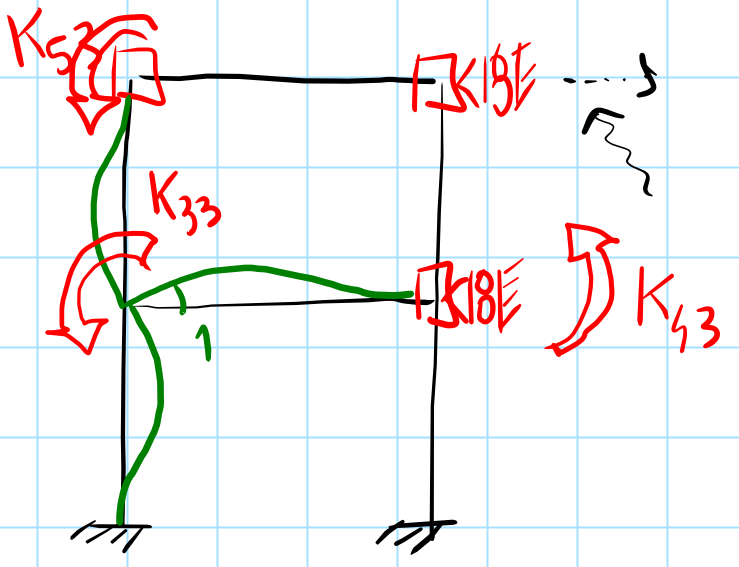
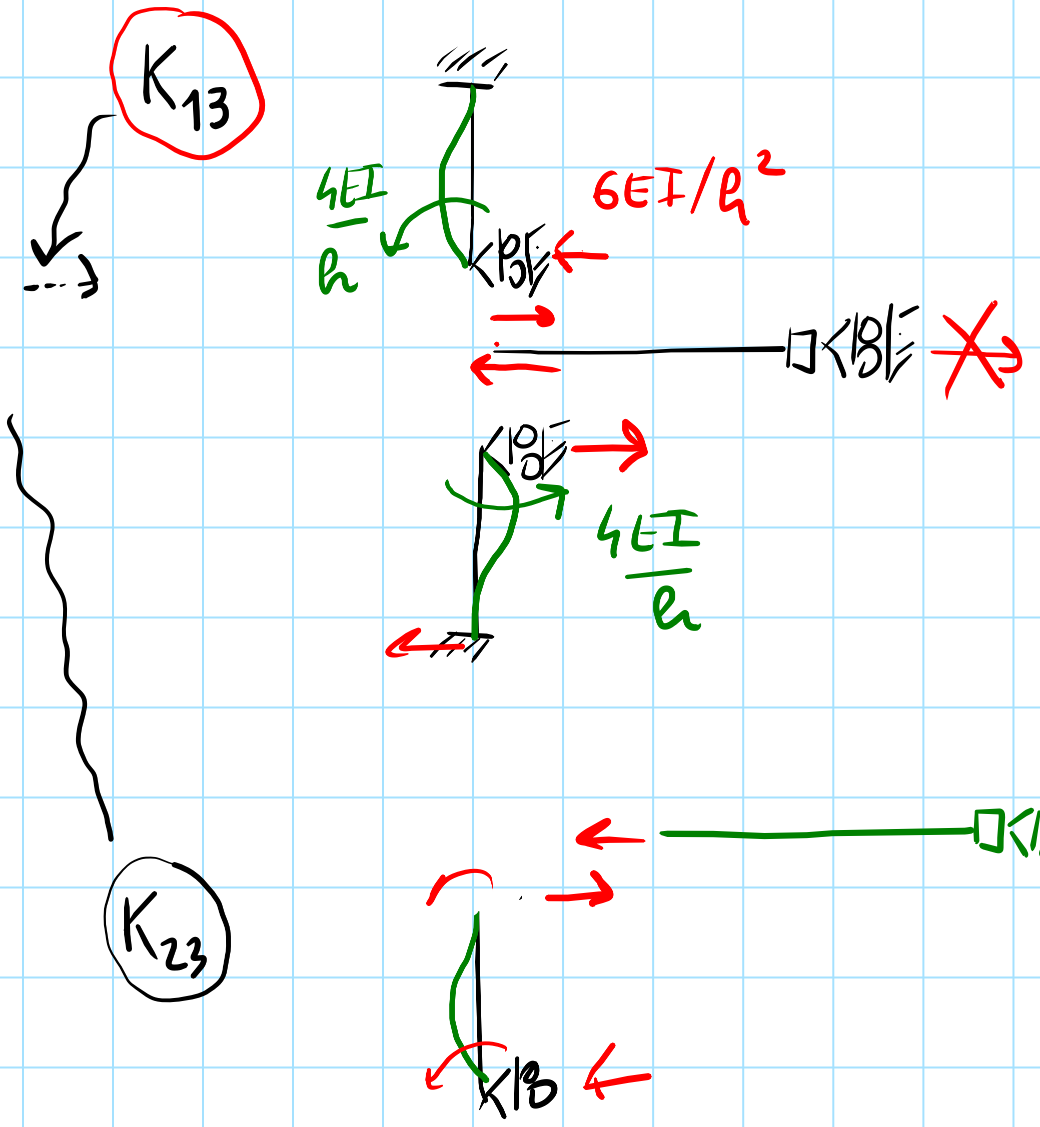
... CONTINUE THE EX

13/5/25

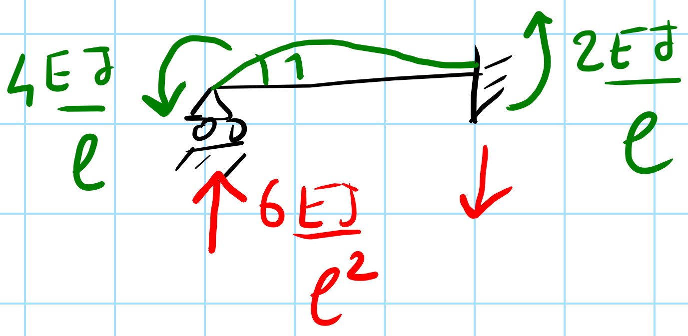
$q_3 = 1, \quad q_1 = q_2 = q_4 = \dots q_6 = 0$ (3rd COLUMN OF $\underset{\sim}{K}$)





$K_{13} = 0$

$K_{23} = + \frac{6EI}{h^2}$



$$K_{33} = 2 \cdot \frac{4EI}{h} + \frac{4EJ}{l}$$

$$K_{43} = + \frac{2EJ}{l}$$

$$K_{53} = + \frac{2EI}{h}$$

$$K_{63} = 0$$

SYM

$$\underset{\sim}{K} = \begin{bmatrix} & & 0 \\ & & +\frac{6EI}{h^2} \\ 0 & & \frac{8EI}{h} + \frac{4EJ}{l} \\ & & 2EJ/l \\ & & 2EI/h \\ & & 0 \end{bmatrix}$$

NOTE: DIMENSIONS OF K_{ij} DEPEND ON THE LINEAR SYSTEM:

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ m_3 \\ m_4 \\ \vdots \end{bmatrix}$$

DISPL (q_1, q_2) — ROTATION ($q_3, q_4, \dots$) — FORCE (F_1, F_2) — MOMENT ($m_3, m_4, \dots$)

DISPL — DISPL — ROTAT.

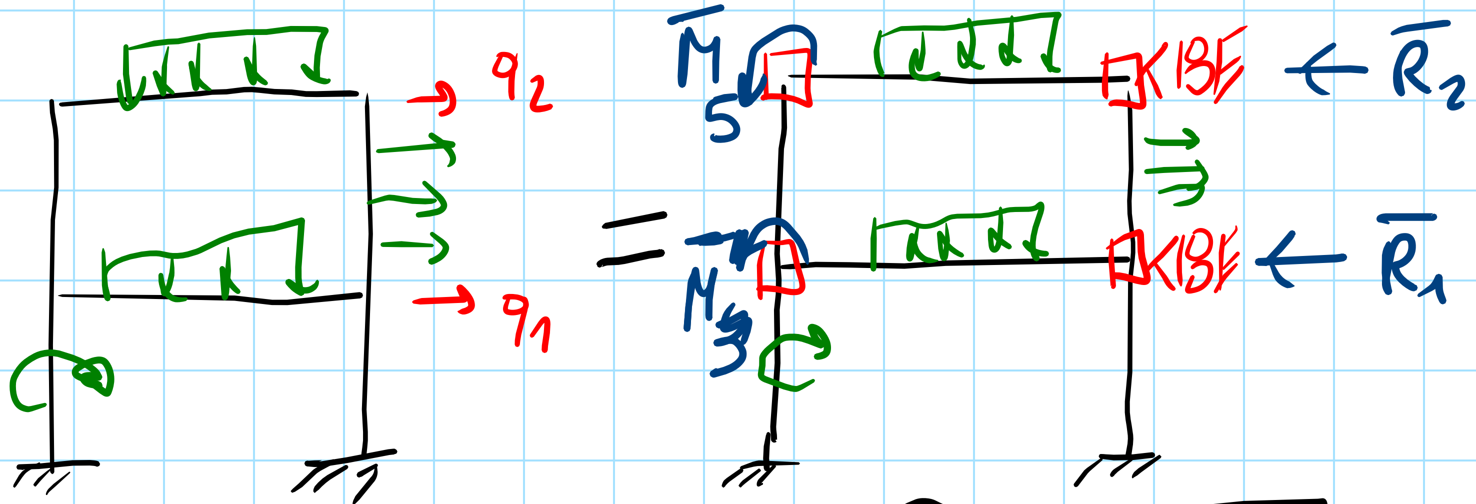
$$K_{11}q_1 + K_{12}q_2 + K_{13}q_3 + \dots = \text{FORCE}$$

$$K_{31}q_1 + K_{32}q_2 + K_{33}q_3 + \dots = \text{MOMENT}$$

IN THE EXAMPLE ABOVE THE SYSTEM $\underset{\sim}{K}\,\underline{q} = \underline{F}$ IS THE "SCHEME II" IN THE DISPLACEMENT METHOD

K_{13} : FORCE

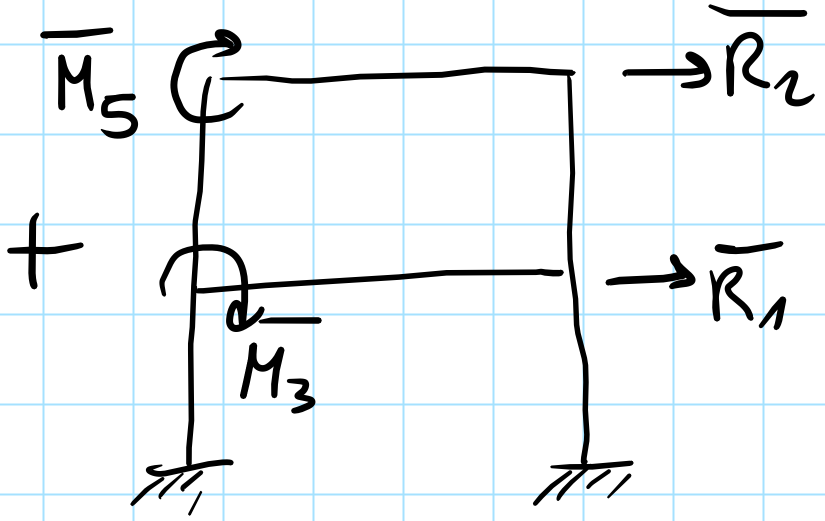
K_{31} : FORCE



→ REACTIONS AT THE AUXILIARY CONSTRAINTS

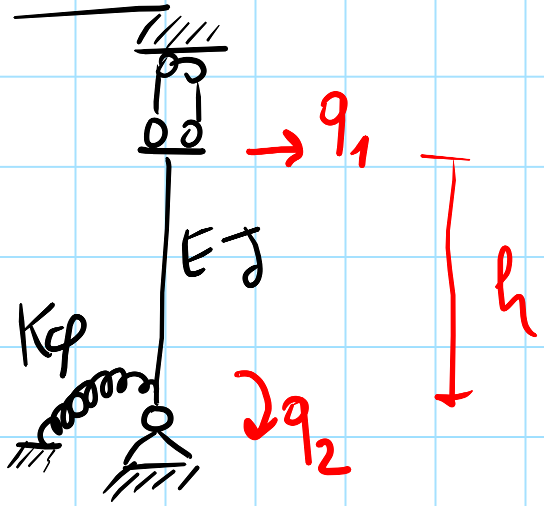
(I) $\boxed{q_i = 0}$

+

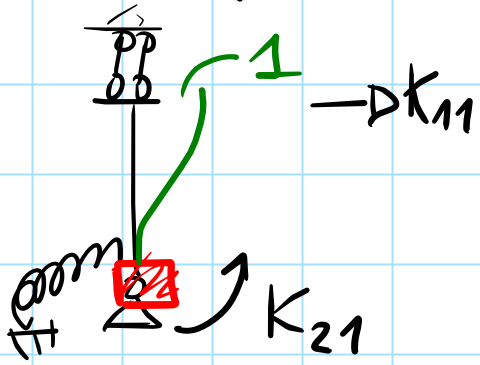


(II) $\underline{q} = \underset{\sim}{K}^{-1}\,\underline{F}$

NOTE: EFFECT OF COMPLIANT CONSTRAINTS

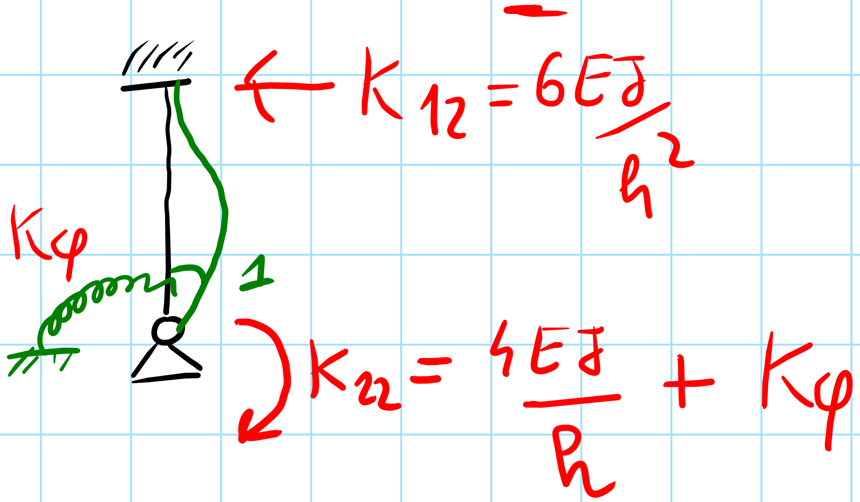


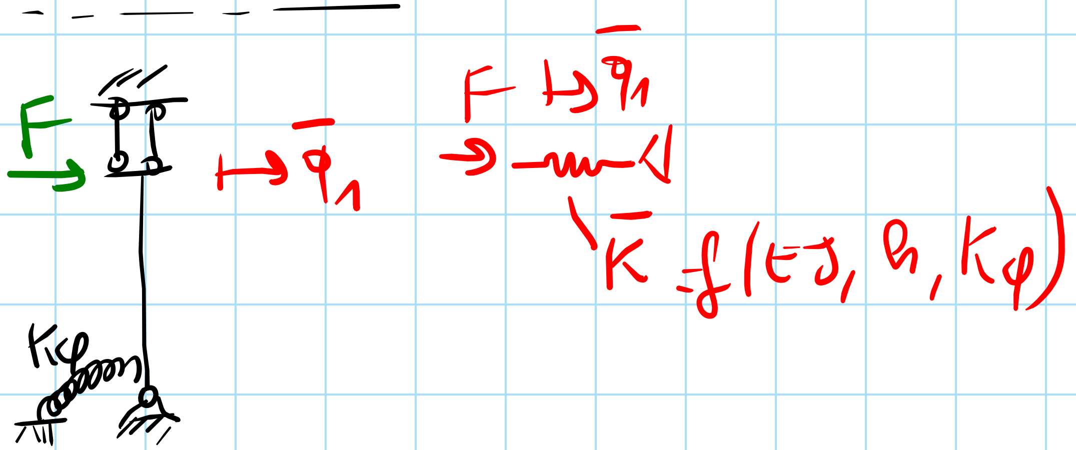
$$\boxed{q_1 = 1,\ q_2 = 0}$$

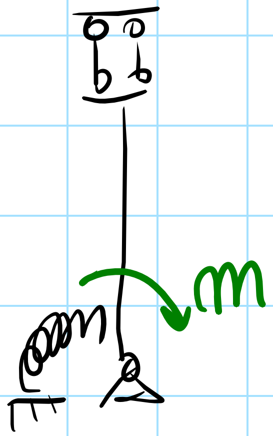


$$\underset{\sim}{K} = \begin{bmatrix} 12\,EJ/h^3 & -6EJ/h^2 \\ -6EJ/h^2 & \frac{4EJ}{h} + K_\varphi \end{bmatrix}$$

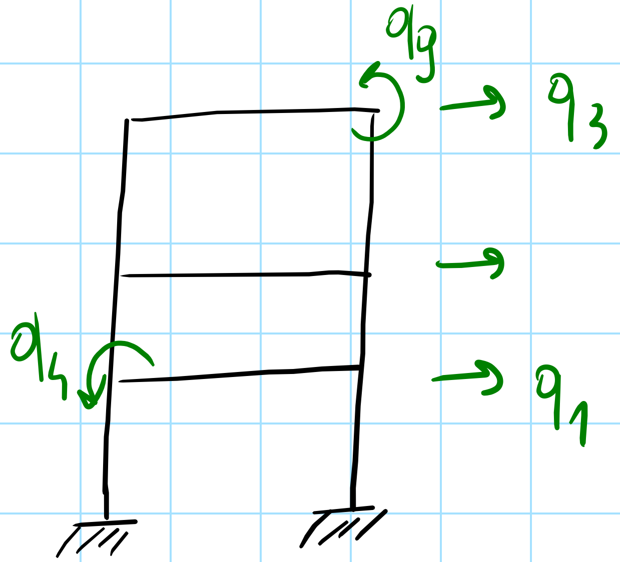
$$\boxed{q_1 = 0,\ q_2 = 1}$$



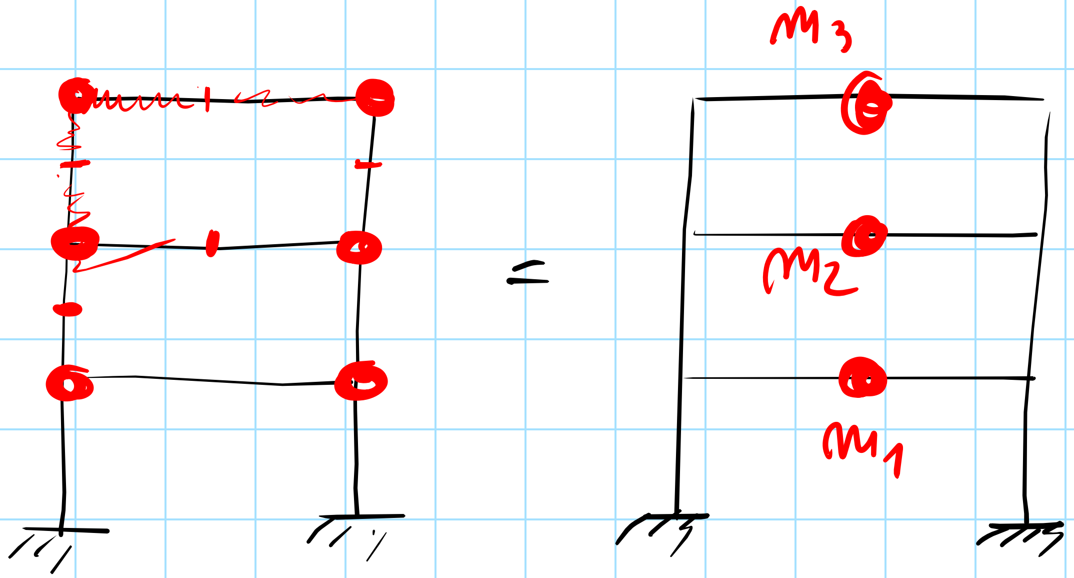




CONDENSATION OF STIFFNESS MATRIX



MASS MATRIX



CONCENTRATED MASSES

$$\underline{q}_A = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} ; \quad \underline{q}_B = \begin{bmatrix} q_4 \\ \vdots \\ q_9 \end{bmatrix}$$

$$\underset{\sim}{M}\, \ddot{\underline{q}} + \underset{\sim}{K}\, \underline{q} = \underset{\sim}{F}(t)$$

$$\left[\begin{array}{c|c} \underset{\sim}{M}_A & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{0} \end{array}\right] \begin{bmatrix} \ddot{\underline{q}}_A \\ \ddot{\underline{q}}_B \end{bmatrix} + \left[\begin{array}{c|c} \underset{\sim}{K}_{AA} & \underset{\sim}{K}_{AB} \\ \hline \underset{\sim}{K}_{BA} & \underset{\sim}{K}_{BB} \end{array}\right] \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix} = \begin{bmatrix} \underline{F}_A(t) \\ \hline 0 \end{bmatrix}$$

$$\underset{\sim}{M}_A = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

WE CAN SHOW THAT THE SYSTEM CAN BE SOLVED ONLY IN TERM OF VECTOR $\underline{q}_A$

$$\underset{\sim}{K}_{BA}\,\underline{q}_A + \underset{\sim}{K}_{BB}\,\underline{q}_B = \underline{0} \quad ; \quad \underline{q}_B = -\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\,\underline{q}_A \quad (*)$$

(BOTTOM SUB SYSTEM)

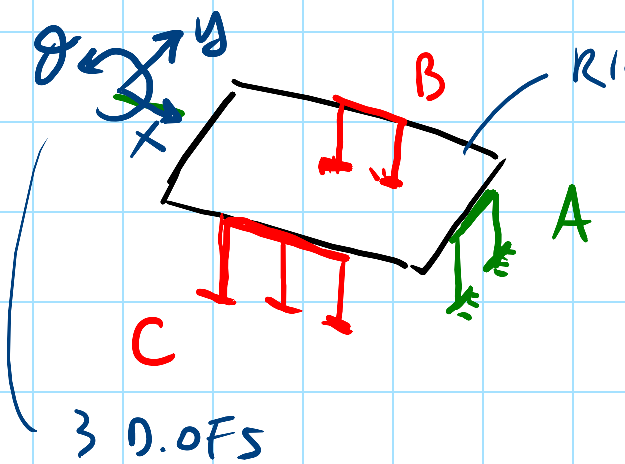
(TOP SUB SYSTEM)

$$\underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underset{\sim}{K}_{AA}\,\underline{q}_A + \underset{\sim}{K}_{AB}\,\underline{q}_B = \underline{F}_A(t) \implies \underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underbrace{\left[\underset{\sim}{K}_{AA} - \underset{\sim}{K}_{AB}\,\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\right]}_{\text{CONDENSED STIFFNESS MATRIX}}\underline{q}_A = \underline{F}_A(t)$$

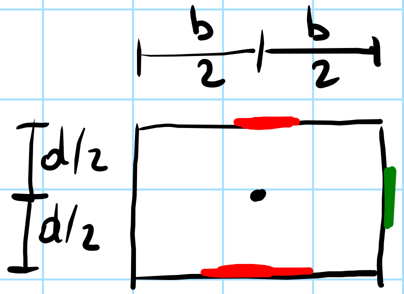
$\Rightarrow$ COMPUTE $\underline{q}_A(t)$

THROUGH (*) $\underline{q}_B$ CAN BE CALCULATED.

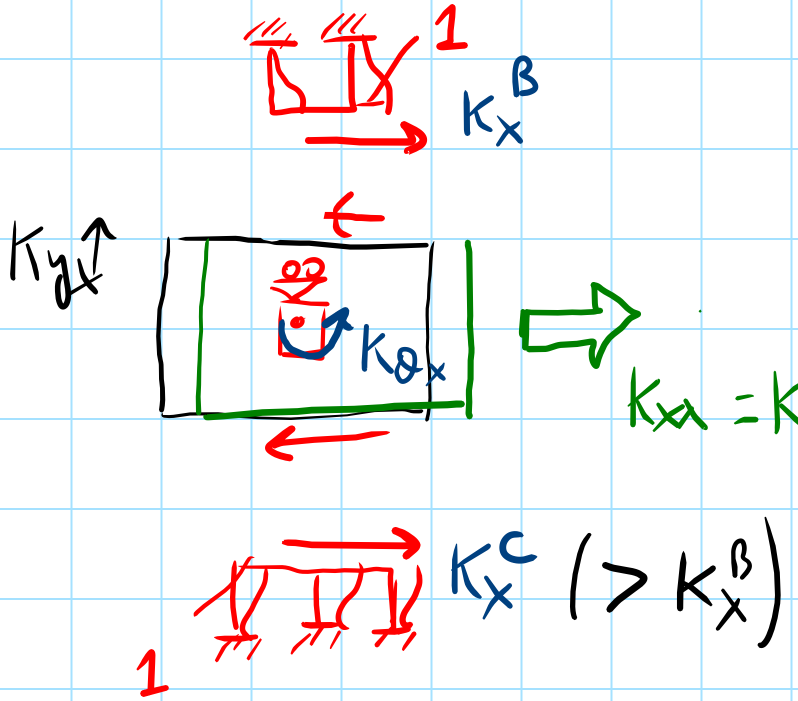
EXAMPLE OF COMPUTAT. OF A STIFFNESS MATRIX IN 3D



$\boxed{x=1,\ y=\theta=0}$



NEGLECT RESISTANCE OF FRAMES OUT OF PLANE



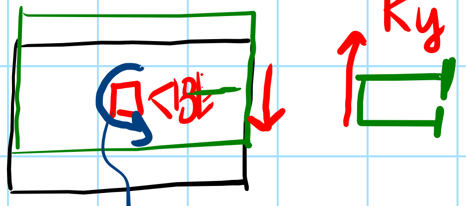
$K_{xx} = K_x^B + K_x^C$

$K_{yx} = 0$

$K_{\theta x}$? $= +K_x^C \frac{d}{2} - K_x^B \frac{d}{2} = (K_x^C - K_x^B)\frac{d}{2}$

$\boxed{y=1,\ x=\theta=0}$

$K_{yy} = K_y^A$



$K_{\theta y} = +K_y^A \frac{b}{2}$

$K_{xy} = 0$