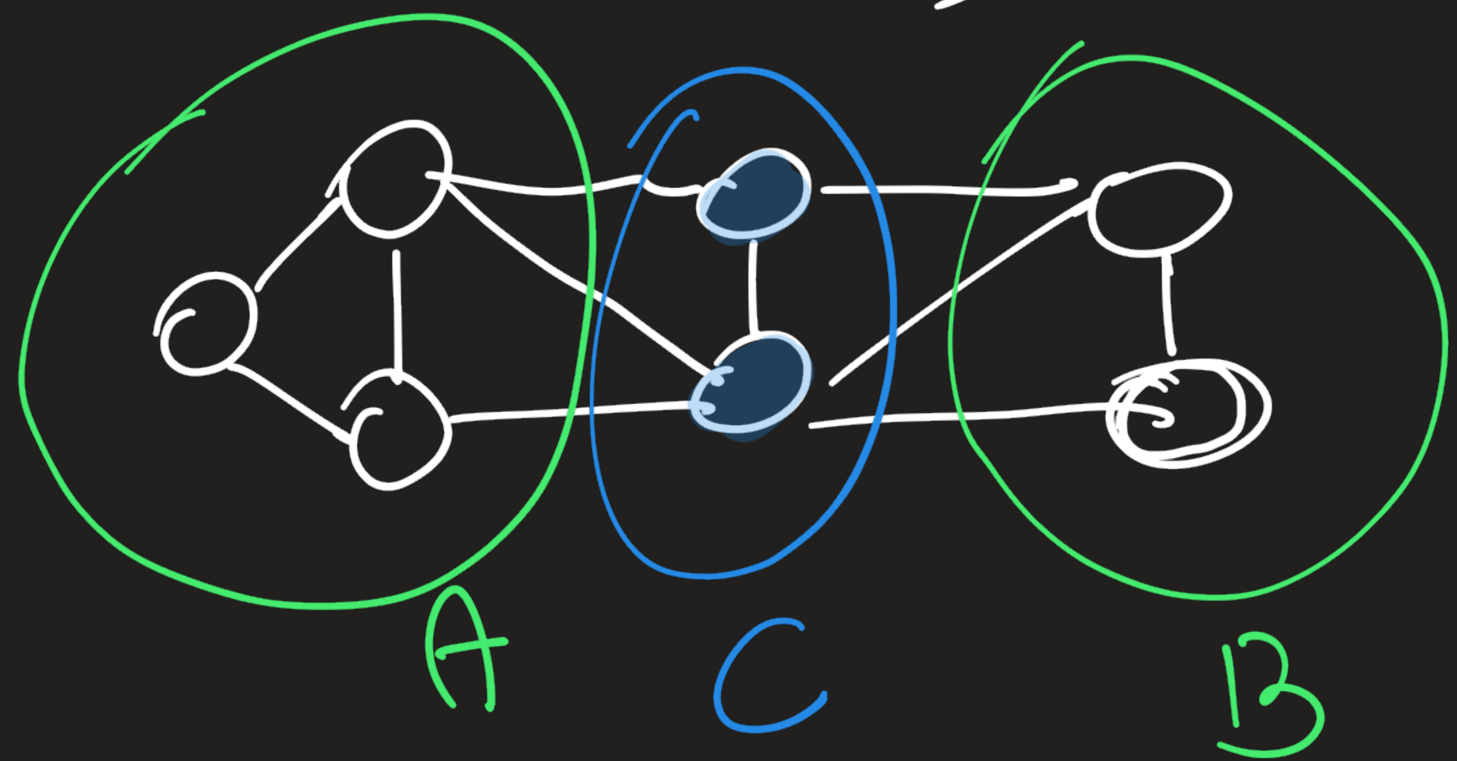
MARKOV RANDOM FIELDS

UNDIRECTED GRAPHS



nodes $\equiv$ variables

edges $\equiv$ dependency

$A \perp\!\!\!\perp B \mid C$ $\Leftrightarrow$ all paths from a node $a \in A$ to any node $b \in B$ pass from C. (or blocked by a node in C)

MARKOV BLANKET of x_i $\equiv$ set of neighbours of x_i

$x_i \perp\!\!\!\perp x_j \mid x_{\setminus\{i,j\}}$?

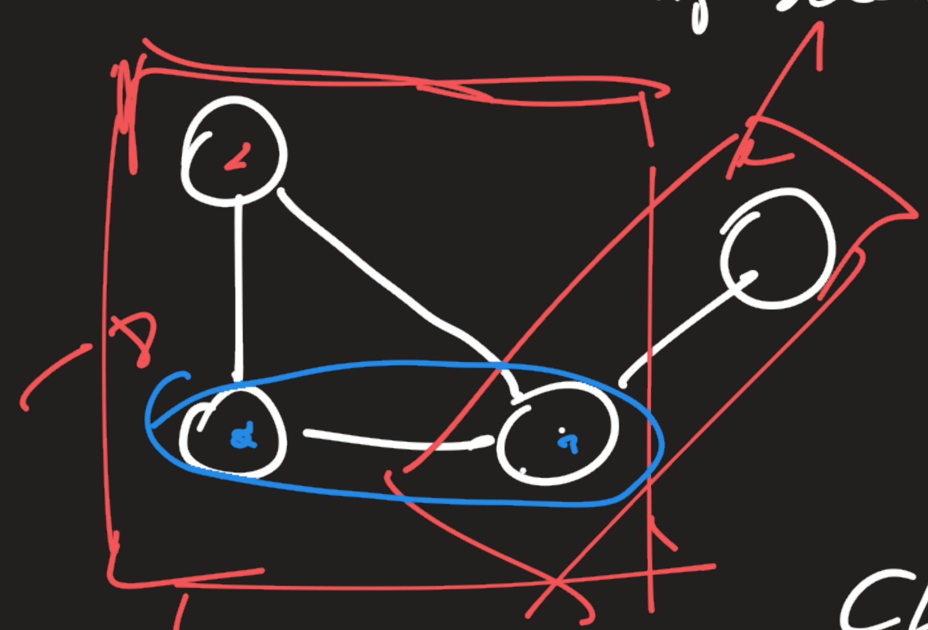
no path from x_i to x_j passing from any other node. $x_i \perp\!\!\!\perp x_j \mid x_{\setminus\{i,j\}}$ iff there is no edge between them.

$\Downarrow$

$$p(x_i, x_j \mid x_{\setminus\{i,j\}}) = p(x_i \mid x_{\setminus\{i,j\}})\, p(x_j \mid x_{\setminus\{i,j\}})$$

$\Downarrow$

x_i, x_j belong to different factors!



FACTORS $\Longleftrightarrow$ MAXIMAL CLIQUES in the GRAPH

CLIQUE : subgraph FULLY CONNECTED

MAXIMAL CLIQUES : cliques that cannot be extended

$\mathcal{C} = \{$ set of maximal cliques $\}$ $\quad x_C \equiv \{x \mid x \in C \in \mathcal{C}\}$

$x = x_1, \dots, x_n$

$$p(x) = \frac{1}{Z} \prod_{c \in \mathcal{C}} \psi_c(x_c)$$

$\underbrace{\psi_c(x_c) \geq 0}_{\text{POTENTIAL FUNCTION}}$ and have a finite integral $\int \psi_c(x_c)\,dx_c < \infty$

$$Z = \int \prod_c \psi_c(x_c)\,dx$$ PARTITION FUNCTION

Z is hard to compute. for $x_i \in \{1 \to K\}$

$$Z = \sum_x \prod_c \psi_c(x_c)$$

on K^n states

- IF we want to compute conditionals, then Z cancels out
- IF we want to compute marginal on few variables, we can work with potentials and normalize at the end.

$$\psi_c(x_c) = \exp(-\underbrace{E(x_c)}_{\text{energy}})$$

low energy = high potential

BOLTZMANN DISTRIBUTION

$$\psi_{c_1}(x_{c_1})\,\psi_{c_2}(x_{c_2}) = \exp(-E_1(x_{c_1}))\exp(-E_2(x_{c_2}))$$

$$= \exp(-E_1(x_{c_1}) - E_2(x_{c_2}))$$