... MEANING OF OFF-DIAGONAL TERMS OF $\underset{\sim}{\varepsilon}$

14/3/25

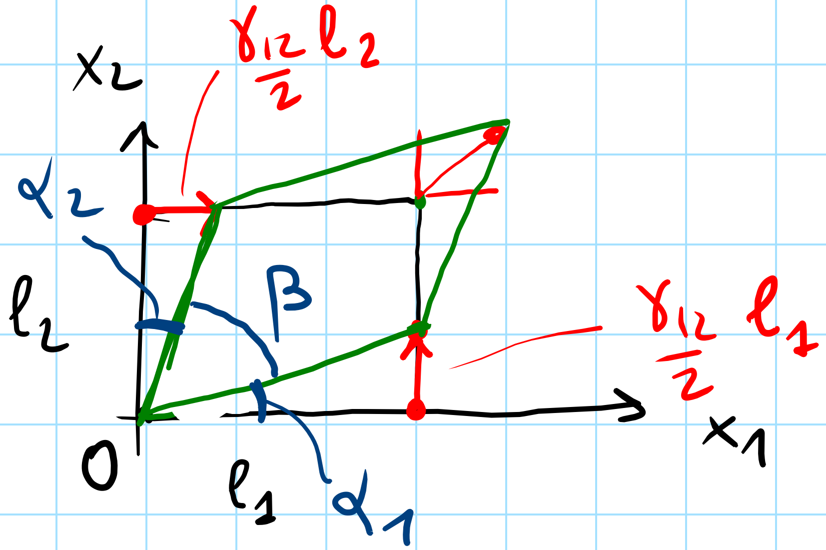
EX:
$$\begin{cases} u_1(x_1,x_2,x_3) = \dfrac{\gamma_{12}}{2} x_2 \\ u_2(\quad) = \dfrac{\gamma_{12}}{2} x_1 \\ u_3(\quad) = 0 \end{cases} \quad , \gamma_{12} > 0 , \gamma_{12} << 1$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$W_{ij} = \frac{1}{2}\left(\quad - \quad \right) = 0$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0 & \gamma_{12}/2 & 0 \\ \gamma_{12}/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq f(x_1,x_2,x_3)$$

HOMOGENEOUS STRAIN



$u_0 = 0$

γ_{12} GOVERNS THE CHANGE IN ANGLES (CHANGE IN SHAPE) OF PARTS OF THE NEIGHBO... OF A POINT

$$\beta + 2\alpha = \frac{\pi}{2} \Rightarrow \beta + \gamma_{12} = \frac{\pi}{2} \Rightarrow \gamma_{12} = \frac{\pi}{2} - \beta$$

γ_{12}: SHEAR STRAIN (SCORRIMENTO)

$\gamma_{12} > 0, \ \beta < \frac{\pi}{2}$

$\gamma_{12} < 0, \ \beta > \frac{\pi}{2}$

$$\alpha_1 \cong \frac{\gamma_{12}}{2}\frac{\ell_1}{\ell_1} = \frac{\gamma_{12}}{2} = \alpha$$

$$\alpha_2 \cong \frac{\gamma_{12}}{2}\frac{\ell_2}{\ell_2} = \frac{\gamma_{12}}{2}$$

$$\begin{cases} u_1 = 0.005 \ x_2 \\ u_2 = \quad '' \quad x_1 \end{cases} \Rightarrow \beta = \frac{\pi}{2} - \gamma_{12} = \frac{\pi}{2} - 0.01$$

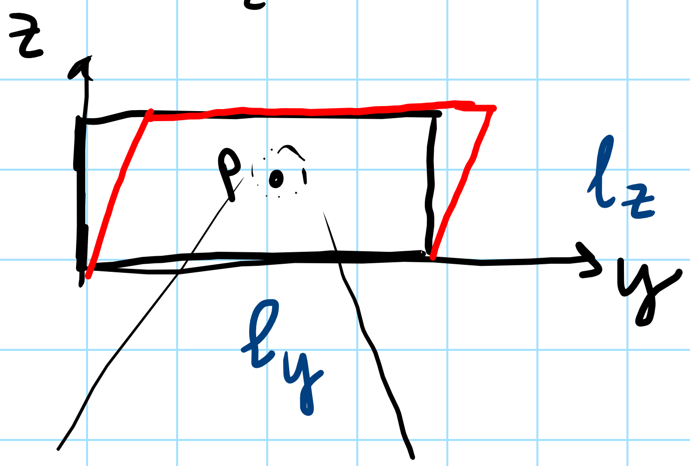
$$= 1.57 - 0.01 \Rightarrow 89^{\circ}.43$$

EX : SIMPLE SHEAR

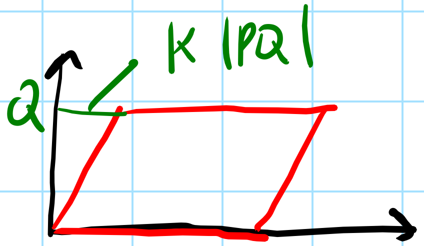
$$\begin{cases} u_x = 0 \\ u_y = Kz \\ u_z = 0 \end{cases} \qquad (0 < K << 1)$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}K \\ 0 & \frac{1}{2}K & 0 \end{bmatrix} \; ; \; [\underset{\sim}{W}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}K \\ 0 & -\frac{1}{2}K & 0 \end{bmatrix}$$

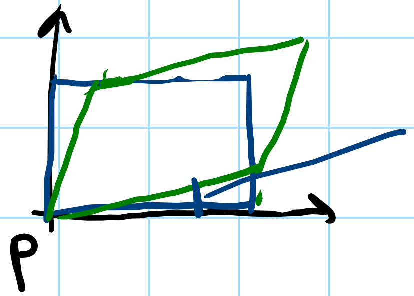
$\underset{\sim}{\varepsilon}, \underset{\sim}{W}$ CONSTANT IN Ω



① + ②



①
$\underset{\sim}{\varepsilon}$

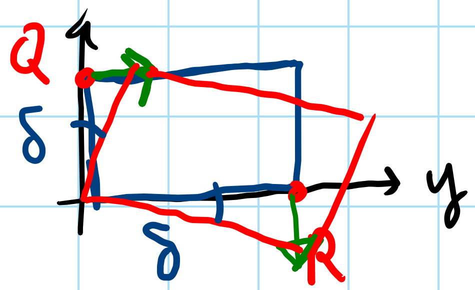


$\alpha = \frac{K}{2}$

$$\underline{u}_Q - \underline{u}_P = \cdots + \begin{bmatrix} & & \\ & & K/2 \\ & -K/2 & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ |PQ| \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{K}{2}|PQ| \\ 0 \end{bmatrix}$$

②
$\underset{\sim}{W}$



RIGID BODY MOTION

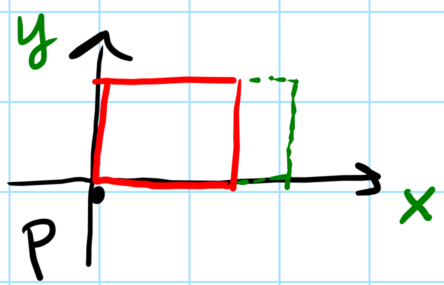
$$\underline{u}_R - \underline{u}_P = \cdots + \underset{\sim}{W}\, PR$$

$$'' = \cdots + \begin{bmatrix} & & \\ & & K/2 \\ & -K/2 & \end{bmatrix} \begin{bmatrix} 0 \\ |PR| \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{K}{2}|PR| \end{bmatrix}$$

$$\delta = \frac{K}{2} |PR| \frac{1}{|PR|} = \frac{K}{2}$$

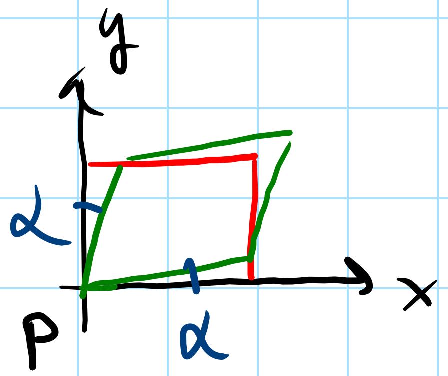
OTHER RELEVANT DEFORMATION / STRAIN STATES

- SIMPLE EXTENSION / ELONGATION



$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- PURE SHEAR



$$[\underset{\sim}{\varepsilon}_1] = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

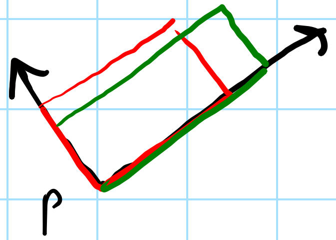
- PLANE STRAIN (x,y)

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(DEFORMAZ. PIANA)

WE DO NOT TALK ABOUT "PRINCIPAL STRAINS" (EIGENVALUES OF $\underset{\sim}{\varepsilon}$) AND "PRINCIPAL DIRECTIONS OF STRAINS" (EIG. VECT)

PURE SHEAR IS EQUIVALENT TO



$$\text{eig}\,[\underset{\sim}{\varepsilon}_1] = \begin{matrix} +* \\ -* \\ 0 \end{matrix}$$

A NOTE ON THE RELATIONSHIP BETWEEN $\underline{u}(P)$ AND $\underset{\sim}{\varepsilon}(P)$

$\underline{u}(P)$ KNOWN $\Longrightarrow$ $\underset{\sim}{\varepsilon}(P) = \frac{1}{2} \operatorname{SYM} \nabla \underline{u}(P)$

3 COMPONENTS

6 COMPONENTS

BUT IN A "SIMULATION" WE COMPUTE FIRST $\underset{\sim}{\varepsilon}$ AND THEN, IF NEEDED, COMPONONETS OF $\underline{u}(P)$ (THROUGH AN "INTEGRATION")

$$\underset{\sim}{\varepsilon} \longrightarrow \underline{u}$$

6 COMPONENTS

3 COMPONENTS

MATH. THEORY HAS SHOWN THAN

ε_{ij} MUST SATISFY :

$$\boxed{\operatorname{rot} \operatorname{rot} \underset{\sim}{\varepsilon} = \underset{\sim}{0}}$$

BIANCHI'S IDENTITY

"INTERNAL COMPATIBILITY"

VIRTUAL WORK THEOREM (FUNDAMENTAL IDENTITY OF MECHANICS)

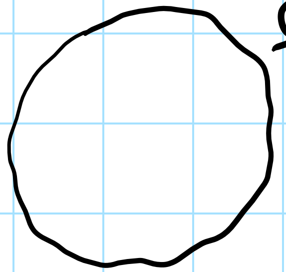
- STATICS (a)

Ω : IN EQUILIBRIUM

$\exists\ \underset{\sim}{\sigma}^a(P)$:
$$\begin{cases} \operatorname{div} \underset{\sim}{\sigma}^a + \underline{b}^a = \underline{0} & \text{in } \Omega \\ \underset{\sim}{\sigma}^a \underline{n} = \underline{p}^a & \text{on } \partial\Omega \end{cases}$$

$\{\underline{b}^a, \underline{p}^a, \underset{\sim}{\sigma}^a(P)\}$ IS A STATICALLY ADMISSIBLE SYSTEM

- KINEMATICS (b)



Ω

$\underline{u}^b(P)$

$\underset{\sim}{\varepsilon}^b(P) = \frac{1}{2}\left(\nabla \underline{u}^b + \nabla \underline{u}^{bT}\right)$

$\{\underline{u}^b, \underset{\sim}{\varepsilon}^b\}$ IS A KINEMATICALLY ADMISS. SYSTEM.

SYSTEMS (a) AND (b) ARE COMPLETELY INDEP.

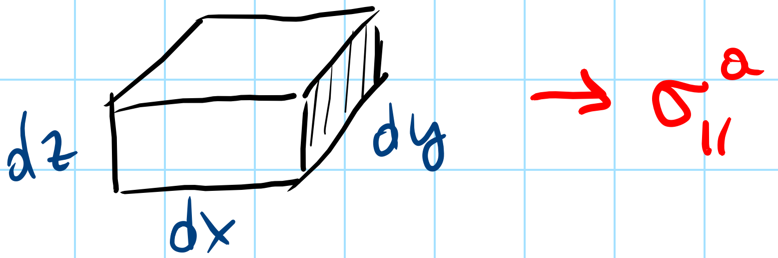
THEOREM: GIVEN A BODY Ω, A SET $\{\underline{b}^a, \underline{p}^a, \underset{\sim}{\sigma}^a\}$ STAT. ADM., A SET $\{\underline{u}^b, \underset{\sim}{\varepsilon}^b\}$ KINEM. ADM. THEN THE IDENTITY $\boxed{L_{v\,ext} = L_{v\,int}}$ WHERE

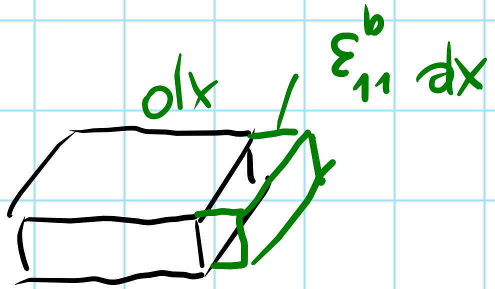
$$L_{v\,ext} = \int_{\Omega} \underline{b}^a \cdot \underline{u}^b \, dV + \int_{\partial\Omega} \underline{p}^a \cdot \underline{u}^b \, dS$$

$$L_{v\,int} = \int_{\Omega} \underset{\sim}{\sigma}^a \cdot \underset{\sim}{\varepsilon}^b \, dV$$

- NOTE THAT FOR RIGID BODIES $\left(\underset{\sim}{\varepsilon}^b = \underset{\sim}{0}\right)$ $L_{v\,int} = 0 \Rightarrow$ IDENITY $\boxed{L_{v\,ext} = 0}$

- DIMENSIONS OF $\sigma_{11}^a \, \varepsilon_{11}^b \, dV$?





WORK: $F \cdot \text{displ} = \sigma_{11}^a \, dy\,dz \cdot \varepsilon_{11}^b \, dx = \sigma_{11}^a \, \varepsilon_{11}^b \, dx\,dy\,dz = \sigma_{11}^a \, \varepsilon_{11}^b \, dV$

GREEN'S LEMMA : $\int_{\partial\Omega} \underset{\sim}{A}\,\underline{n}\cdot\underline{u}\,dS = \int_{\Omega} \operatorname{div}\underset{\sim}{A}\cdot\underline{u} + \underset{\sim}{A}\cdot\nabla\underline{u}\,dV$

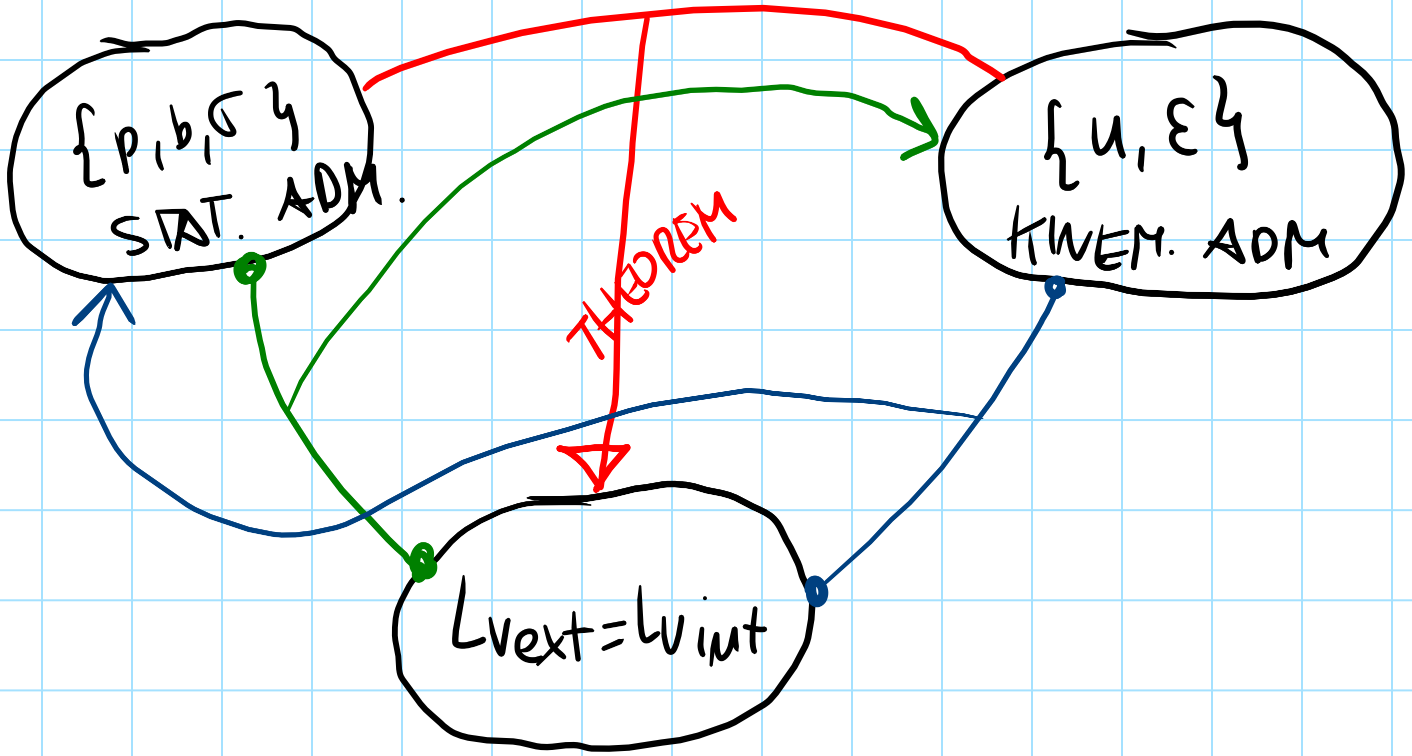
PROOF :

$$\int_{\Omega} \underline{b}\cdot\underline{u}\,dV + \int_{\partial\Omega} \underline{f}\cdot\underline{u}\,dS = \int_{\Omega} \qquad + \int_{\partial\Omega} \underset{\sim}{\sigma}\,\underline{n}\cdot\underline{u}\,dS = \int_{\Omega} \underline{b}\cdot\underline{u}\,dV + \int_{\Omega} \operatorname{div}\underset{\sim}{\sigma}\cdot\underline{u} + \int_{\Omega} \underset{\sim}{\sigma}\cdot\nabla\underline{u}\,dV$$

$$= \int_{\Omega} \underset{\sim}{\sigma}^{a}\cdot\nabla\underline{u}^{b}\,dV \quad ; \quad \underset{\sim}{\sigma}^{a} \text{ is SYM} \quad \Longrightarrow \quad = \int_{\Omega} \underset{\sim}{\sigma}^{a}\cdot\underset{\sim}{\varepsilon}^{b}\,dV$$

↳ SYM $\nabla\underline{u}^{b}$

CHART OF VIRTUAL WORK PRINCIPLE



- NO ASSUMPTIONS ON BEHAVIOUR OF THE MATERIAL

- BASIC TOOL TO FORMULATE STRUCTURAL MODELS IN ENGINEERING.