HYDROST. STRESS AND DEVIATORIC STRESS 13/3/25

$$\underset{\sim}{\sigma} = \sigma_m \underset{\sim}{I} + \underset{\sim}{\sigma}^{DEV} \quad ; \quad \sigma_m = \frac{tr\,\underset{\sim}{\sigma}}{3} \quad ; \text{ "HYDROSTATIC PRESSURE" } \in \mathbb{R}$$

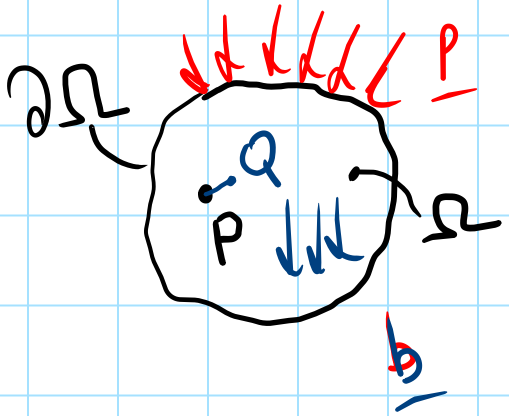
$$\underset{\sim}{\sigma}^{DEV} = \underset{\sim}{\sigma} - \sigma_m \underset{\sim}{I} \quad ; \quad tr\,\underset{\sim}{\sigma}^{DEV} = 0 \quad ; \quad tr\,\underset{\sim}{\sigma}^{DEV} = tr\,\underset{\sim}{\sigma} - \sigma_m 3 = tr\,\underset{\sim}{\sigma} - tr\,\underset{\sim}{\sigma} = 0$$

THIS DECOMPOSITION OF $\underset{\sim}{\sigma}$ IS FUNDAMENTAL TO FORMULATE FAILURE CRITERIA FOR MOST OF THE MATERIALS.

$$[\underset{\sim}{\sigma}] = \begin{bmatrix} 3 & 9 & 1 \\ 9 & 5 & 0 \\ 1 & 0 & -10 \end{bmatrix} MPa \Rightarrow \sigma_m = -\frac{2}{3} MPa \quad , \quad [\underset{\sim}{\sigma}^{DEV}] = \begin{bmatrix} 3-\left(-\frac{2}{3}\right) & 9 & 1 \\ 9 & 5-\left(-\frac{2}{3}\right) & 0 \\ 1 & 0 & -10-\left(-\frac{2}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 11/3 & & \\ & 17/3 & \\ & & -28/3 \end{bmatrix}$$

BALANCE EQUATIONS (EQUILIBRIUM, CHANGE IN $\underset{\sim}{\sigma}$ AT A CHANGE OF THE POINT)

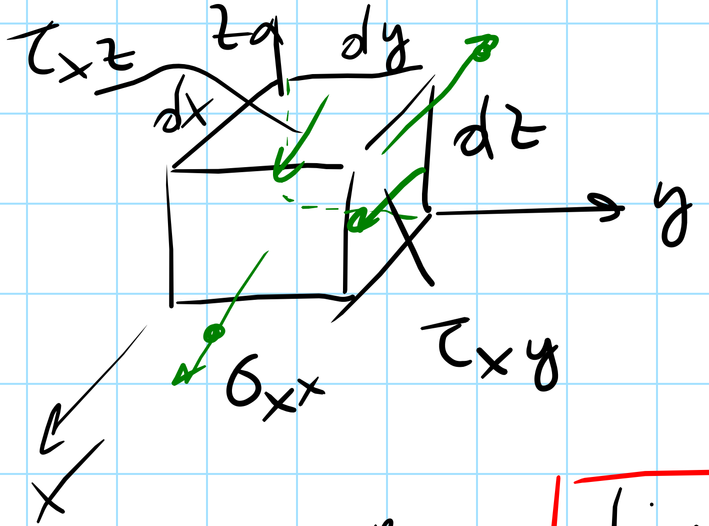


$$\underset{\sim}{\sigma}(P)$$

$$\underset{\sim}{\sigma}(Q) = \underset{\sim}{\sigma}(P) + d\underset{\sim}{\sigma}$$

$$\underline{t} = \underset{\sim}{\sigma}\,\underline{n}$$

SKETCH OF A PROOF:



3 SCALAR BALANCE EQS ALONG THE 3 AXES.

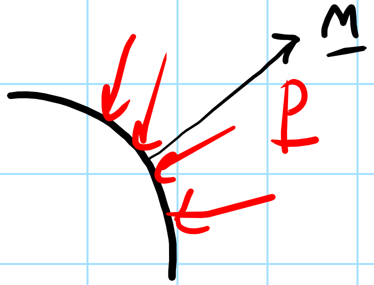
$$\begin{cases} \sigma_{xx,x} + \tau_{xy,y} + \tau_{xz,z} + b_x = 0 \\ \tau_{yx,x} + \sigma_{yy,y} + \tau_{yz,z} + b_y = 0 \\ \tau_{zx,x} + \tau_{zy,y} + \sigma_{zz,z} + b_z = 0 \end{cases}$$

GIVEN

OR $\boxed{\operatorname{div} \underset{\sim}{\sigma} + \underline{b} = \underline{0}}$ in Ω

UNKNOWN GIVEN

WHAT ABOUT B.C.'s?



$\boxed{\underset{\sim}{\sigma}\,\underline{n} = \underline{p}}$ on $\partial\Omega$

$$\begin{cases} \operatorname{div} \underset{\sim}{\sigma} + \underline{b} = \underline{0} & (\Omega) \\ \underset{\sim}{\sigma}\,\underline{n} = \underline{p} & (\partial\Omega) \end{cases}$$

$$(\underset{\sim}{\sigma} = \underset{\sim}{\sigma}^T)$$

(RECALL THAT $\underset{\sim}{\sigma} = \underset{\sim}{\sigma}^T$ FROM BALANCE OF MOMENTA)

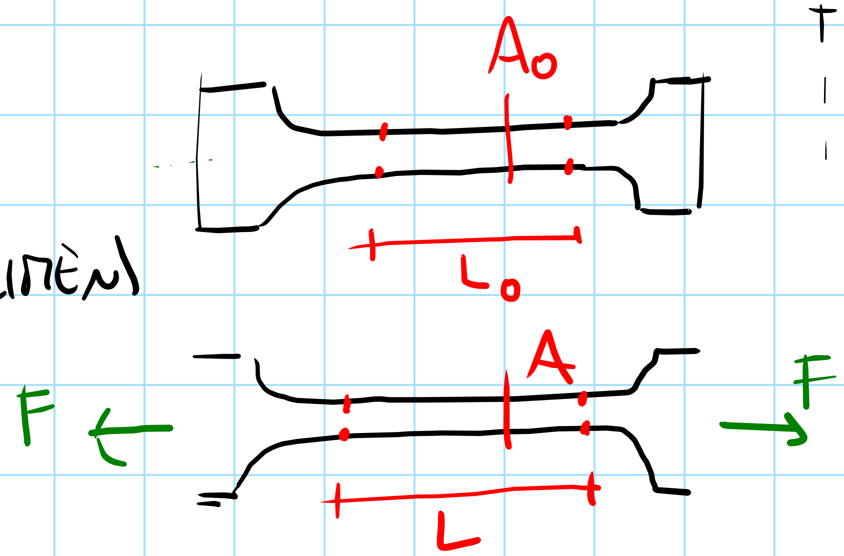
KINEMATICS ; STRAIN, DEFORMATION

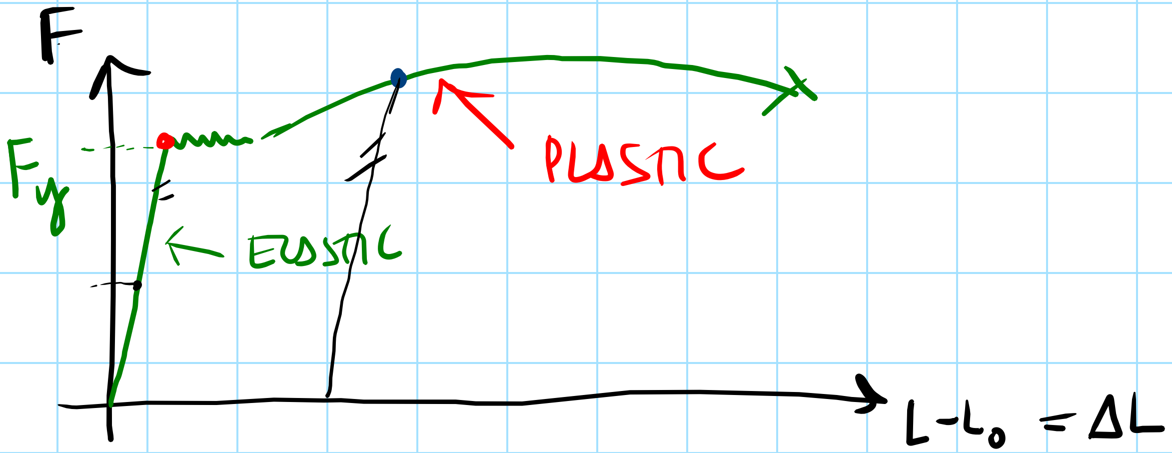
CONSTRUCTION STEEL $\sigma_y \simeq 300$ MPa

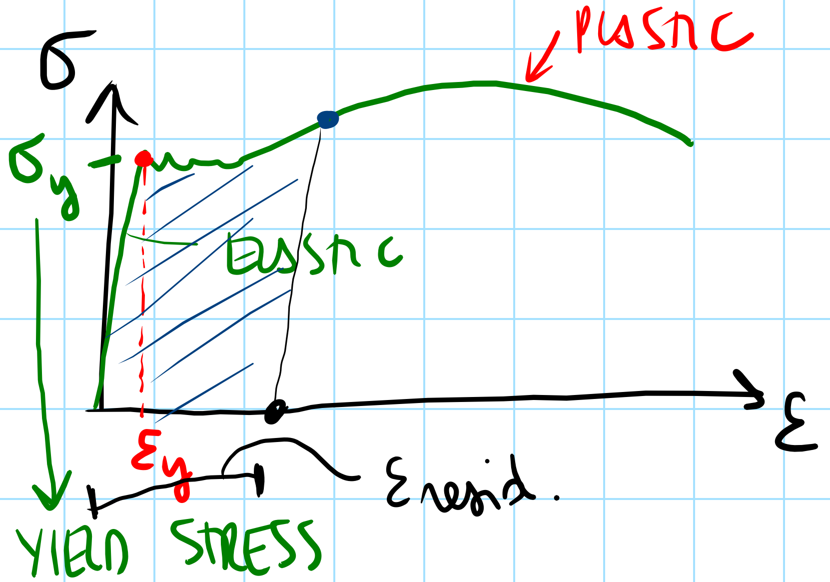
$\varepsilon_y \simeq 2‰$

UNIAXIAL TEST.

(STEEL SPECIMEN)



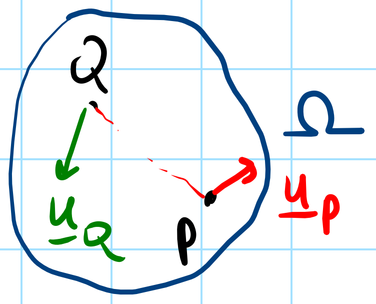




F_y: FORCE AT YIELDING

$$\sigma = \frac{F}{A} \simeq \frac{F}{A_0}$$

$$\varepsilon = \frac{\Delta L}{L_0}$$: LONGITUDINAL STRAIN [-]

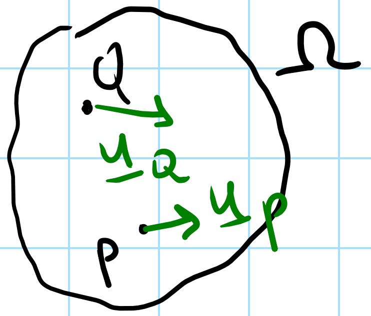


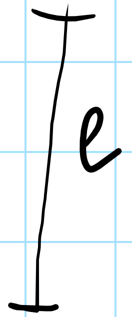
RIGID-BODY MOTION

$$\underline{u}_Q = \underline{u}_P + \underset{\sim}{W}\, PQ$$

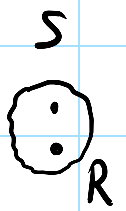
$\underset{\sim}{W} = -\underset{\sim}{W}^T$; ANTISYMMETRIC

HYPOTHESES OF ANALYSIS OF STRAIN

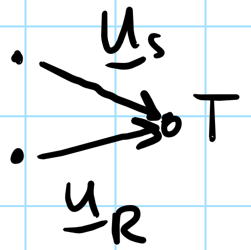




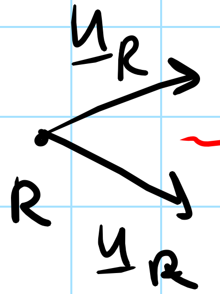
- SMALL DISPLACEMENTS $|\underline{u}_P| \ll \ell \quad \forall P$
- SMALL DISPLACEMENT GRADIENTS $|\nabla \underline{u}_P| \ll 1 \quad \forall P$



IN THE NEIGHBOOR...



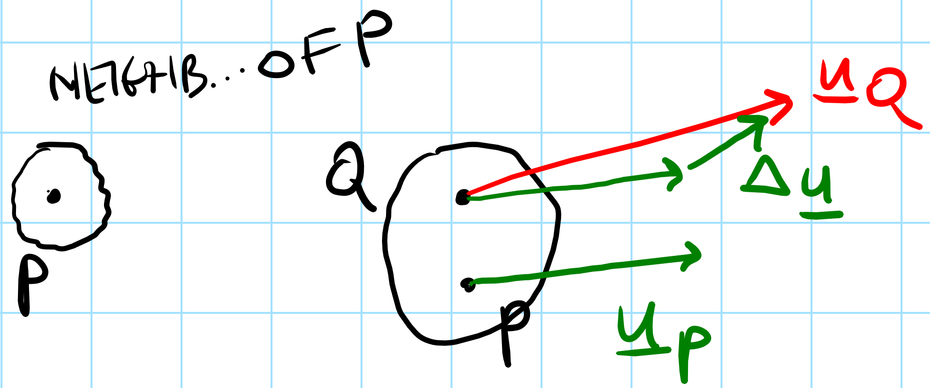
NO



CREATION OF A CRACK

NO

STRAIN AT A POINT $P \in \Omega$



$$\underline{u}_Q = \underline{u}_p + \Delta \underline{u}$$

TAYLOR EXPANSION

$$= u_p + \underbrace{\nabla \underline{u}(P)\, PQ}_{\text{LINEAR TERM}} + o(|PQ|)$$

u_p IS A DISPLACEMENT FIELD

SYM $\nabla \underline{u}_p$

SKEW $\nabla \underline{u}_p$

$$\underline{u}_Q \simeq \underline{u}_p + \nabla \underline{u}_p\, PQ = \underline{u}_p + (\underset{\sim}{\varepsilon} + \underset{\sim}{W})\, PQ$$

$$= u_p + \underbrace{\underset{\sim}{\varepsilon}\, PQ}_{\text{STRAIN}} + \underbrace{\underset{\sim}{W}\, PQ}_{\text{RIGID BODY}}$$

SMALL-

$\underset{\sim}{\varepsilon}$: STRAIN TENSOR

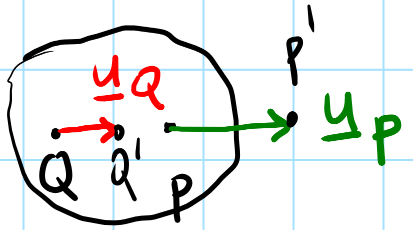
$(\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^T)$

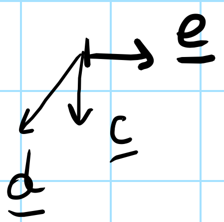
$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\varepsilon_{11} = u_{1,1}$$
$$\varepsilon_{22} = u_{2,2}$$
$$\varepsilon_{33} = u_{3,3}$$
$$\varepsilon_{13} = \frac{1}{2}(u_{1,3} + u_{3,1})$$

$$\underline{u}_q - \underline{u}_p = \underset{\sim}{\varepsilon}\, PQ$$ (RIGID-BODY MOTIONS APART)

MEANING OF THE COMPONENTS OF $\underset{\sim}{\varepsilon}$





$\{\underline{e}, \underline{c}, \underline{d}\}$ ORTHONORM BASIS

$$\underset{\sim}{\varepsilon} = \varepsilon_{11}\, \underline{e} \otimes \underline{e} + \varepsilon_{12}\, \underline{e} \otimes \underline{c} + \varepsilon_{13}\, \underline{e} \otimes \underline{d} \ldots\ldots$$

$|PQ| = L_0$

$|P'Q'| = L$

$|P'Q'| = 20 + 0.5 - 0.2$
$= 20.3$

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{0.3}{20} = +0.015$$

$\underline{u}_P = 0.5\, \underline{e}$

$\underline{u}_Q = 0.2\, \underline{e}$

$PQ = -20\, \underline{e}$

IN $[\underset{\sim}{\varepsilon}]$, DIAGONAL TERMS ARE THE DILATIONS ALONG THE AXIS OR VECTORS OF THE ORTHONORMAL BASIS.

$\underline{u}_Q - \underline{u}_P = \underset{\sim}{\varepsilon}\, PQ$

$\underline{e}(0.2 - 0.5) = \underset{\sim}{\varepsilon}\, \underline{e}\, (-20)$

$\underline{e}\, (-0.3) = \varepsilon_{11}\, \underline{e}\, (-20)$

$$\varepsilon_{11} = \frac{-0.3}{-20} = +0.015$$

$\varepsilon_{11} > 0$: ELONGATION ALONG DIRECTION 1

$\varepsilon_{11} < 0$: CONTRACTION ALONG DIRECTION 1