Titolo:

$$[M] = [X]^T \underline{\underline{[B]}} [X]$$

Affinché si abbia una matrice $[M]$ diagonale si considera uno smorzamento proporzionale

$$[B] = 2\alpha [C] + 2\beta [A] \longrightarrow \text{la matrice } [M] \text{ sarà diagonale}$$

$$m_{ij} = \begin{cases} 0 & i \neq j \\ b_i^* & i = j \end{cases} \qquad b_i = 2\alpha k_i^* + 2\beta m_i^*$$

- $m_i^* \ddot{p}_i + b_i^* \dot{p}_i + k_i^* p_i = 0 \qquad \omega_i^2 = \dfrac{k_i^*}{m_i^*}$

$$\downarrow$$

$$\ddot{p}_i + 2\nu_i^* \omega_i \dot{p}_i + \omega_i^2 p_i = 0 \qquad \nu_i^* = \frac{b_i^*}{2\sqrt{k_i^* m_i^*}} = \alpha \omega_i + \frac{\beta}{\omega_i}$$

Soluzione $\nu^* < 1$

$$p_i(t) = e^{-\nu_i^* \omega_i t} \left(A_i \operatorname{sen} \Omega_i t + B_i \cos \Omega_i t \right) \qquad \Omega_i = \omega_i \sqrt{1 - \nu_i^{*2}}$$

– Come ricavare α e β

$\times \quad \nu_i^* = \alpha \omega_i + \dfrac{\beta}{\omega_i}$

Risolviamo una struttura 4 GL

$\omega_1 = 38{,}16$ r/s $\qquad \omega_2 = 98{,}63$ rad/s

$\omega_3 = 140{,}6$ rad/s $\qquad \omega_4 = 204{,}05$ rad/s

Fissiamo per 2 modi i e $j \longrightarrow \nu_i^*$ e ν_j^*

$$\nu_1^* = \nu_3^* = 5\% \qquad \left.\begin{aligned} &\bullet\ 38{,}16^2 \alpha + \beta = 0{,}05 \cdot 38{,}16 \\ &\bullet\ 140{,}6^2 \alpha + \beta = 0{,}05 \cdot 140{,}6 \end{aligned}\right\} \begin{aligned} &\alpha = 2{,}7 \cdot 10^{-4} \\ &\beta = 1{,}514 \end{aligned}$$

Titolo:

$\nu_2^* = 4,2\%$ $\nu_4^* = 6,34\%$

Oscillazioni prodotte da forze pulsanti

$$[A]\{\ddot{q}\} + [b]\{\dot{q}\} + [C]\{q\} = \{F\}\sin\bar{\omega}t \qquad \{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_n \end{Bmatrix}$$

costante

Premoltiplichiamo per $[X]^T$

$$\{F^*\}\sin\bar{\omega}t = [X]^T\{F\}\sin\bar{\omega}t$$

$$f_i^*\sin\bar{\omega}t = \{\varphi^i\}^T\{F\}\sin\bar{\omega}t$$

- $m_i^*\ddot{p}_i(t) + b_i^*\dot{p}_i(t) + k_i^* p_i(t) = f_i^*\sin\bar{\omega}t$

$$p_{p_i}(t) = p_{st_i} N_i \,\text{sen}(\bar{\omega}t - \varphi_i)$$

$$p_{st_i} = \frac{f_i^*}{k_i^*} \qquad N_i = \frac{1}{\sqrt{(1-\beta_i^2)^2 + (2\nu_i^*\beta_i)^2}} \qquad \beta_i = \frac{\bar{\omega}}{\omega_i} \qquad \nu_i^* = \frac{b_i^*}{2\sqrt{k_i^* m_i^*}}$$

$$p_i(t) = p_{o_i}(t) + p_{p_i}(t) \qquad p_i(t) \simeq p_{p_i}(t)$$

transitorio

$$\{q\} = \sum_{i=1}^{n}\{\varphi^i\}p_i(t) \simeq \sum_{i=1}^{n}\{\varphi^i\}p_{st_i}N_i\,\text{sen}(\bar{\omega}t - \varphi_i)$$

trascurato il regime transitorio

Titolo:

Spostamento variabile della base

$y(t) = \bar{y} \operatorname{sen} \bar{\omega} t \qquad \ddot{y}(t) = -\bar{\omega}^2 \bar{y} \operatorname{sen} \bar{\omega} t$

Spostamento alla base

Masse saranno soggette $\{\ddot{q}(t)\} + \{1\} \ddot{y}(t)$

$[A]\left(\{\ddot{q}(t)\} + \{1\}\ddot{y}(t)\right) + [B]\{\dot{q}(t)\} + [C]\{q(t)\} = \{0\}$

$[A]\{\ddot{q}(t)\} + [B]\{\dot{q}(t)\} + [C]\{q(t)\} = -[A]\{I\}\ddot{y}(t)$

$[A]\{\ddot{q}(t)\} + [B]\{\dot{q}(t)\} + [C]\{q(t)\} = \bar{\omega}^2 [A]\{I\}\bar{y} \operatorname{sen} \bar{\omega} t$

$\{F\} = \bar{\omega}^2 [A]\{1\}\bar{y}$

Forza di trascinamento

- Forza variabile nel tempo $\{F\} s(t)$

$[A]\{\ddot{q}\} + [b]\{\dot{q}\} + [C]\{q\} = \{F\} s(t)$

$[L]\{\ddot{p}\} + [M]\{\dot{p}\} + [N]\{p\} = [X]^T\{F\} s(t)$

$m_i^* \ddot{p} + b_i^* \dot{p}_i + k_i^* p_i = f_i^* s(t)$

$\{q_0\} = \{0\} \rightarrow \{p_0\} = \{0\}$

$\{\dot{q}_0\} = \{0\} \quad \{\dot{p}_0\} = \{0\}$

$p_i(t) = \dfrac{f_i^*}{m_i^* \Omega_i} \int_0^t s(t_1) e^{-\nu_i \omega_i (t - t_1)} \sin \Omega_i (t - t_1)\, dt_1$

Titolo:

Spostamento della base sia $y(t) = s(t)$

$$[A]\{\ddot{q}\} + [b]\{\dot{q}\} + [c]\{q\} = -[A]\{1\}\ddot{s}(t)$$

$$[L]\{\ddot{p}\} + [M]\{\dot{p}\} + [N]\{p\} = -[X]^T[A]\{1\}\ddot{s}(t)$$

- $m_i^* \ddot{p} + b_i^* \dot{p}_i + k_i^* p_i = -\{\psi^i\}^T[A]\{1\}\ddot{s}(t)$

  dividiamo per m_i^*

$$\ddot{p}_i(t) + 2r_i^* \omega_i \dot{p}_i + \omega_i^2 p_i = -\frac{\{\psi^i\}^T[A]\{1\}}{\underbrace{\{\psi^i\}^T[A]\{\psi^i\}}_{m_i^*}}\ddot{s}(t)$$

$$\ddot{p}_i(t) + 2r_i^* \omega_i \dot{p}_i + \omega_i^2 p_i = -g_i\ddot{s}(t)$$

$$g_i = \frac{\{\psi^i\}^T[A]\{1\}}{\{\psi^i\}^T[A]\{\psi^i\}}$$ Fattore di partecipazione

g_i si riducono al crescere dell'indice i

$$\{q(t)\} = \Sigma\{\psi^i\}p_i(t)$$ Ridurre la sommatoria ai modi significativi

- Forzante generica $\{F(t)\}$

$$[A]\{\ddot{q}\} + [b]\{\dot{q}\} + [c]\{q\} = \{F(t)\} \qquad \vec{F} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ \vdots \\ F_m(t) \end{Bmatrix}$$

Titolo:

Soluzione

$$m_i^* \ddot{p}_i + b_i^* \dot{p}_i + k_i^* p = \sum_{j=1}^{n} \psi_j^i F_j(t)$$

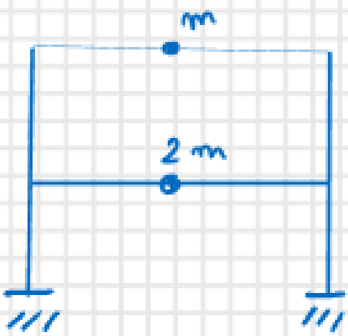
$$p_i(t) = p_{i,1}(t) + p_{i,2}(t) + \dots + p_{i,n}(t)$$

- $p_{i,j}$ soluzione dell'equazione i esima dovuta al j esimo termine

$$p_{i,j}(t) = \frac{\psi_j^i}{m_i^* \Omega_i^*} \int_0^t F_j(t) e^{-\nu_i^* \omega_i (t-t_1)} \sin \Omega_i (t-t_1)\, dt_1$$

$$\{q\} = [X]\{p\}$$

Es



$$\underbrace{\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}}_{[A]} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}}_{[C]} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \{0\}$$

$$X = \begin{array}{c} \begin{bmatrix} 1 & 1 \\ \sqrt{2} & -\sqrt{2} \end{bmatrix} \\ \psi^1 \quad \psi^2 \end{array}$$

$$L = [X]^T [A] [X] = \begin{bmatrix} 4m & 0 \\ 0 & 4m \end{bmatrix}$$

$$N = [X]^T [C] [X] = \begin{bmatrix} 1{,}17k & 0 \\ 0 & 6{,}82k \end{bmatrix}$$

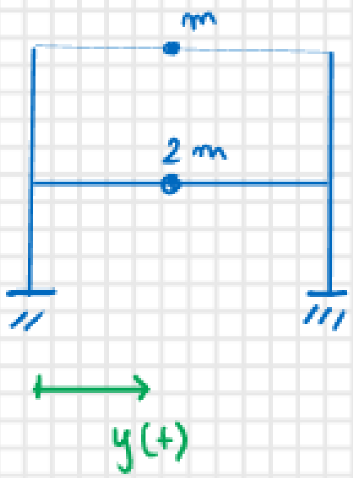
Titolo:

$$4m\,\ddot{p}_1 + 1{,}17K\,p_1 = 0 \longrightarrow \omega_1^2 = \frac{1{,}17K}{4m}$$

$$4m\,\ddot{p}_2 + 6{,}82K\,p_2 = 0$$

$$m = 10 \qquad K = 1000 \qquad \omega_1^2 = 29{,}29$$

Imprimiamo uno spostamento alla base y(t)



$$[A]\{\ddot{q}\} + [C]\{q\} = -[A]\{1\}\ddot{y}(t) = \{F(t)\}$$

$$[L]\{\ddot{p}\} + [N]\{p\} = -[X]^T[A]\{1\}\ddot{y}(t)$$

$$\{F(t)\} = -[A]\{1\}\ddot{y}(t) = -\begin{Bmatrix} 2m\,\ddot{y}(t) \\ m\,\ddot{y}(t) \end{Bmatrix}$$

$$\{\bar{F}(t)\} = -[X]^T[A]\{1\}\ddot{y}(t) = [X]^T\{F(t)\} = \begin{Bmatrix} -3{,}41\,m\,\ddot{y}(t) \\ -0{,}58\,m\,\ddot{y}(t) \end{Bmatrix}$$

$$\begin{cases} 4m\,\ddot{p}_1 + 1{,}17K = -3{,}41\,m\,\ddot{y}(t) \\ 4m\,\ddot{p}_2 + 6{,}82K = -0{,}58\,m\,\ddot{y}(t) \end{cases}$$

Sistema disaccoppiato

* Autovettore normalizzato

$$\{\psi^i\}\{\psi^i\} = 1 \longrightarrow \{\tilde{\psi}^i\} = \frac{1}{|\{\psi^i\}|}\{\psi^i\}$$

Modulo

$$\{\psi^1\} = \begin{Bmatrix} 1 \\ \sqrt{2} \end{Bmatrix} \qquad \{\tilde{\psi}^1\} = \begin{Bmatrix} 1/\sqrt{3} \\ \frac{\sqrt{2}}{\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} 0{,}577 \\ 0{,}8164 \end{Bmatrix}$$