PSYCHROMETRICS

HVAC SYSTEM DESIGN

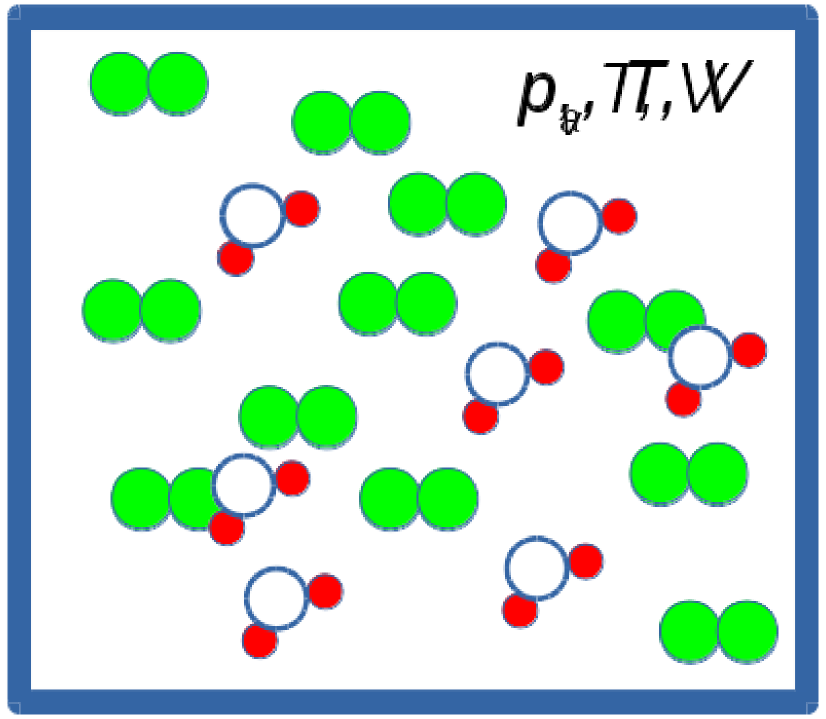
Engineering for the Energy Transition

March 2025

Moist Air

- Moist Air: The term moist air is a mixture of dry air and water vapor in which the dry air is treated as a pure substance.
- Moist air as a perfect gas: $T_{air} >> T_{critical}$ e $p << p_{critical}$
- Mixture of perfect gases: thermodynamic system consisting of a homogeneous mixture of *n* non-reacting components in the gaseous state, each of which behaves as a perfect gas.
- At equilibrium, the state of the system is determined by the value of two intensive/specific properties and the composition of the system.
- Specific quantities referred to the mass of dry air

Moist Air- Dalton



- Partial pressure of a gaseous component *i* of a mixture: pressure to which the *i-th* component would be subjected if it alone occupied the entire volume *V* occupied by the mixture at the same temperature *T*.

$$p = \frac{n \cdot \bar{R} \cdot T}{V}$$

$$p_{da} = \frac{n_{da} \cdot \bar{R} \cdot T}{V} = \frac{m_{da} \cdot R_{da} \cdot T}{V}$$

$$p_{vap} = \frac{n_{vap} \cdot \bar{R} \cdot T}{V} = \frac{m_{vap} \cdot R_{vap} \cdot T}{V}$$

$$p = p_{da} + p_{vap}$$

Moist Air– mole fraction

Mole fraction: ratio of the number of moles of a component in the mixture to the total number of moles.

$$p_{da} = \frac{n_{da} \cdot \bar{R} \cdot T}{V} = \frac{m_{da} \cdot R_{da} \cdot T}{V}$$

$$p_{vap} = \frac{n_{vap} \cdot \bar{R} \cdot T}{V} = \frac{m_{vap} \cdot R_{vap} \cdot T}{V}$$

$$p = \frac{n \cdot \bar{R} \cdot T}{V} \qquad n = n_{da} + n_{vap} \qquad m = m_{da} + m_{vap}$$

Mole fraction of dry air

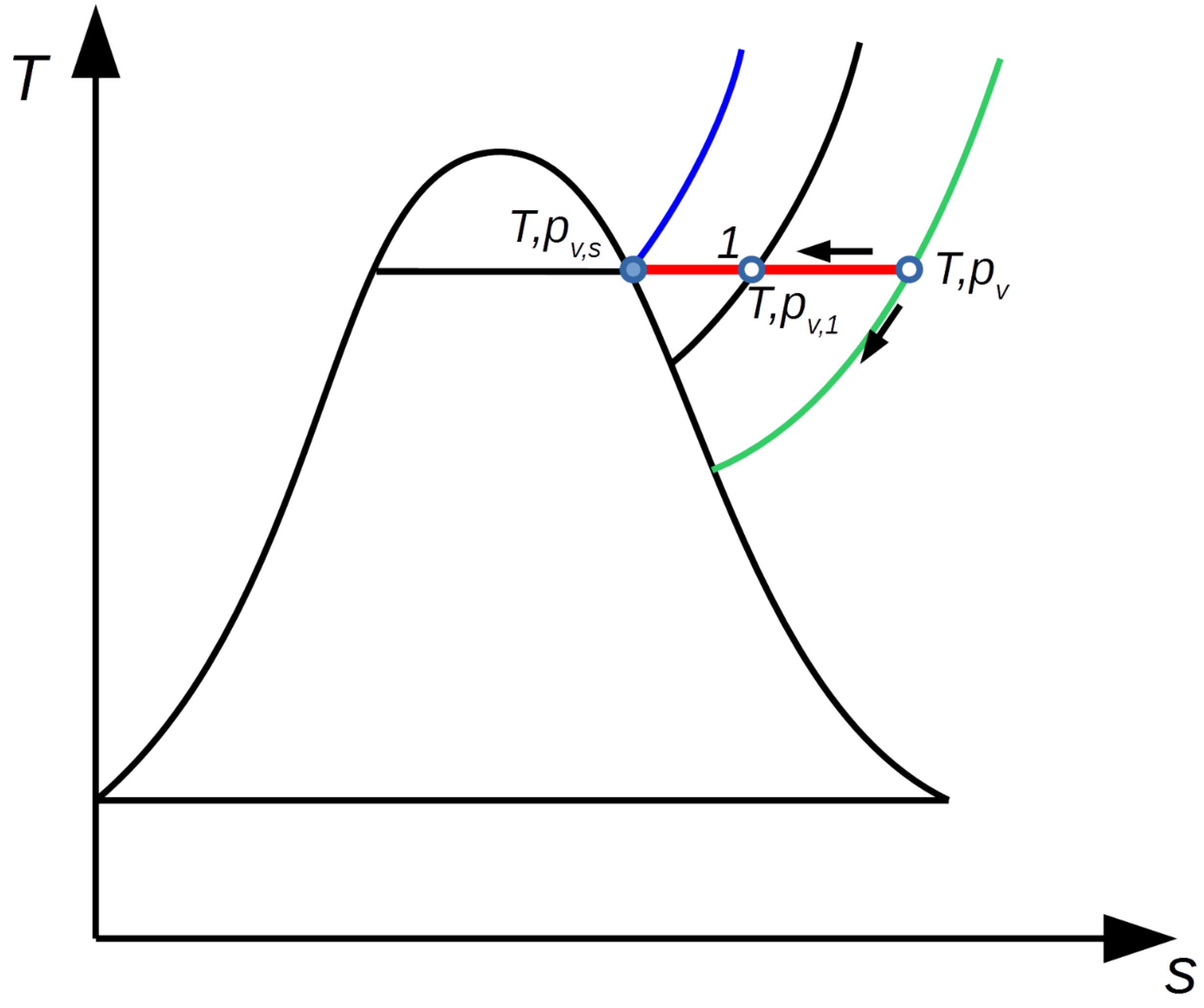
$$\frac{p_{da}}{p} = \frac{n_{da} \cdot \bar{R} \cdot T/V}{n \cdot \bar{R} \cdot T/V} = \frac{n_{da}}{n} = X_{da}$$

Mole fraction of water vapor

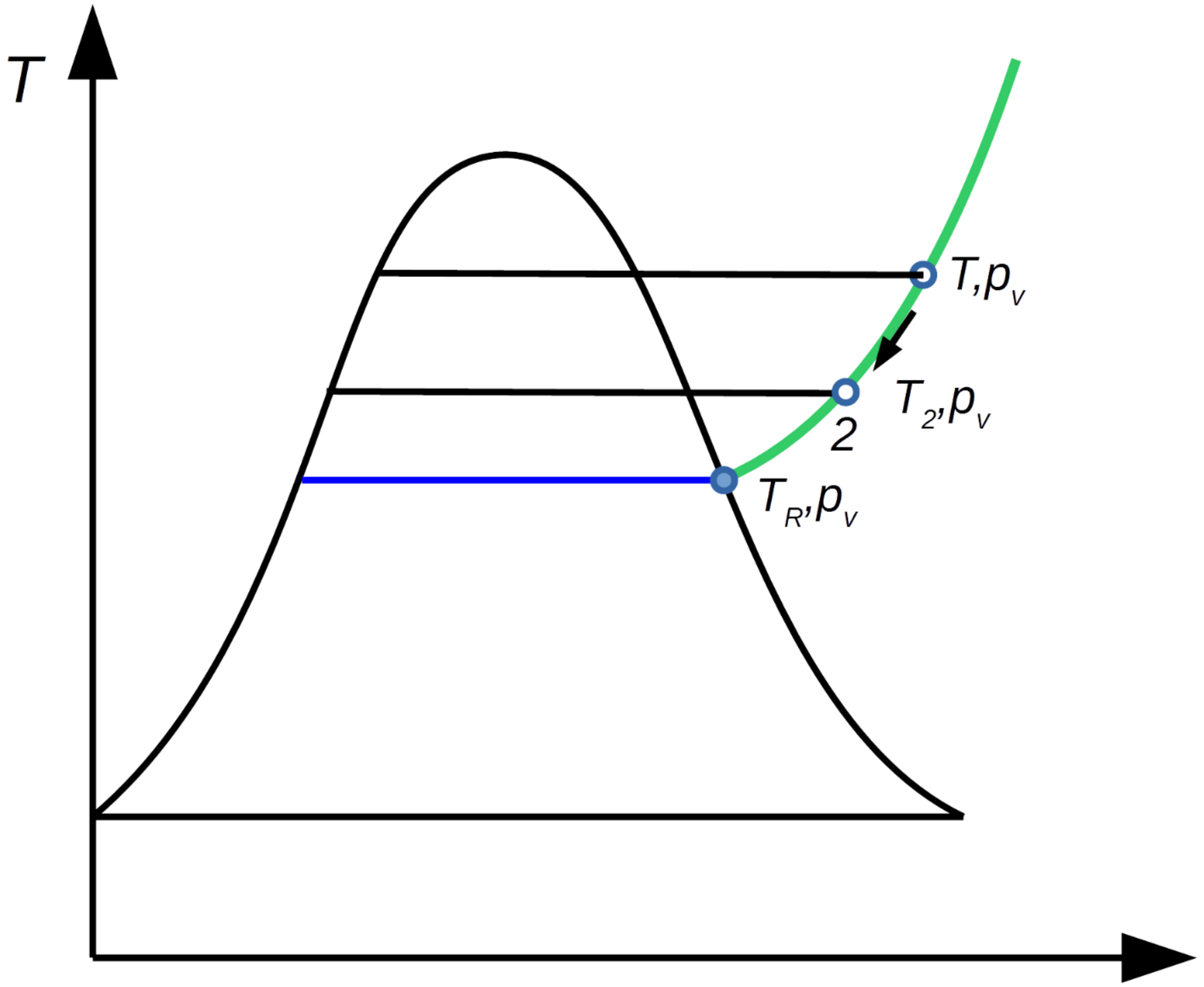
$$\frac{p_{vap}}{p} = \frac{n_{vap} \cdot \bar{R} \cdot T/V}{n \cdot \bar{R} \cdot T/V} = \frac{n_{vap}}{n} = X_{vap}$$

Vapor Condensation

Increase of vapor mole



Air cooling



Humidity Ratio

$$w = \frac{m_{vap}}{m_{da}} = \frac{M_{vap} \cdot p_{vap} \cdot V/(\bar{R} \cdot T)}{M_{da} \cdot p_{da} \cdot V/(\bar{R} \cdot T)} = \frac{M_{vap} \cdot p_{vap}}{M_{da} \cdot p_{da}}$$

$$w = 0.622 \cdot \frac{p_{vap}}{p - p_{vap}}$$

$$m_{vap} = M_{vap} \cdot n_{vap}$$

$$p_{vap} \cdot V = n_{vap} \cdot \bar{R} \cdot T \Rightarrow n_{vap} = \frac{p_{vap} \cdot V}{\bar{R} \cdot T}$$

$$m_{da} = M_{da} \cdot n_{da}$$

$$p_{da} \cdot V = n_{da} \cdot \bar{R} \cdot T \Rightarrow n_{da} = \frac{p_{da} \cdot V}{\bar{R} \cdot T}$$

Relative humidity

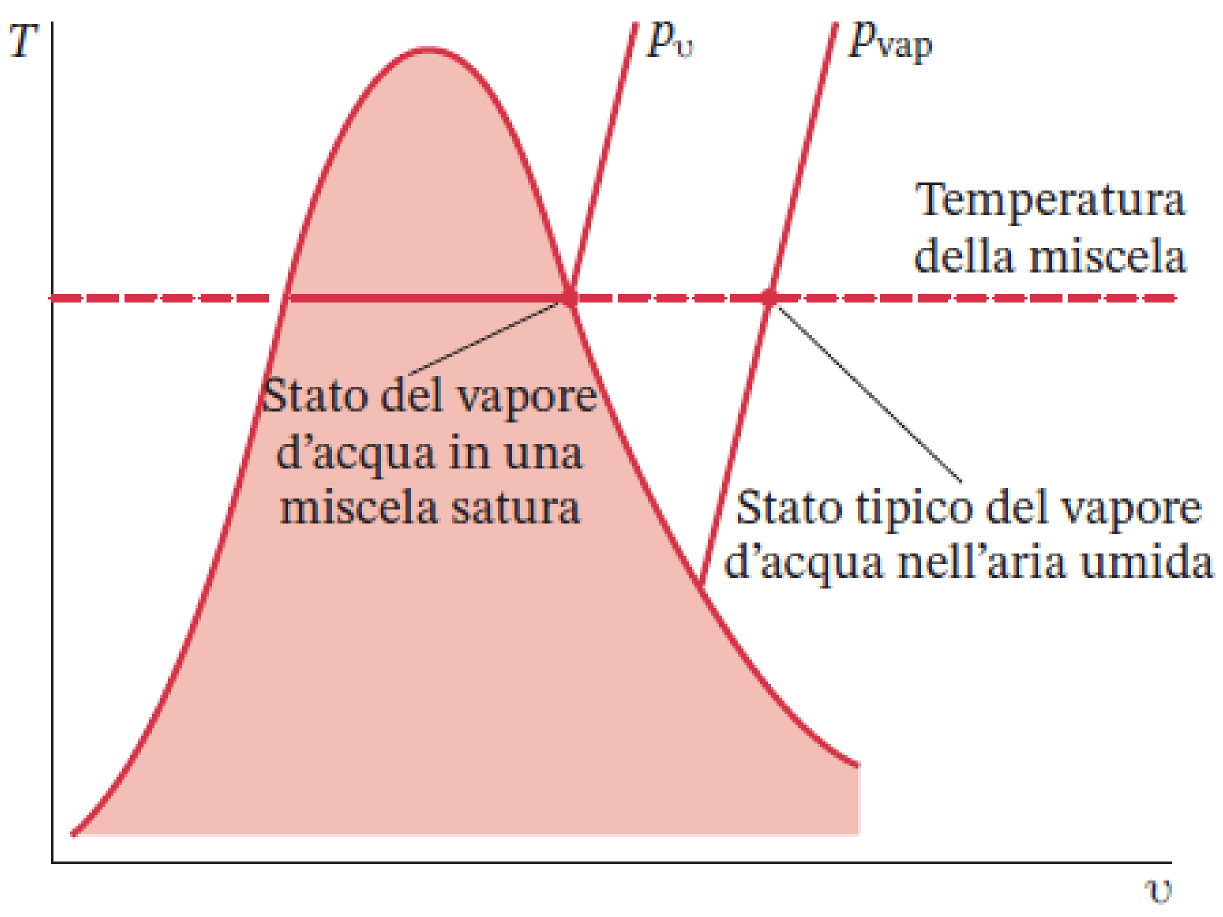
$$\phi = \frac{m_{vap}}{m_{vs}} = \frac{M_{vap} \cdot p_{vap} \cdot V/(\bar{R} \cdot T)}{M_{vap} \cdot p_{vs} \cdot V/(\bar{R} \cdot T)} = \frac{p_{vap}}{p_v}$$

$$m_{vap} = M_{vap} \cdot n_{vap}$$

$$p_{vap} \cdot V = n_{vap} \cdot \bar{R} \cdot T \Rightarrow n_{vap} = \frac{p_{vap} \cdot V}{\bar{R} \cdot T}$$

$$m_{vs} = M_{vap} \cdot n_{vs}$$

$$p_{vs} \cdot V = n_{vs} \cdot \bar{R} \cdot T \Rightarrow n_{vs} = \frac{p_{vs} \cdot V}{\bar{R} \cdot T}$$



$$\phi = \left.\frac{p_{vap}}{p_{vs}(T)}\right|_T$$

$$w = 0{,}622 \cdot \frac{\phi \cdot p_{vs}(T)}{p - \phi \cdot p_{vs}(T)}$$

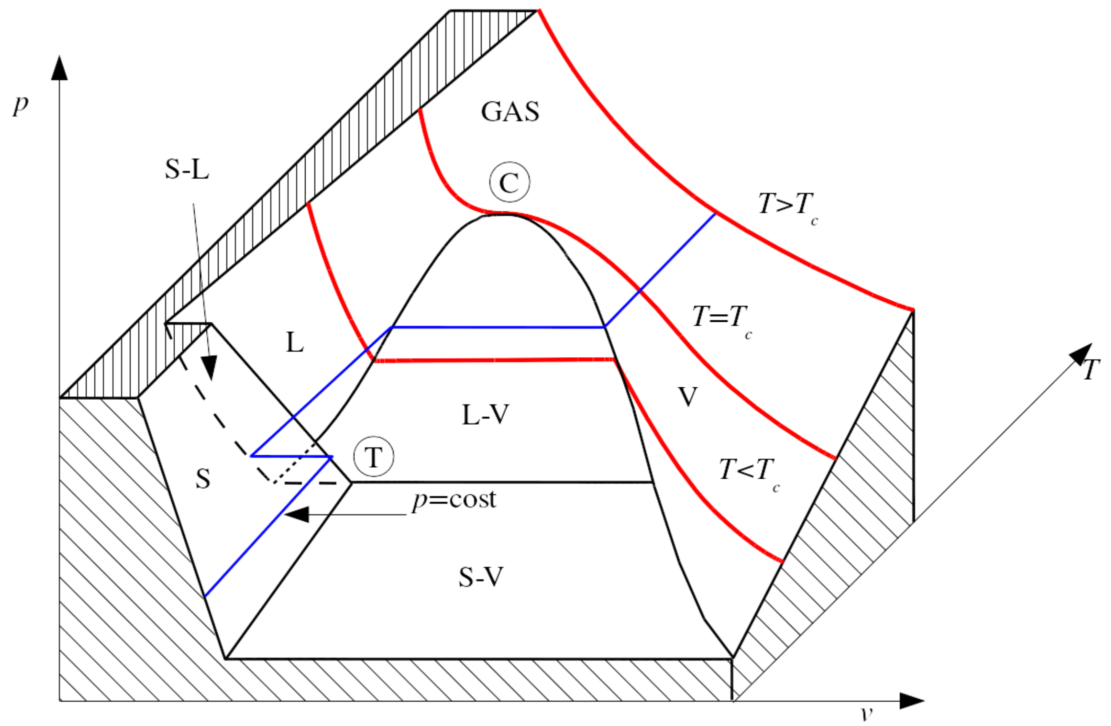
Saturation pressure

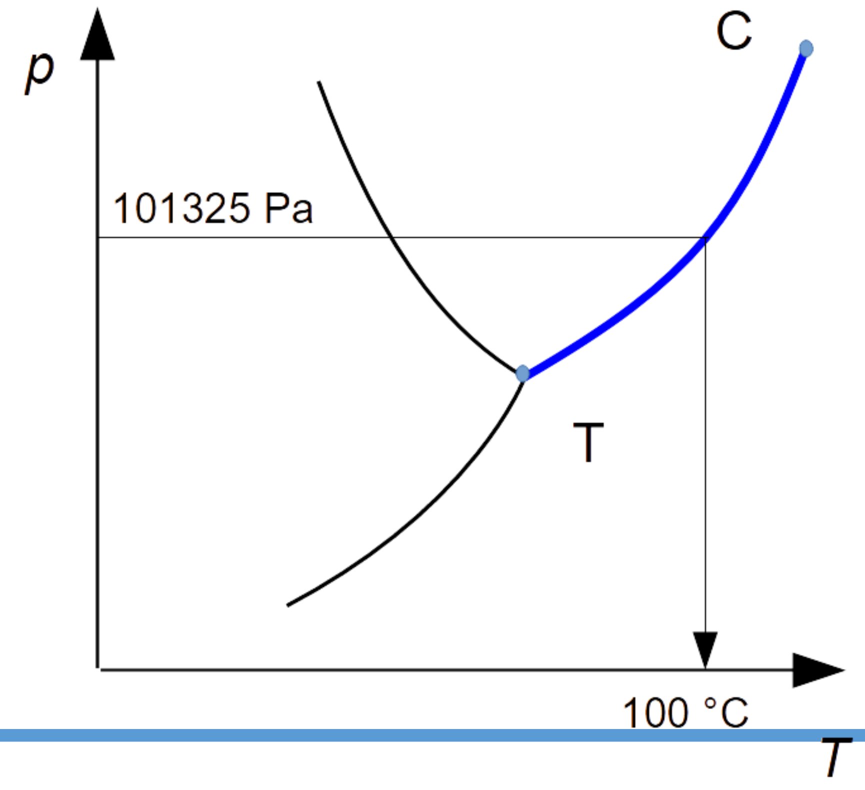
- Saturation pressure depends only on temperature
- In Italian standards the saturation pressure can be computed with the following expressions

$$p_v(t) = 610{,}5 \cdot e^{\frac{17{,}269 \cdot t}{237{,}3+t}} \; per \; t \geq 0\,°\mathrm{C}$$

$$p_v(t) = 610{,}5 \cdot e^{\frac{21{,}875 \cdot t}{265.5+t}} \; per \; t < 0\,°\mathrm{C}$$

- t [°C], p_v [Pa]





Enthalpy of moist air

Enthalpy of mixture (H)

$$H = H_{da} + H_{vap} = m_{da} \cdot h_{da} + m_{vap} \cdot h_{vap}$$

Specific enthalpy (*h*)

$$h = \frac{H}{m_{da}} = h_{da} + \frac{m_{vap}}{m_{da}} \cdot h_{vap} = h_{da} + w \cdot h_{vap} \left(\frac{\mathrm{kJ}}{\mathrm{kg_{da}}}\right)$$

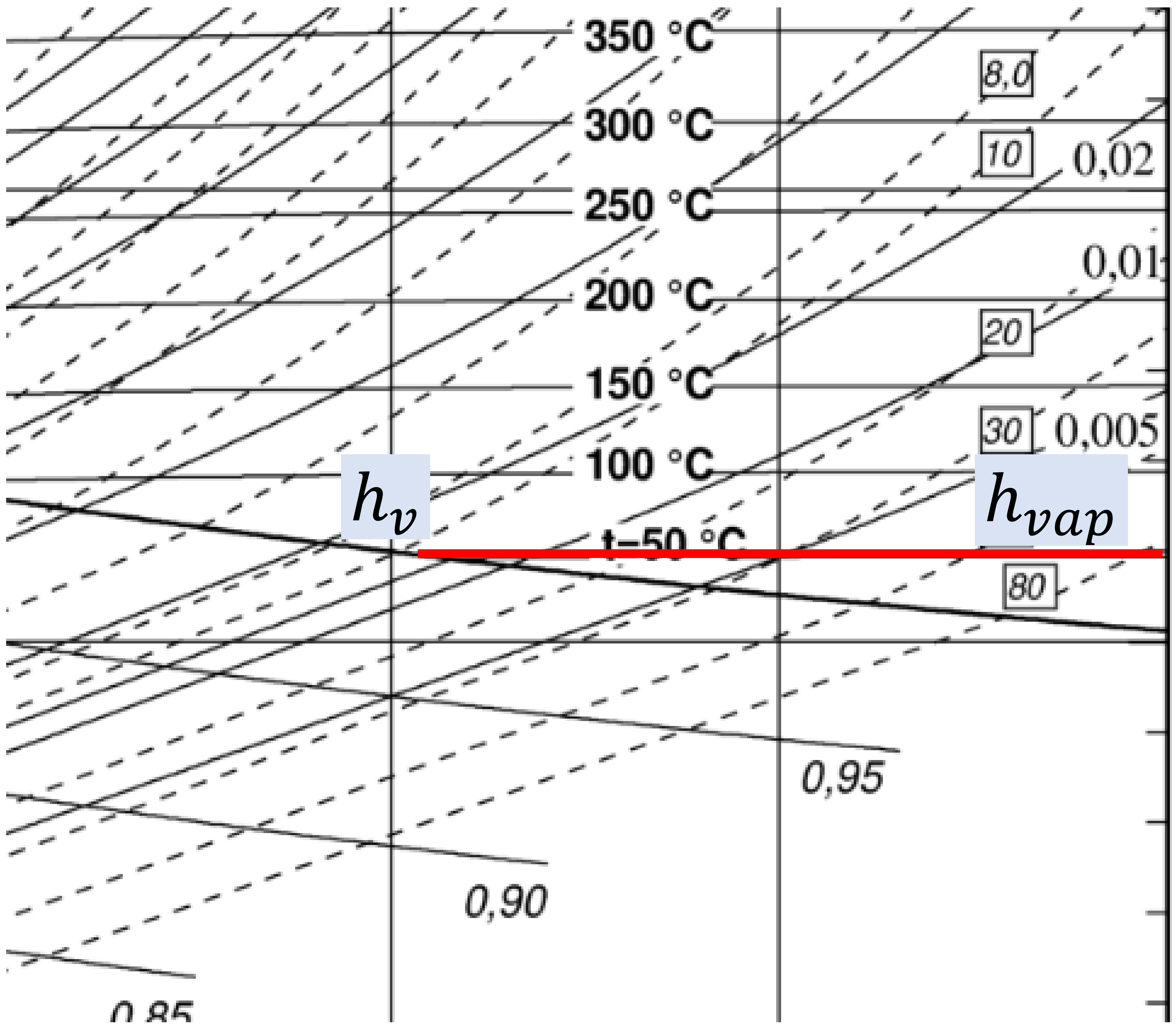
$$h_{da} = c_{p,da}(T - 273.15) = c_{p,da} \cdot \theta$$

Conditions:

- Superheated vapor
- Low pressures (<10kPa)
- Specific values referred to dry air

Enthalpy of vapor

- Low partial pressure of the vapor
- The behavior is of an ideal gas
- The enthalpy is a function of the temperature only $h_{vap} = h_v$
- Enthalpy of the vapor h_{vap}
- Enthalpy of the saturated vapor at the same temperature h_v



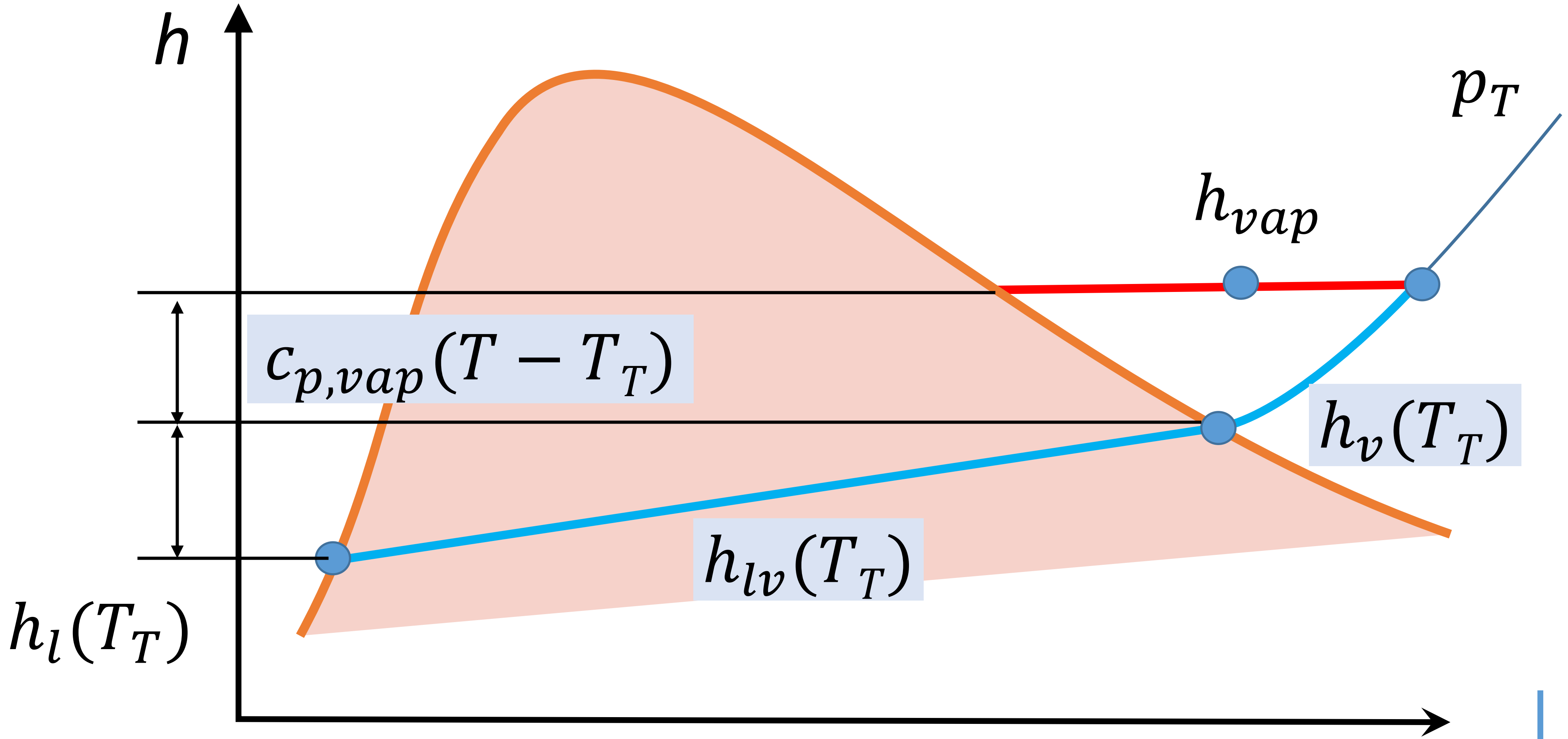
Vapor enthalpy

- The behavior is of an ideal gas
- The enthalpy is a function of temperature
- I construct a transformation from a reference state to a state at the same temperature
- $h_l(0°C) = 0$ enthalpy on the lower limit curve
- $h_{lv}(0°C)$ latent heat of vaporization
- $c_{p,vap}$ specific heat of vapor

$$h_{lv} = 2501{,}3\ kJ/kg$$

$$c_{p,vap} = 1{,}875\ kJ/(kg\ K)$$

$$h_{lv} = h_v - h_l$$



$$h_{vap} = h_l(T_T) + h_{lv}(T_T) + c_{p,vap}(T - T_T) \quad h_l(T_T) = 0 \quad \Rightarrow \quad h_{vap} = h_{lv} + c_{p,vap} \cdot t$$

Specific enthalpy of moist air

$$h = \frac{H}{m_{as}} = h_{da} + \frac{m_{vap}}{m_{da}} \cdot h_{vap} = h_{da} + w \cdot h_{vap} \left(\frac{\mathrm{kJ}}{\mathrm{kg_{as}}}\right)$$

$$h = c_{p,as} \cdot \theta + w \cdot (c_{p,vap} \cdot \theta + h_{lv})$$

$$h = 1{,}005 \cdot t + w \cdot (1{,}875 \cdot t + 2501{,}3)\ \frac{kJ}{kg}$$

Condizioni:

- Superheated vapor
- Low vapor pressure(<10kPa)
- Specific quantities referred to dry air

Dew point temperature

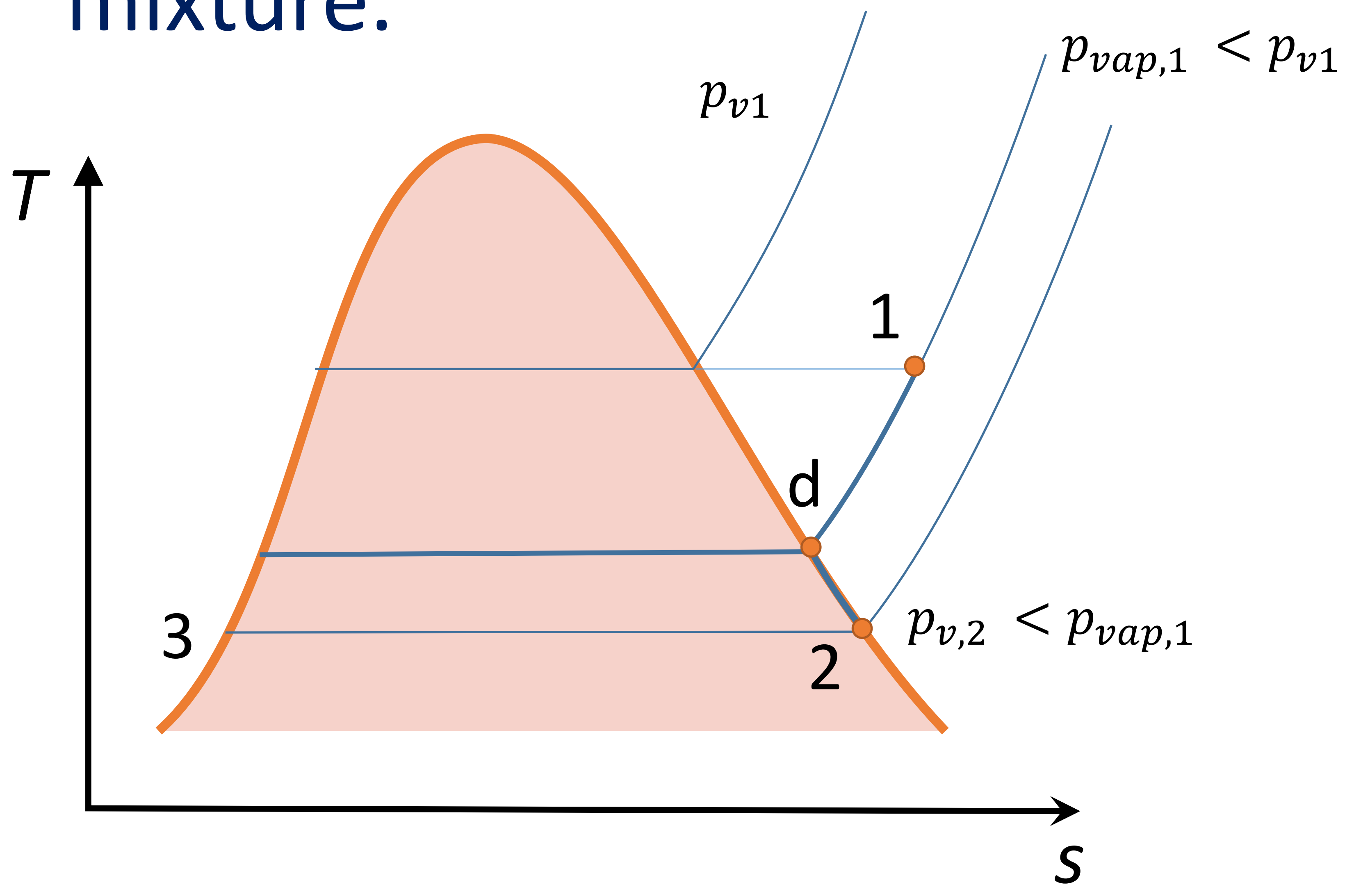
- Dew point temperature is the temperature of saturated moist air of the same moist air sample, having the same humidity ratio, and at the same atmospheric pressure of the mixture.
- Two moist air samples at the same T_{dew} will have the same humidity ratio *w* and the same partial pressure of water vapor p_w.

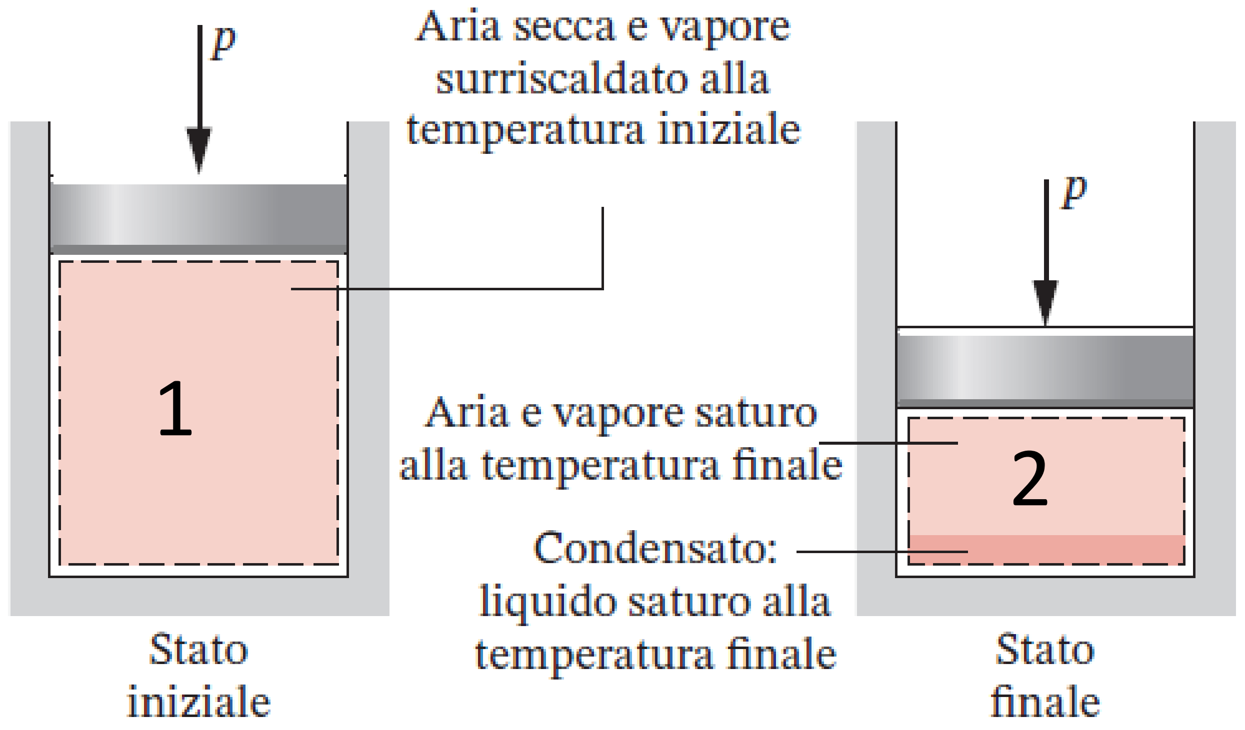
$$w_s(p_{at}, T_{dew}) = w$$

- where w_s humidity ratio of saturated moist air. At a specific atmospheric pressure,
- the dew-point temperature determines the humidity ratio *w* and the water vapor pressure p_w of the moist air.

Dew point temperature

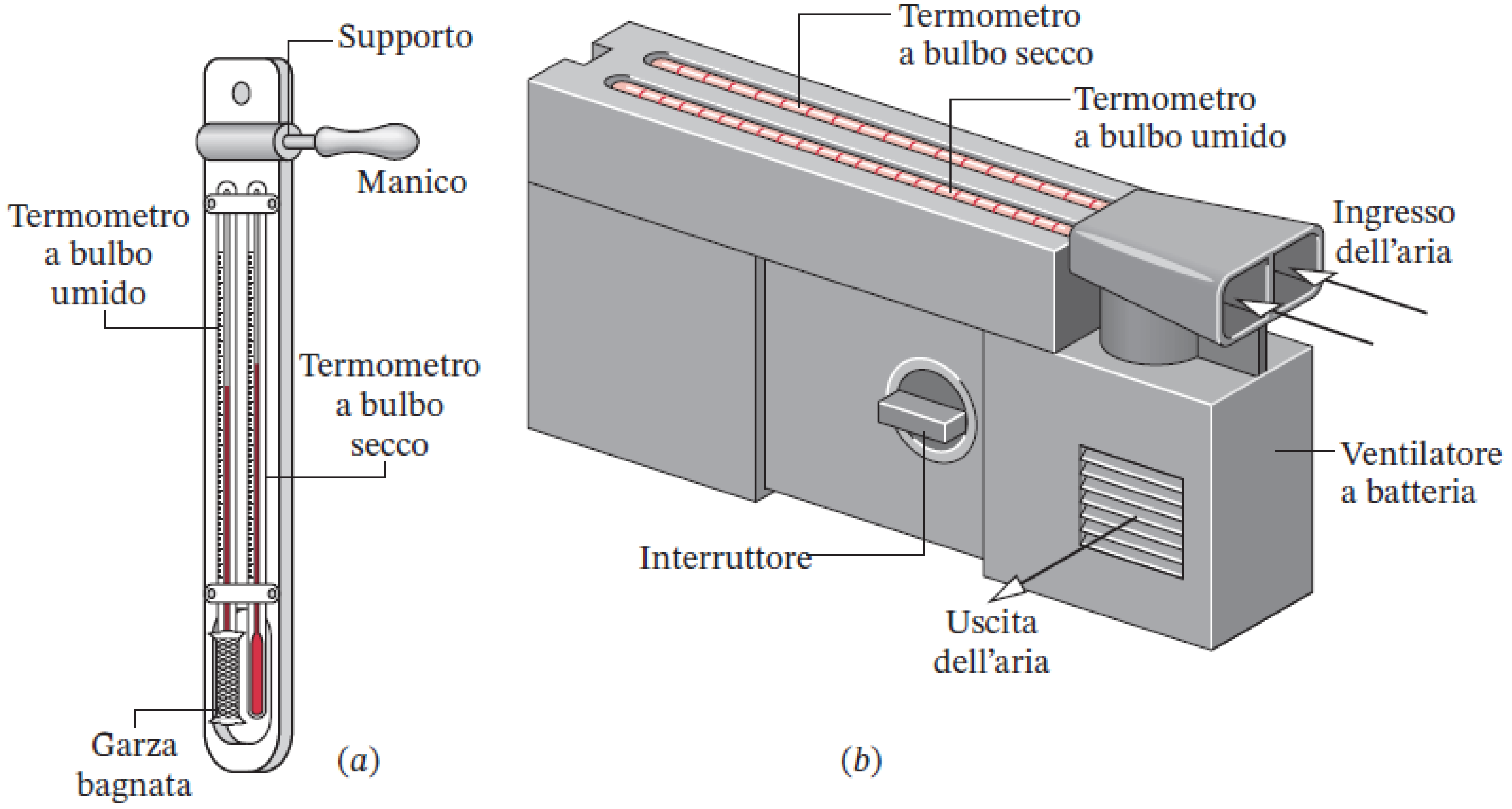
- The dew point temperature is the saturation temperature of water vapor corresponding to the partial pressure of vapor in the mixture.





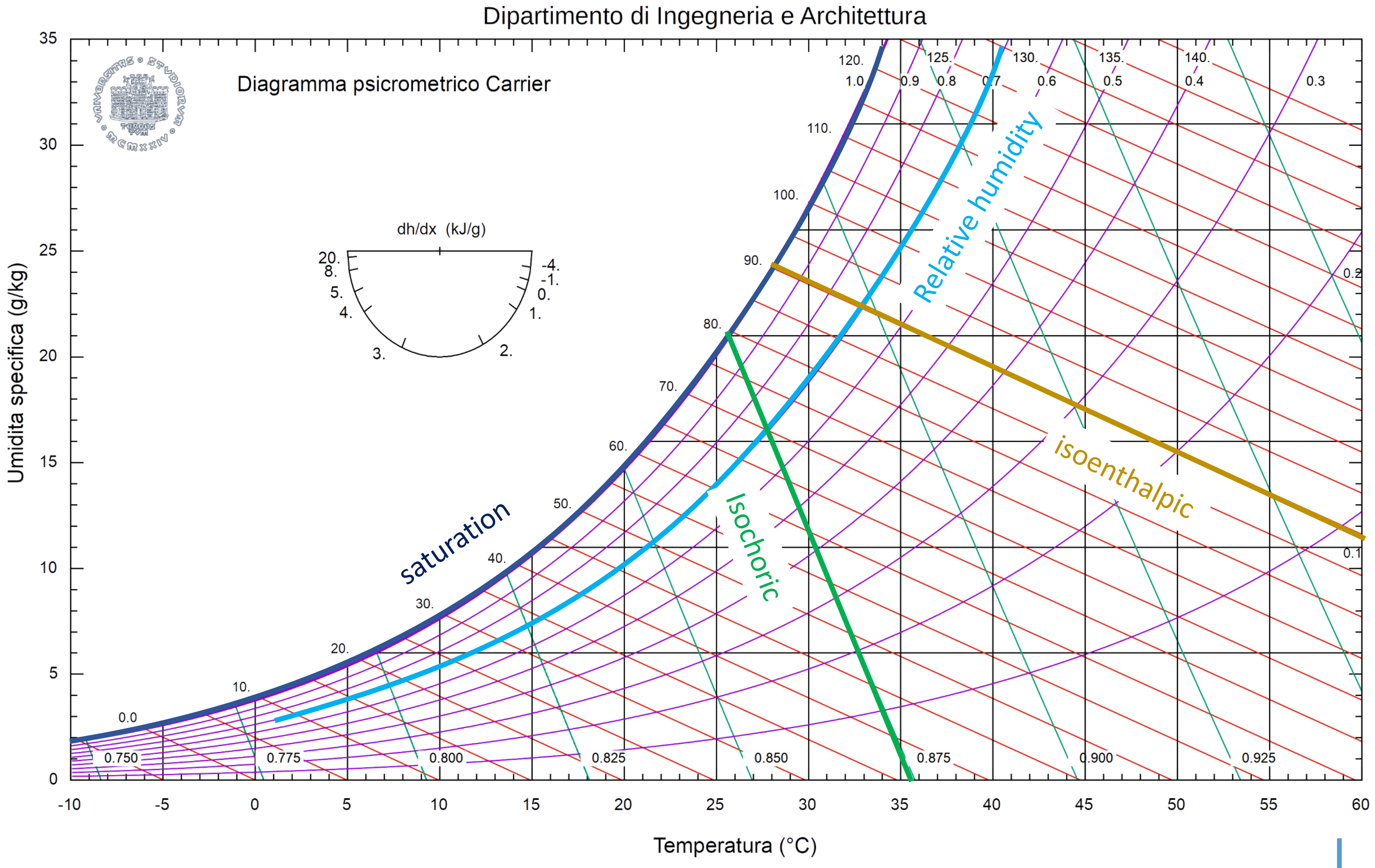
Dry and wet bulb temperatures

- Wet bulb temperature: temperature measured with a thermometer whose sensor, wrapped in a moist gauze, is ventilated.
- Dry bulb temperature: temperature measured with a thermometer immersed in the air-vapor mixture.



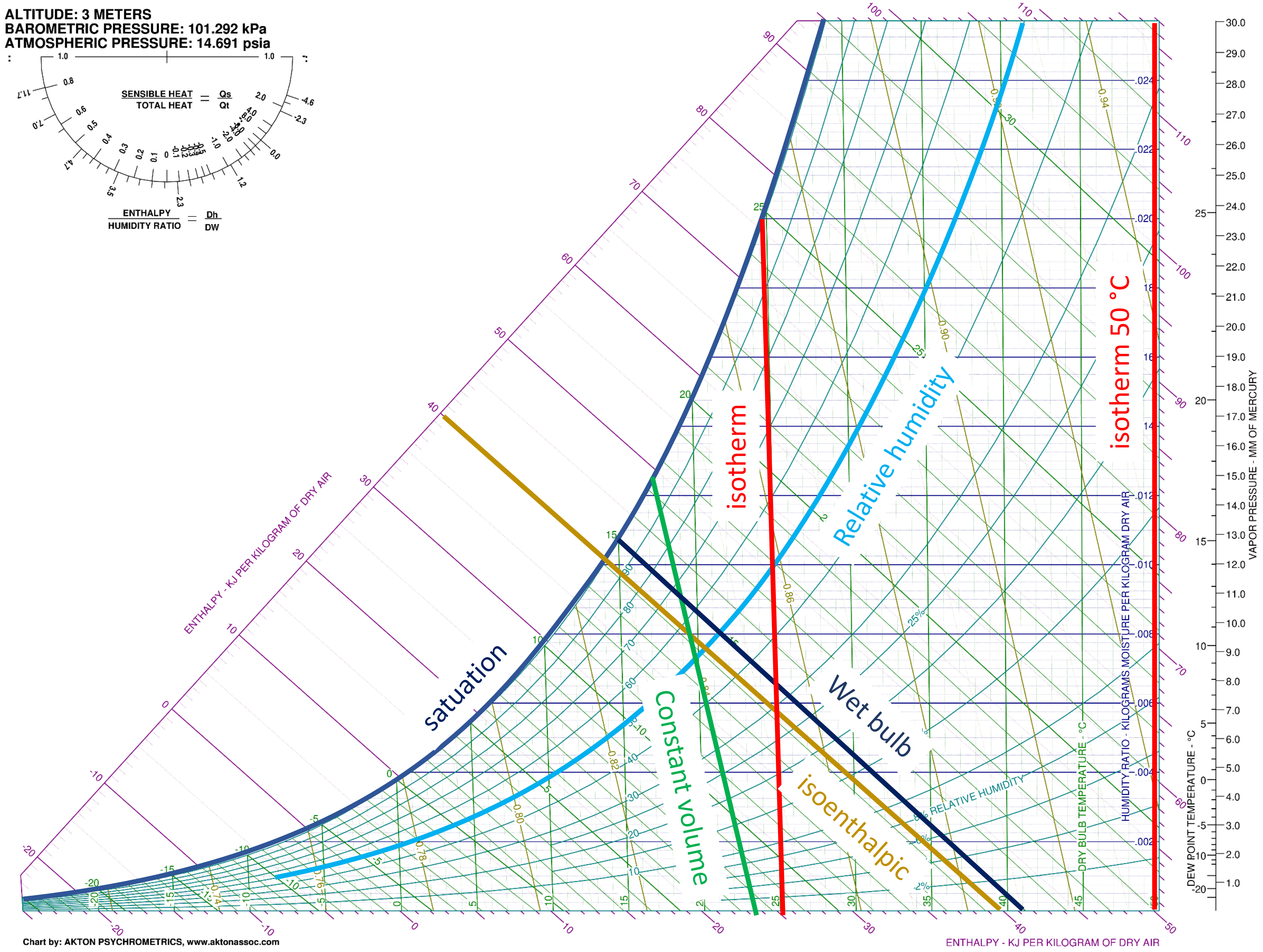
Carrier Psychrometric chart

- Constant relative humidity
- Saturation curve ϕ =1
- Isoenthalpic
- Constant volume
- In the Carrier diagram, enthalpies and constant wet bulb curves coincide



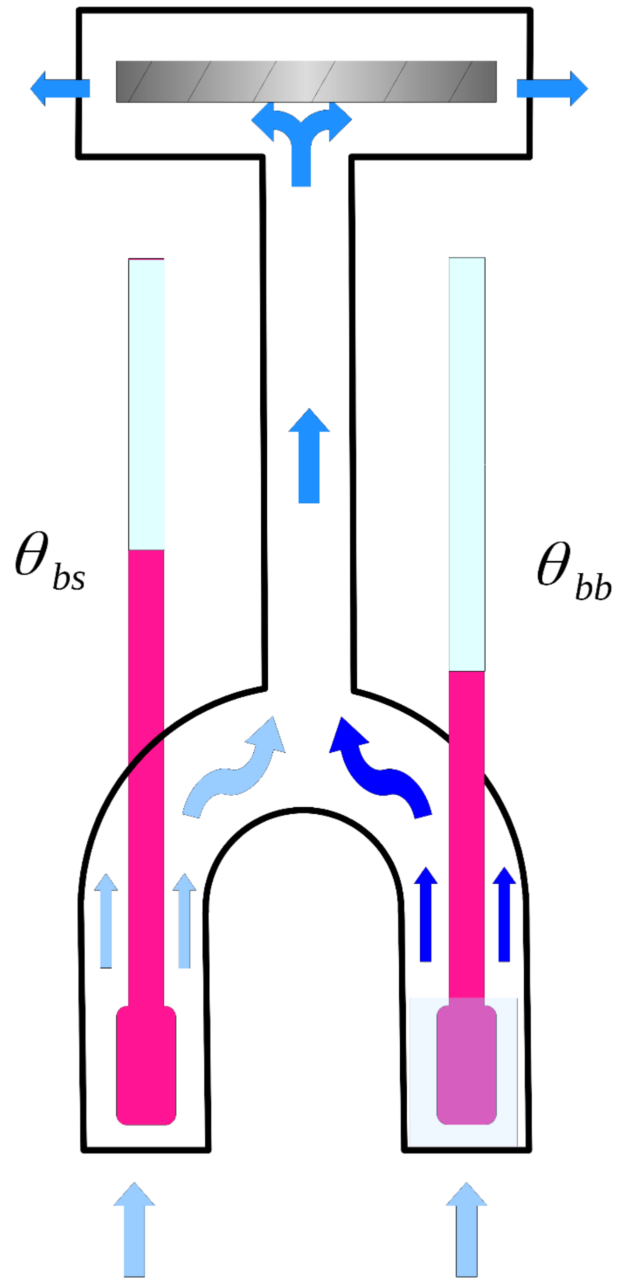
Ashrae chart

- **American Society of Heating, Refrigerating and Air-Conditioning Engineers**
- Isotherm
- Not vertical, only 50 °C isotherm is vertical
- Constant relative humidity
- Saturation curve ϕ=1
- Isoenthalpic
- Wet bulb
- Isochoric



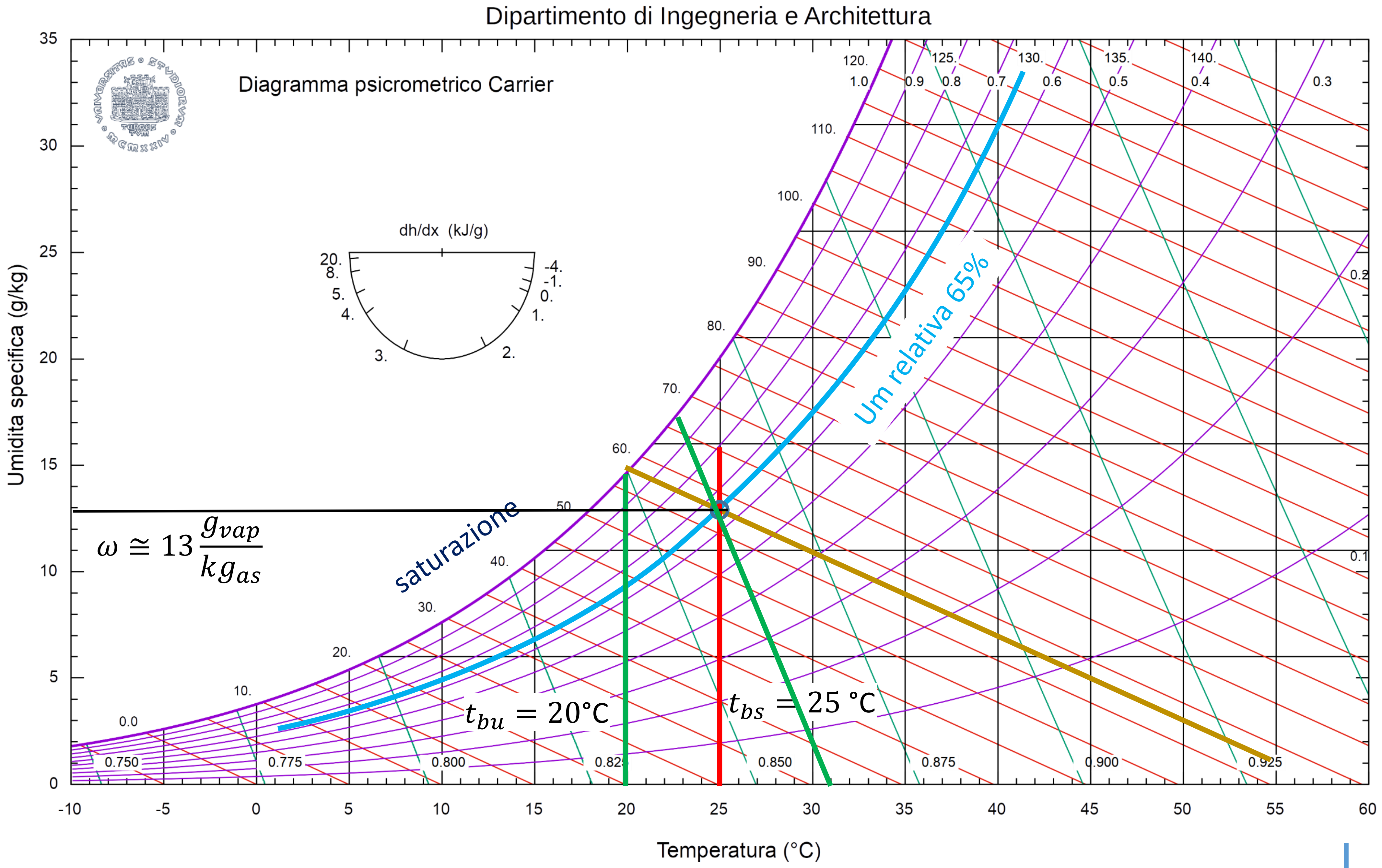
Hasman psychrometer

- Simple method to obtain the state of the system
- measures the dry bulb and wet bulb temperature
- The temperature difference allows the determination of the state of moist air



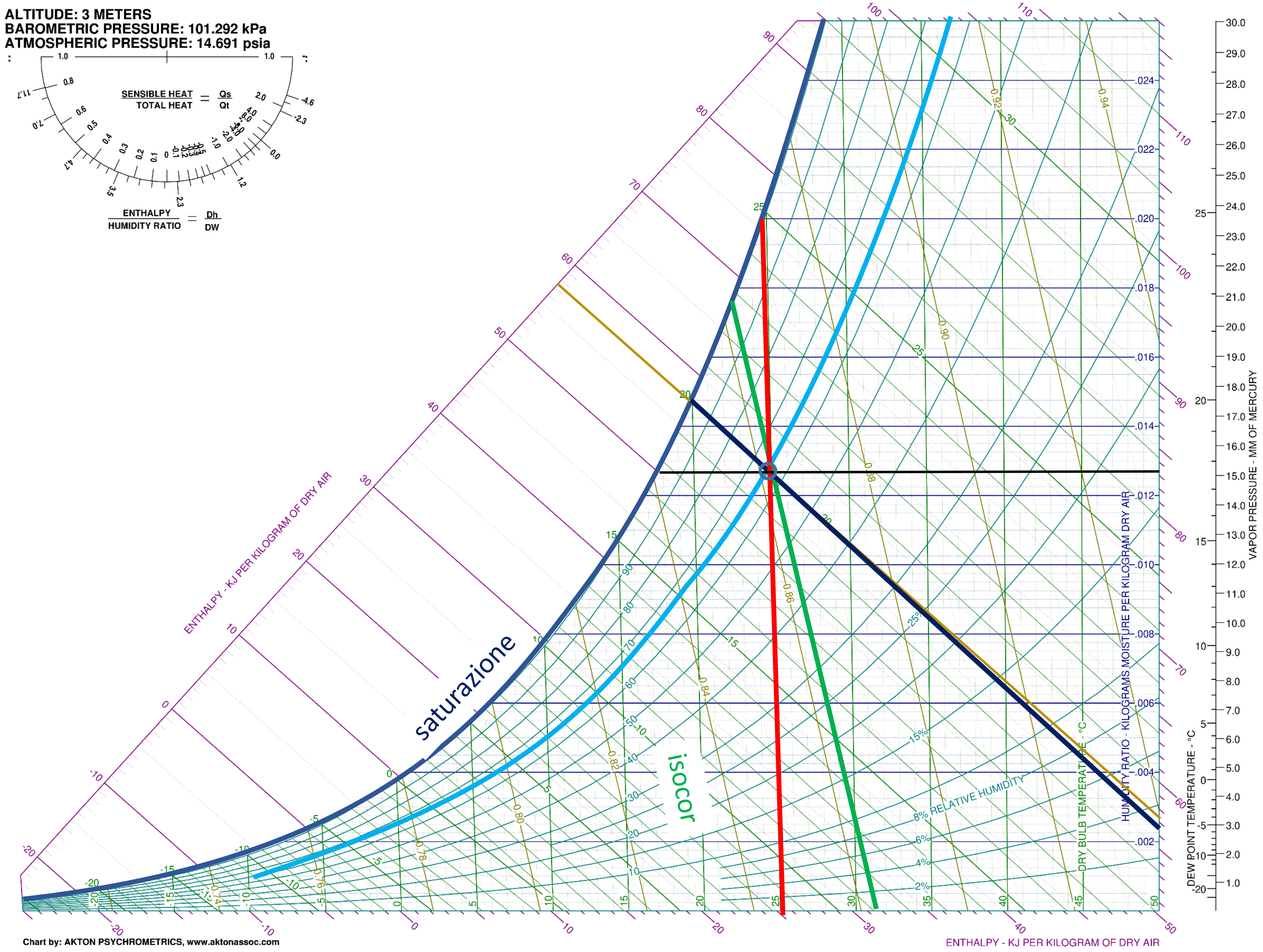
Assman Psychrometer

- example
  - $t_{db} = 25\,°C$
  - $t_{wb} = 20\,°C$
- I measure the dry bulb temperature t_{db}
- I measure the wet bulb temperature t_{wb}
- consider the isenthalpic and wet bulb temperature to be coincident
- Intersection identifies the state
- Then can be read the
  - Relative humidity
  - Specific humidity
  - Specific volume



Assman Psychrometer

- example
  - $t_{bs} = 25\ °C$
  - $t_{bu} = 20\ °C$
- I measure the dry bulb temperature t_{db}
- I measure the wet bulb temperature t_{wb}
- consider the isenthalpic and wet bulb temperature to be coincident
- Intersection identifies the state
- Then can be read the
  - Relative humidity
  - Specific humidity
  - Specific volume



Moist air balance equations

Dry air mass balance

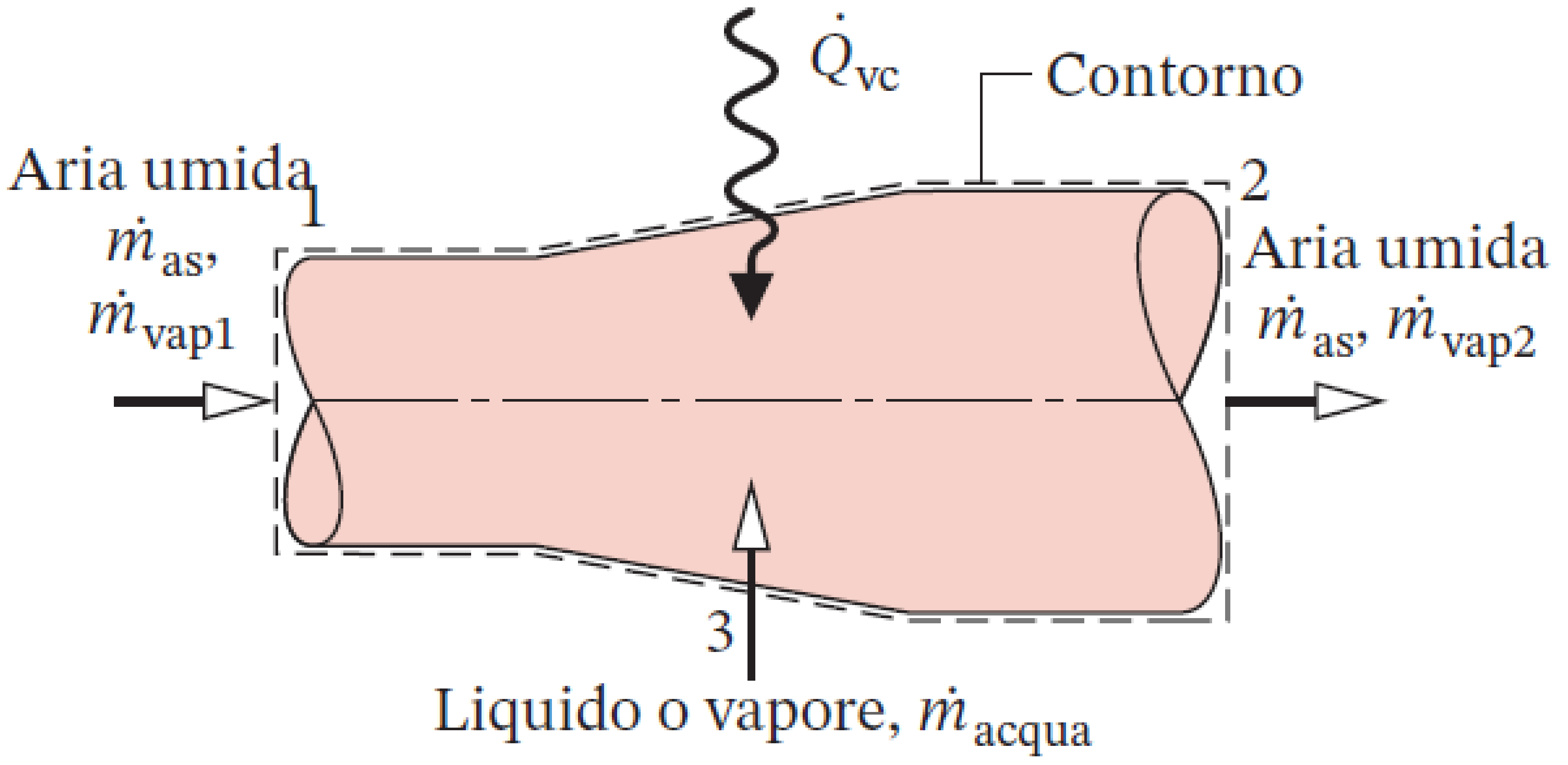
$$\dot{m}_{da} = \dot{m}_{da1} = \dot{m}_{da2}$$

Vapor mass balance

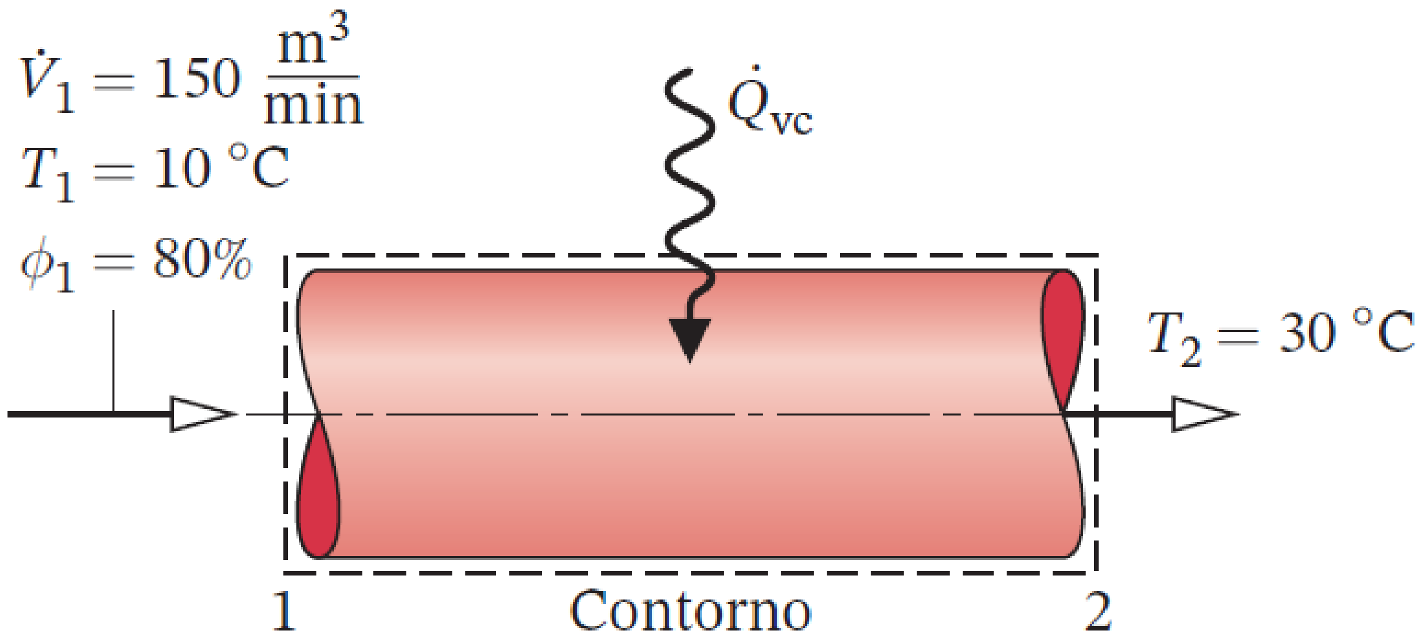
$$\dot{m}_{vap2} = \dot{m}_{water} + \dot{m}_{vap1}$$

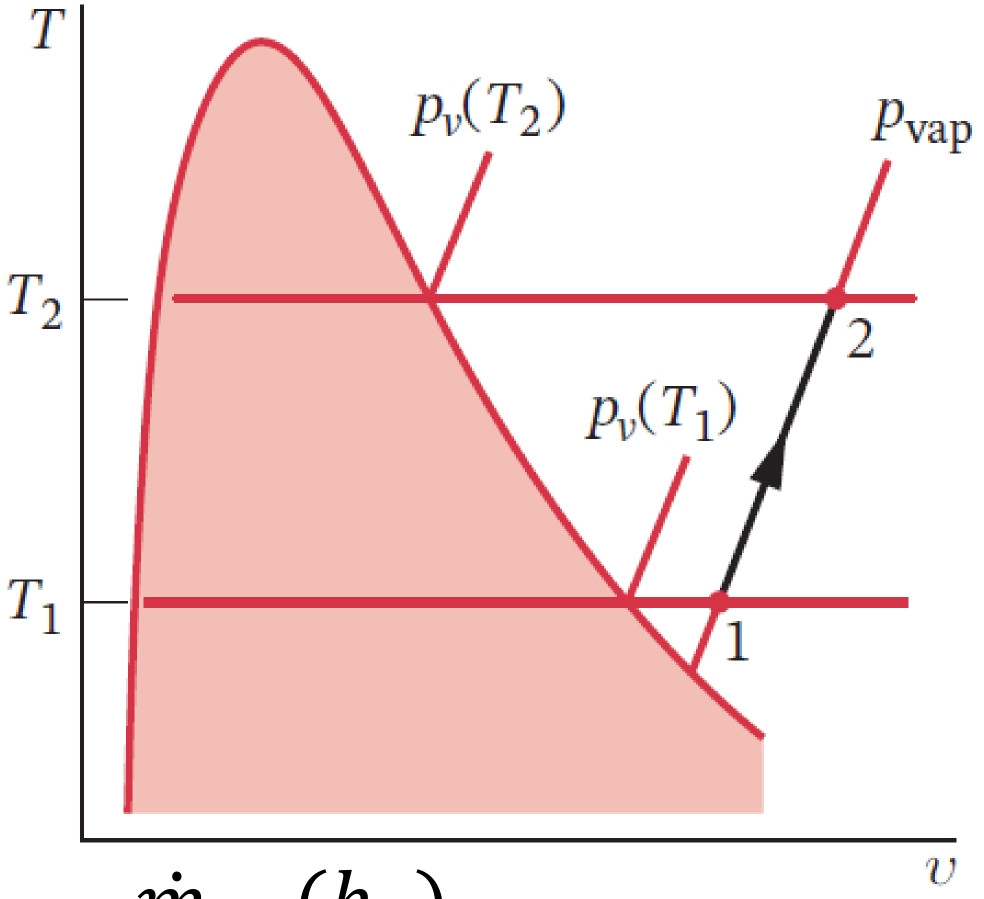
$$\dot{m}_{vap1} = w_1 \cdot \dot{m}_{da}$$
$$\dot{m}_{vap2} = w_2 \cdot \dot{m}_{da}$$

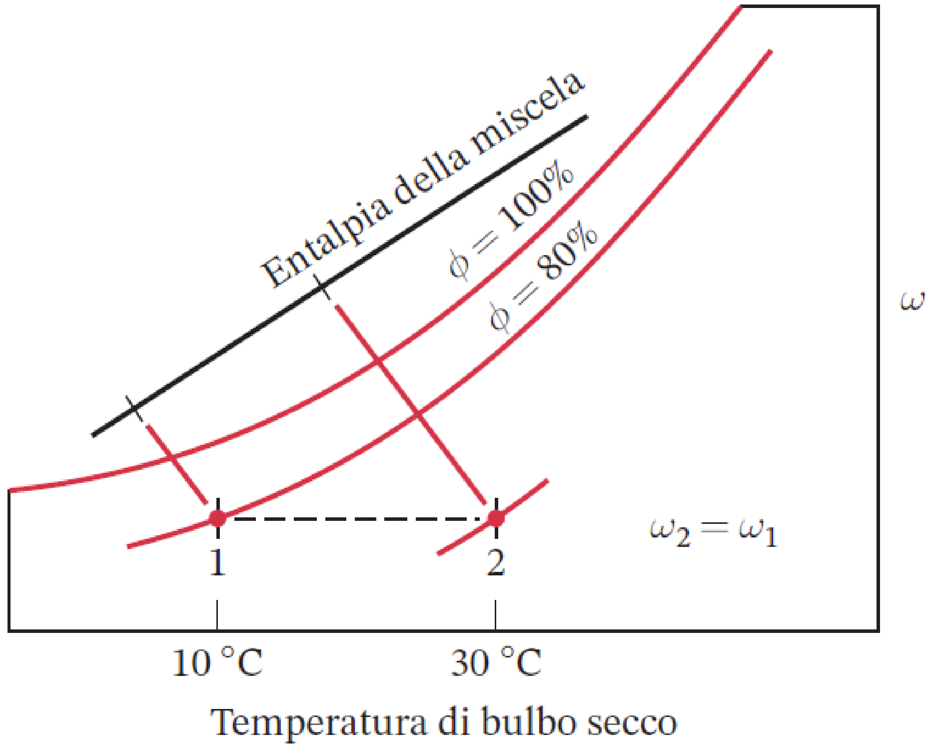
$$\dot{m}_{wate} = (w_2 - w_1) \cdot \dot{m}_{da}$$



Heating of a moist air







$$\omega_2 = \omega_1 \Rightarrow \dot{m}_{water} = 0$$

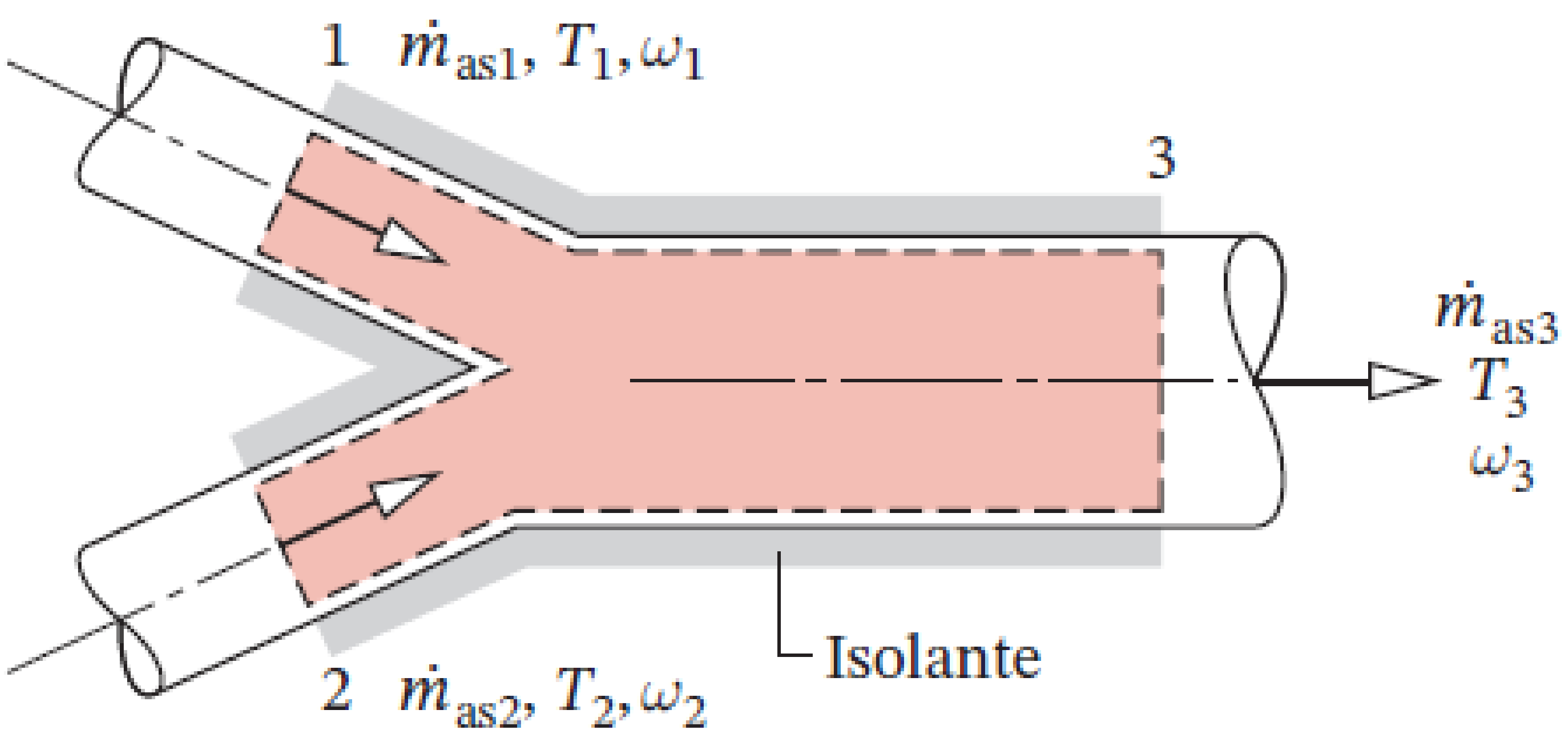
$$0 = \dot{Q}_{vc} + \dot{m}_{da}(h_1) + \cancel{\dot{m}_{water} \cdot h_{acqua}} - \dot{m}_{da}(h_2)$$

$$\dot{Q}_{vc} = \dot{m}_{da} \cdot (h_2 - h_1) = \dot{m}_{da} \cdot [c_{p,da} \cdot t_2 + w_2 \cdot (c_{p,v} \cdot t_2 \cancel{+ h_{lv}}) - c_{p,da} \cdot t_1 - w_1 \cdot (c_{p,v} \cdot t_1 \cancel{+ h_{lv}})]$$

$$\dot{Q}_{vc} = \dot{m}_{da} \cdot \boxed{(c_{p,da} + w_2 \cdot c_{p,v})} \cdot (t_2 - t_1) = \dot{m}_{da} \cdot \boxed{c_{p,a}} \cdot (t_2 - t_1)$$

specific heat of moist air

Adiabatic mixing

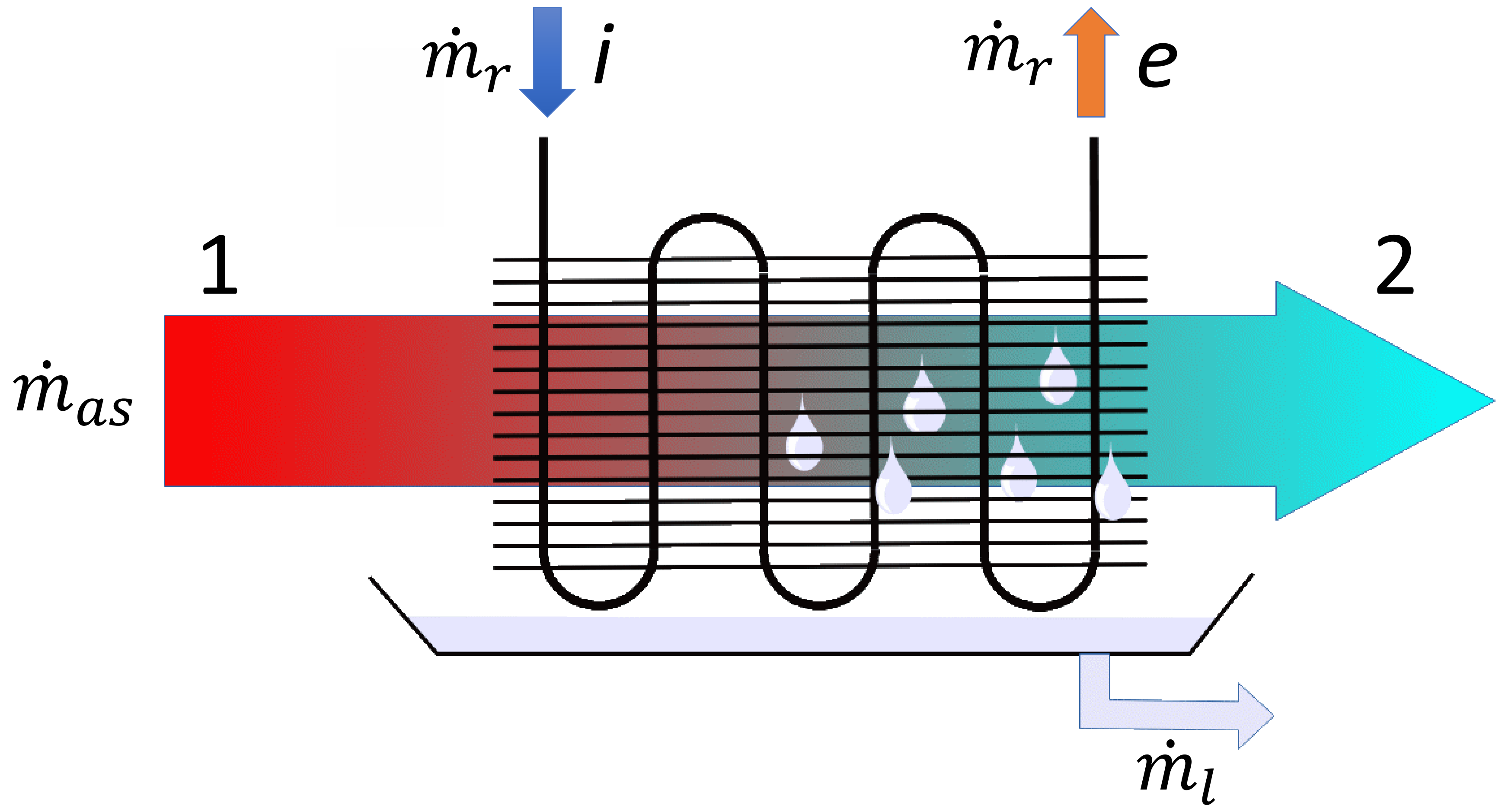


$$\dot{m}_{da1} \cdot (h_{da1} + w_1 h_{vap,1}) + \dot{m}_{da2} \cdot (h_{da2} + w_2 h_{vap,2}) = \dot{m}_{da3} \cdot (h_{da3} + w_3 \cdot h_{v3})$$

$$w_1 \dot{m}_{da1} + w_2 \dot{m}_{da2} = w_3 \dot{m}_{da3}$$
$$\dot{m}_{da1} + \dot{m}_{da2} = \dot{m}_{da3}$$

Raffreddamento con deumidificazione

- The air flow, in condition 1, pass through a cold surface (coil)
- The coil is at an average temperature t_s lower than the dew point temperature of the air
- The air flow cools and dehumidifies
- The condensed water $\dot{m}_l$ must be collected and drained
- The coil is cooled by a cold fluid flow that enters at condition *i* and exits at condition *e*
- The heat flow to be removed is q_{cool}



Cooling and dehumidification

- The exit condition can be represented as a point positioned in a line joining the air inlet condition 1 and the point S placed on the saturation curve at the average surface temperature of the cold battery t_S
- the thermal flux needed is $\left|\dot{Q}_{12}^{-}\right|$

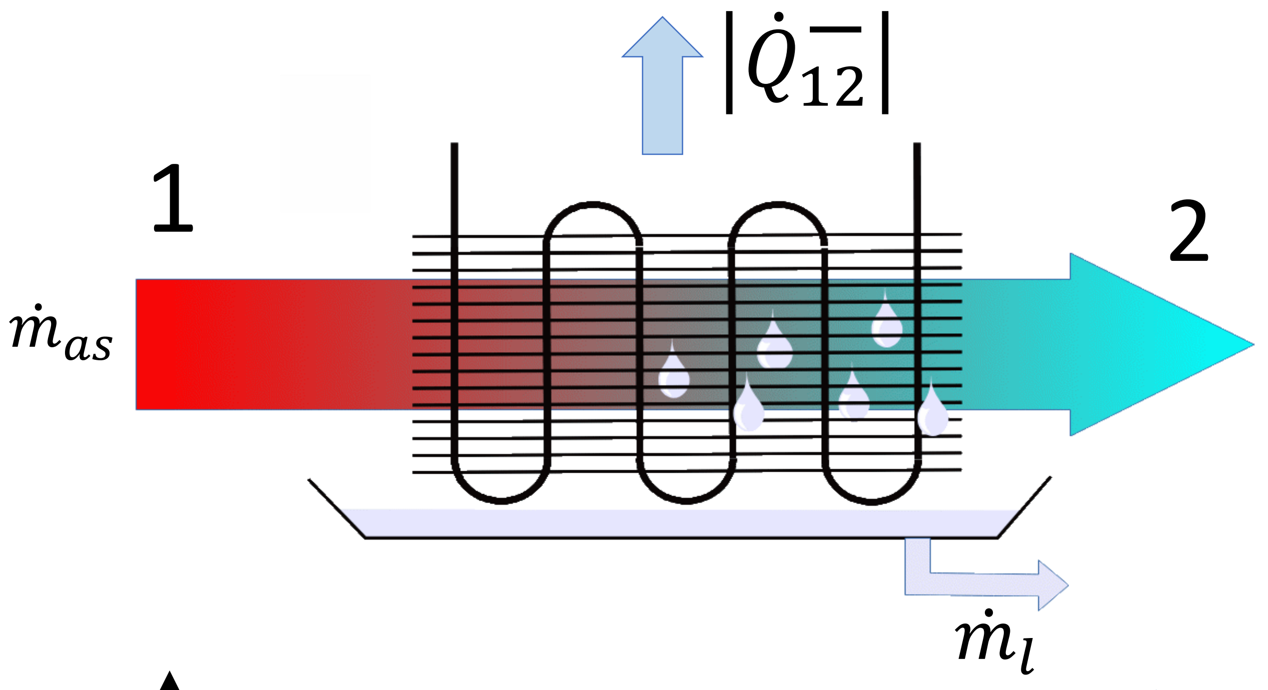
$$\left|\dot{Q}_{12}^{-}\right| = \dot{m}_{da} \cdot (h_1 - h_2) + \dot{m}_l \cdot h_s$$

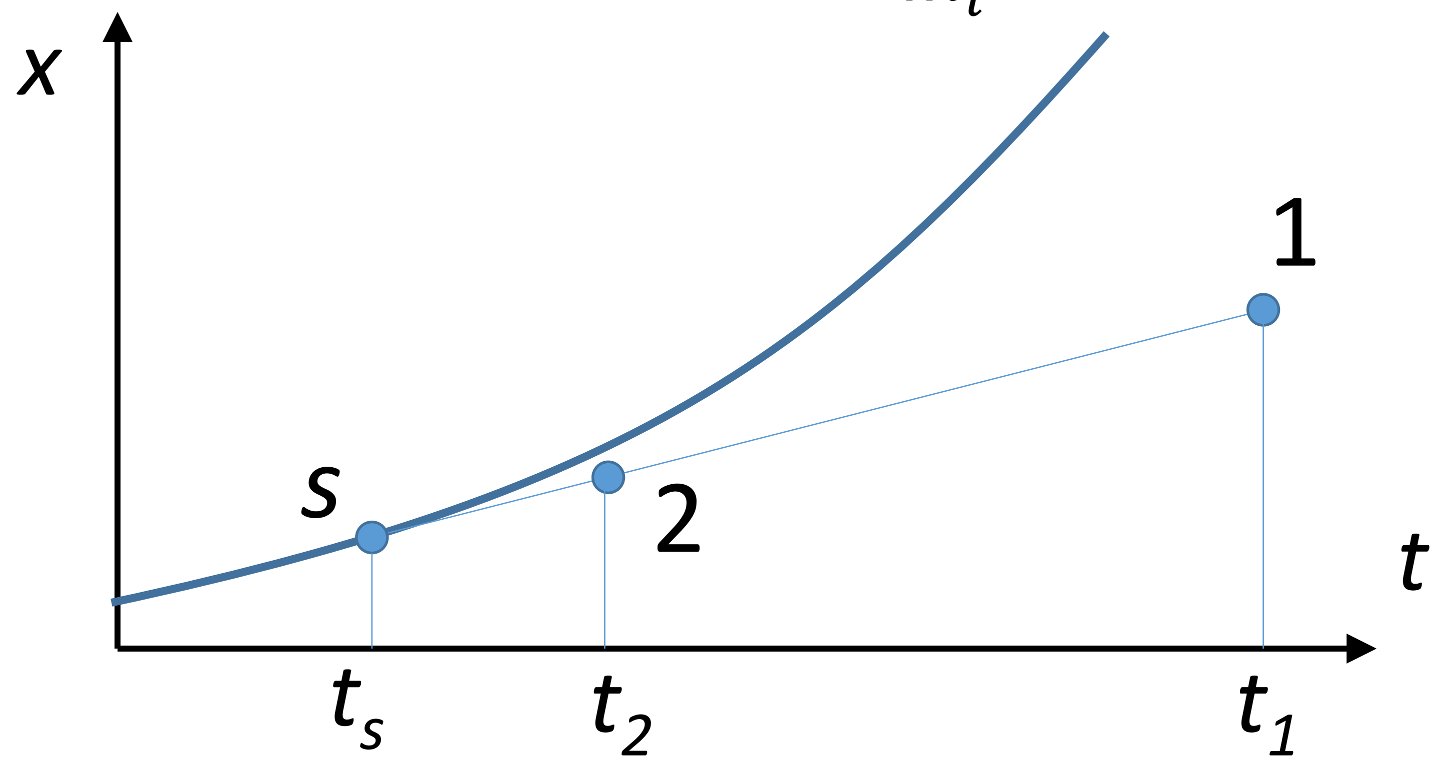
$$\dot{m}_l = \dot{m}_{da} \cdot (w_1 - w_2)$$

$$\left|\dot{Q}_{12}^{-}\right| = \dot{m}_{da} \cdot (h - h_2) + \dot{m}_{da} \cdot (w_1 - w_2) \cdot h_s$$

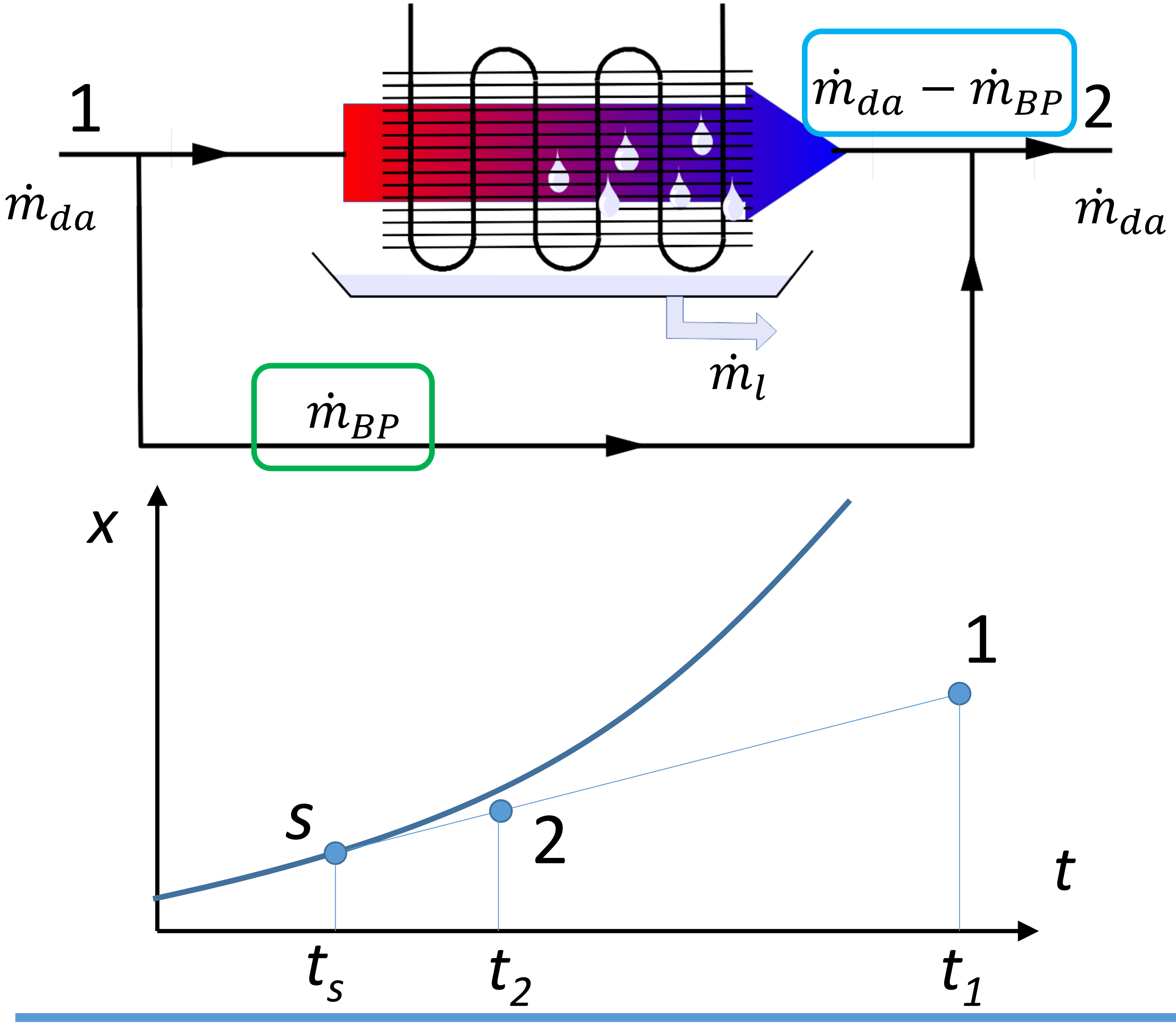
$$h_s = c_l \cdot t_S \, , h_1 \, , h_2 \;\; same\ order\ of\ magnitude$$

$$\left|\dot{Q}_{12}^{-}\right| \cong \dot{m}_{da} \cdot (h_1 - h_2)$$



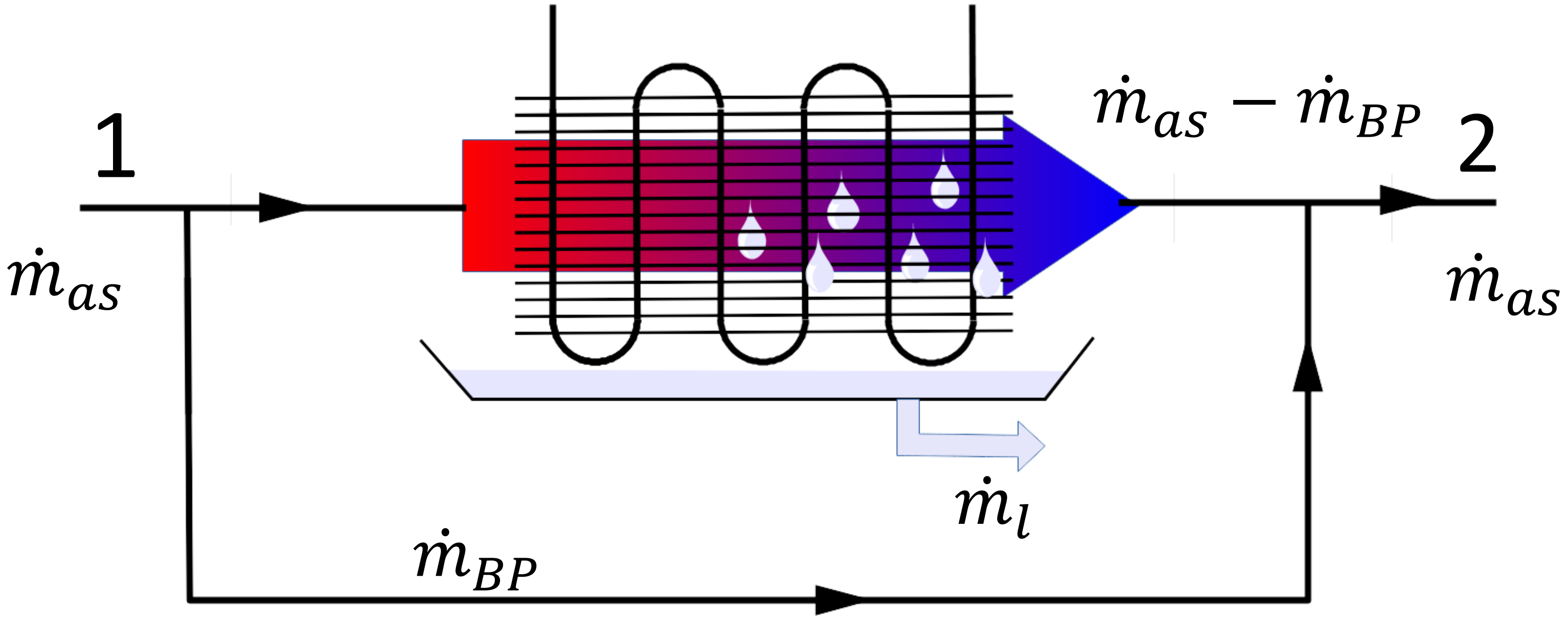


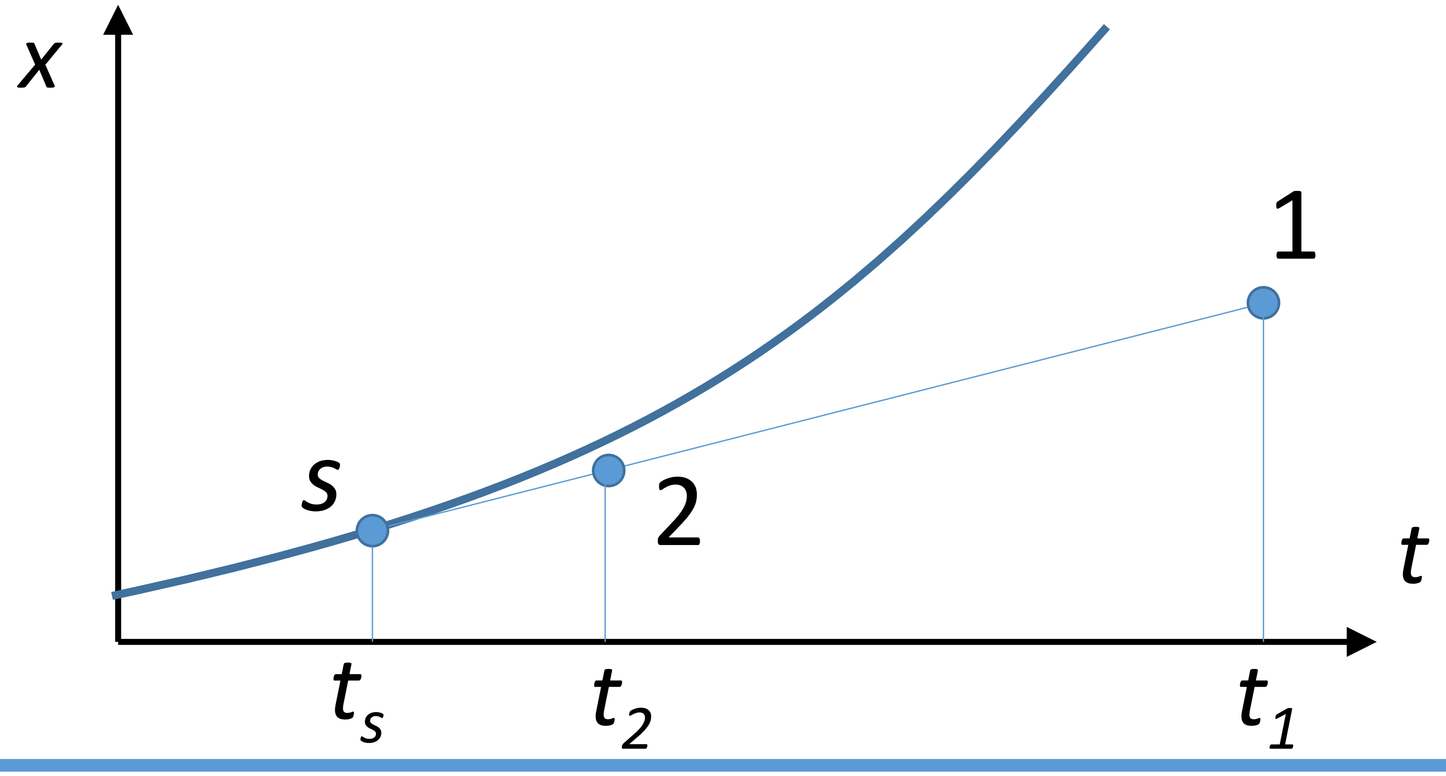
Bypass Factor



- The process can be represented as the mixing of two air flows
  - One mass flow bypasses the coil $\dot{m}_{BP}$
  - The mss flow $\dot{m}_{da} - \dot{m}_{BP}$ exits the coil in saturate condition S at temperature t_S
- Bypass Factor $\mathrm{BPF} = \dfrac{\dot{m}_{BP}}{\dot{m}_{as}}$

Bypass Factor





$$h_2 = \frac{(\dot{m}_{da} - \dot{m}_{BP}) \cdot h_S + \dot{m}_{BP} \cdot h_1}{\dot{m}_{da}}$$

$$h_2 = (1 - BPF) \cdot h_S + BPF \cdot h_1$$

$$BPF = \frac{h_2 - h_S}{h_1 - h_S}$$

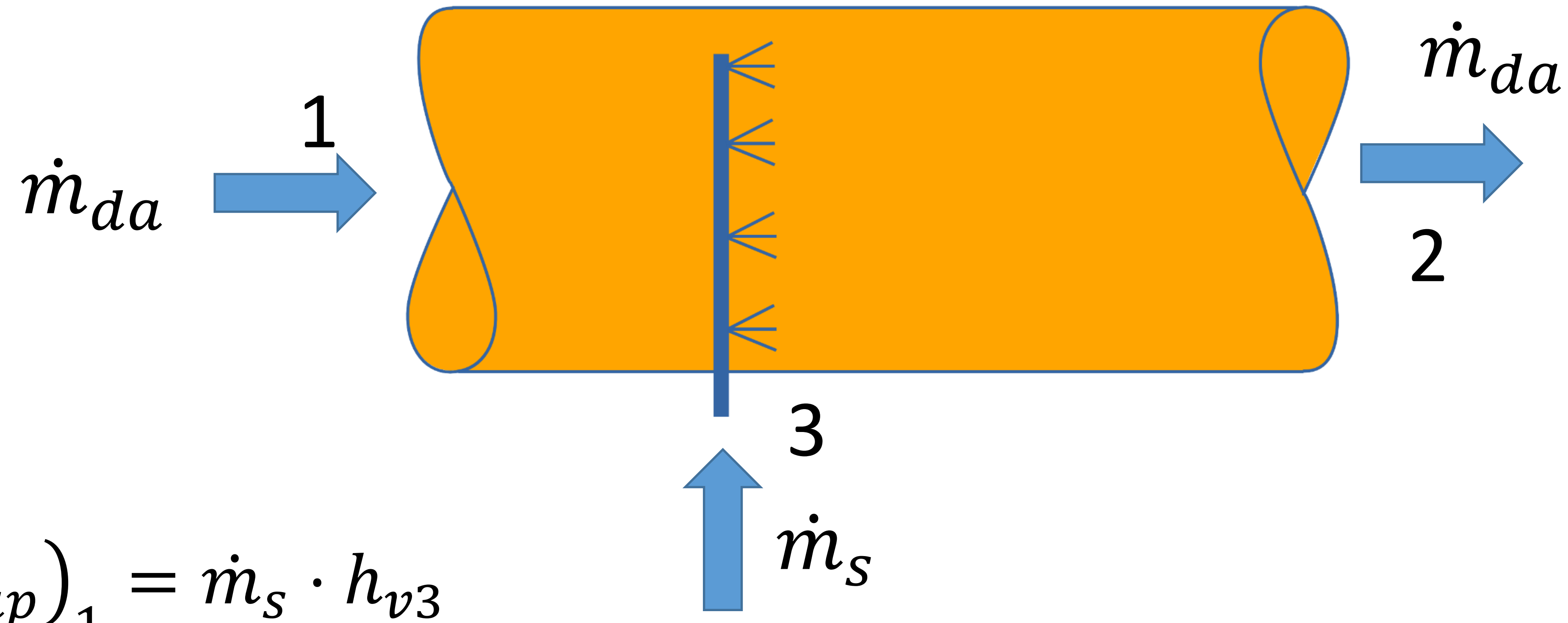
Same result with humidity ratio

$$BPF = \frac{\omega_2 - \omega_S}{\omega_1 - \omega_S}$$

Steam Humidifiers

$$\dot{m}_{da} \cdot (w_2 - w_1) = \dot{m}_s$$

$$w_2 = w_1 + \frac{\dot{m}_s}{\dot{m}_{da}}$$

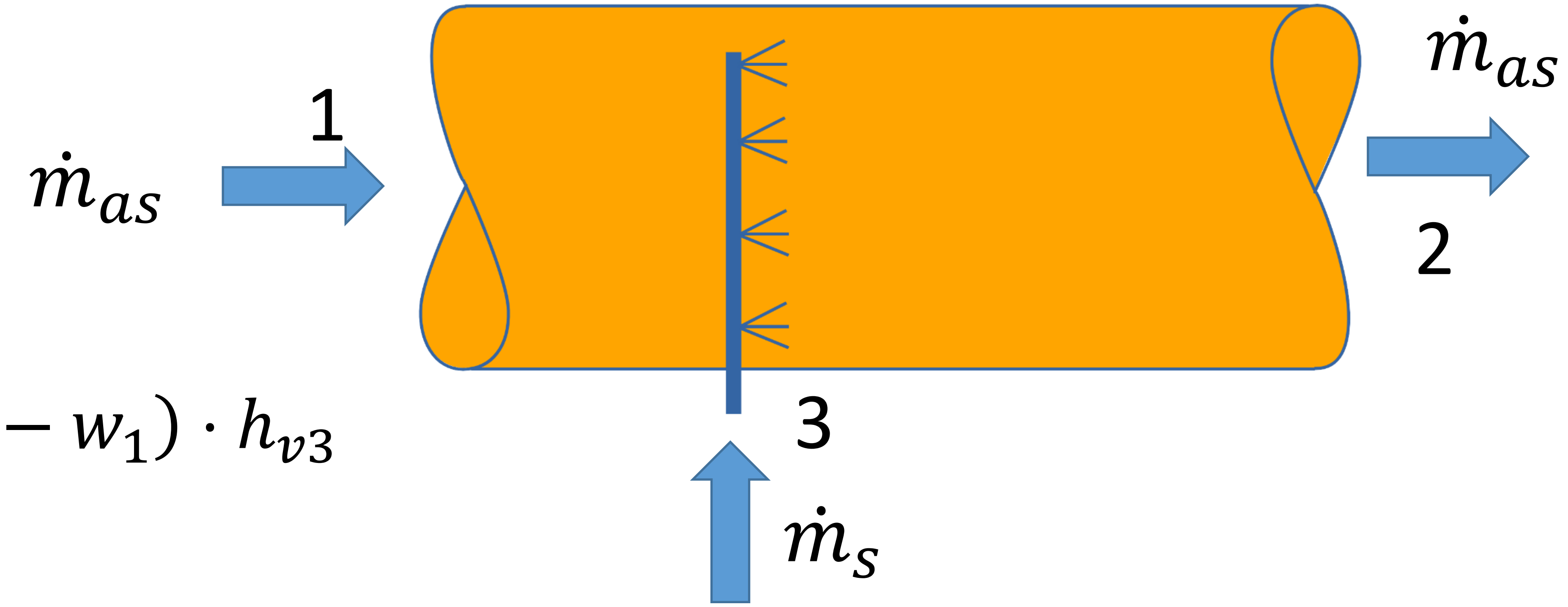


$$\dot{m}_{da} \cdot \left(h_{da} + w\, h_{vap}\right)_2 - \dot{m}_{as} \cdot \left(h_{da} + w\, h_{vap}\right)_1 = \dot{m}_s \cdot h_{v3}$$

$$\left(h_{da} + w\, h_{vap}\right)_2 - \left(h_{da} + w\, h_{vap}\right)_1 = (w_2 - w_1) h_{v3}$$

$$h_{da,2} + w_2\, h_{vap,2} - h_{da,1} - w_1\, h_{vap,1} = (w_2 - w_1) h_{v3}$$

Steam Humidifiers



$$h_{da,2} + w_2 h_{vap,2} - h_{da,1} - w_1 h_{vap,1} = (w_2 - w_1) \cdot h_{v3}$$

$$h_{da,2} - h_{da,1} + \overline{h_{vap,12}} \cdot (w_2 - w_1) = (w_2 - w_1) h_{v3}$$

$$h_{da,2} - h_{da,1} = (h_{v3} - \overline{h_{vap,12}}) \cdot (w_2 - w_1)$$

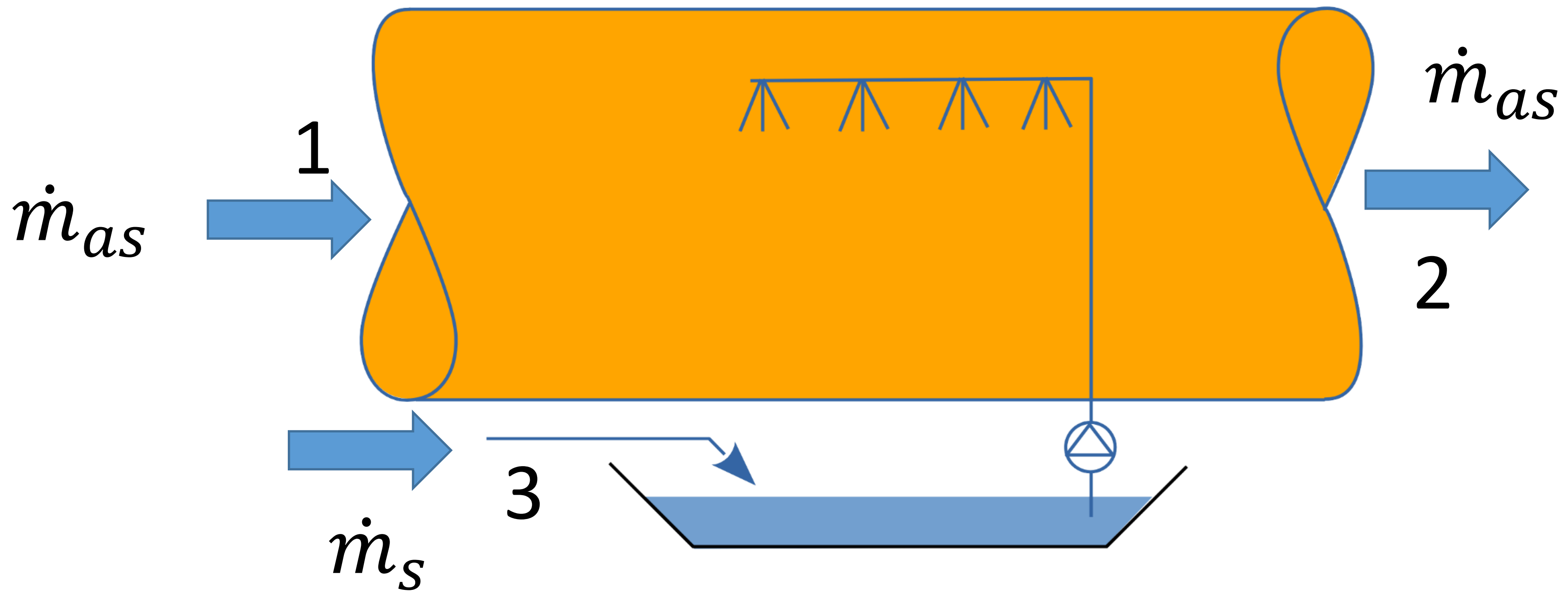
$$c_{p,da} \cdot (t_2 - t_1) = (h_{v3} - \overline{h_{vap,12}}) \cdot (w_2 - w_1) = c_{p,v} \cdot (t_3 - \overline{t_{12}}) \cdot (w_2 - w_1)$$

$$(t_2 - t_1) = {}^{c_{p,v}}/_{c_{p,da}} \cdot (t_3 - \overline{t_{12}}) \cdot (w_2 - w_1) \cong 0$$

Air Washer

$$\dot{m}_{da} \cdot (w_2 - w_1) = \dot{m}_s$$

$$w_2 = w_1 + \frac{\dot{m}_s}{\dot{m}_{da}}$$



$$\dot{m}_{da}\left(h_{da} + w\,h_{vap}\right)_2 - \dot{m}_{da}\left(h_{da} + w\,h_{vap}\right)_1 = \dot{m}_s \cdot h_{l3}$$

$$\left(h_{da} + w\,h_{vap}\right)_2 - \left(h_{da} + w\,h_{vap}\right)_1 = (w_2 - w_1)h_{l3} \cong 0$$

$$\left(h_{da} + w\,h_{vap}\right)_2 - \left(h_{da} + w\,h_{vap}\right)_1 = (w_2 - w_1)h_{l3} \cong 0$$

$$h_2 - h_1 = (w_2 - w_1)h_l \cong 0$$

Performance of an Air Washer

- For a humidification process along the thermodynamic wet-bulb temperature line, the performance of an air washer can be illustrated by the saturation

$$\varepsilon_{sat} = \frac{w_2 - w_1}{w_s^* - w_1} \approx \frac{T_2 - T_1}{T_s^* - T_1}$$

where T_1,T_2 temperature of air entering and leaving air washer

w_1, w_2 humidity ratio of air entering and leaving air washer

w_s^*, T_s^* humidity ratio and temperature of saturated air at thermodynamic wet-bulb temperature,

Air Washer

$$h_2 = (w_2 - w_1)h_l + h_1$$

Enthalpy of liquid.

$$\varepsilon_{sat} = \frac{w_2 - w_1}{w_s^* - w_1} \approx \frac{T_2 - T_1}{T_s^* - T_1}$$