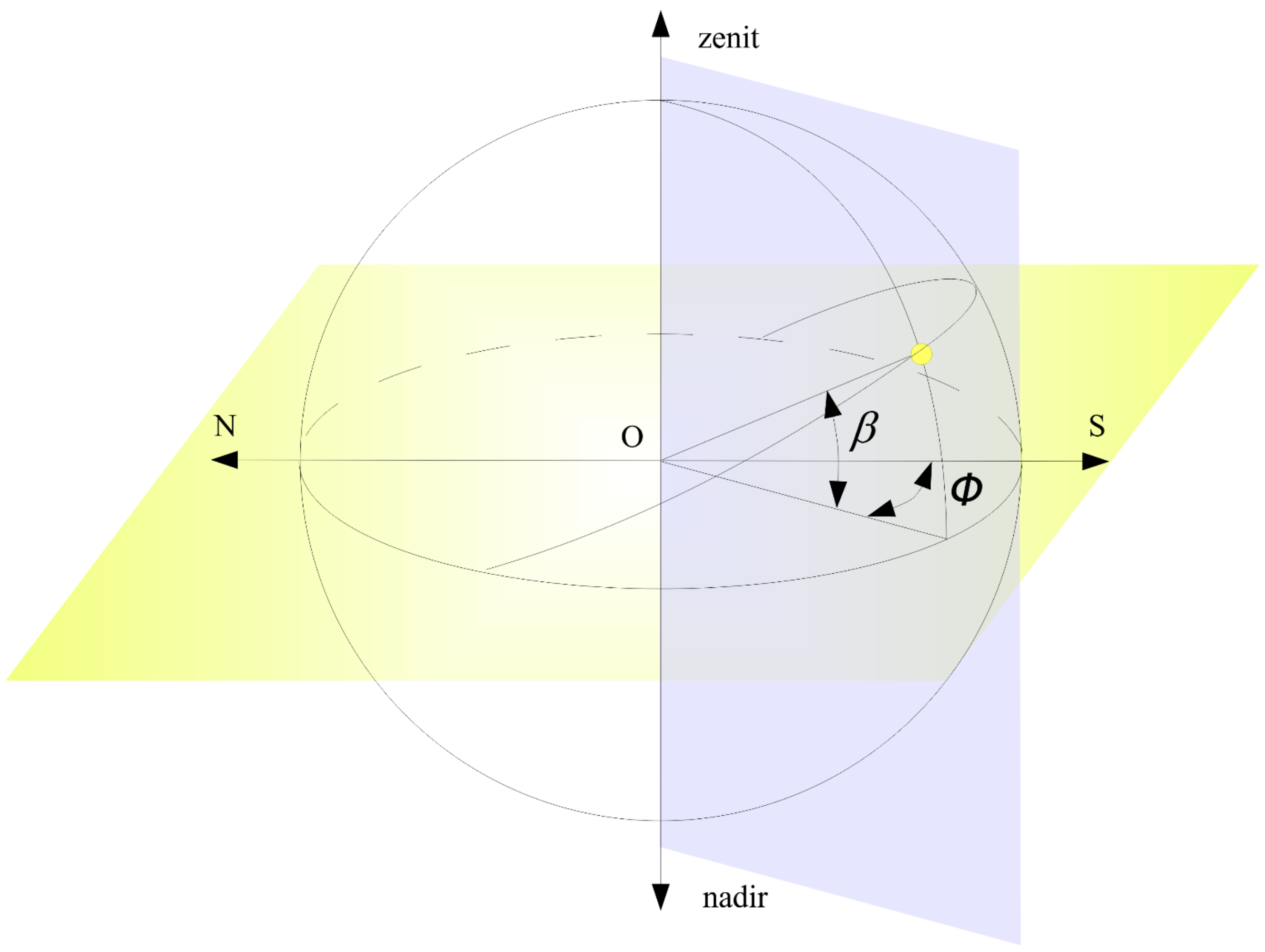
BUILDINGS HVAC SYSTEM
Solar Angles

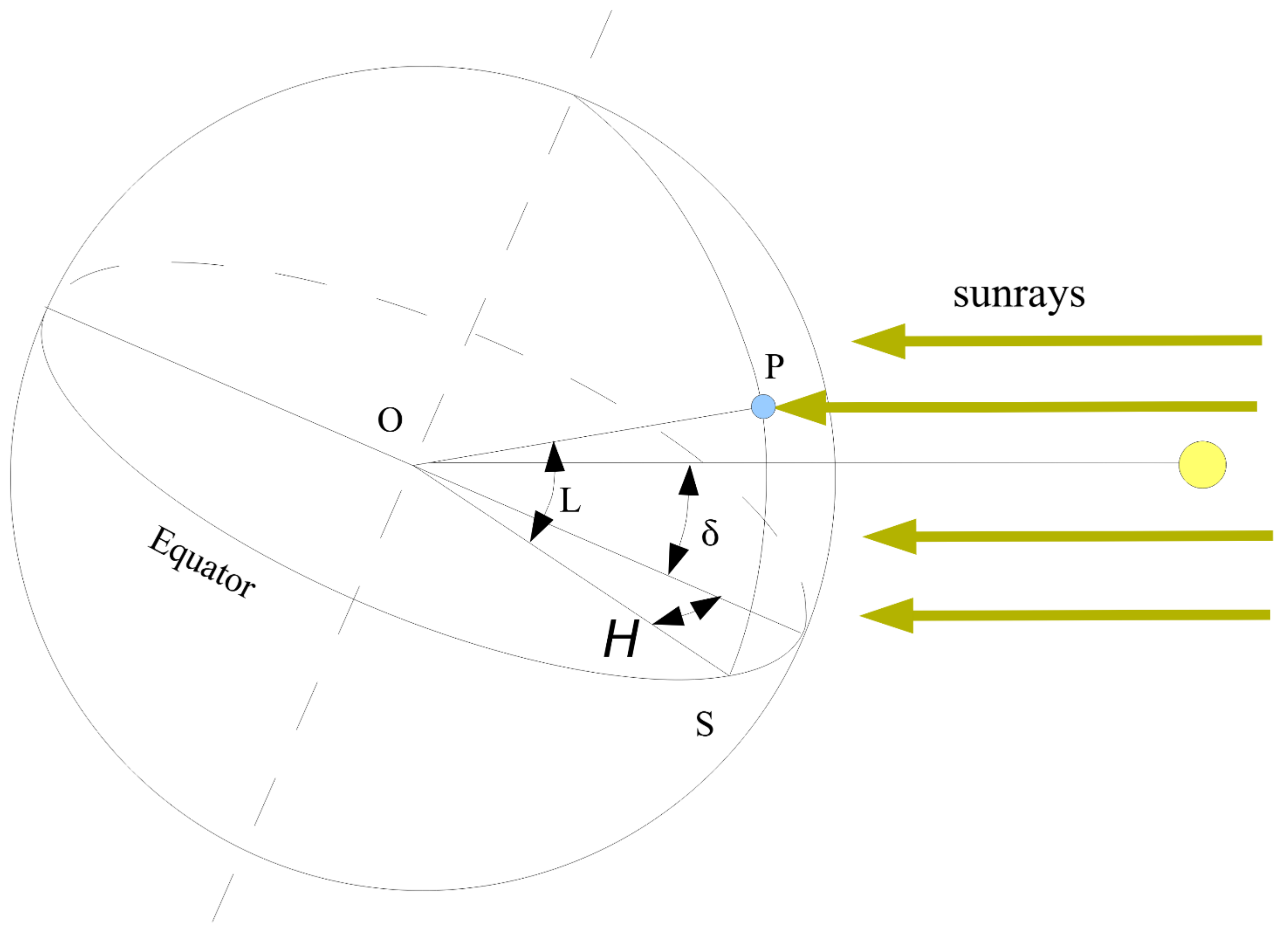
Solar angles, horizontal coordinates

- The reference is the observation horizontal plan
- We identify the solar angles
- θ_z : zenith angle
- β : solar height
- ϕ: solar azimuth, positive in west direction

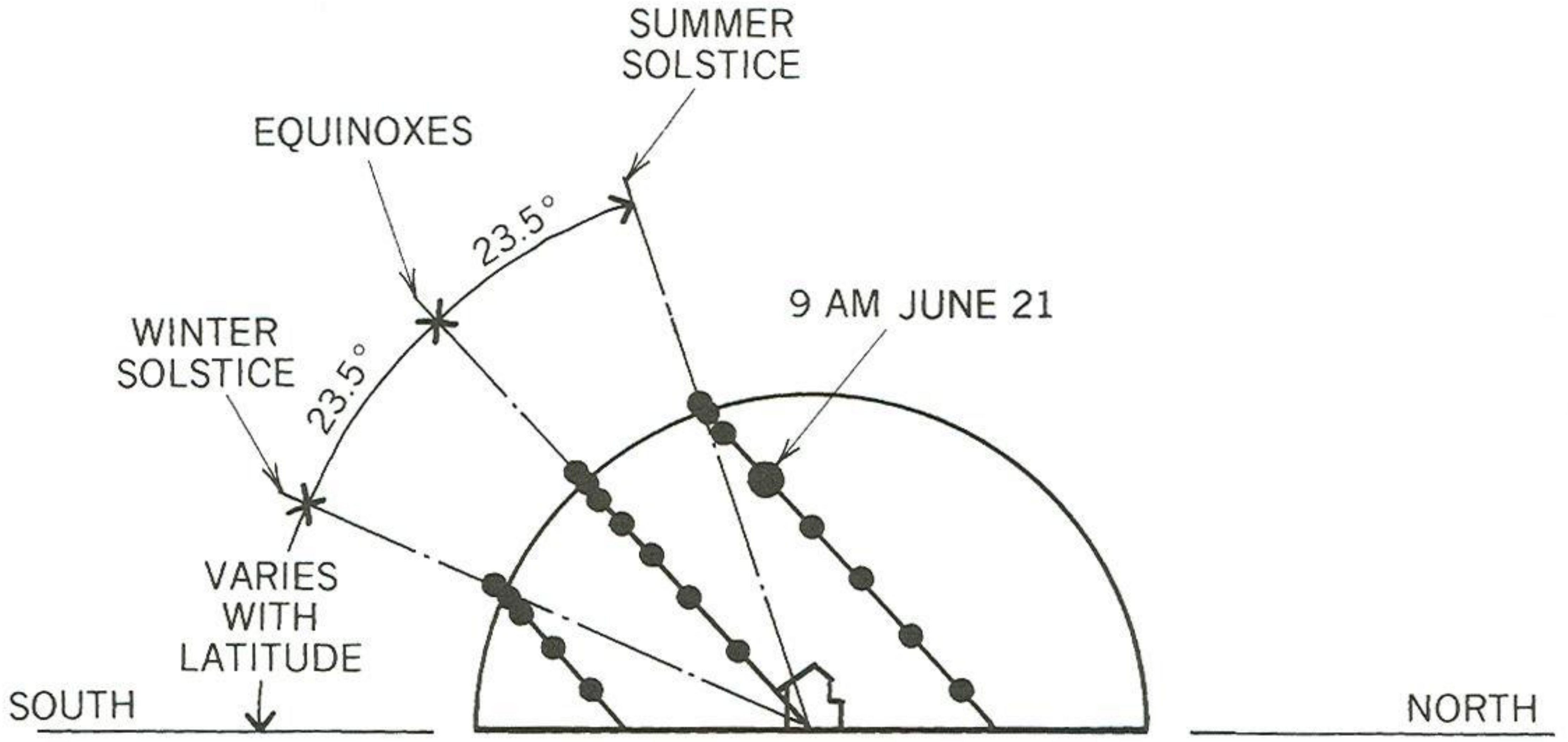


Geographic coordinates

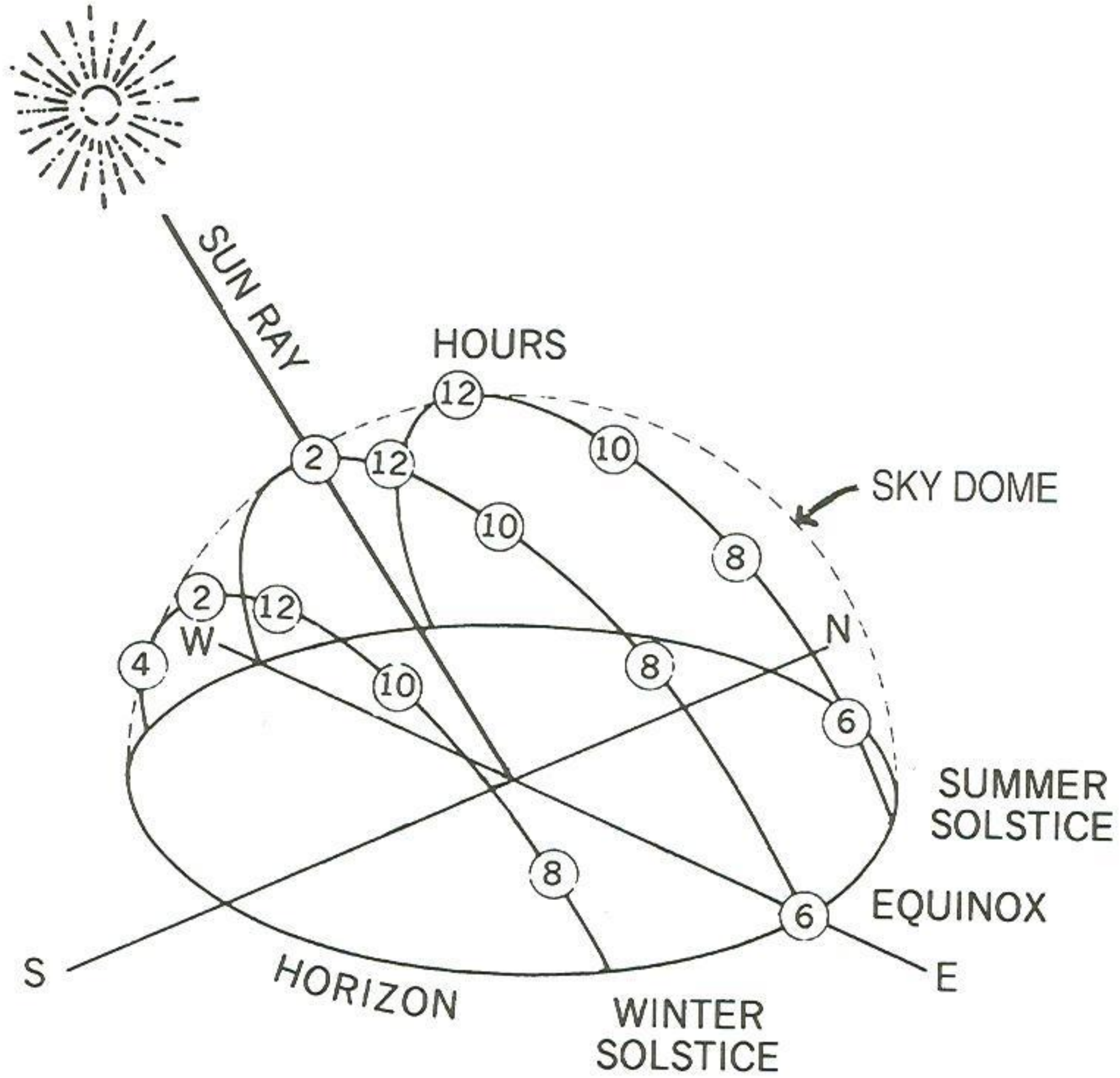
- L : latitude
- δ: declination
- H : hour angle
- n : day of the year

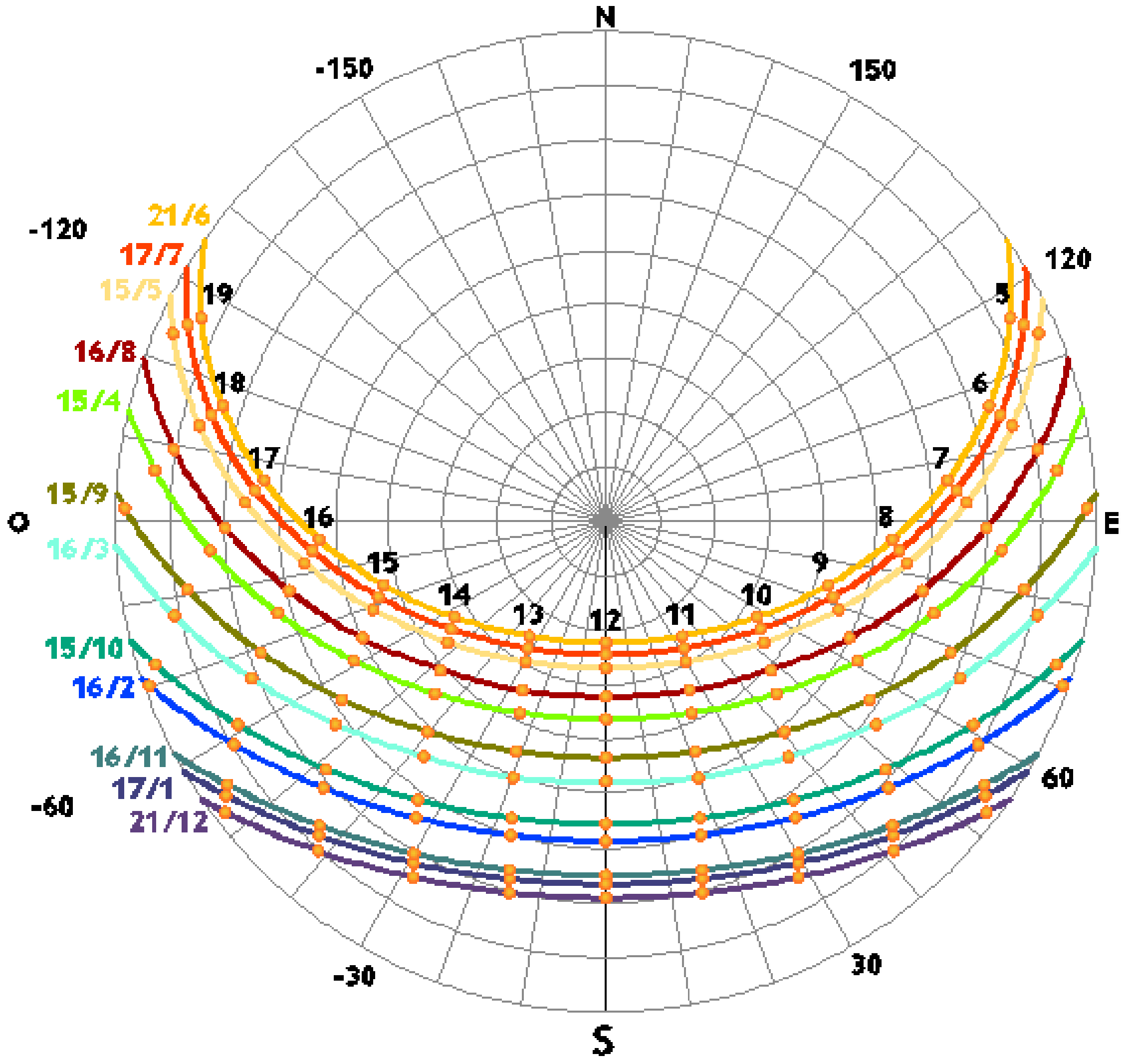


Sun and seasons



Solar path


SUN RAY
HOURS
SKY DOME
SUMMER SOLSTICE
EQUINOX
WINTER SOLSTICE
HORIZON
N
S
E
W
12
2
10
8
6
4


N
S
E
O
-150
150
-120
120
-60
60
-30
30
21/6
17/7
15/5
16/8
15/4
15/9
16/3
15/10
16/2
16/11
17/1
21/12
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5

Solar time

- When computing solar position and solar related quantities all must be referenced using the true solar time
- But solar time differs from the one measured by a clock
- Solar time must be recovered in function of the solar angles and the coordinates of the location
- The difference between true solar time and local time is defined as “Equation of time”
- We define the orbit angle Ω

$$\Omega = 2 \cdot \pi \cdot \frac{n-1}{365}$$

Solar time and equation of time

- The difference between true solar time and local time is defined as “Equation of time”

$$\frac{Et}{hour} = \left(\frac{24}{2\,\pi}\right) \times [a_1 + a_2 \cdot \cos(\Omega) + a_3 \times \sin(\Omega) + a_4 \times \cos(2\Omega) + a_5 \times \sin(2\Omega)]$$

a_1=	0,0000075	a_4=	-0,014615
a_2=	0,001868	a_5=	-0,040849
a_3=	-0,032077		

- For Italy (hour defined with reference at the meridian at east of Greenwich
- $t_{sa} = t_{is} + Et + \frac{G-RM}{15} + DST$ [hour]

  G longitude in degree, positive east
  t_{is} local standard time
  RM = reference meridian in italy RM=15°
  DST daylight saving time, usually -1 h

Position of sun

- Hour angle *H*
  - Positive in the afternoon
  - Negative in the morning
  - t_{sa} solar time in hours

$$H = 15° \times (t_{sa} - 12)$$

- Height of the sun

$$\sin(\beta) = \cos(L) \times \cos(\delta) \times \cos(H) + \sin(L) \times \sin(\delta)$$

- Maximum height at solar noon *H=0*

$$\beta_{max} = 90° - |L - \delta|$$

Position of the sun, solar azimuth

- Solar azimuth
- Solar azimuth is positive facing west
- Can be obtained using the following relations

$$\sin\phi = \sin H \cdot \cos\delta / \cos\beta$$

$$\cos\phi = (\cos \mathrm{H} \cdot \cos\delta \cdot \sin L - \sin\delta \cdot \cos L)/\cos\beta$$

$$\cos\phi = \frac{\sin\beta \cdot \sin L - \sin\delta}{\cos\beta \cdot \cos L}$$

Earth orbit deviation and declination

- the hearth orbit is inclined therefore the declination changes during the year

$$\delta = \frac{360°}{2\pi} \times \left[a_0 + a_1 \times \cos(\Omega) + a_2 \times \sin(\Omega) + a_3 \times \cos(2\Omega) + a_4 \times \sin(2\Omega) + a_5 \times \cos(3\Omega) + a_6 \times \sin(3\Omega) \right]$$

$a_0 = 0{,}006918$

$a_1 = -0{,}399912$

$a_2 = 0{,}070257$

$a_3 = -0{,}006758$

$a_4 = 0{,}000907$

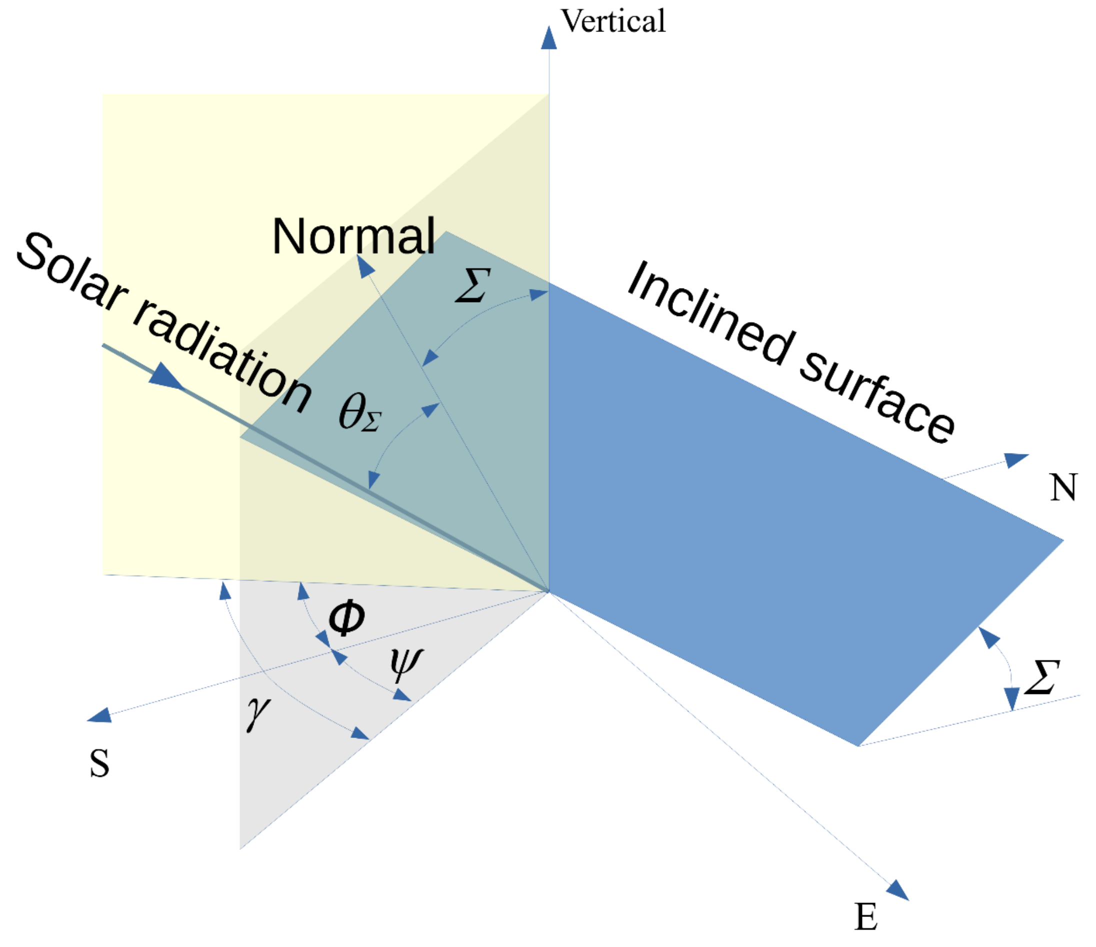
$a_5 = -0{,}002697$

$a_6 = 0{,}00148$

- Simple relation $$\delta = 23.45 \cdot \sin\left(360\,° \cdot \frac{n + 284}{365}\right)$$

Surface and solar radiation

- Tilt angle is the angle between the surface and the horizontal plane. Its value lies between 0 and 180°.
- The surface azimuth ψ is defined as the displacement from south of the projection, on the horizontal plane, of the normal to the surface, negative if faces east.
- The surface-solar azimuth angle γ is defined as the angular difference between the solar azimuth ω and the surface azimuth ψ
- Values of γ greater than 90° or less than –90° indicate that the surface is in the shade.

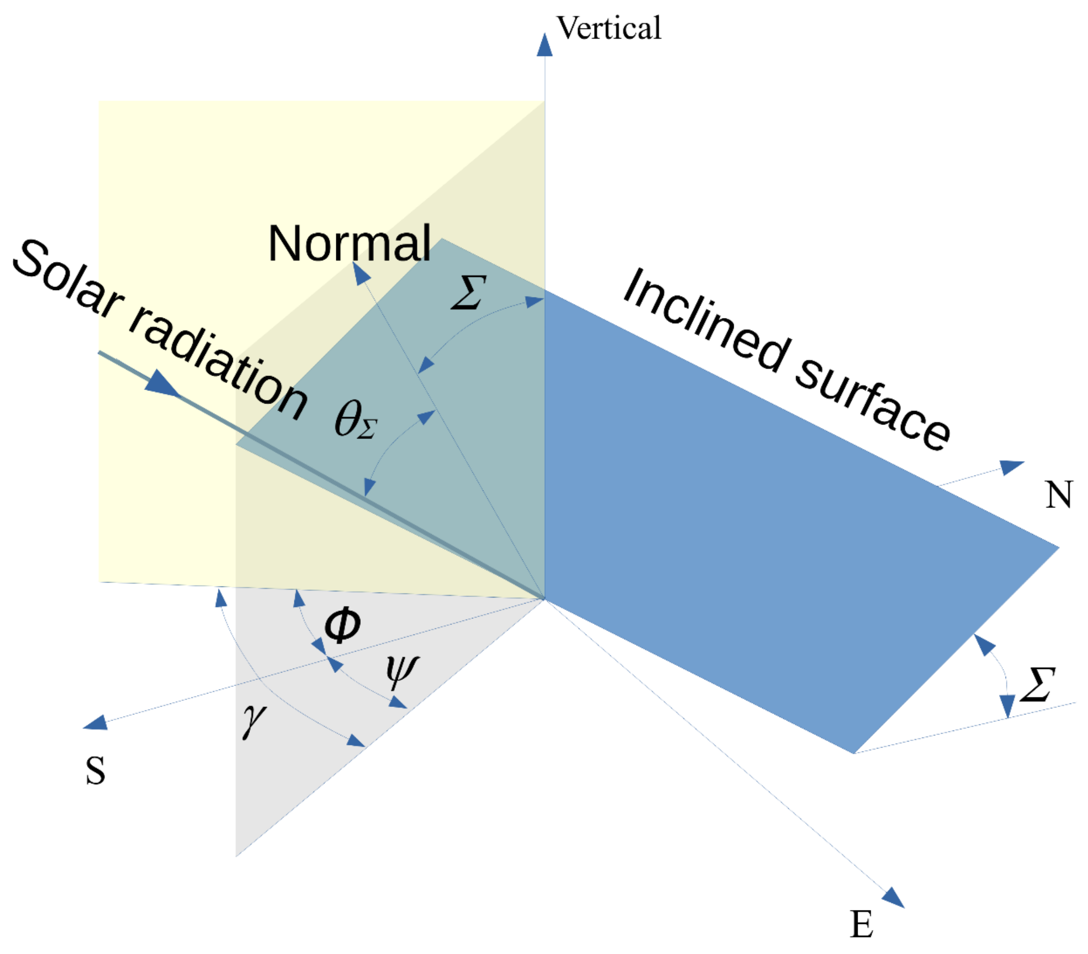


$$\gamma = |\phi - \psi|$$

Angle of incidence

- angle between the line normal to the irradiated surface and the earth-sun line θ_Σ

$$\cos\theta_\Sigma = \cos\ \beta\cdot\cos\gamma\cdot\sin\Sigma + \sin\beta\cdot\cos\Sigma$$



- For vertical surfaces

$$\cos\theta_\Sigma = \cos\beta\cdot\cos\gamma$$

- Horizontal

$$\theta = 90\ - \beta$$

Exernal solar radiation

- External normal solar G_0 radiation can be computed as:

$$G_0 = G \times \ [a_0 + a_1 \times \cos(\Omega) + a_2 \times \sin(\Omega) + a_3 \times \cos(2\Omega) + a_4 \times \sin(2\Omega)]$$
$$a_0 = 1{,}000110$$
$$a_1 = 0{,}034221$$
$$a_2 = 0{,}001280$$
$$a_3 = 0{,}000719$$
$$a_4 = 0{,}000077$$

- solar costant, $G = 1367 \frac{\mathrm{W}}{\mathrm{m}^2}$
- External global radiation on a plane parallel to the horizontal is computed as

$$I_{\mathrm{ho}} = G_0 \times \sin(\beta)$$

Solar radiation

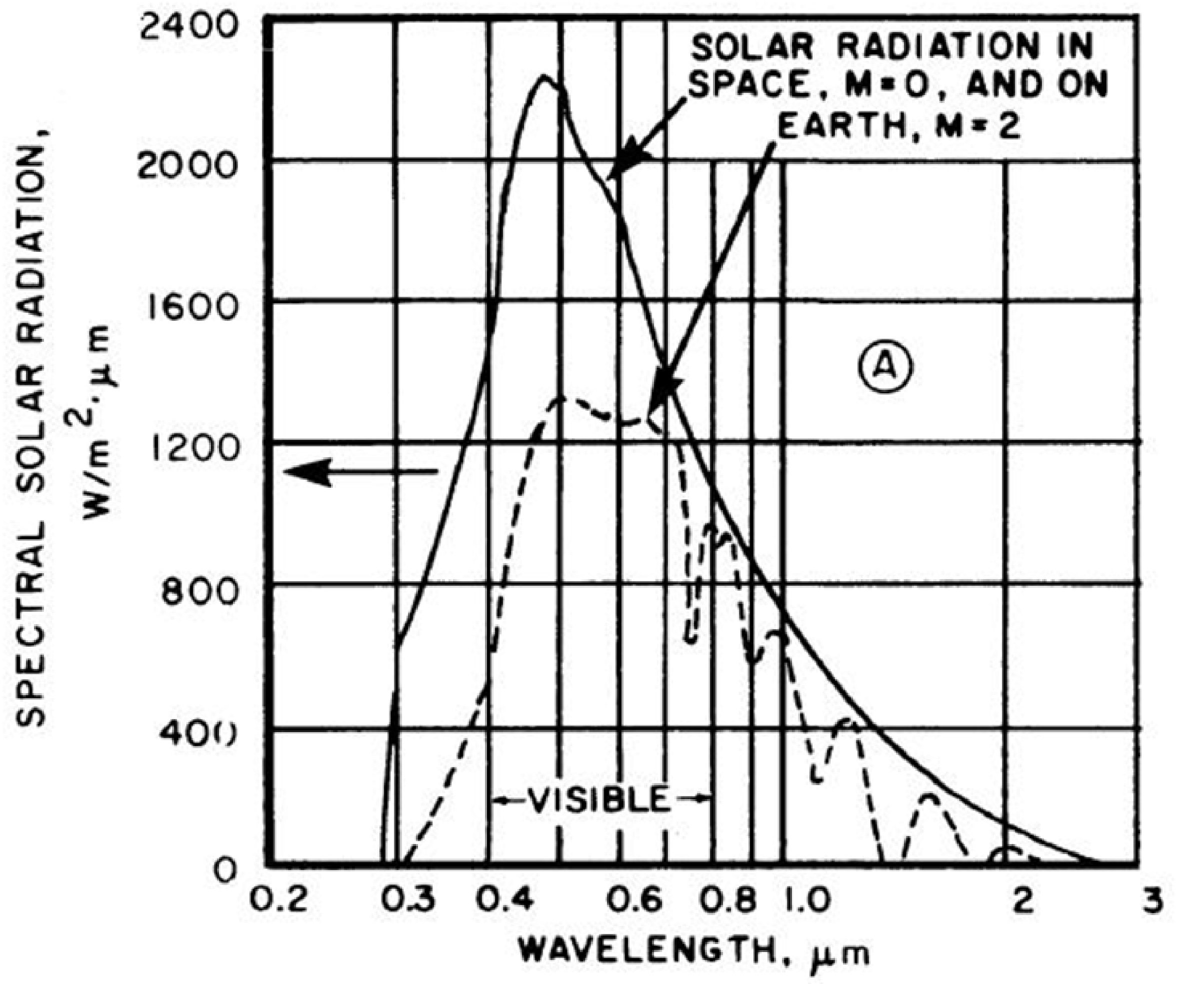


Fig. 7 Terrestrial and Extraterrestrial Solar Spectral Irradiances

- Solar radiation is absorbed
- The spectrum changes
- Solar radiation reaches the ground with two components
- Direct component
- Diffuse component

Air Mass

- The relative air mass *m* is the ratio of the mass of atmosphere in the actual earth/sun path to the mass that would exist if the sun were directly overhead. Air mass is solely a function of solar altitude and is obtained from

$$m = \frac{1}{[\sin\beta + 0.50572 \cdot (6.07995 + \beta)^{-1.6364}}$$

β in degrees

Clear-Sky Solar Radiation

- Solar radiation on a clear day is defined with direct (beam) and diffuse components
- $E_b = E_0 \cdot \exp(-\tau_b \cdot m^{ab})$
- $E_d = E_0 \cdot \exp(-\tau_d \cdot m^{ad})$
- E_b normal irradiance measured in the direction of sun rays
- E_d diffuse radiation on a horizontal plane
- m air mass
- τ_b *and* τ_d beam and diffuse optical depth

Air mass exponents

$$ab = 1.219 - 0.043 \cdot \tau_b - 0.151 \cdot \tau_d - 0.204 \cdot \tau_b \cdot \tau_d$$

$$ad = 0.202 - 0.852 \cdot \tau_b - 0.007 \cdot \tau_d - 0.357 \cdot \tau_b \cdot \tau_d$$

		Annual	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Clear Sky Solar Irradiance	taub		0.344	0.369	0.417	0.461	0.465	0.476	0.465	0.456	0.44	0.419	0.376	0.34
	taud		2.401	2.32	2.196	2.101	2.14	2.159	2.198	2.238	2.247	2.27	2.336	2.43
	Ebn at noon		748	795	802	799	809	800	804	795	771	725	694	711
	Edn at noon		74	97	125	150	149	147	140	129	118	101	79	66

Fenestration

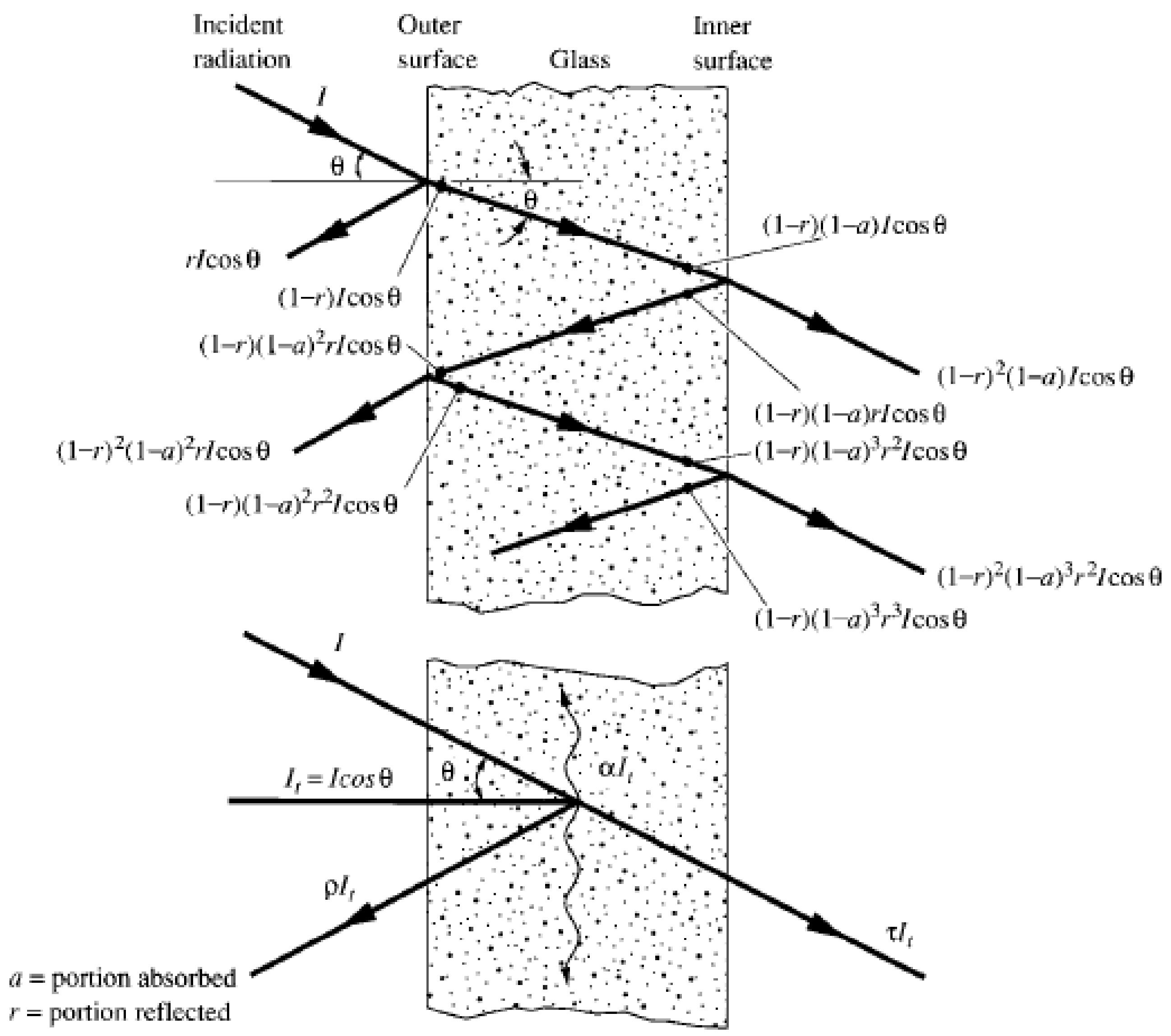
- *Clear plate or sheet glass or plastic*. Clear plate glass permits good visibility and transmits more solar radiation than other types.
- *Tinted heat-absorbing glass*. Tinted heat-absorbing glass is fabricated by adding small amounts of selenium, nickel, iron, or tin oxides. These produce colors from pink to green, including gray or bluish green, all of which absorb infrared solar heat and release a portion of this to the outside atmosphere through outer surface convection and radiation. Heat-absorbing glass also reduces visible light transmission.
- *Insulating glass*. Insulating glass consists of two panes—an outer plate and a inner plate—or three panes separated by metal, foam, or rubber spacers around the edges and hermetically sealed in a stainless-steel or aluminum-alloy structure. The dehydrated space between the glass panes usually has a thickness of 0.125 to 0.75 in. (3.2–19 mm) and is filled with air, argon, or other inert gas. Air- or gas-filled space increases the thermal resistance of the fenestration.
- *Reflective coated glass*. Reflective glass has a microscopically thin layer of metallic or ceramic coating on one surface of the glass, usually the inner surface of a single-pane glazing or the outer surface of the inner plate for an insulating glass. For a single pane, the coating is often protected by a layer of transparent polyester. The chromium and other metallic coatings give excellent reflectivity in the infrared regions but reduced transmission of visible light compared to clear plate and heat-absorbing glass. Reflections from buildings with highly reflective glass may blind drivers, or even kill grass in neighboring yards.
- *Low-emissivity (low-E) glass coatings*. Glazing coated with low-emissivity, or low-*E*, films has been in use since 1978. It is widely used in retrofit applications. A low-emissivity film is usually a vacuum-deposited metallic coating, usually aluminum, on a polyester film, at a thickness of about 4 107 in. (0.01 m).

Optical properties

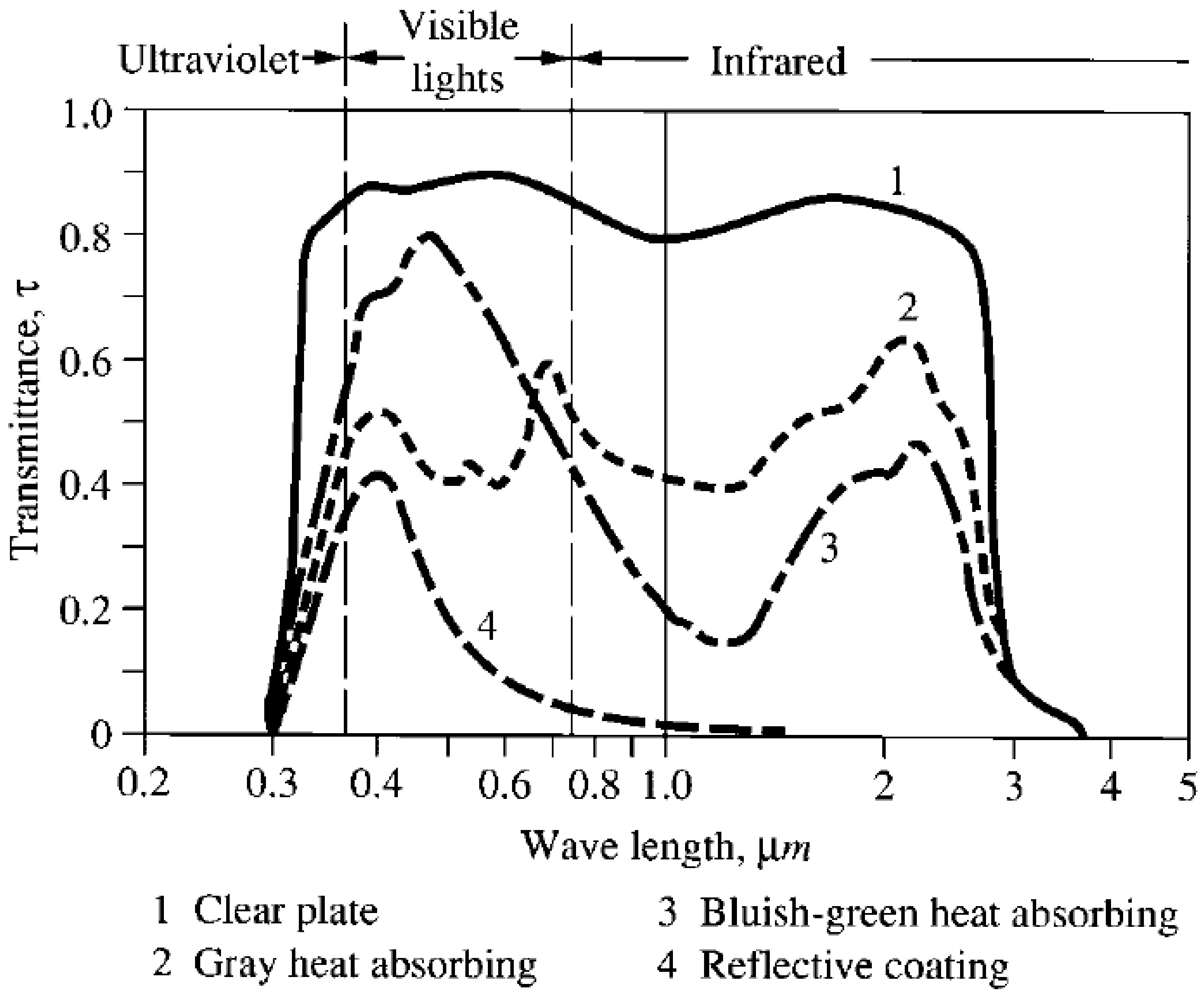
- Solar radiation is
  - Transmitted
  - Reflected
  - Absorbed

$$\tau + \alpha + \rho = 1$$

$$I \cdot \tau + I \cdot \alpha + I \cdot \rho = I$$



Spectral transmittance of window glasses



- Different glasses perform in different way
- Each glass has a spectral transmittance
- Spectral transmittance can be modified also using films

Heat trough windows

- $Heat\ gain\ through\ window = \ solar\ radiation\ transmitted\ + \ inward\ heat\ flow\ from\ glass\ inner\ surface$

$$\frac{Q_{wi}}{A_s} = \frac{\tau \cdot I_t + Q_{RCi}}{A_s}$$

- Q_{RCi} inward heat flow from inner surface

Single glazing

- Q_{RCi} inward absorbed radiation + conductive heat transfer

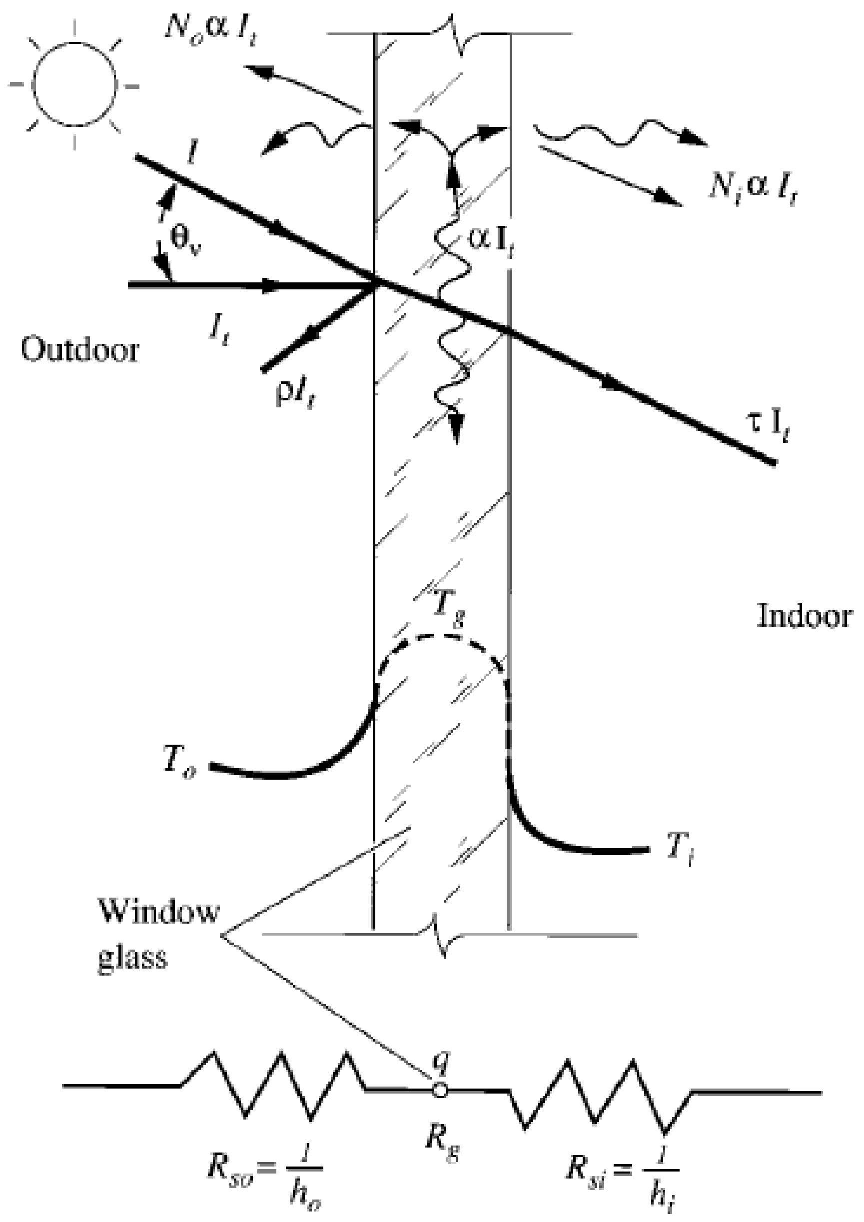
$$Q_{RCi} = U \cdot A_s \cdot \left(\frac{\alpha \cdot I_t}{h_o} + T_o - T_i \right)$$

- Heat admitted through a unit area of the single-glazing

$$\frac{Q_{wi}}{A_s} = \tau I_t + U \cdot \left(\frac{\alpha \cdot I_t}{h_o} + T_o - T_i \right)$$

- Solar heat gain coefficient (SHGC) ratio of solar heat gain entering the space to the incident solar radiation

$$SHGC = \frac{Q_{ws}}{I_t \cdot A_s} = \tau + \frac{U \cdot \alpha}{h_o}$$



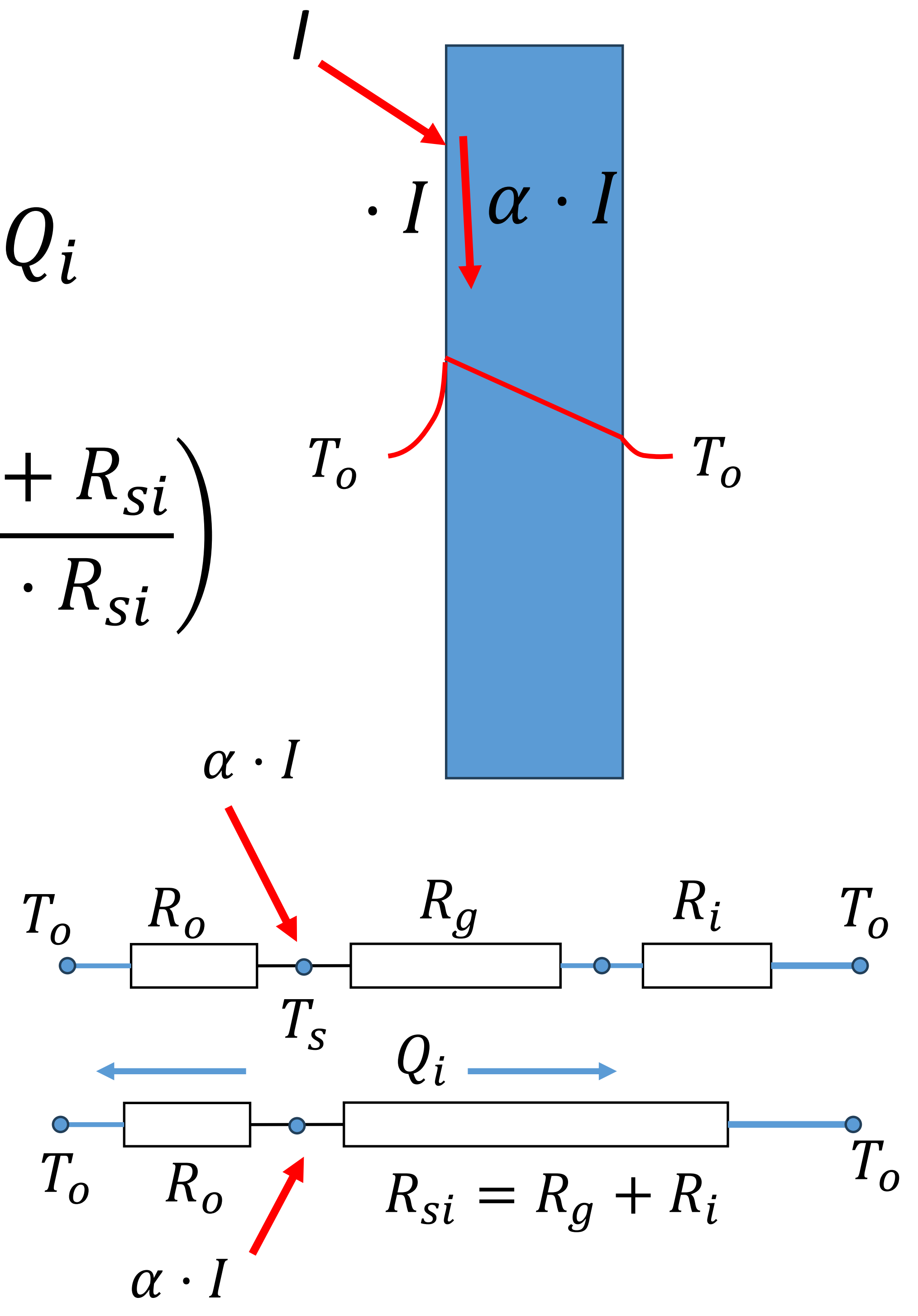
Radiation absorbed and transferred at the internal side

$$Q_o = \frac{T_s - T_o}{R_o} \qquad Q_i = \frac{T_s - T_o}{R_{tot} - R_o} = \frac{T_s - T_o}{R_{si}} \qquad \alpha \cdot I_t = Q_o + Q_i$$

$$\alpha \cdot I_t = \frac{T_s - T_o}{R_o} + \frac{T_s - T_o}{R_{si}} = (\mathrm{T_s - T_o}) \cdot \left(\frac{1}{R_o} + \frac{1}{R_{si}}\right) = (T_s - T_o) \cdot \left(\frac{R_o + R_{si}}{R_o \cdot R_{si}}\right)$$

$$Q_i \cdot R_{si} = T_s - T_o \qquad \alpha \cdot I_t = Q_i \cdot \cancel{R_{si}} \cdot \left(\frac{R_o + R_{si}}{R_o \cdot \cancel{R_{si}}}\right)$$

$$Q_i = \alpha \cdot I_t \cdot \frac{R_o}{R_o + R_{si}} \qquad Q_i = \alpha \cdot I_t \cdot \frac{R_o}{R_{tot}} = \alpha \cdot I_t \cdot R_o \cdot U$$

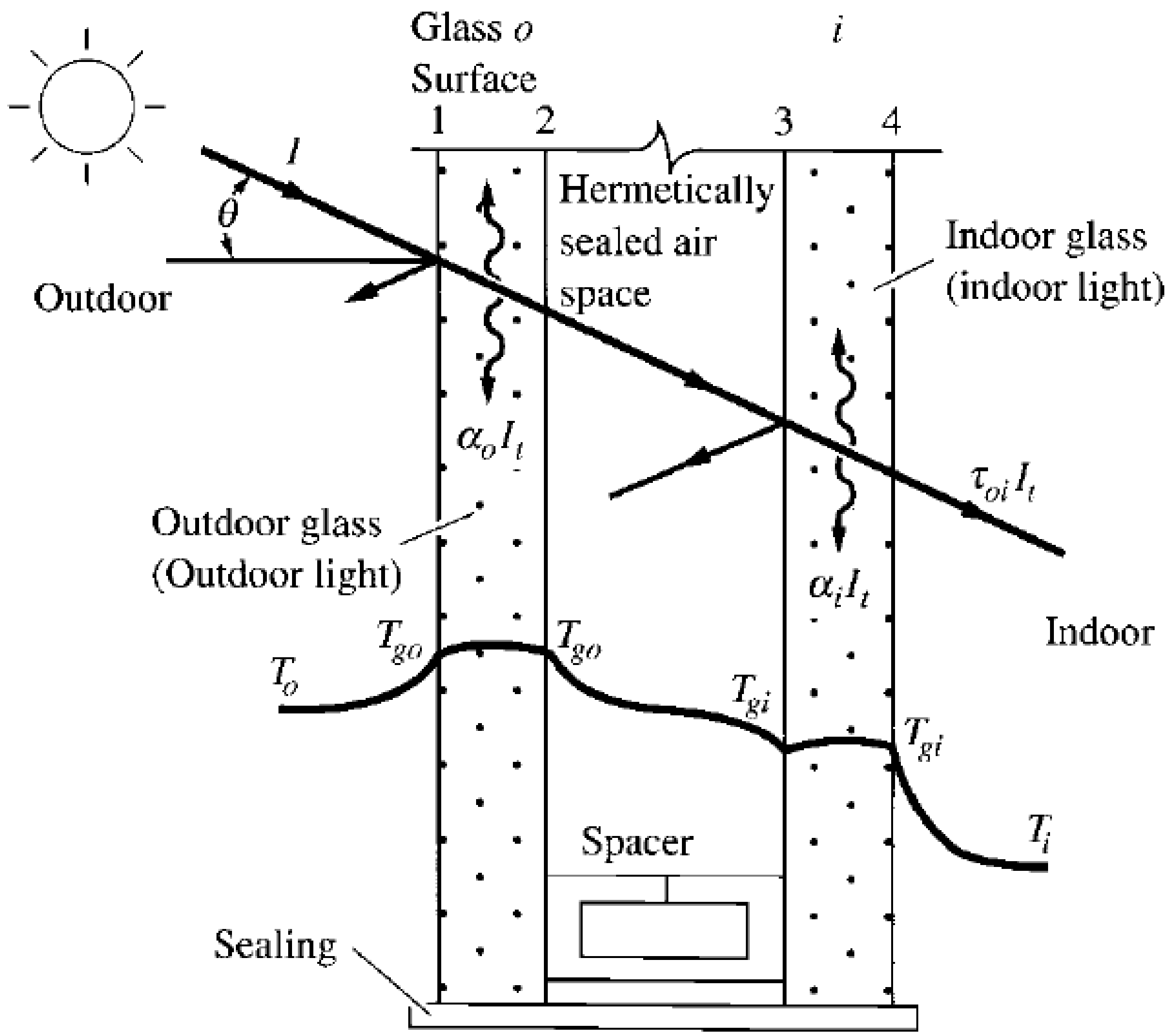


Double Glazing

$$\frac{Q_{woi}}{A_s} = \tau_{oi} \cdot I_t + U \cdot \left(\frac{\alpha_o}{h_o} + \frac{\alpha_i}{h_o} + \frac{\alpha_i}{h_o}\right) + I_t + U \cdot (T_o - T_i)$$

$$\frac{Q_{woi}}{A_s} = SHGC_{oi} \cdot I_t + U \cdot (T_o - T_i)$$

$$\tau_{oi} = \frac{\tau_o \cdot \tau_i}{1 - \rho_2 \cdot \rho_3}$$



Shading Coefficient

- The Shading Coefficient SC is defined as the solar heat gain of the specified glass over the shading coefficient of a double-strenght sheet glass

$$SC = \frac{SHGC_W}{SHGC_{DSA}} = \frac{SHGC_W}{0.87} = 1.15 \cdot SHGC_w$$

Shading Coefficients

Type of glass	Thickness of glass, in.	Solar transmittance	Glass	Venetian blinds Med.*	Venetian blinds Light†	Roller shade Opaque, white	Roller shade Translucent	Draperies Med.‡	Draperies Light§
Clear	$\frac{3}{32}$	0.87 to 0.79		0.74	0.67	0.39	0.44	0.62	0.52
Heat-absorbing	$\frac{3}{16}$ or $\frac{1}{4}$	0.46		0.57	0.53	0.30	0.36	0.46	0.44
	$\frac{3}{8}$	0.34		0.54	0.52	0.28	0.32		
		0.24		0.42	0.40	0.28	0.31	0.38	0.36
Reflective-coated			0.30	0.25	0.23				
			0.40	0.33	0.29				
			0.50	0.42	0.38				
Insulating glass		Outer Inner							
Clear out	$\frac{3}{32}$, or $\frac{1}{8}$	0.87 0.87		0.62	0.58	0.35	0.40		
Clear in									
Heat-absorbing out	$\frac{1}{4}$	0.46 0.80		0.39	0.36	0.22	0.30		
Clear in									
Reflective glass			0.20	0.19	0.18				
			0.30	0.27	0.26				
			0.40	0.34	0.33				

*Med. indicates medium color.

†Light indicates light color.

‡Draperies Med. represents draperies of medium color with a fabric openness of 0.10 to 0.25 and yarn reflectance of 0.25 to 0.50.

§Draperies Light repesents draperies of light color with a fabric openness below 0.10 and yarn reflectance over 0.50.