VC DIMENSION and RADEMACHER COMPLEXITY

$\mathcal{H}$ IS PAC LEARNABLE iff $\forall \varepsilon, \delta \in (0,1)$, $\forall p(x,y)$
$\exists m_{\varepsilon,\delta} \in \mathbb{N}$ s.t. $\forall m \geq m_{\varepsilon,\delta}$, $\forall D \sim p^m$, $|D| = m$,

$$P_D\left(R(h_D^*) \leq \min_{h' \in \mathcal{H}} R(h') + \varepsilon \right) \geq 1 - \delta$$

VC-dimension.

$\mathcal{H} = \{ h : \mathcal{X} \to \{0,1\} \}$, $C \subseteq \mathcal{X}$, $C = \{c_1, \dots, c_m\}$.

$\mathcal{H}_C = \{ (h(c_1), \dots, h(c_m)) \mid h \in \mathcal{H} \}$. $\mathcal{H}$ SHATTERS C iff $|\mathcal{H}_C| = 2^m$

$\mathrm{VCdim}(\mathcal{H}) = \max \{ m \mid \exists C \subseteq \mathcal{X}, |C| = m \text{ and } \mathcal{H} \text{ SHATTERS } C \}$

$$C_1 \frac{\mathrm{VCdim}(H) + \log\left(\frac{1}{\delta}\right)}{\varepsilon^2} \leq m_{\varepsilon,\delta} \leq C_2 \frac{\mathrm{VCdim}(\mathcal{H}) + \log\left(\frac{1}{\delta}\right)}{\varepsilon^2}$$

$p(x,y)$, $D \sim p^m$ $\quad \mathcal{H} = \{ h : \mathcal{X} \to \{-1, 1\} \}$

RADEMACHER DISTRIBUTION: $\sigma = (\sigma_1, \dots, \sigma_m)$ $\sigma_i \in \{-1, 1\}$ $p(\sigma_i = +1) = \frac{1}{2}$

$$\hat{R}_D(\mathcal{H}) = \mathbb{E}_{\sigma}\left[\sup_{h \in \mathcal{H}} \underbrace{\frac{1}{m} \sum_{i=1}^{m} \underbrace{\sigma_i \cdot h(x_i)}_{\pm 1}}_{\in [-1, 1]} \right]$$

data dependent

$$R_m(\mathcal{H}) = \mathbb{E}_{D \sim p^m}\left[\hat{R}_D(\mathcal{H}) \right]$$

$$P_D\left(R(h) \leq \hat{R}_D(h) + \overbrace{\hat{R}_D(\mathcal{H}) + 3\sqrt{\frac{\log(2/\delta)}{2m}}}^{\varepsilon} \right) \geq 1 - \delta$$