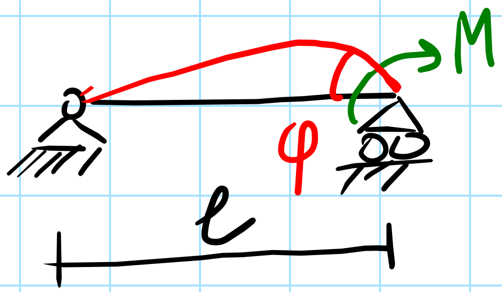
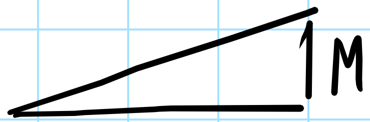
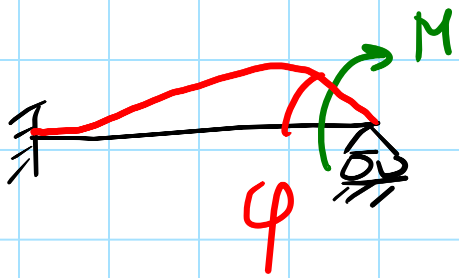
RELEVANT SCHEMES FOR DISPLACEMENT METHODS



$$\varphi = \frac{Ml}{3EJ} \quad , \quad M = \frac{3EJ}{l}\varphi$$

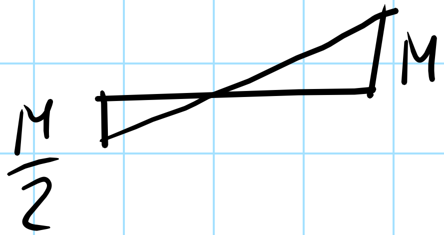
M

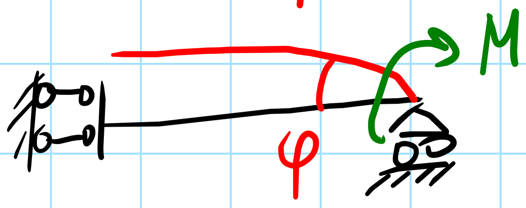




$$\varphi = \frac{Ml}{4EJ} \quad ; \quad M = \frac{4EJ}{l}\varphi$$

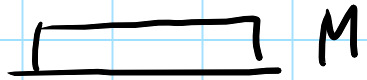
M

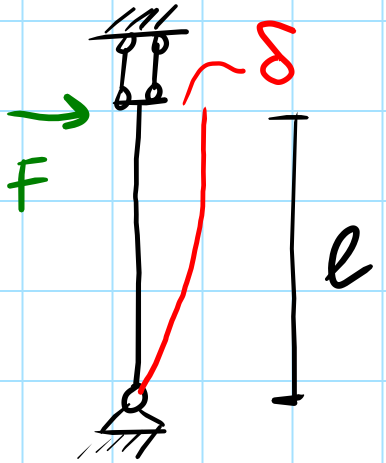




$$\varphi = \frac{Ml}{EJ} \quad ; \quad M = \frac{EJ}{l}\varphi$$

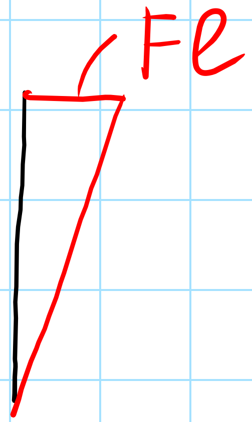
M



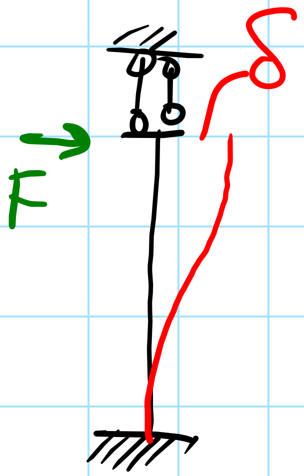


$$\delta = \frac{Fl^3}{3EJ}$$

$$F = \frac{3EJ}{l^3}\delta$$

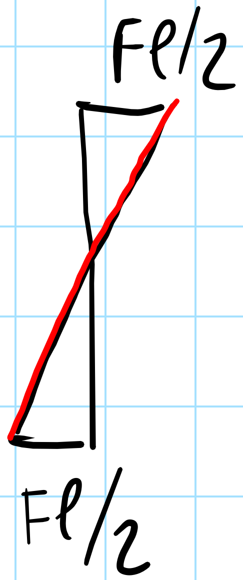


M



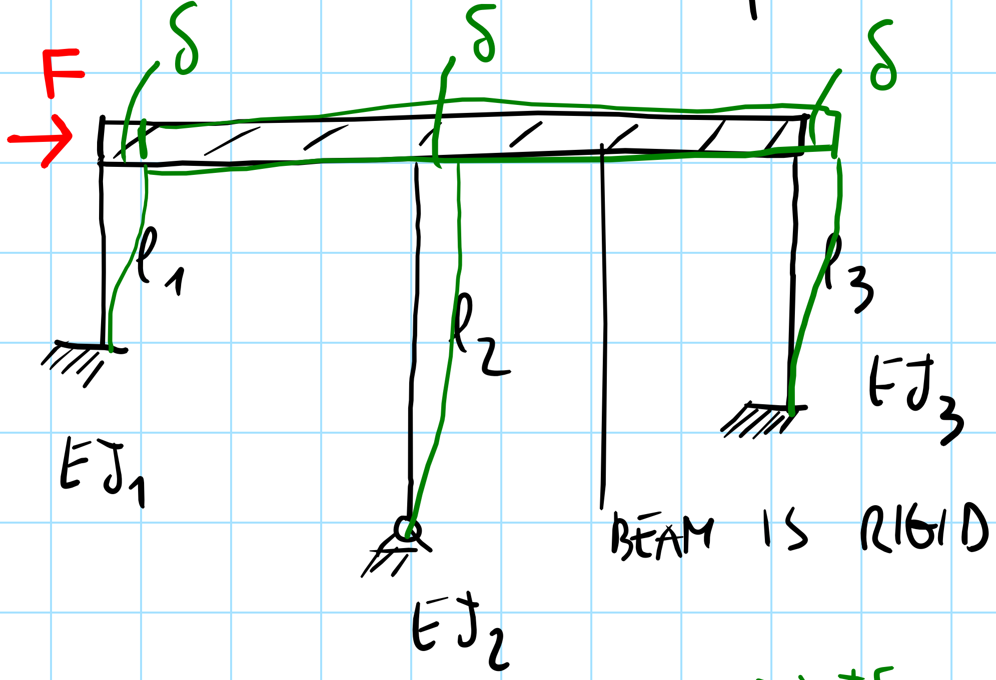
$$\delta = \frac{Fl^3}{12EJ}$$

$$F = \frac{12EJ}{l^3}\delta$$



M

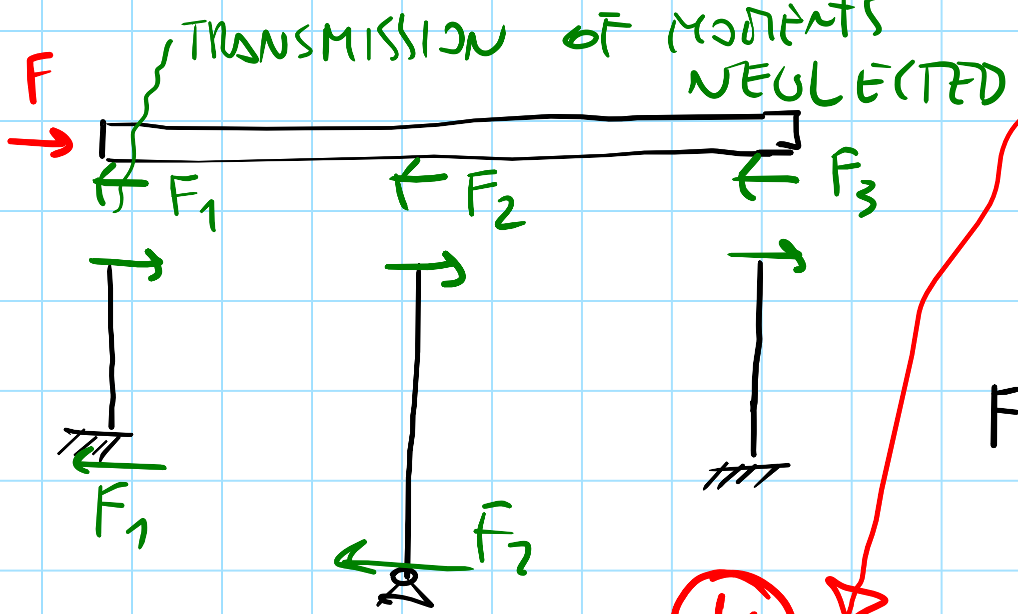
SHEAR-TYPE FRAME (REVIEW OF DISPLACEMENT METHOD)



δ: 1 D.O.F. OF THE STRUCTURE

? $F = K\delta$

? $(EJ_1, \dots, l_1 \dots)$



EQUIL. OF BEAM:

$$F - F_1 - F_2 - F_3 = 0 \Rightarrow \delta!$$

$$F - \frac{12EJ_1}{l_1^3}\delta - \frac{3EJ_2}{l_2^3}\delta - \frac{12EJ_3}{l_3^3}\delta = 0$$; δ IS THE UNKNOWN!

$$F = \overbrace{\left(\frac{12EJ_1}{l_1^3} + \frac{3EJ_2}{l_2^3} + \frac{12EJ_3}{l_3^3}\right)}^{K}\delta$$

STIFFNESS OF THE STRUCTURE

HOW TO COMPUTE $F_1, F_2, \dots$

$$F_1 = \frac{12EJ_1}{l_1^3}\underbrace{\frac{F}{K}}_{\delta} = \rho_1 F$$

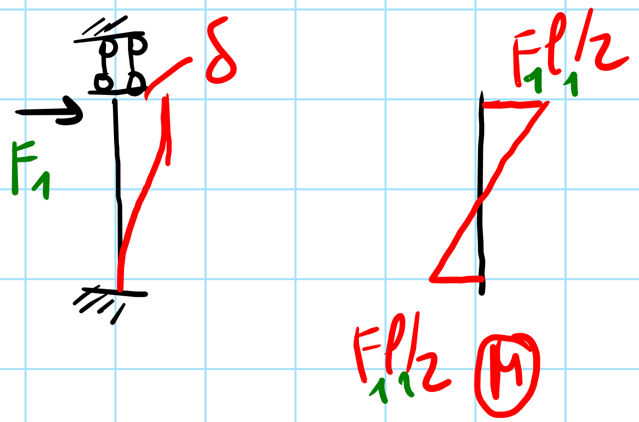
COEFF. OF DISTRIB.

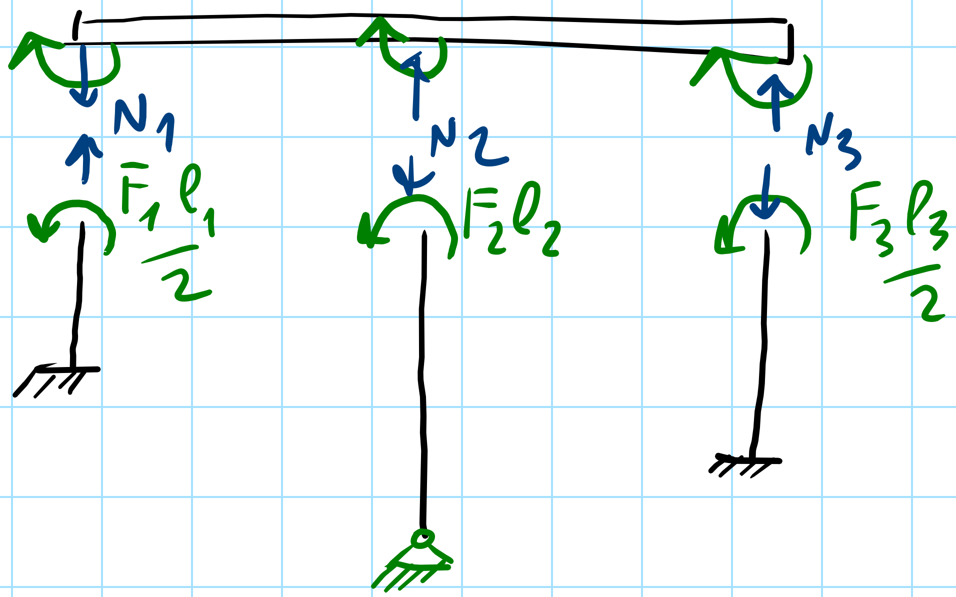
$$\rho_1 = \frac{12EJ_1}{l_1^3}\frac{1}{K}$$

$F_2 = \rho_2 F$; $F_3 = \rho_3 F$

$$\boxed{\rho_1 + \rho_2 + \rho_3 = 1}$$

WHAT ABOUT THE MOMENTS AT THE CONNECTING SECTIONS?





UP TO NOW, WE KNOW:

- M, T IN THE PILLARS
- N IN THE BEAM

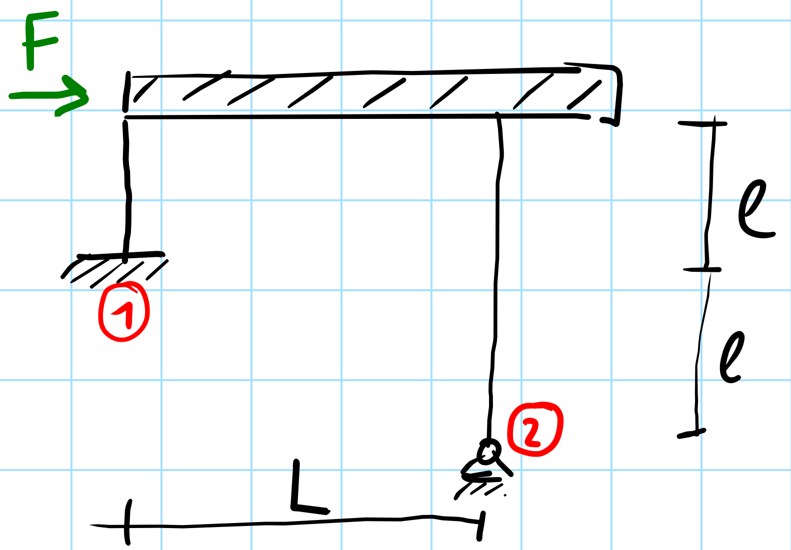
NOTE THAT THE BEAM IS NOT IN EQUIL!! (ROTATION) $\Rightarrow$ WE HAVE NOT CONSIDERED AXIAL FORCES IN PILLARS!

THE PROBLEM OF FINDING N_i IS, IN GENERAL, REDUNDANT.

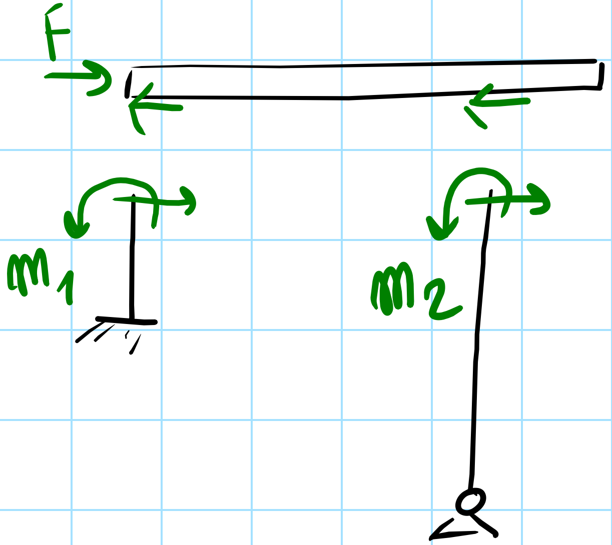
N_1, N_2, N_3 WILL GUARANTEE ROTATION. EQUIL OF THE BEAM.

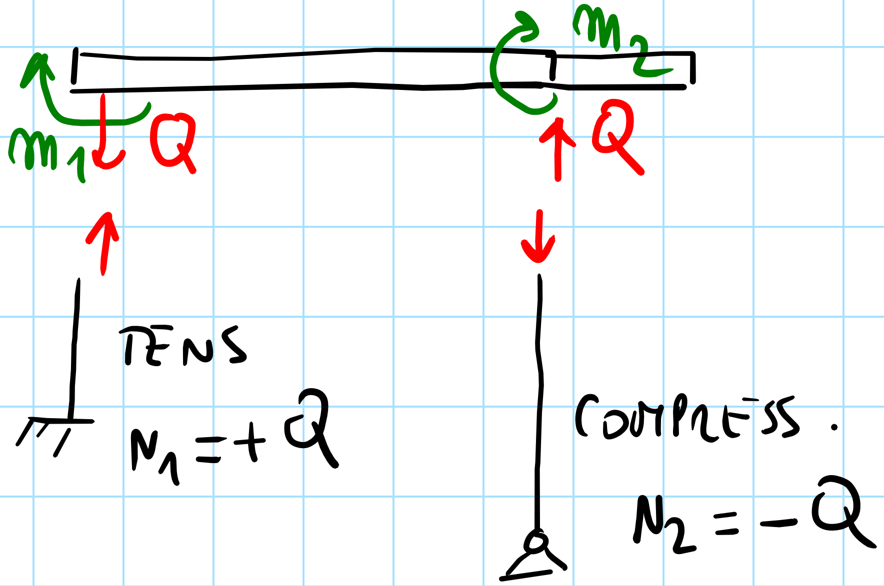
$N_1, N_2, N_3 \Rightarrow$ SHEAR DIAGR. OF THE BEAM (AND BENDING MOMENT)

EX.



EJ: cost





$$F = K\delta = \Big(\overbrace{12\,\frac{EJ}{\ell^3} + 3\,\frac{EJ}{(2\ell)^3}}^{K}\Big)\delta$$

$$F_1 = 12\,\frac{EJ}{\ell^3}\cdot\frac{F}{K} \quad ; \quad M_1 = \frac{F_1\,\ell_1}{2}$$

$$F_2 = 3\,\frac{EJ}{(2\ell)^3}\,\frac{F}{K} \quad ; \quad M_2 = F_2\,(2\ell_2)$$

2 PILLARS! Q IS STATICALLY DETERMINED

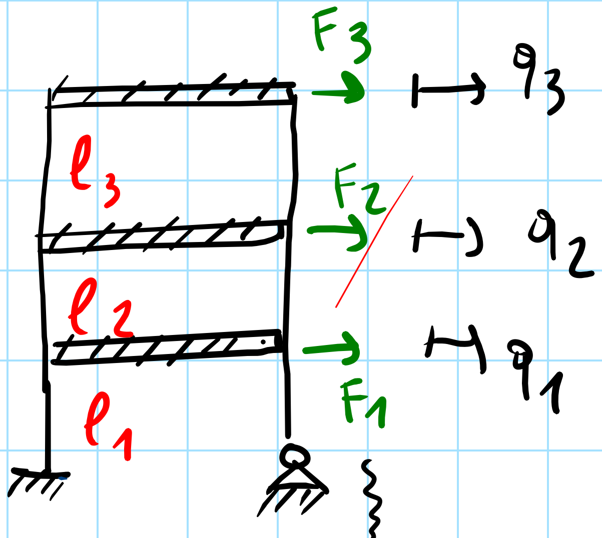
$$QL = M_1 + M_2 \Rightarrow Q$$

POZZATI (TEORIA ... STRUTTURE), vol 1, vol 2^1, vol 2^2

EXAMPLE OF PRACTICAL COMPUTATION OF A 3x3 STIFFNESS MATRIX

HOW TO COMPUTE K_{ij}? K_{ij} IS THE "FORCE" RELATED TO THE "DISPLACEMENT" i WHEN THE D.O.F. j IS EQUAL TO 1 AND ALL THE OTHER ARE NULL.

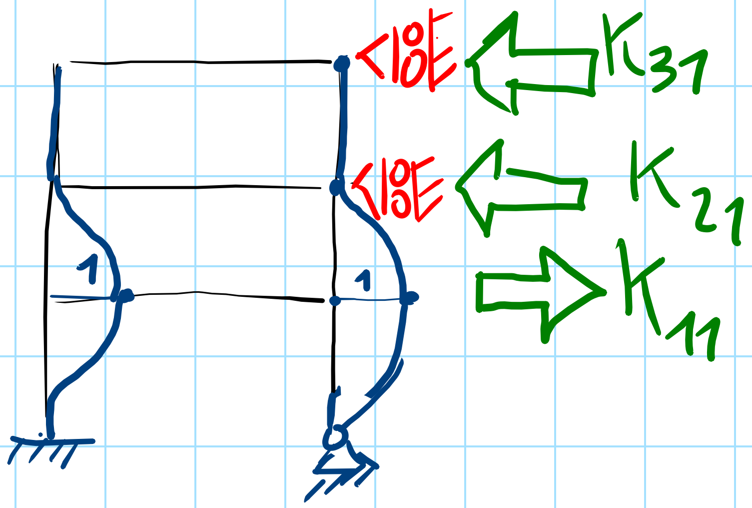
EJ cost



$$\underset{\sim}{K}\,\underline{q} = F$$

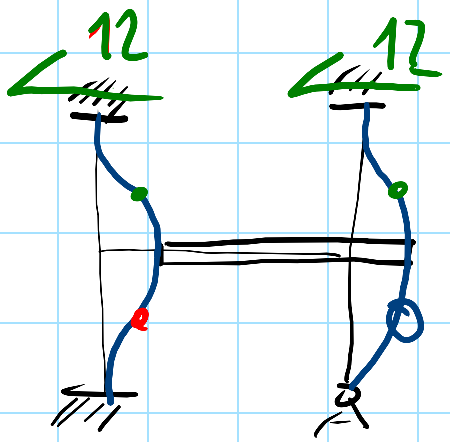
↑ STIFFNESS MATRIX

$$\boxed{q_1 = 1,\ q_2 = q_3 = 0} \Rightarrow K_{11},\ K_{21},\ K_{31}$$



K_{31}, K_{21}: REACTION OF AUXILIARY CONSTRAINTS

K_{11} : FORCE TO PROVIDE $q_1 = 1$



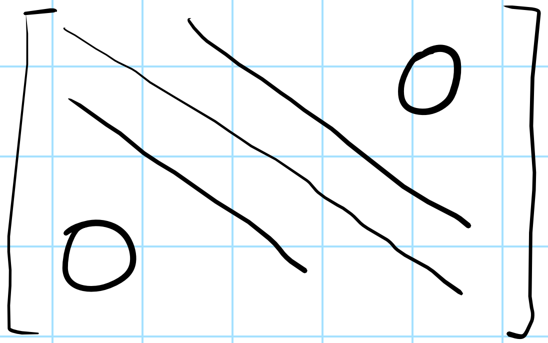
$$K_{21} = -\frac{24EJ}{l_2^3}$$

$$K_{31} = 0$$

$$K_{11} = \underbrace{\frac{12EJ}{l_2^3} + \frac{12EJ}{l_2^3}}_{24} + \underbrace{\frac{12EJ}{l_1^3} + \frac{3EJ}{l_1^3}}_{15}$$

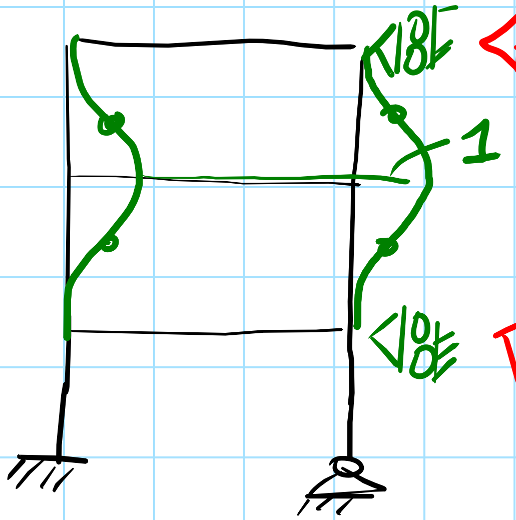
$$\underset{\sim}{K} = \begin{bmatrix} 15\frac{EJ}{\ell_1^3} + 24\frac{EJ}{\ell_2^3} & -24\frac{EJ}{\ell_2^3} & \boxed{0} \\ -24EJ/\ell_2^3 & 24 \dots + 24 \dots & -24EJ/\ell_3^3 \\ \boxed{0} & -24EJ/\ell_3^3 & 24EJ/\ell_3^3 \end{bmatrix}$$

SYMM. MATRIX



TRIDIAGONAL STRUCT. OF STIFFN. MATRIX

$q_2 = 1, \; q_1 = q_3 = 0$

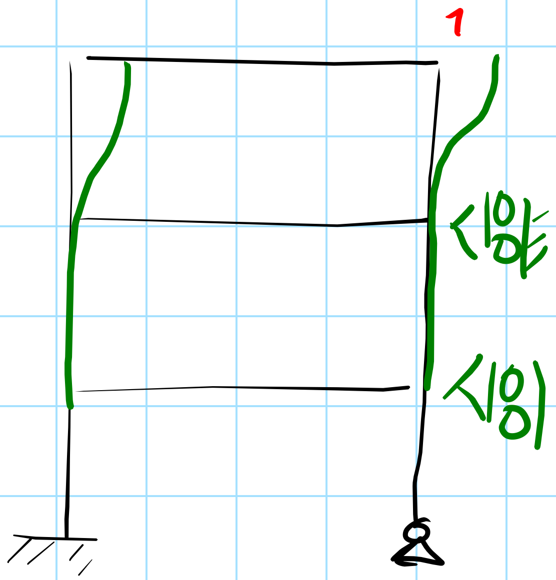


$K_{32} = -24\frac{EJ}{\ell_3^3}$

$K_{22} = 24\frac{EJ}{\ell_3^3} + 24\frac{EJ}{\ell_2^3}$

$K_{12} = -24\frac{EJ}{\ell_2^3}$

$q_3 = 1, \; q_1 = q_2 = 0$

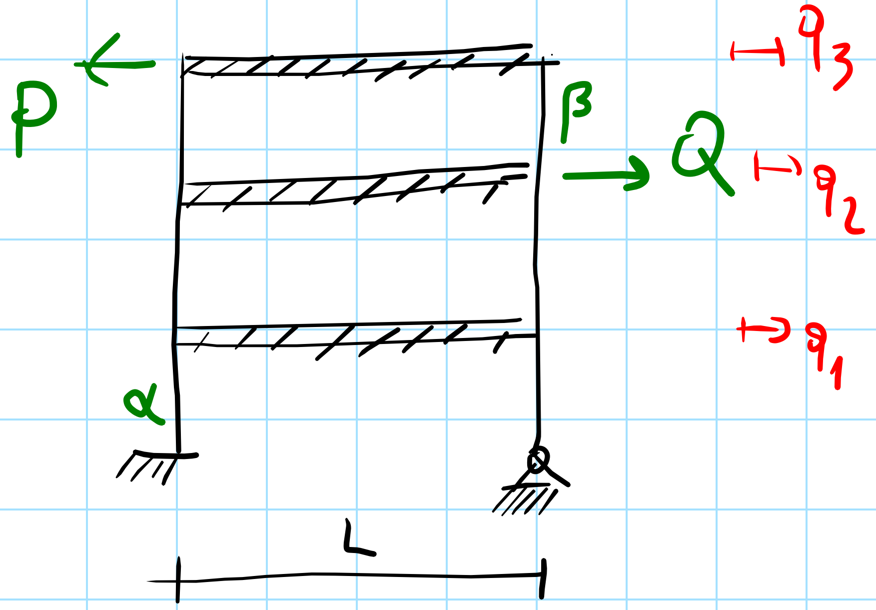


$K_{33} = +24\frac{EJ}{\ell_3^3}$

$K_{23} = -24EJ/\ell_3^3$

$K_{13} = 0$

EX



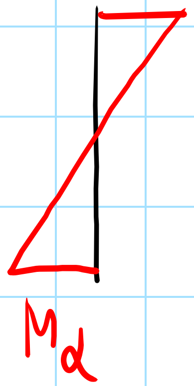
$$\left[\;\underset{\sim}{K}\;\right]\underbrace{\begin{bmatrix}q_1\\q_2\\q_3\end{bmatrix}}_{\underline{q}}=\begin{bmatrix}0\\+Q\\-P\end{bmatrix}\Longrightarrow \underline{q}=\underset{\sim}{K}^{-1}\begin{bmatrix}0\\Q\\-P\end{bmatrix}\Longrightarrow \begin{array}{l}q_1>0\\q_2>0\\q_3>0\end{array}$$

$(q_3<q_2)$

PILLAR α

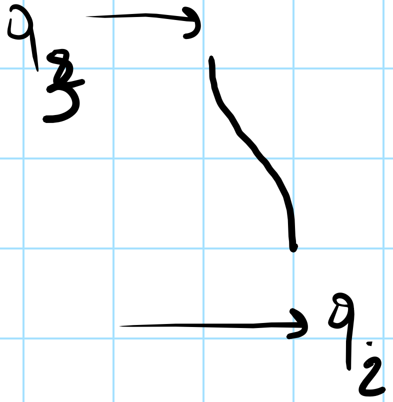
$F_\alpha = 12\dfrac{EJ}{\ell_1^3}q_1$

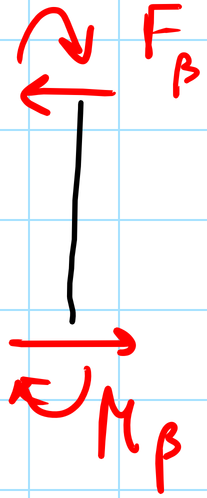
$M_\alpha = 6\dfrac{EJ}{\ell_1^2}q_1$



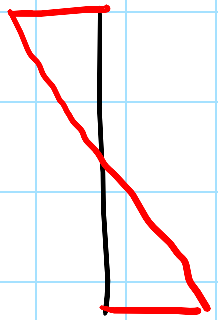
$$M_\alpha = F_\alpha\frac{\ell_1}{2} = 12\frac{EJ}{\ell_1^3}q_1\frac{\ell_1}{2} = 6\frac{EJ}{\ell_1^2}q_1$$

PILLAR β





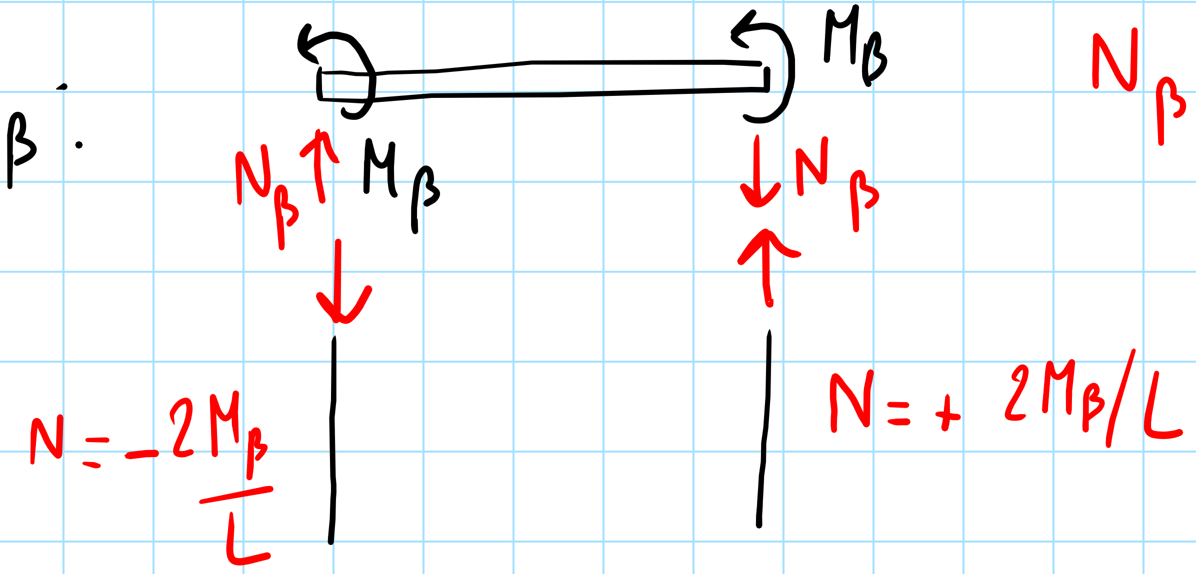
$F_\beta = 12\dfrac{EJ}{\ell_3^3}(q_2-q_3)$



$M_\beta = F_\beta\dfrac{\ell_3}{2}$

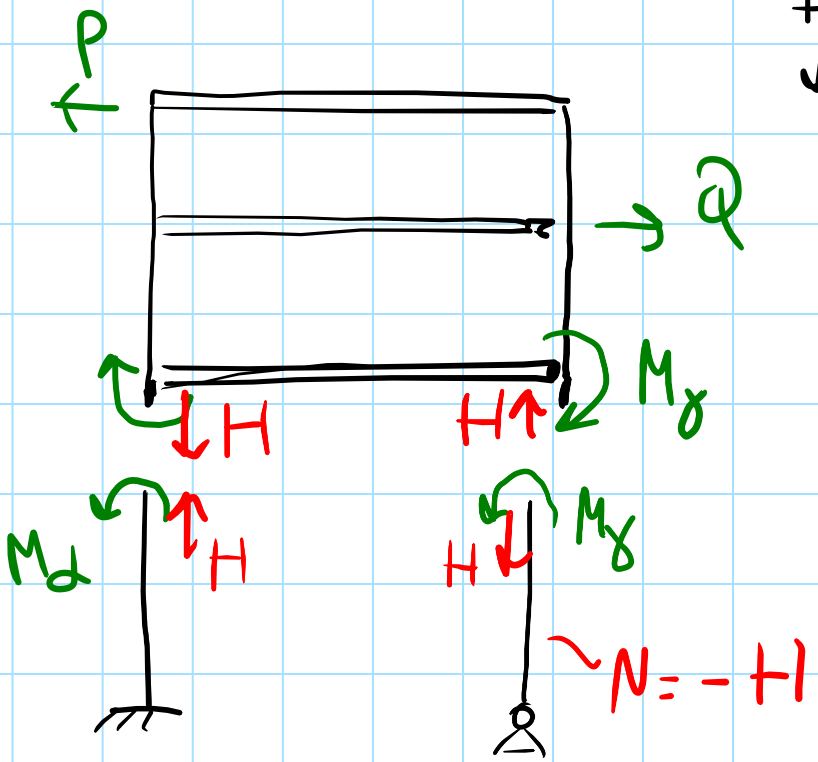
N_α, N_β ?

N_β:



$N_\beta \cdot L = 2M_\beta$

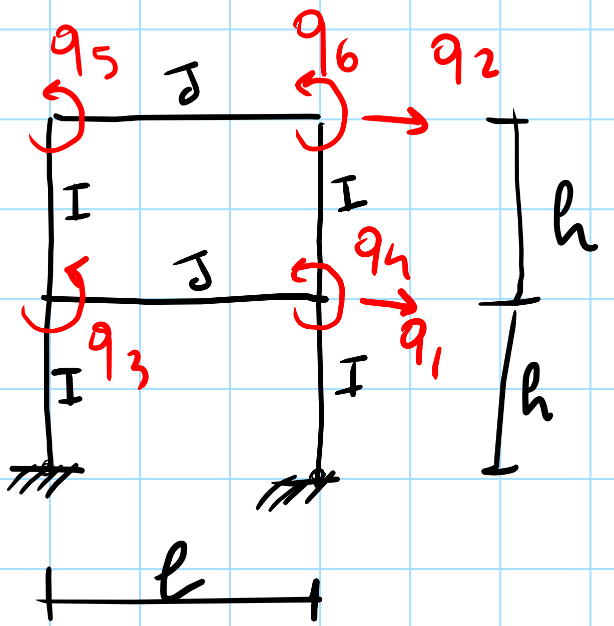
N_α:



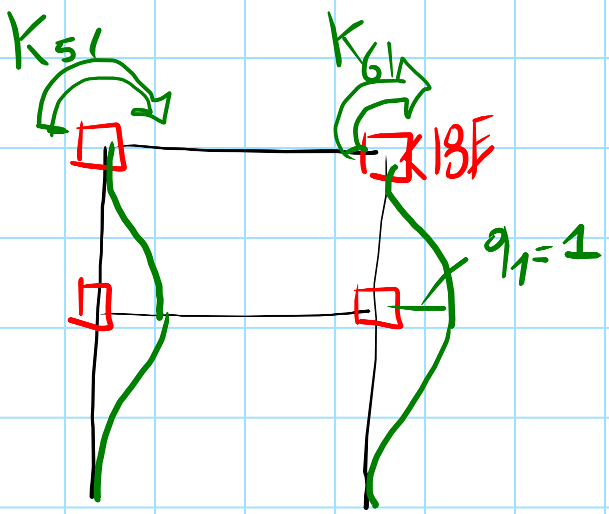
$$\overset{+}{\curvearrowleft}\; HL - M_\alpha - M_\gamma - Q\,l_2 + P(l_2 + l_3) = 0 \;\rightarrow H > 0$$

$N_\alpha = +H$

STIFFNESS MATRIX OF A FRAME WITH BOTH DOFS ROTATIONS AND DISPLACEMENTS



$$\boxed{q_1 = 1,\ q_2 \cdots = q_6 = 0}$$



$\Leftarrow K_{21} = -\dfrac{24EI}{h^3}$

$\Rightarrow K_{11} = 48\dfrac{EI}{h^3}$

$K_{51} = K_{61} = -\dfrac{6EI}{h^2}$

E: CONSTANT

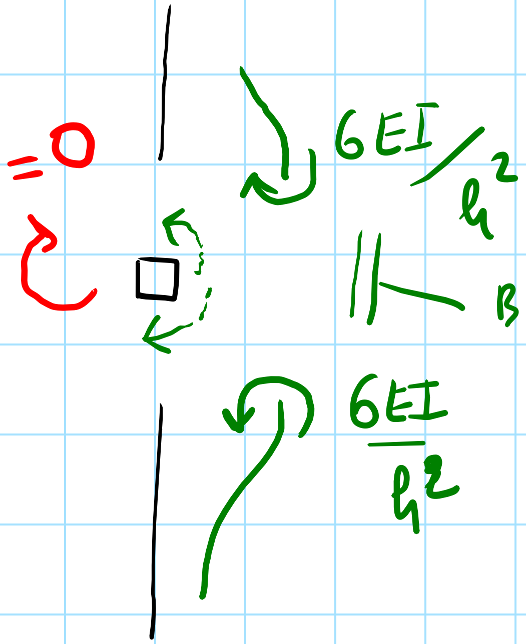
6 DOFS: $6 \times 6\ \underset{\sim}{K}$

WHY 6 DOFS:

BECAUSE WE NEGLECT AXIAL DEFORMABILITY!

GOOD FOR ELASTIC FRAME.

$K_{31} = 0$



|| BECAUSE $h_1 = h_2 = h$ (PARTICULAR CASE)

ALSO $K_{41} = 0$ FOR THE SAME REASONS

$$\underset{\sim}{K} = \left[\begin{array}{c} 48\,EI/l^3 \\ -24\,EI/l^3 \\ 0 \\ 0 \\ -6EI/l^2 \\ -6EI/l^2 \end{array} \right.$$