G. or. 6

Funzione mottimole di Hordy Littlewood.

fe L¹eoc (TR^d) r >0

 $A_{r}f(x) = \frac{1}{|B(x,r)|} \int f(y)dy$

 $A_{rfe}C^{\circ}(\mathbb{R}^{d},\mathbb{C})$

Mf(x) = supr Ar[f]

Mf à lover semicontinuous.

subbaddictiv

 $M(f+g) \leq M(f) + M(g).$ $|A_r f(x)| \leq \frac{1}{|B(x,r)|} \int_{B(x,r)} |f| dx$

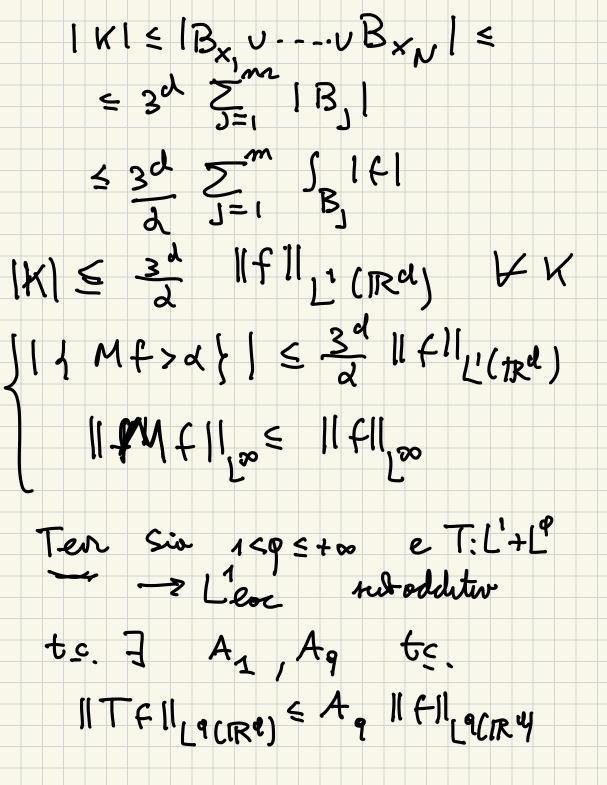
 \Rightarrow $Wf(x) \leq ||f||^{2}$ $M: L^{\infty}(\mathbb{R}^{d}) \supseteq$ $M: L^2 \rightarrow L^2 \quad in \mathbb{R}^d$ Kcc Rd in coro t_{c} . $B(o, c_{o}) > K$ Allow per (x1 > co) de B(x,21x1) > B(o, 1x1) > K $M\chi_{K}(x) = \sup_{r \geq 0} \frac{|B(x, r) \wedge K|}{C_{d} r d} \ge \frac{|K|}{C_{0} 2^{d} |x|^{d}}$ $\forall x t: |x| > c_0$ $M \mathscr{X}_{K}(x) \ge C \xrightarrow{1}_{I \times 1^{d}} \mathscr{L}^{1}(\mathbb{R}^{d})$ (0, c)

Si disnorthe ou che poro $\forall 1$ Mfll F(Rd) ≤ Cp llfll P(Rd) Ricordionned Cheb. $\forall g \in L^1(\mathbb{R}^4)$ 1 8 L'(R4) |dx: |g(x) |> 2 | = $\forall d > 0.$ $T: L^{1}(\mathbb{R}^{d}) \rightarrow L'(\mathbb{R}^{d})$ Ovvionmente sle che woldrif $\|Tf\|_{L^{1}} \leq A \|f\|_{L^{1}} \forall fel^{1}(\mathbb{R}^{4})$ vole onche $|\{x: |Tf(x)| > d | \leq \frac{\|Tf\|_{l}}{d}$ $\leq \frac{A}{d} \|f\|_{l^1} \quad \forall d > 0$ $e \forall f \in L^1(\mathbb{R}^d)$ Risulta che Muddifo

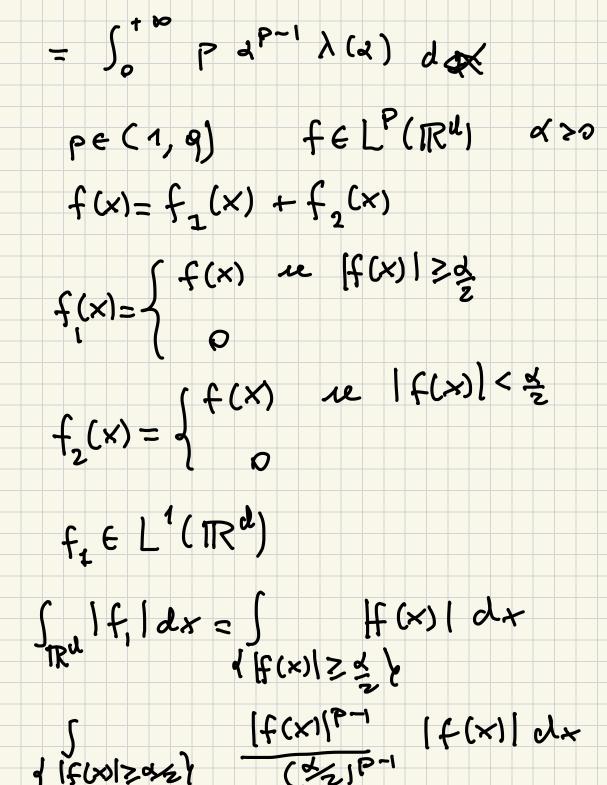
FC250 tr.

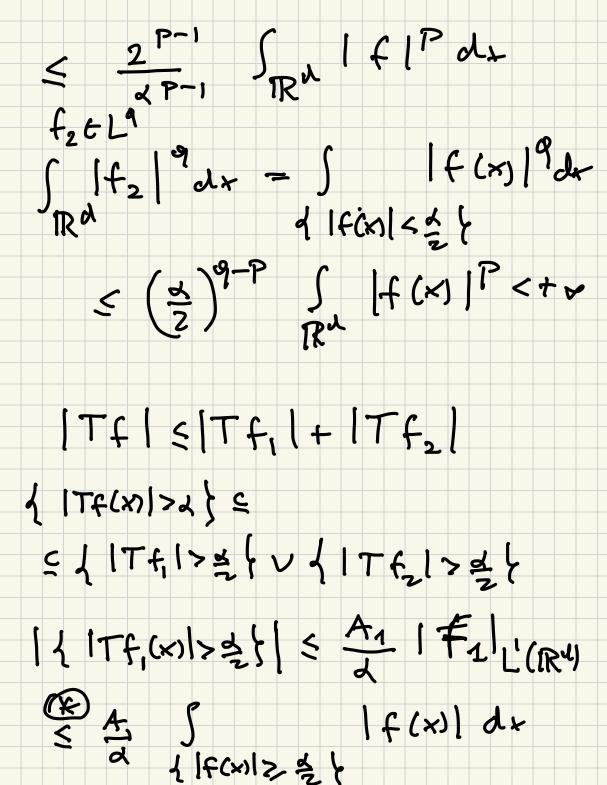
1{x: [Mf(x)]>d = ∀ d>v e ∀ fel'(R^y $\leq \frac{\zeta_{2}}{\alpha} \|f\|_{1}$ Ca= 3^d voddufo Me (1,1) veale bounded Lemmo Sio Bx, ..., Bx, uno formglue di pulle in TRd. $f \{B_{1}, \dots, B_{m}\} \in \{B_{\chi_{1}}, \dots, B_{\chi_{n}}\}$ a due a due disjunte te $|B_{x_1}\cup\cdots\cup B_{x_N}| \leq 3^d \sum_{j=1}^{\infty} |B_j|$

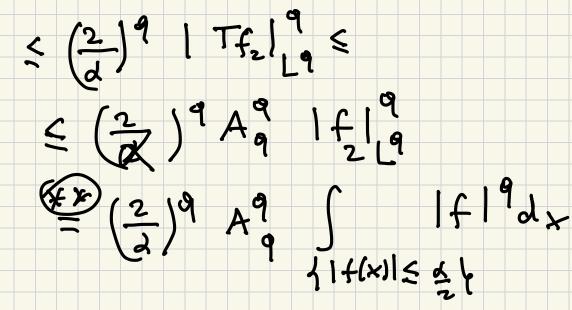
|{x: [Mf(x)]>2 |= $\leq \frac{C_{\lambda}}{d} \|f\|_{l^{1}} \quad \forall d > 0$ $C_d = 3^d$ Siven KCC {X: [Mf(x)]>d} $\forall x \in K \qquad Mf(x) = \sup_{t > 0} \frac{\int_{B(x,r)} |f|}{|B(x,r)|}$ 3 rx 70 ts. $\int |f| > d |B(x, r_{x})|$ B(x, r_{x}) K C Bx, U --- · U BxN |B(x, r,)| $\frac{1}{d} \int \frac{|f|}{B(x_{j}, r_{x_{j}})}$ B1, --, Bm

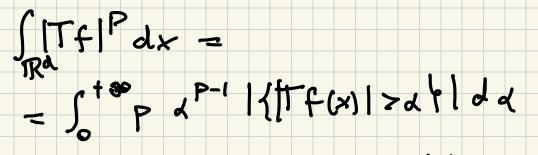


 $|\{x: |Tf(x)| > d_{y}| \leq \frac{A_{1}}{d} |f_{1}| \lfloor^{1}(\mathbb{R}^{d})$ ¥ d> Allow 34 1 CP<9 3 Ap tc. $\|\mathbf{T}f\|_{\mathcal{P}} \leq A_{\mathcal{P}} \|f\|_{\mathcal{P}} \neq f \in \mathcal{L}^{p}.$ Din of: Rd -> R misurelle $\lambda(a): |\{\beta(x)|>a\}| \neq$ $\lambda: [0, +\infty) \rightarrow [0, +\infty]$ gel^P(IR^d) $\int_{\mathbb{R}} |g(x)|^{P} dx =$ $= \int_{\mathbb{R}} dx \int_{0} |g(x)|$ $= \int_{\mathbb{R}} dx \int_{0} P d' dd$ = 5° da papi 5 dx (18(x))>4







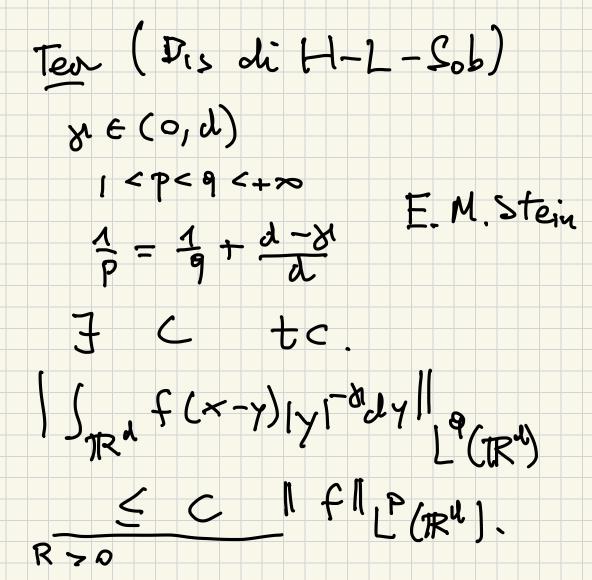


< 5 P 2 P-1 1 { [[x]] > 2 4 1 d 2 4

 $\int_{0}^{+\infty} P^{-1} \left| \left\{ \left| T_{f_{2}}(x) \right| > d_{f_{2}}(x) \right| \right| d d$ = $I_{1} + I_{2}$

 $T_{1} \leq 2A_{1} \int_{0}^{P} p d^{P-2} \int_{X: 1 \in \{X\} \leq \frac{1}{2}} |f(x)| dx$ $= 2A_{1}P \int dx [f(x)] \int d^{p-2} dd$ $= \frac{2AP}{P-1} \int_{\mathbb{R}^d} |f(x)|^P dx$ $T_2 = \int_0^{+\infty} P^{-1} \left[\left(\left[T_{f_2}(x) \right] \right) d_2 \left[dd \right] \right]$ $\leq 29 A_{q}^{q} \int P d^{p-1-q} \int dr |f(x)|^{q}$ イベ: マイトのにかり $= 2^{q}A^{q}P\int dx |f(x)|^{q} \int t^{\infty} dP - q - 1$ $= 2^{q}A^{q}P\int dx |f(x)|^{q} \int t^{\infty} dP - q - 1$ $= \frac{2^{4}}{p - q} A^{q}_{q} P \int dx |f(x)| dx$

 $\int [Tf|^{P} dx \leq I_{1} + I_{2}$ $\leq C \qquad \int [f|^{P} dx$



SR& f(x-y) 1y/-8 dy $= \int f(x-y)|y|^{-d}dy + \int dy f(x-y)|y|^{d}$ $|y| \leq R$ $|y| \leq R$

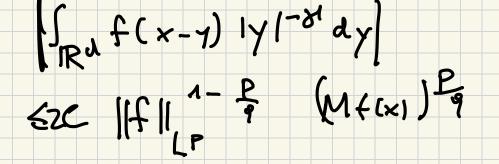
 $\left| \begin{array}{c} \int f(x-y) |y|^{-d} dy \right| \leq \\ |y| \leq R \\ \leq M f(x) \int |y|^{-d} dy \\ |y| \leq R \\ = C A R^{d-d} M f(x) \end{array} \right|$

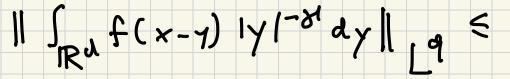
 $\left| \begin{array}{c} \int f(x-y) |y|^{-\delta'} dy \Big|_{\leq} \\ |y| \geq R \\ \leq \left| f \right|_{L^{p}} \left| \left| y \right|^{-\delta'} dy \Big|_{\leq} \\ & y| \geq d \\ \end{array} \right|$

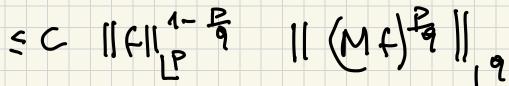
= C I F R q

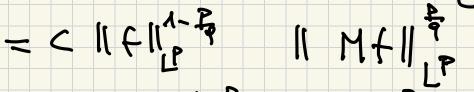
(JRd f(x-y) 1y1-dr dy)

 $\leq C \left(\left[f \right]_{P} R^{-\frac{m}{q}} \right)$ $R^{d-d} Mf(x)$ $\frac{Mf(x)}{\|f\|_{L^{p}}} = R^{-\frac{\alpha}{p}}$









 $= C_1 \| f \|_{P}$

| S f(x-y)|y|-dy|< $\leq Mf(x) \int_{|y|\leq R} |y|^{-\delta'} dy$ ✓ q ∈ L¹(R^d) positiva g vodièle
è decrevente $|\int_{\mathbb{R}^{d}} f(x-y) \phi(y) dy| \leq \int_{\mathbb{R}^{d}} \phi(y) dy$ $\leq Mf(x) \int_{\mathbb{R}^{d}} \phi(y) dy$ $\phi(\gamma) = \chi_{B(0,R)}^{1} |\gamma|^{-\delta^{1}}$ Si comincina con $\phi = \sum_{j} \alpha_{j} \chi_{B_{j}}$ | S f(x-y) Z ay XB(y) dy | < Z @ [B] [f(x-7) [ZB(Y) dy

 $\leq Z \alpha_{1}B_{1}$ Mf(x) Sø dr Per denuti ogni op $\phi \in L^{1}(\mathbb{R}^{d})$ positiva e vodièle e decremente e equimilile $\phi_{n} = \phi_{n}$