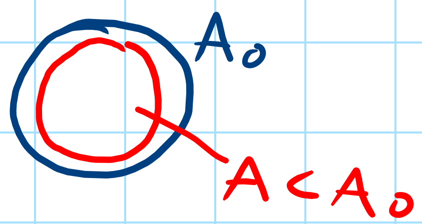
SOLID MECHANICS

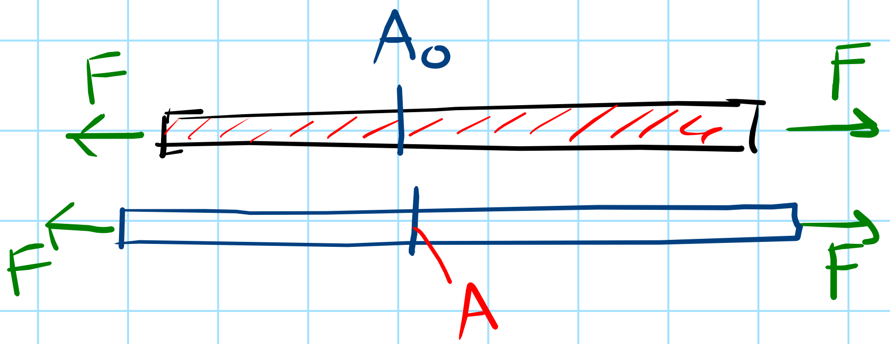
CSM 5/13/25

(EQUILIBRIUM, DEFORMATION (STRAIN, CONSTITUTIVE BEHAVIOUR)

STRESS THEORY

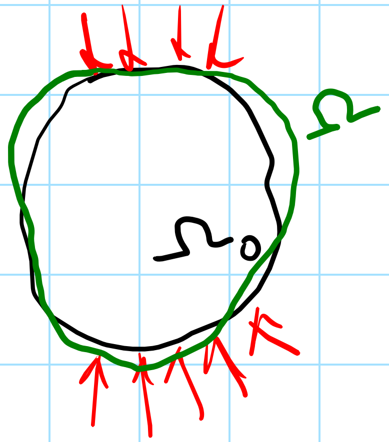


MAIN TEST: UNIAXIAL TEST



ΔL : LENGTHENING

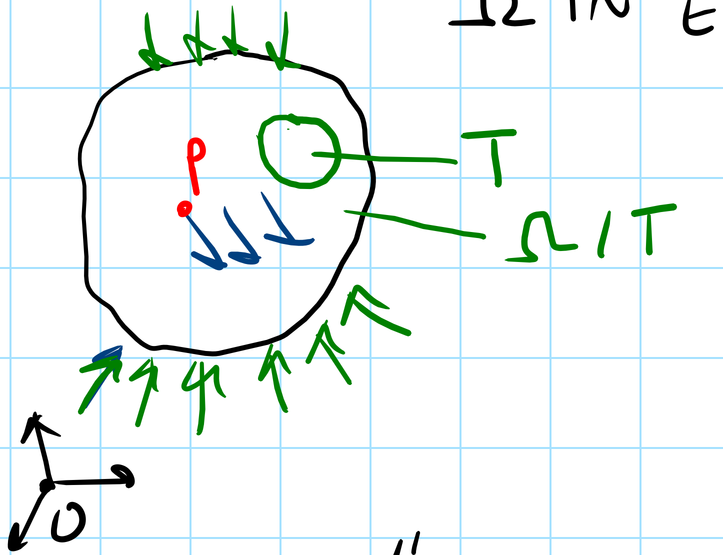
HYPOTHESIS: WHEN CONSIDERING EQUILIBRIUM, WE ASSUME THE CURRENT CONFIGURATION COINCIDENT WITH THE INITIAL CONFIGURATION (WITH SMALL ERROR)



TYPES OF FORCES

$\underline{b}$: BODY FORCES (FORCES × UNIT VOLUME) $\in \Omega$

$\underline{p}$: SURF. FORCES (" × UNIT AREA) $\in \partial\Omega$

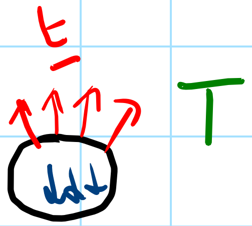


Ω IN EQUILIBRIUM :

$$\int_{\partial\Omega} \underline{p}\, dS + \int_{\Omega} \underline{b}\, dV = \underline{0} \qquad \left(\sum_i \underline{F}_i = \underline{0}\right)$$

$$\int_{\partial\Omega} OP \times \underline{p}\, dS + \int_{\Omega} OP \times \underline{b}\, dV = \underline{0} \qquad \left(\sum_i \underline{M}_{O i} = \underline{0}\right)$$

TO BUILD "STRESS THEORY" WE HAVE ALSO TO POSTULATE THE FOLLOWING:
EQUILIBRIUM MUST BE SATISFIED FOR ALL PARTS OF BODY Ω

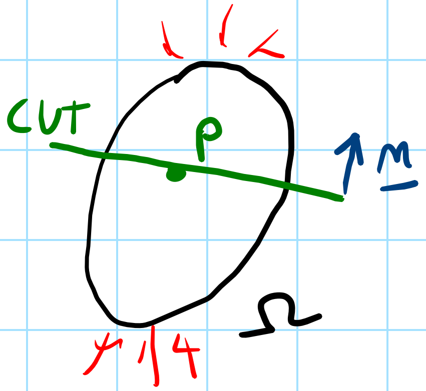


EQUIL. (T) : $\displaystyle\int_{\partial T} \underline{t}\, dS + \int_{T} \underline{b}\, dV = \underline{0} \qquad (\forall T)$

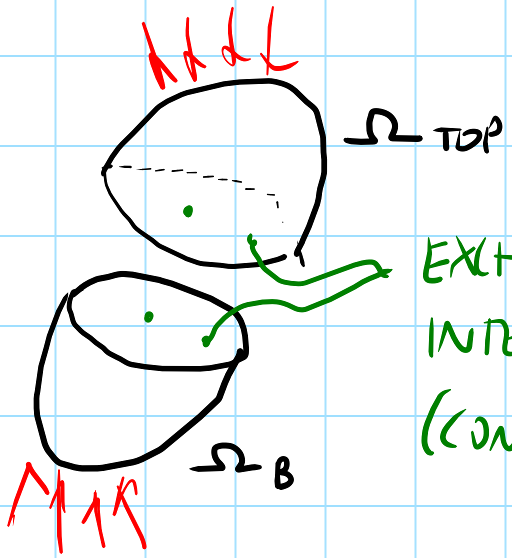
also (Ω/T) is in EQUILIBRIUM

$\underline{t}$: CONTACT FORCES (INTERNAL)

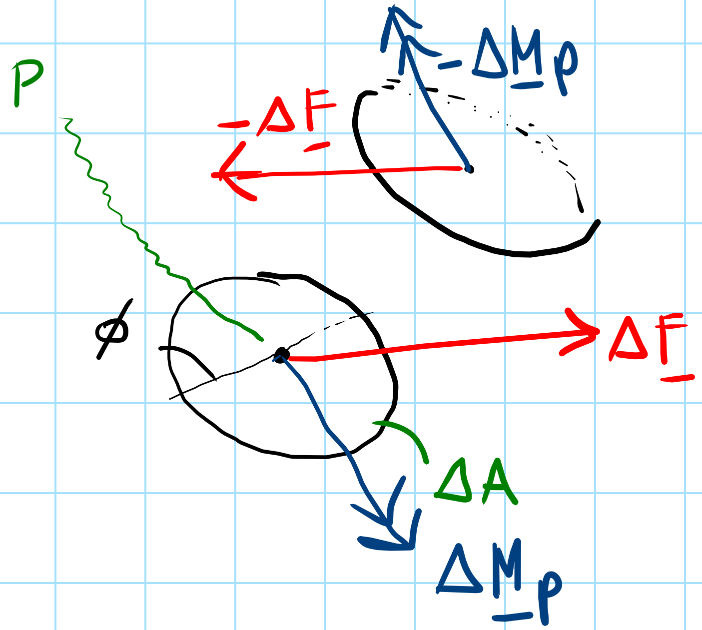
LOCAL ANALYSIS OF STRESS



$|\underline{n}| = 1$



EXCHANGE OF INTERNAL SURFACE (CONTACT FORCES?)



TRACTION VECTOR $\underline{t}(P, \underline{n})$

$$\underline{t}(P,\underline{n}) = \lim_{\phi \to 0} \frac{\Delta \underline{F}}{\Delta A}$$ (FINITE QUANTITY)

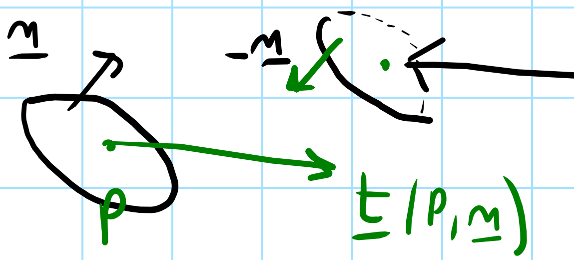
$$\lim_{\phi \to 0} \frac{\Delta \underline{M}_P}{\Delta A} = \underline{0}$$ (NO CONTACT MOMENTS)

CAUCHY'S THEORY (1827) OF STRESS (SIMPLE MATERIALS)

$$\lim_{\phi \to 0} \frac{\Delta \underline{M}_P}{\Delta A} = \underline{m}(P,\underline{n})$$ (COSSERAT'S THEORY) 1909

(MICROSTRUCTURED MATERIALS)

OBS : $\underline{t}(P,\underline{n})$ $\qquad \underline{t}(P,-\underline{n}) = -\underline{t}(P,\underline{n})$



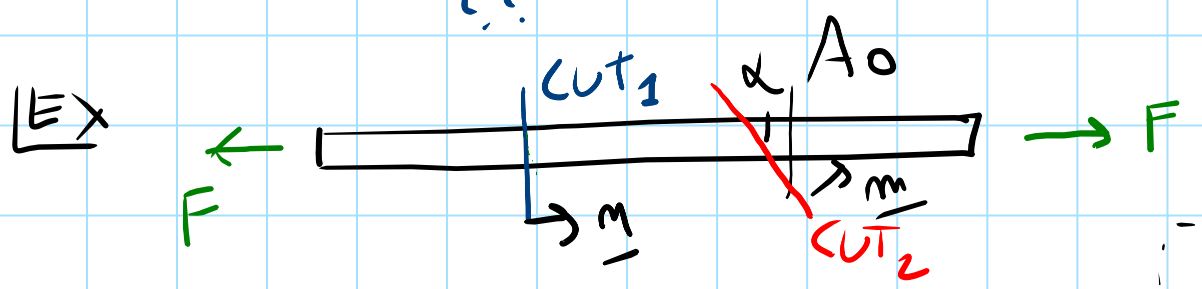
$-\underline{t}(P,\underline{n}) = \underline{t}(P,-\underline{n})$

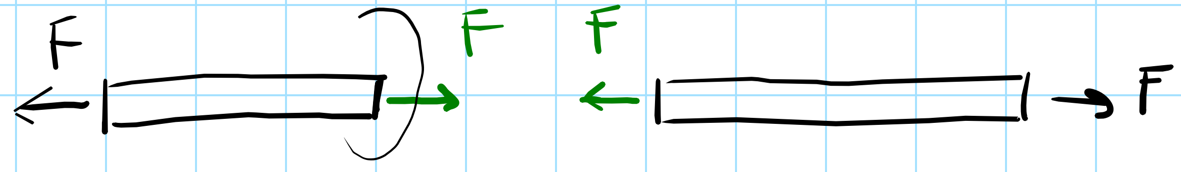
VERY IMPORTANT :

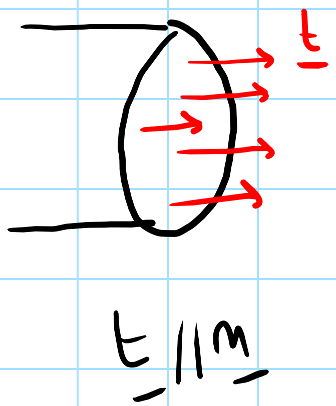
$\underline{t}(P,\underline{n}) \neq \underline{t}(P,\underline{m})$, $\underline{m} \neq \underline{n}$

??

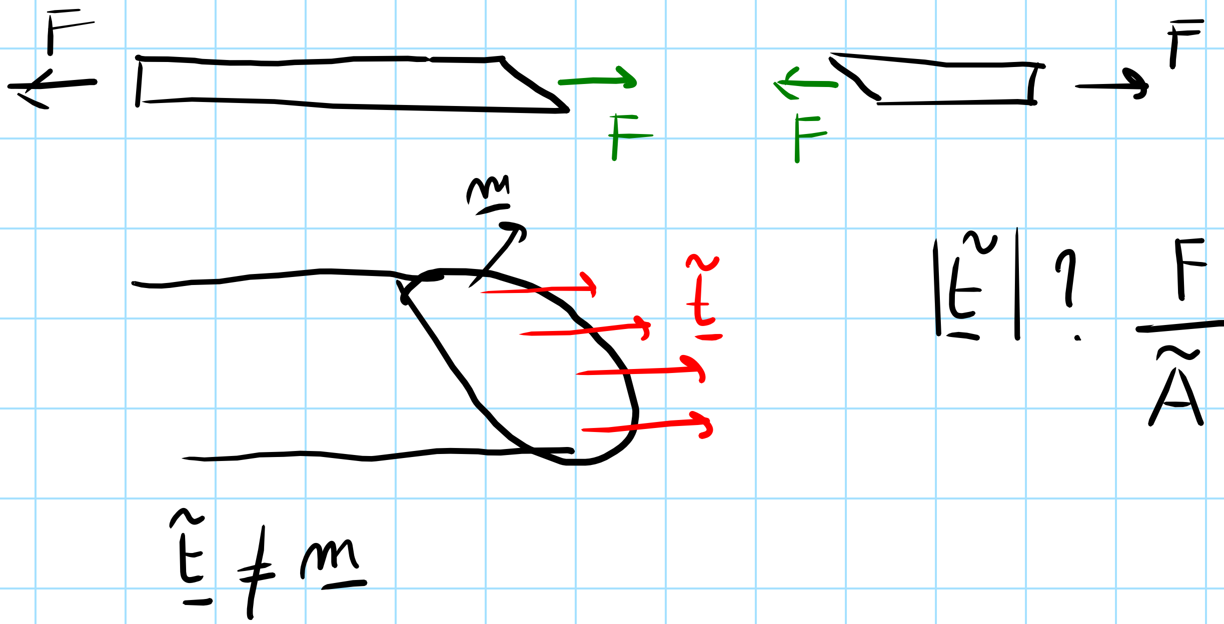
EX







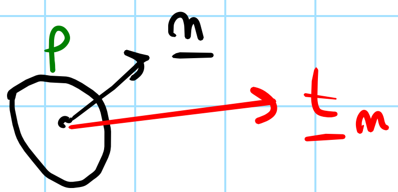
$|\underline{t}| \overset{?}{=} \dfrac{F}{A_0}$



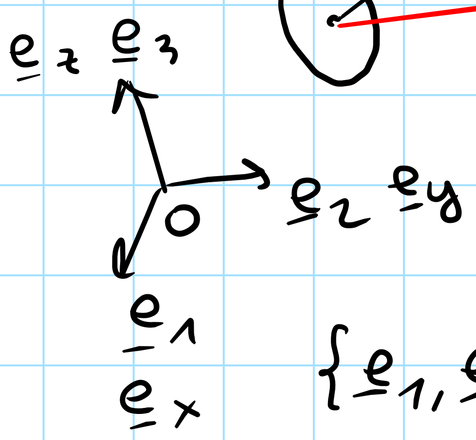
$|\tilde{\underline{t}}| \; ? \; \dfrac{F}{\tilde{A}}$

$\tilde{A} = A \sin\alpha$

CARTESIAN COMPONENTS OF TRACTION VECTOR



$$t_{n_i} = \underline{t}_n \cdot \underline{e}_i$$

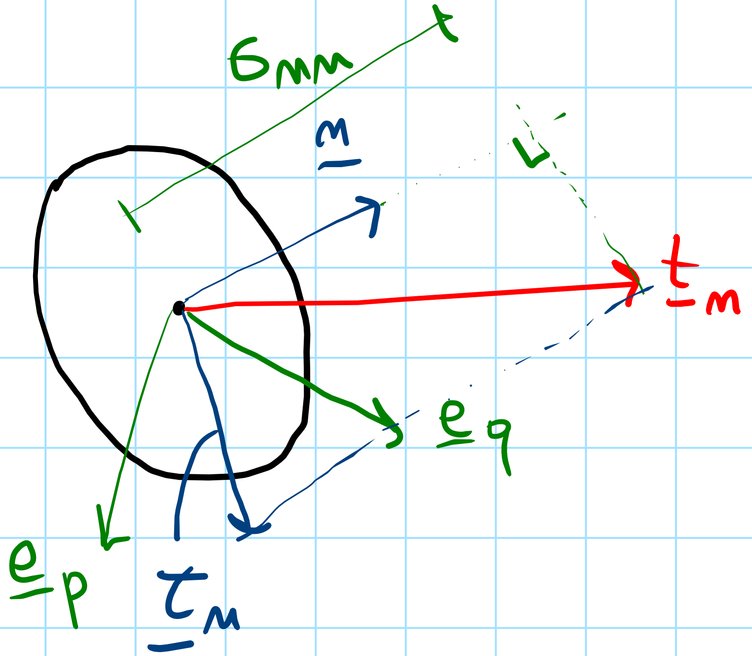


$$\underline{t}_n = \begin{bmatrix} t_{n1} \\ t_{n2} \\ t_{n3} \end{bmatrix}$$

$\{\underline{e}_1, \underline{e}_2, \underline{e}_3\}$

ORTHONORMAL BASIS

SPECIAL COMPONENTS OF TRACTION VECTOR



$\sigma_{nn} = \underline{t}_n \cdot \underline{n}$: NORMAL STRESS

> 0 : TENSION

< 0 : COMPRESSION

$$\underline{t}_n = \sigma_{nn}\,\underline{n} + \underline{\tau}_n \Rightarrow \underline{\tau}_n = \underline{t}_n - \sigma_{nn}\,\underline{n}$$

(SHEAR STRESS VECTOR)

$$\left.\begin{aligned} \tau_{np} &= \underline{\tau}_n \cdot \underline{e}_p \\ \tau_{nq} &= \underline{\tau}_n \cdot \underline{e}_q \end{aligned}\right\} \text{SHEAR STRESSES}$$

$\{\underline{n}, \underline{e}_p, \underline{e}_q\}$: ORTHON. BASIS