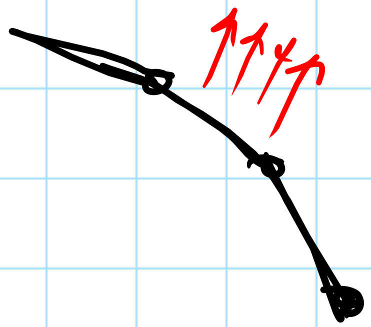
RECALL THAT: $\Pi(\underline{u}) = \frac{1}{2}\int_\Omega \tilde{\varepsilon}\cdot \tilde{C}\,\tilde{\varepsilon}\, dV - \int_{\partial\Omega} \underline{p}\cdot\underline{u}\, dS - \int_\Omega \underline{b}\cdot\underline{u}\, dV$

24/3/25



$$\tilde{\Pi} = \sum_{e=1}^{m} \Big(\overbrace{\frac{1}{2}\int_{\Omega_e} \tilde{B}\underline{D}_e \cdot \tilde{C}\tilde{B}\underline{D}_e\, dV - \int_{\partial\Omega_e} \underline{p}\cdot \tilde{N}\underline{D}_e\, dS - \int_{\Omega_e} \underline{b}\cdot\tilde{N}\underline{D}_e\, dV}^{\tilde{\Pi}_e} \Big)$$

$$\delta\tilde{\Pi} = \frac{\partial\tilde{\Pi}}{\partial\underline{D}}\delta\underline{D} = \sum_{e=1}^{m} \boxed{\frac{\partial\tilde{\Pi}_e}{\partial\underline{D}_e}}\,\delta\underline{D} = 0 \quad ; \quad \boxed{\frac{\partial\tilde{\Pi}_e}{\partial\underline{D}_e} = \underline{0}} \quad e = 1,\dots,m$$

$$\Phi = \frac{1}{2}\tilde{A}\underline{D}_e\cdot\underline{D}_e$$

$$\frac{\partial\Phi}{\partial\underline{D}_e} = \tilde{A}\underline{D}_e$$

$$\frac{\partial\tilde{\Pi}_e}{\partial\underline{D}_e} = \underbrace{\int_{\Omega_e} \tilde{B}^T\tilde{C}\tilde{B}\, dV}_{\text{STIFFNESS MATRIX } \tilde{K}_e}\,\underline{D}_e - \underbrace{\int_{\partial\Omega_e} \tilde{N}^T\underline{p}\, dS}_{\text{EQUIV. SURFACE FORCE VECTOR}} - \underbrace{\int_{\Omega_e} \tilde{N}^T\underline{b}\, dV}_{\text{EQUIVALENT FREE-BODY FORCE VECTOR}} = \underline{0} \rightarrow \tilde{K}_e\underline{D}_e - \underline{P}_e - \underline{B}_e = \underline{0}$$

DIMENSIONS CHECK

$\tilde{K}_e$: $3m\times 3m$: $(3m\times 6)(6\times 6)(6\times 3m)$

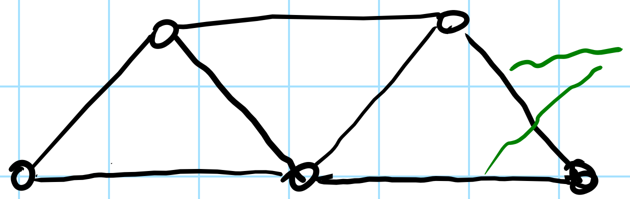
$\underline{P}_e, \underline{B}_e$: $3m$: $(3m\times 3)\,3$

VERY IMPORTANT: $\tilde{K}_e$: SYMMETRIC ($\tilde{C}$ IS SYMM.)

$\tilde{K}_e$: POSITIVE DEFINITE ($\tilde{C}$ IS POSIT. DEFINITE)

$\tilde{K}_e$ HAS GOT THE PROPERTIES OF $\tilde{C}$

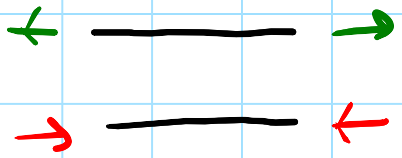
TRUSS STRUCTURES (PLANE)



MEMBERS / BARS

NODES (HINGES)

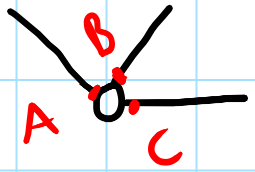
BEHAVIOUR OF A BAR



TIE-ROD (TIRANTE)

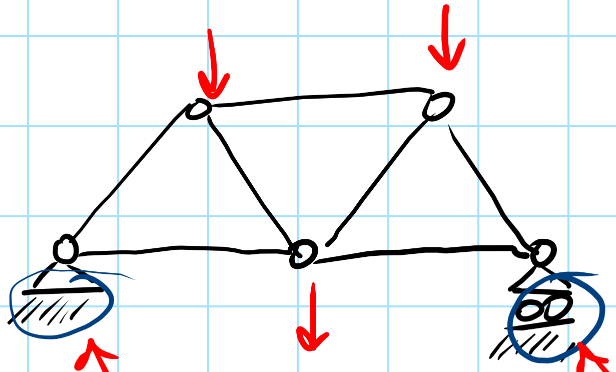
STRUT (PUNTONE)

HINGE: CERNIERA



$\underline{u}_A = \underline{u}_B = \underline{u}_C$

ROTATIONS AT THE HINGE ARE FREE FOR EACH BAR



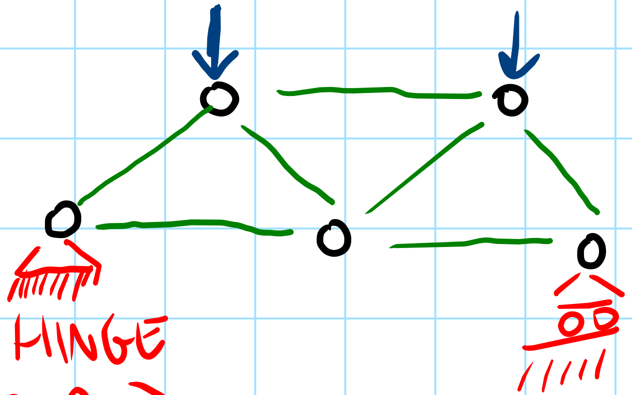
LOADS: FORCES APPLIED TO NODES

EXTERNAL CONSTRAINTS

SUPPORT, ROLLER

EXTERNAL HINGE

DEGREES OF FREEDOM OF A TRUSS STRUCTURE



HINGE (−2)

ROLLER (−1)

5 x 2 = 10 dofs

7 bars = −7 dofs

] 3 dofs remaining (RIGID BODY IN THE PLANE)

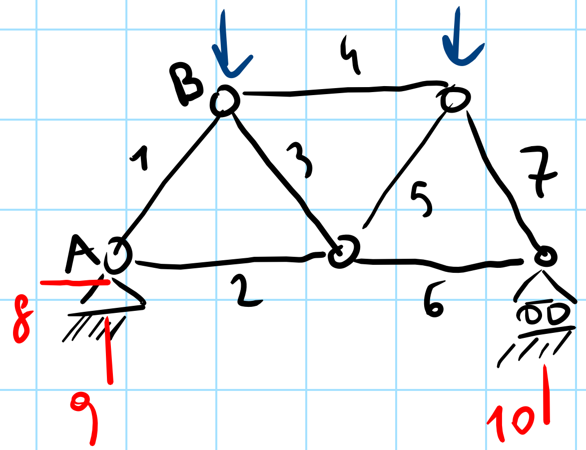
EXTERN CONSTRAINTS = −3 dofs

0 dofs : MINIMUM No. OF CONSTRAINTS

STRUCTURE IS FIXED

ISOSTATIC STRUCTURE

HOW MANY UKNOWNS FOR THE PROBLEM IN THE PREVIOUS PAGE?



2 TYPES: → NODAL DISPLACEMENTS (KINEMATICS) (a)

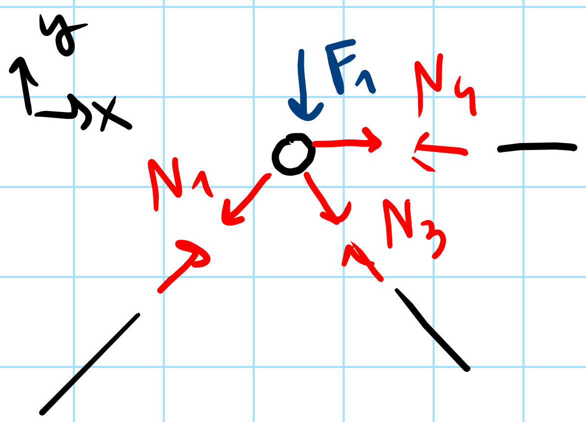
→ FORCES IN BARS AND EXTERNAL REACTIONS (STATICS) (b)

FOCUS ON (b): HOW MANY? 10

FOR EACH NODE WE CAN WRITE 2 SCALAR EQUATIONS → 10 EQS TOTAL

FOR AN ISOSTATIC STRUCTURE THE STATIC UKNOWNS CAN BE DETERMINED ONLY FROM THE MAXIMAL SYSTEM OF EQUILIBRIUM EQUATIONS

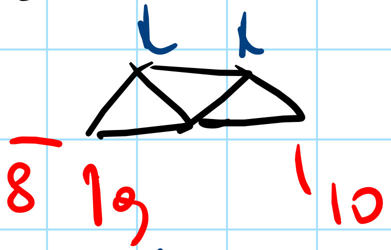
(EX: NODE B:



$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
)

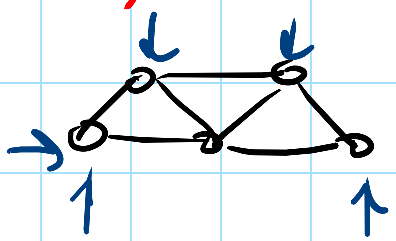
USUALLY, FOR AN ISOST. STRUCT., WE CAN FIRST SOLVE EXTERNAL CONSTRAINTS (3), THEN, 2° STEP, SOLVE INTERNAL UKNOWNS (7).

1°



3 GLOBAL EQUATIONS → 8, 9, 10

2°



7 REMAINING EQS → 1, 2, ..., 7

TAKE OUT MESSAGE FROM ISOST. STRUCTURES:

m_N : No. OF NODES
m_A : No. OF BARS
v_E : No. OF EXTERNAL CONSTRAINTS

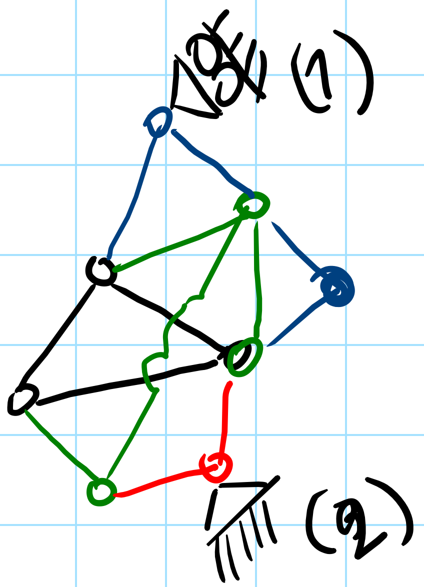
$\Rightarrow$ A NECESSARY CONDITION FOR A STRUCT. TO BE ISOSTATIC IS

MAXWELL RULE

$$\boxed{2m_N = m_A + v_E}$$

ISOSTATIC STRUCTURES ARE ALSO CALLED "DETERMINED" STRUCTURES

A TRICK TO BUILD A RIGID BODY COMPOSED OF NODES AND BARS



$m_N : 8$

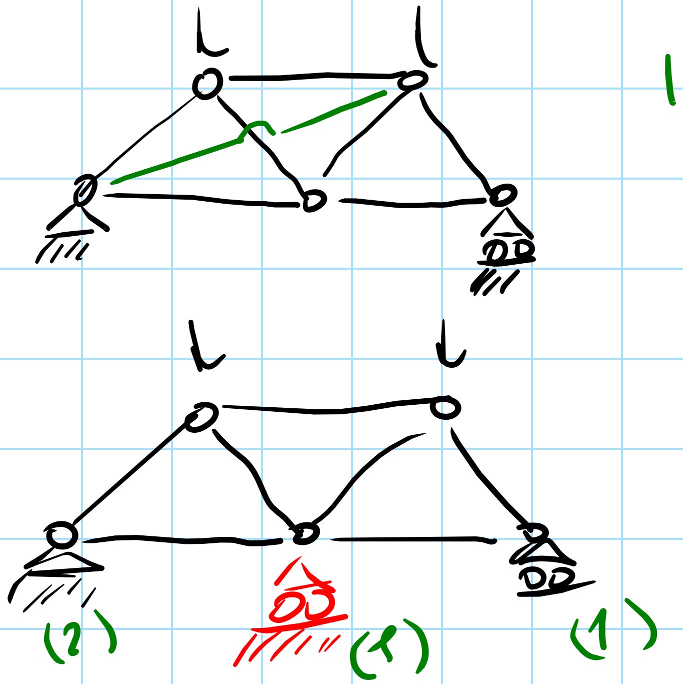
$m_A : 13$

$v_E : 3$

$16 = 13+3$

OK

OVERCONSTRAINED STRUCTURES — IPERSTATIC STR / REDUNDANT STR.



ISOSTATIC STR + INTERNAL BAR

$10 < 8+3$

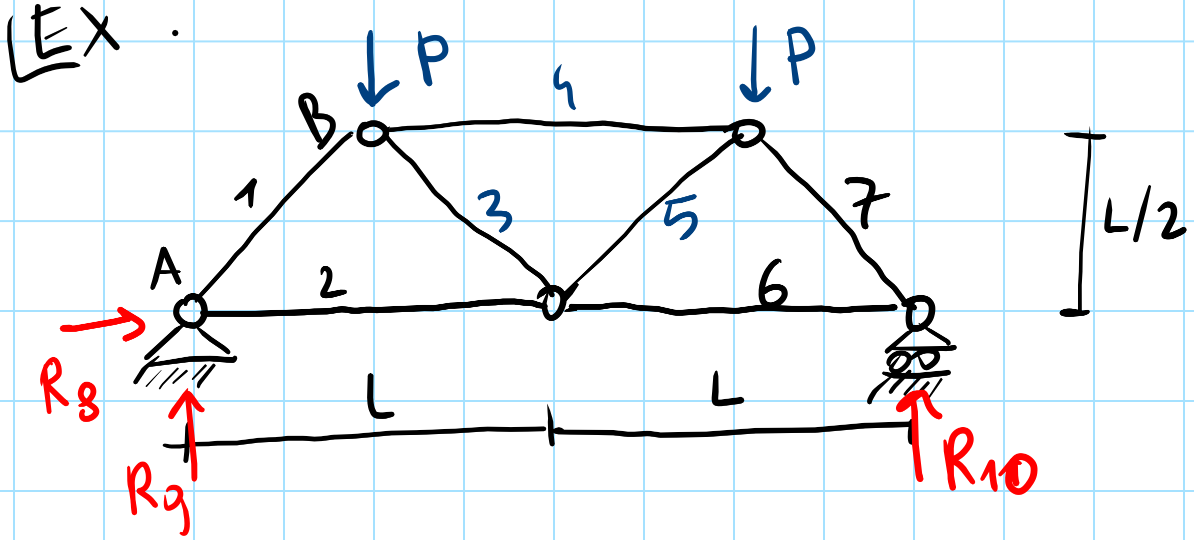
ISOST. STR. + EXTERNAL CONSTRAINT

$10 < 7+4$

IN BOTH CASES I HAVE 11 UNKNOWS $\rightarrow$ 10 EQUILIBR. EQS

+1 ?? MUST INVOLVE ELASTICITY OF THE SYSTEM

1 EX .



RESULTS:

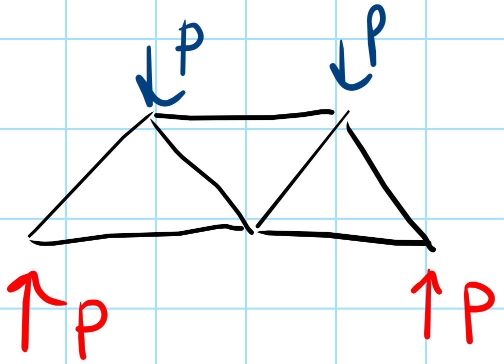
$N_1 = -P\sqrt{2}$: STRUT

$N_2 = +P$: TIE-ROD

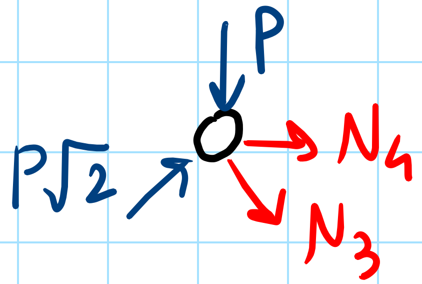
$N_6 = +P$: " "

$N_7 = -P\sqrt{2}$: STRUT

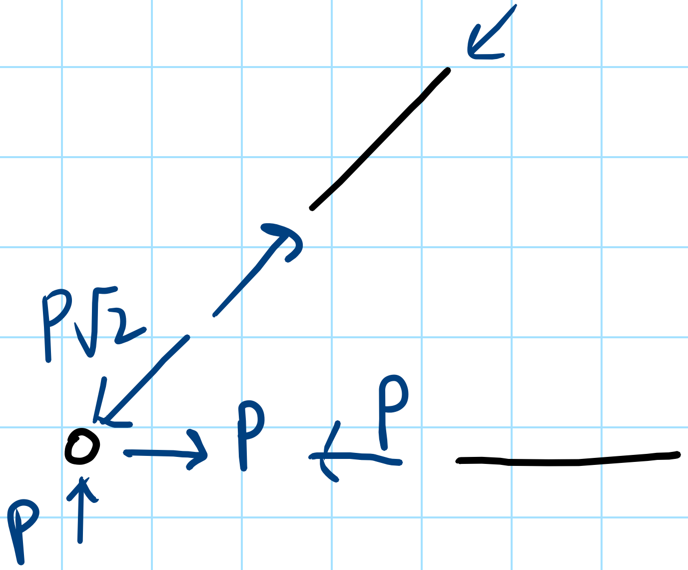
1°)



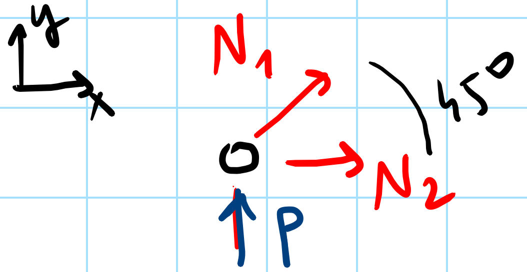
3°) NODE B



..... WE CAN SOLVE



2°) NODE A



$$\begin{cases} \Sigma F_x = 0 : N_2 + N_1 \frac{1}{\sqrt{2}} = 0 \\ \Sigma F_y = 0 : P + N_1 \frac{1}{\sqrt{2}} = 0 \end{cases}$$

$N_1 = -P\sqrt{2}$; $N_2 = +P$