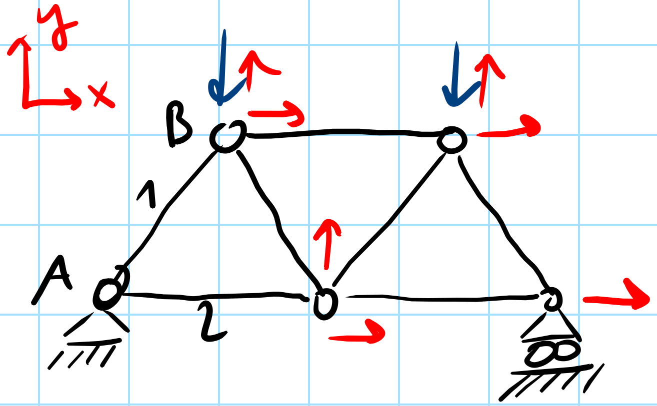
25/03/25

TRUSS STRUCTURES: SOLUTION BASED ON "METHOD OF DISPLACEMENTS"



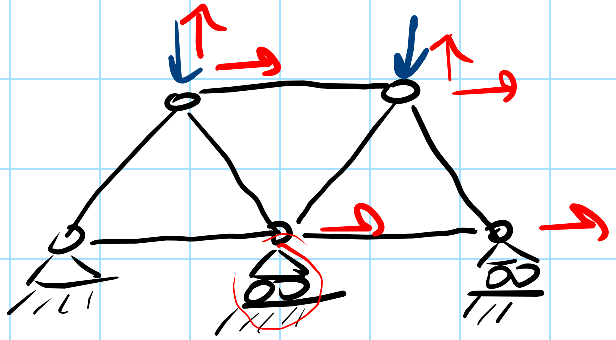
NODAL DISPLACEMENTS ARE NOW THE UNKNOWNS

(7 IN OUR EXERCISE). IN ORDER TO SOLVE THE PROBLEM, WE CAN WRITE A SYSTEM OF THE 7 EQUIL. EQS ASSOCIATED WITH THE DIRECTIONS OF THE UNKNOWNS. TO ELIMINATE FORCES OF BARS WE INTRODUCE THE ELASTIC CONSTITUTIVE LAW $(N_i = f(u_j))$

$\Rightarrow$ SYSTEM AS A FUNCTION OF THE 7 DISPLACEMENTS.

THERE ARE STILL 3 EQUIL EQS THAT ALLOW THE CALCULATION OF THE 3 EXTERNAL REACTIONS.

THE ADVANTAGE OF THE METHOD IS THAT IT CAN BE APPLIED IN THE SAME WAY FOR A REDUNDANT STRUCTURE:

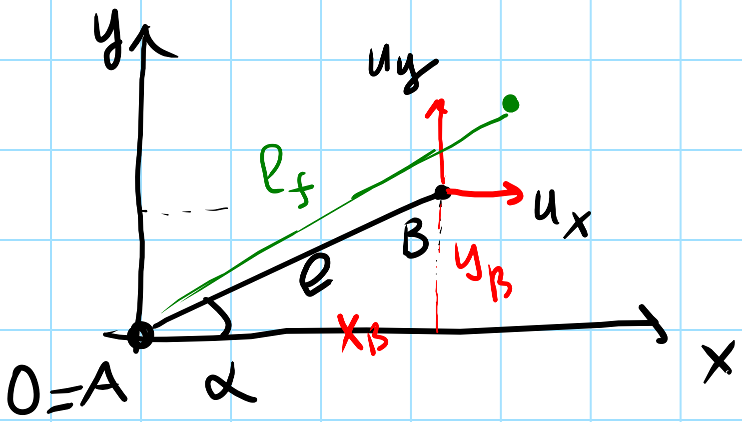


6 UNKNOWNS $\Rightarrow$ 6 EQUIL. EQS + CONSTITUTIVE LAW

LATER

4 EXTERNAL REACTIONS WITH THE REMAINING 4 EQUIL. EQS

INTRODUCTION OF DISPLACEMENT METHODS FOR TRUSS STRUCTURES



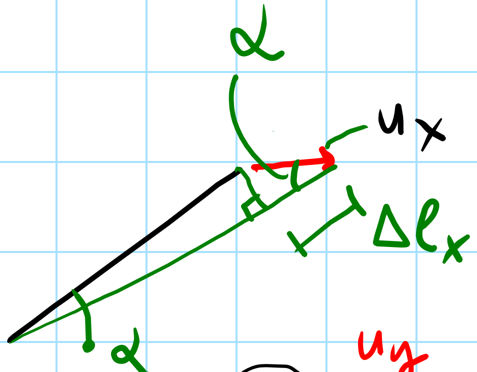
$$\ell = \sqrt{x_B^2+y_B^2} \quad ; \quad \ell_f = \sqrt{(x_B+u_x)^2+(y_B+u_y)^2} = \ell_f(u_x,u_y)$$

$$\Delta\ell = \ell_f - \ell = \Delta\ell_x + \Delta\ell_y$$

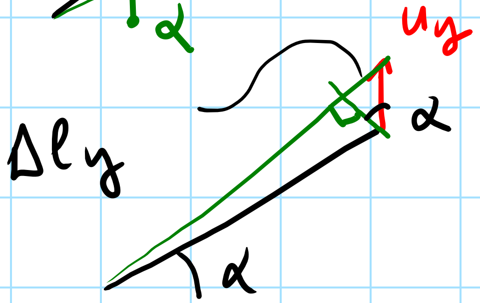
EXPAND ℓ_f TO THE FIRST ORDER IN u_x AND u_y

$$\ell_f = \ell_f(0,0) + \left.\frac{\partial \ell_f}{\partial u_x}\right|_{(0,0)} u_x + \left.\frac{\partial \ell_f}{\partial u_y}\right|_{(0,0)} u_y + O\left(\sqrt{u_x^2+u_y^2}\right)$$

$$\ell_f \cong \ell + \left[\frac{1}{\cancel{2}\sqrt{\ }}\cancel{2}(x_B+u_x)\right]_{(0,0)} u_x + \left[\frac{1}{\cancel{2}\sqrt{\ }}\cancel{2}(y_B+u_y)\right]_{(0,0)} u_y = \ell + \underbrace{\overset{\cos\alpha}{\frac{x_B}{\ell}}\, u_x}_{\Delta\ell_x} + \underbrace{\overset{\sin\alpha}{\frac{y_B}{\ell}}\, u_y}_{\Delta\ell_y}$$



$$\Delta\ell_x = u_x \cos\alpha$$



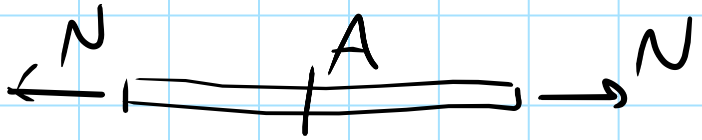
$$\Delta\ell_y = u_y \sin\alpha$$

$$\sigma = E\varepsilon$$

$$\frac{N}{A} = E\frac{\Delta\ell}{\ell}$$

$$N = \frac{EA}{\ell}\Delta\ell$$

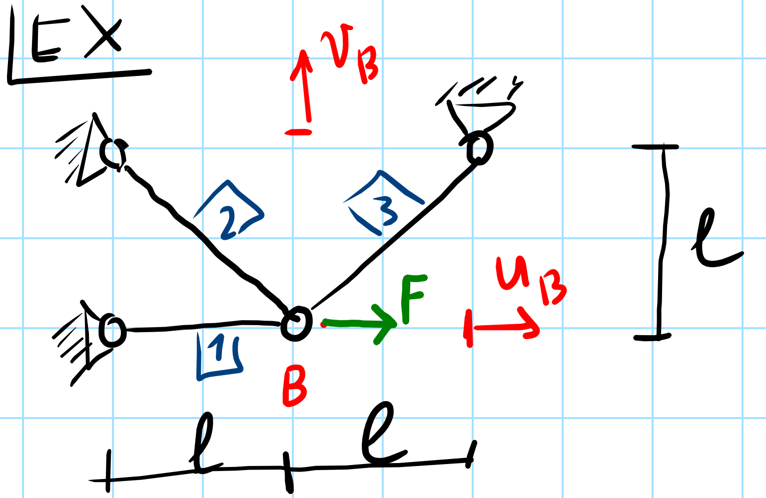
K: EQUIV. SPRING STIFFNESS



RECALL THAT THE CONSTITUTIVE EQUATION FOR A BAR IS

$$N = \frac{EA}{\ell}\Delta\ell$$

EX



E: CONST

$A_1 = A$; $A_2 = A_3 = \sqrt{2}\,A$

$\ell_1 = \ell$; $\ell_2 = \ell_3 = \sqrt{2}\,\ell$

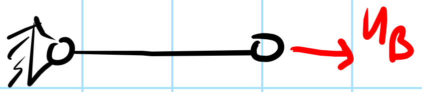
u_B, v_B: UNKNOWNS

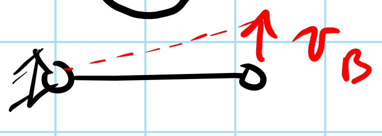
CONSTITUTIVE EQS: $N_i = N_i(u_B, v_B)$ $\quad K = EA/\ell$

$$N_1 = \frac{EA}{\ell}\Delta\ell_1 \;;\; N_2 = \frac{E\cancel{\sqrt{2}}A}{\cancel{\sqrt{2}}\,\ell}\Delta\ell_2 \;;\; N_3 = \frac{E\cancel{\sqrt{2}}A}{\cancel{\sqrt{2}}\,\ell}\Delta\ell_3$$

u_B $\qquad$ v_B

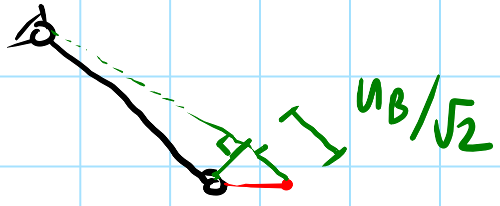
$\Delta\ell_1)$

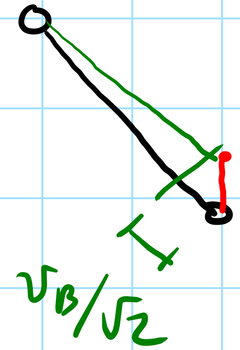




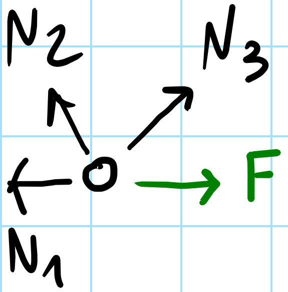
$\Delta\ell_1 = +u_B$

$\Delta\ell_2)$

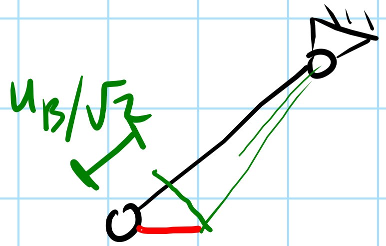


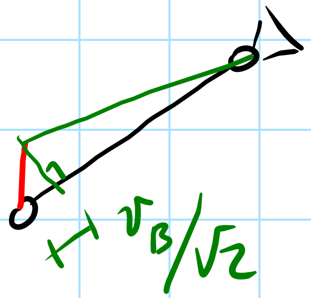


$$\Delta\ell_2 = +\frac{u_B}{\sqrt{2}} - \frac{v_B}{\sqrt{2}}$$



$\Delta\ell_3)$





$$\Delta\ell_3 = -\frac{u_B}{\sqrt{2}} - \frac{v_B}{\sqrt{2}}$$

2 EQUIL. EQS

$$\overset{+}{\rightarrow}:\; -N_1 - N_2\frac{1}{\sqrt{2}} + N_3\frac{1}{\sqrt{2}} + F = 0$$

$$+\uparrow:\; N_2\frac{1}{\sqrt{2}} + N_3\frac{1}{\sqrt{2}} = 0$$

$$-K\,u_B - \frac{K}{\sqrt{2}\sqrt{2}}(u_B - v_B) + \frac{K}{\sqrt{2}\sqrt{2}}(-u_B - v_B) + F = 0$$

$$\frac{K}{2}(u_B - v_B) + \frac{K}{2}(-u_B - v_B) = 0$$

$$-K u_B - \frac{K}{2}(u_B - v_B) - \frac{K}{2}(u_B + v_B) = -F \; ;$$

$$\frac{K}{2}(u_B - v_B) - \frac{K}{2}(u_B + v_B) = 0$$

$$\begin{bmatrix} -2K & 0 \\ 0 & -K \end{bmatrix} \begin{bmatrix} u_B \\ v_B \end{bmatrix} = \begin{bmatrix} -F \\ 0 \end{bmatrix} \; ; \quad u_B = \frac{F}{2K}, \quad v_B = 0$$

$$\Downarrow$$

$$\underbrace{\begin{bmatrix} 2K & 0 \\ 0 & K \end{bmatrix}}_{\text{STIFFNESS MATRIX } [K]} \begin{bmatrix} u_B \\ v_B \end{bmatrix} = \begin{bmatrix} +F \\ 0 \end{bmatrix}$$

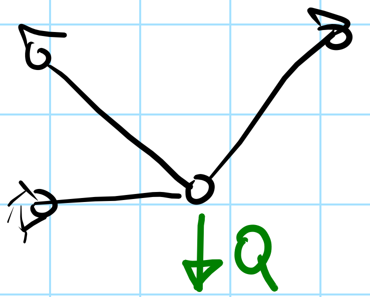
(SYMMETRIC AND POSITIVE DEFINITE)

$$N_1 = K u_B = F/2$$

$$N_2 = \frac{K}{\sqrt{2}}(u_B - v_B) = \frac{F}{2\sqrt{2}}$$

$$N_3 = -\frac{K}{\sqrt{2}}(u_B + v_B) = -\frac{F}{2\sqrt{2}}$$

COMMENT: (1)



SYSTEM

$$\begin{bmatrix} 2K & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} u_B \\ v_B \end{bmatrix} = \begin{bmatrix} 0 \\ -Q \end{bmatrix}$$

SOLUTION $u_B = 0$, $v_B = -\frac{Q}{K}$ $\begin{pmatrix} N_1 = 0 \\ \vdots \end{pmatrix}$

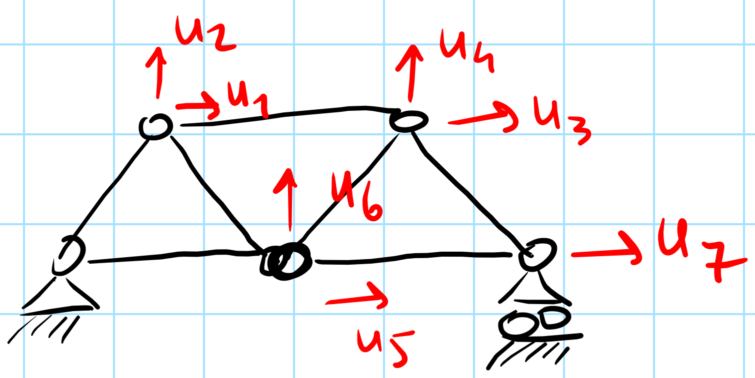
(2) MEANING OF K_{11}

first line of the system: $K_{11} u_B + K_{12} v_B = F$

TAKE $u_B = 1$ AND $v_B = 0$ $\Rightarrow K_{11} = F$

TAKE $u_B = 0$ AND $v_B = 1$ $\Rightarrow K_{12} = F$

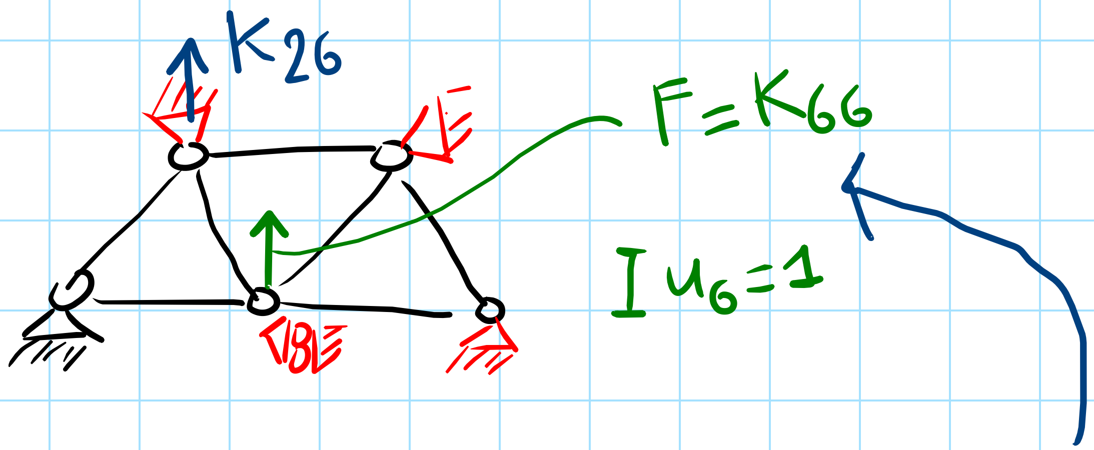
K_{ij} IS THE "FORCE" ASSOCIATED WITH DOF i WHEN DOF $j = 1$ AND THE REMAING DOFS ARE ALL NULL.



K_{66} ?

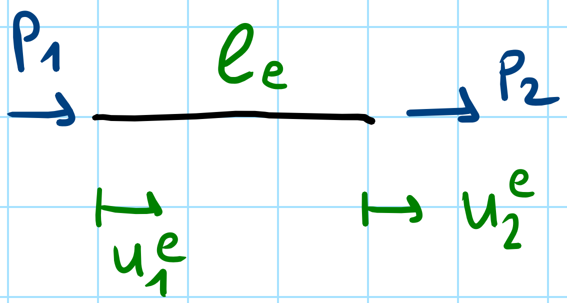
$u_6 = 1$

$u_i \ (i \neq 6) = 0$



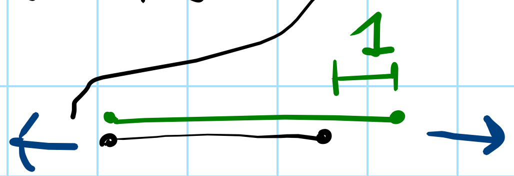
K_{26} : REACTION ALONG DIRECTION 2 OF THE LOADED SYSTEM

LOCAL STIFFNESS MATRIX OF A BAR



$u_2^e = 1$

$u_1^e = 0$



$K_{12} = -\frac{EA}{\ell_e} \cdot 1$

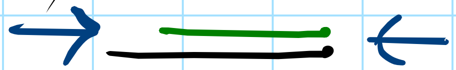
$K_{22} = \frac{EA}{\ell_e} \cdot 1$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}$$

$$= \underbrace{\frac{EA}{\ell_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\text{LOCAL STIFFNESS MATRIX } \underset{\sim}{K}_e}$$

$u_1^e = 1$

$u_2^e = 0$



$K_{11} = \frac{EA}{\ell_e} 1$

$K_{21} = -\frac{EA}{\ell_e} 1$