqrt{2hkt}\; tanh(mL)\left(T_b - T_\infty\right)$$

with

$$m = \sqrt{2h/kt}$$

References

[1] Y. Huang, X.-F. Li, Exact and approximate solutions of convective-radiative fins with temperature-dependent thermal conductivity using integral equation method, *Int. J. Heat Mass Transfer*, **150**, 119303, 2020.

[2] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).