Trova $\langle r \rangle$ per elettrone in stato ψ_{100}

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a}}$$, ma conviene qui vederlo come

$$\psi_{100} = R_{10} Y_\ell^m(\theta,\varphi) = \frac{2}{\sqrt{a_0^3}} e^{-\frac{r}{a_0}} \cdot \frac{1}{\sqrt{4\pi}}$$

Ora,

$$\langle r \rangle = \langle \psi_{100} | r | \psi_{100} \rangle$$

$$= \int d\vec{r} \, |\psi_{100}|^2 r$$

reale

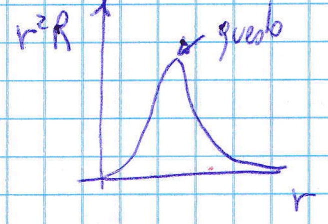
$$= \underbrace{\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta \, |Y_\ell^m(\theta,\varphi)|^2}_{=1} \int_0^{\infty} r^3 |R_{10}|^2 dr$$

$$= \frac{4}{a_0^3} \int_0^{\infty} r^3 e^{-\frac{2r}{a_0}}$$

$$\int_0^{\infty} x^n e^{-\frac{x}{a}} dx = n! \, a^{n+1}$$

$$= \frac{4}{a_0^3} 3! \left(\frac{a_0}{2}\right)^4 = \frac{3}{2} a_0$$

!! invece, qual è il valore di r più probabile?



$\Rightarrow$ $r^2 R$ massimo x $\frac{d}{dr} r^2 R^2 = 0$

$$\propto \frac{d}{dr}\left(r^2 e^{-\frac{2r}{a}}\right) = e^{-\frac{2r}{a}}\left(2r - \frac{2r^2}{a_0}\right)$$

$= 0$ x $\boxed{r = a_0}$ raggio di Bohr...