X-ray interaction with matter

The 4 major interactions of x-ray and gamma-ray photons with matter are:

- Photoelectric effect
- Rayleigh scattering
- Compton scattering
- Pair production (above 1.022 MeV)

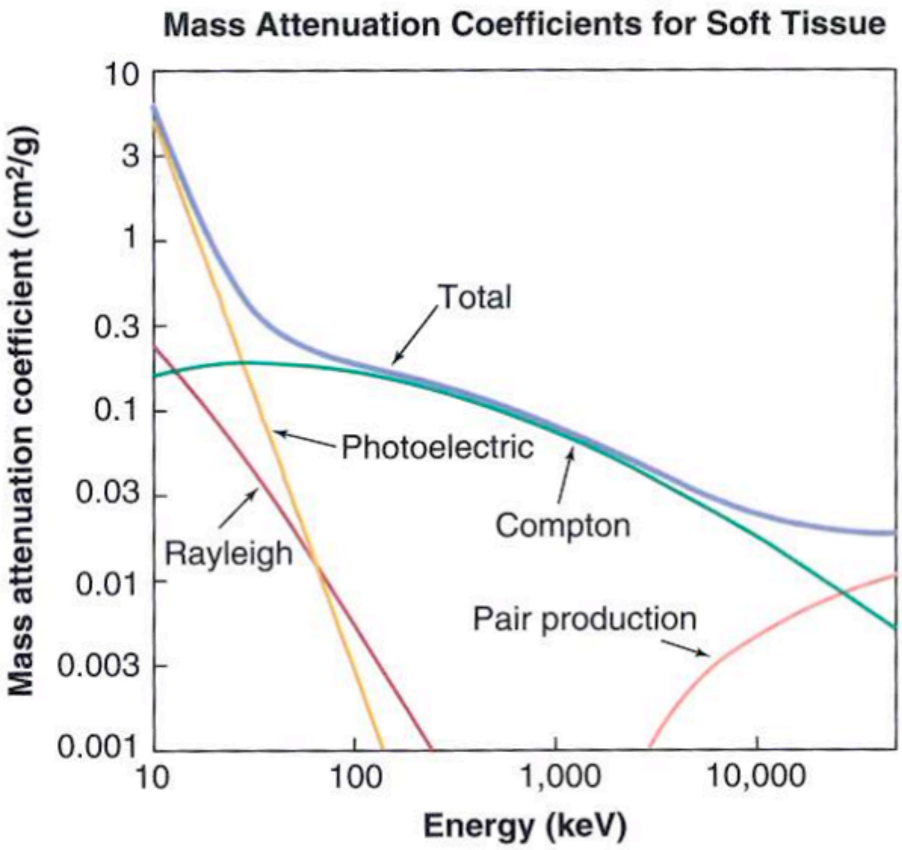


FIGURE 3-13 Graph of the Rayleigh, photoelectric, Compton, pair production, and total mass attenuation coefficients for soft tissue ($Z \approx 7$) as a function of photon energy.

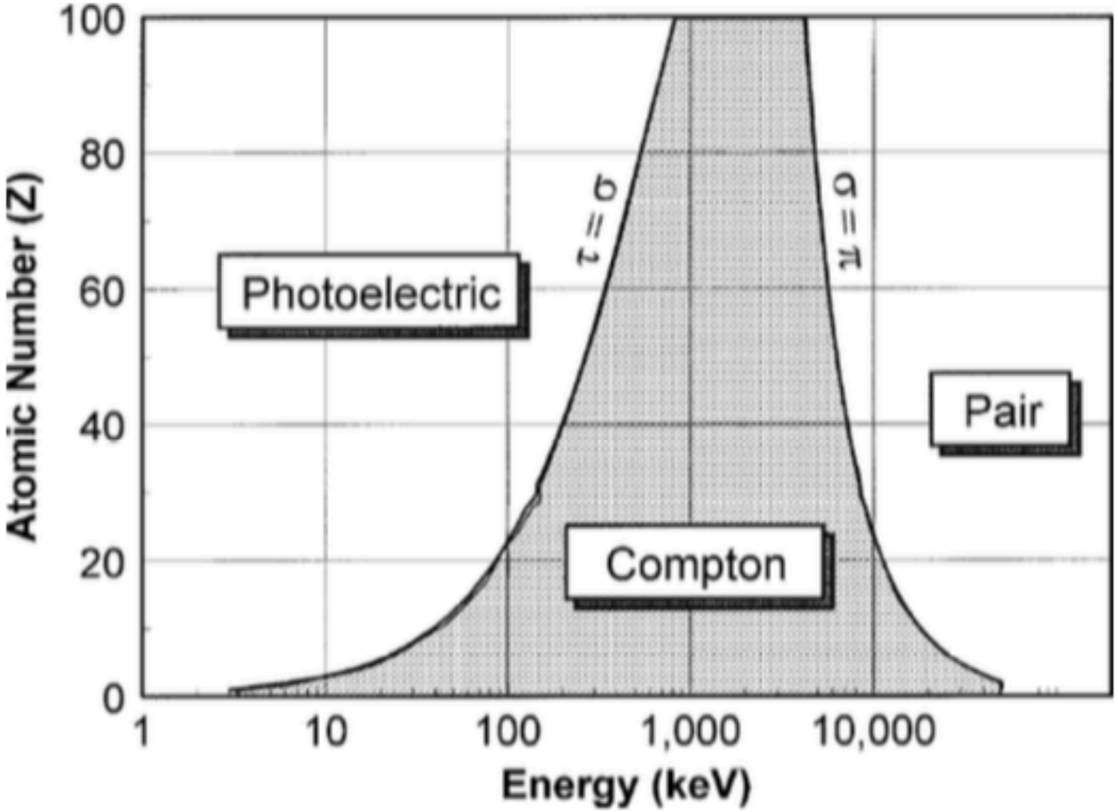
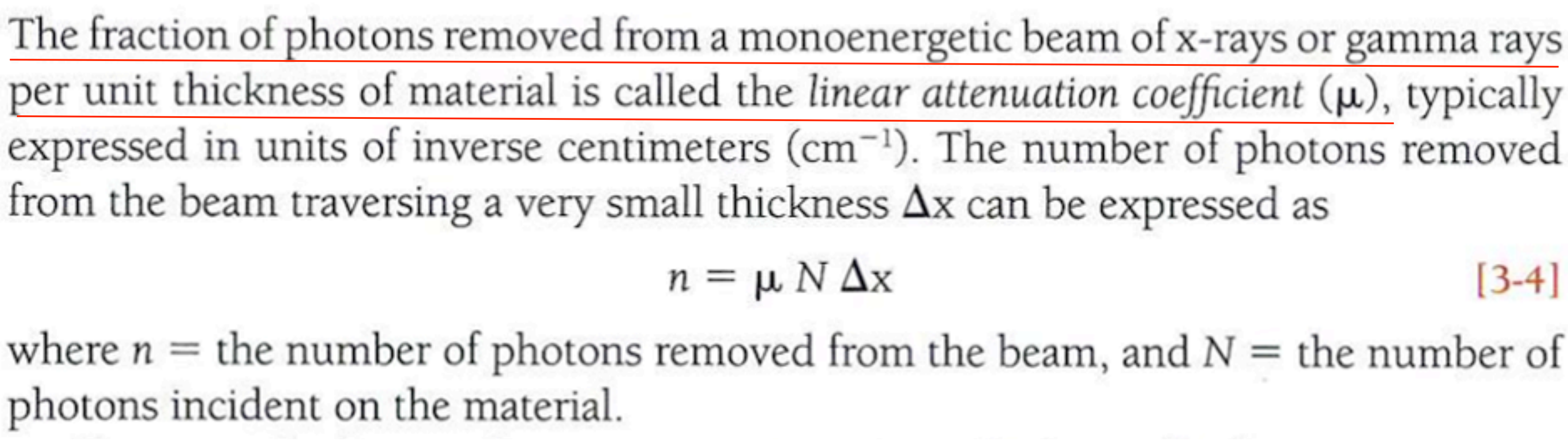


Figure 1.27: The region where each x-ray interaction process is most likely is shown as a function of atomic number and x-ray energy. The transition zones between regions correspond to the two cross sections being equal ($\tau = \sigma$ and $\sigma = \pi$).

Linear attenuation coefficient



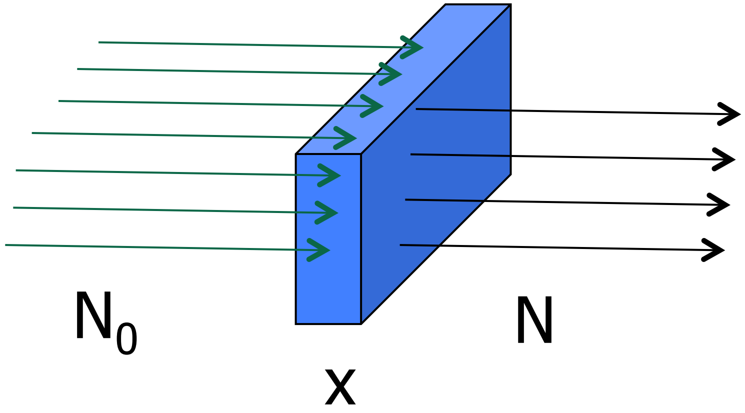
The fraction of photons removed from a monoenergetic beam of x-rays or gamma rays per unit thickness of material is called the *linear attenuation coefficient* (μ), typically expressed in units of inverse centimeters (cm^{-1}). The number of photons removed from the beam traversing a very small thickness Δx can be expressed as

$$n = \mu N \Delta x \qquad [3\text{-}4]$$

where n = the number of photons removed from the beam, and N = the number of photons incident on the material.

The relationship for thin slab ΔX, can be integrated. If N_0 is the number of the incident photons and N the transmitted photons through a thickness x without any interaction (primary):

$$N = N_0 e^{-\mu x}$$



Linear attenuation coefficient mathematical model

Let's consider monoenergetic photons through an absorber of infinitesimal thickness dx of the same material and the same density
If n is the number of photons removed from the beam: $n = -dN$

where $-dN$ is the variation in photon number; minus sign indicating that the intensity is reduced by the absorber

$$-dN = \mu N dx$$

Dividing by N

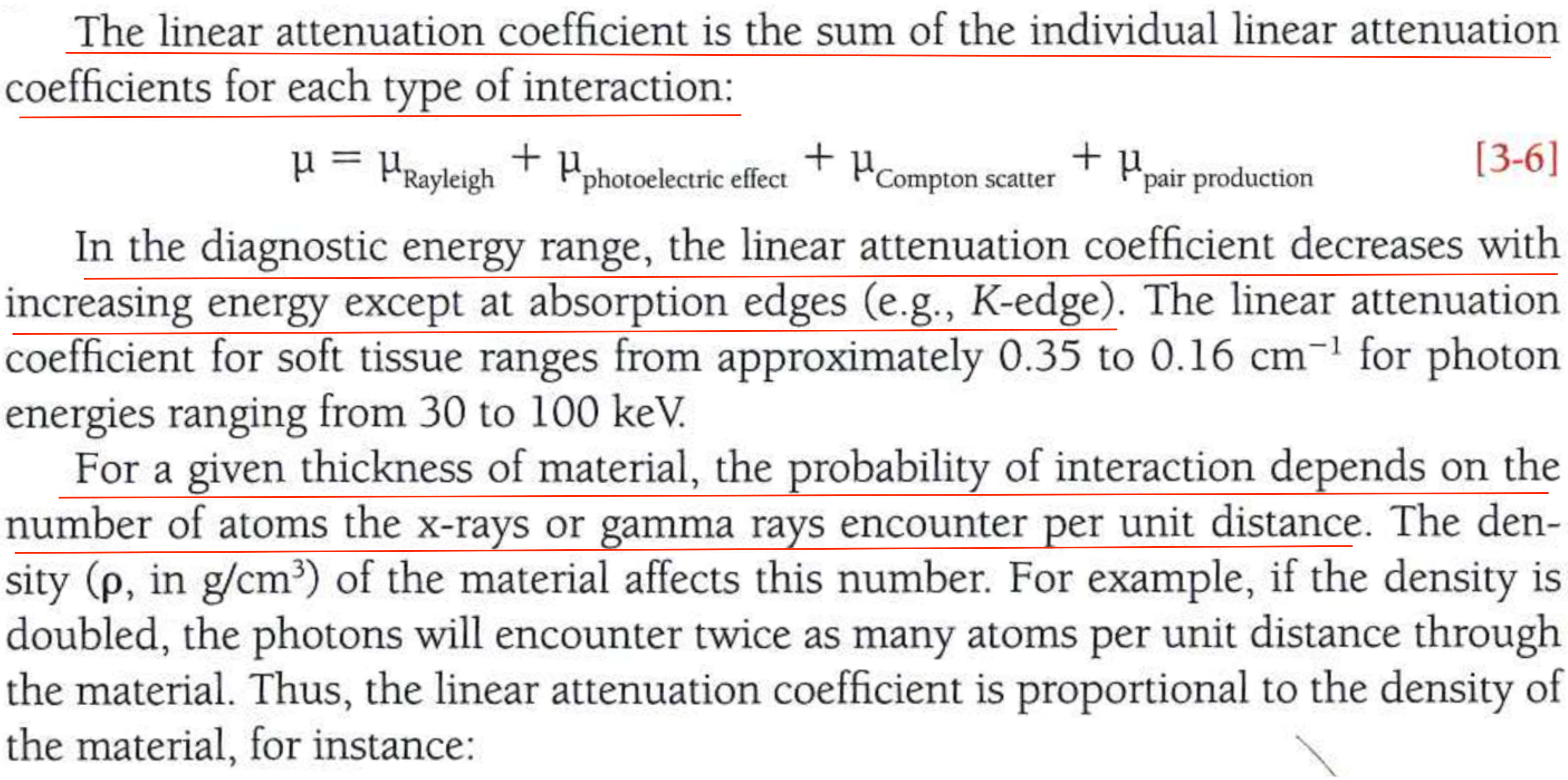
$$-\frac{dN}{N} = \mu \, dx$$

This equation describes the situation for thin absorber with thickness dx.
For the thickness x of an absorber we integrate the above equation.
From thickness 0 to x, the radiation intensity will decrease from N_0 to N:

$$-\int_{N_0}^{N} \frac{dN}{N} = \mu \int_0^x dx \qquad \ln\left(\frac{N}{N_0}\right) = -\mu \, x$$

$$\boxed{\frac{N}{N_0} = \exp(-\mu \, x)}$$

Linear attenuation coefficient



The linear attenuation coefficient is the sum of the individual linear attenuation coefficients for each type of interaction:

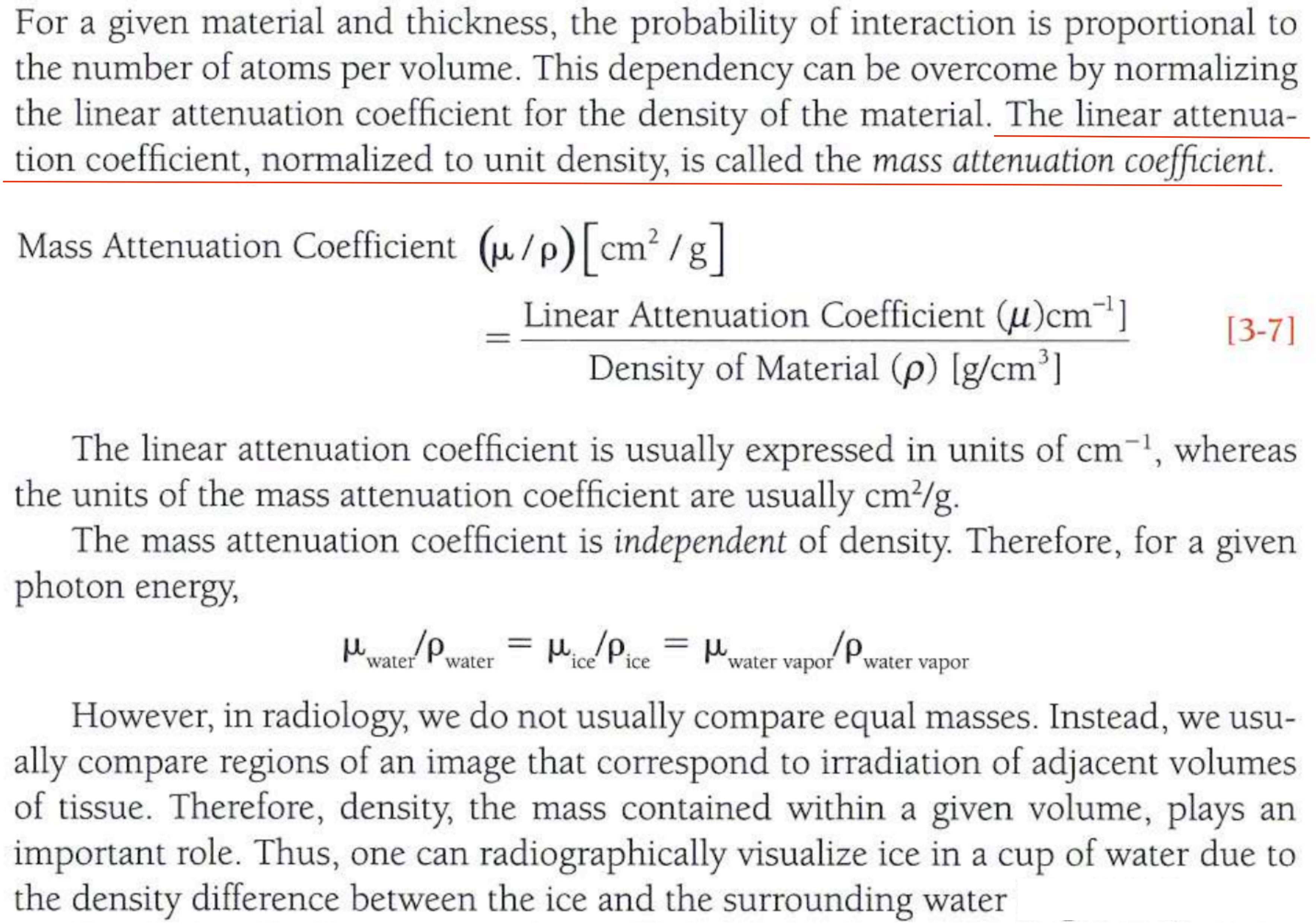
$$\mu = \mu_{\text{Rayleigh}} + \mu_{\text{photoelectric effect}} + \mu_{\text{Compton scatter}} + \mu_{\text{pair production}} \qquad [3\text{-}6]$$

In the diagnostic energy range, the linear attenuation coefficient decreases with increasing energy except at absorption edges (e.g., *K*-edge). The linear attenuation coefficient for soft tissue ranges from approximately 0.35 to 0.16 cm^{-1} for photon energies ranging from 30 to 100 keV.

For a given thickness of material, the probability of interaction depends on the number of atoms the x-rays or gamma rays encounter per unit distance. The density (ρ, in g/cm^3) of the material affects this number. For example, if the density is doubled, the photons will encounter twice as many atoms per unit distance through the material. Thus, the linear attenuation coefficient is proportional to the density of the material, for instance:

$$\mu_{\text{water}} > \mu_{\text{ice}} > \mu_{\text{water vapor}}$$

Mass attenuation coefficient



For a given material and thickness, the probability of interaction is proportional to the number of atoms per volume. This dependency can be overcome by normalizing the linear attenuation coefficient for the density of the material. The linear attenuation coefficient, normalized to unit density, is called the *mass attenuation coefficient*.

$$\text{Mass Attenuation Coefficient } (\mu/\rho)\left[\text{cm}^2/\text{g}\right] = \frac{\text{Linear Attenuation Coefficient } (\mu)\text{cm}^{-1}]}{\text{Density of Material } (\rho)\ [\text{g/cm}^3]} \qquad [3\text{-}7]$$

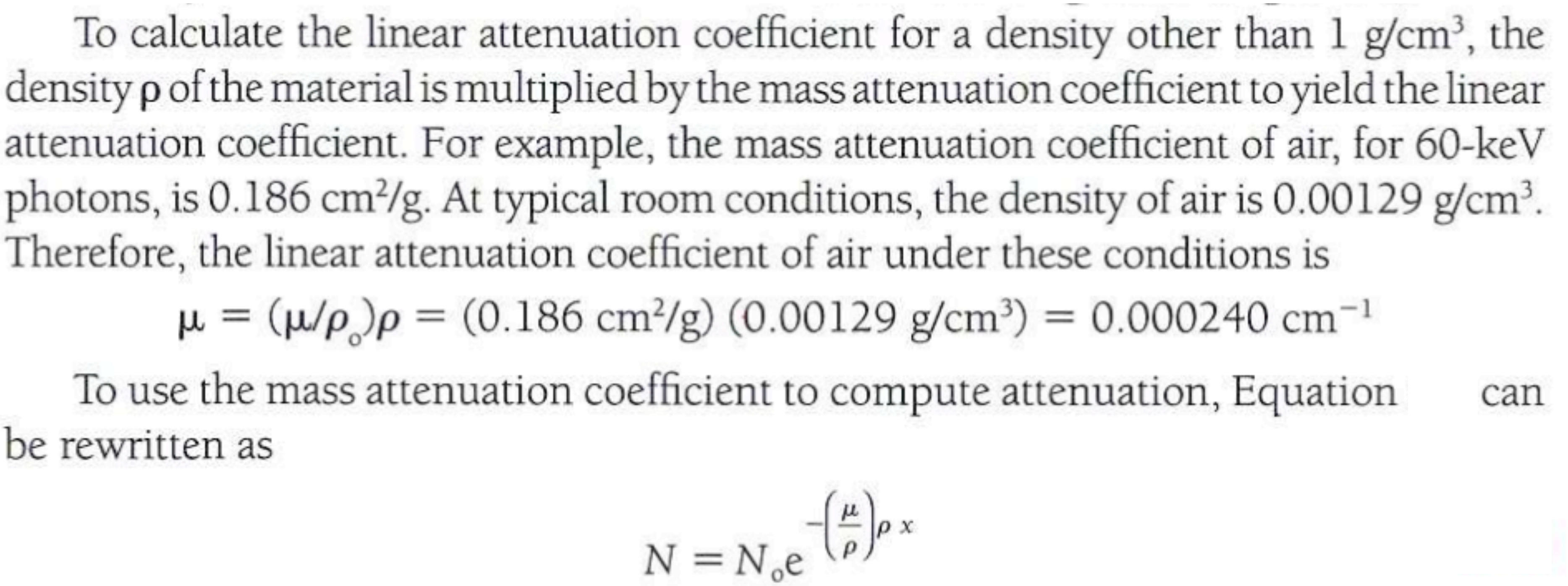
The linear attenuation coefficient is usually expressed in units of cm^{-1}, whereas the units of the mass attenuation coefficient are usually cm^2/g.

The mass attenuation coefficient is *independent* of density. Therefore, for a given photon energy,

$$\mu_{\text{water}}/\rho_{\text{water}} = \mu_{\text{ice}}/\rho_{\text{ice}} = \mu_{\text{water vapor}}/\rho_{\text{water vapor}}$$

However, in radiology, we do not usually compare equal masses. Instead, we usually compare regions of an image that correspond to irradiation of adjacent volumes of tissue. Therefore, density, the mass contained within a given volume, plays an important role. Thus, one can radiographically visualize ice in a cup of water due to the density difference between the ice and the surrounding water

Mass attenuation coefficient



To calculate the linear attenuation coefficient for a density other than 1 g/cm^3, the density ρ of the material is multiplied by the mass attenuation coefficient to yield the linear attenuation coefficient. For example, the mass attenuation coefficient of air, for 60-keV photons, is 0.186 cm^2/g. At typical room conditions, the density of air is 0.00129 g/cm^3. Therefore, the linear attenuation coefficient of air under these conditions is

$$\mu = (\mu/\rho_o)\rho = (0.186\ \mathrm{cm}^2/\mathrm{g})\ (0.00129\ \mathrm{g/cm}^3) = 0.000240\ \mathrm{cm}^{-1}$$

To use the mass attenuation coefficient to compute attenuation, Equation can be rewritten as

$$N = N_o e^{-\left(\frac{\mu}{\rho}\right)\rho x}$$

Linear attenuation coefficient

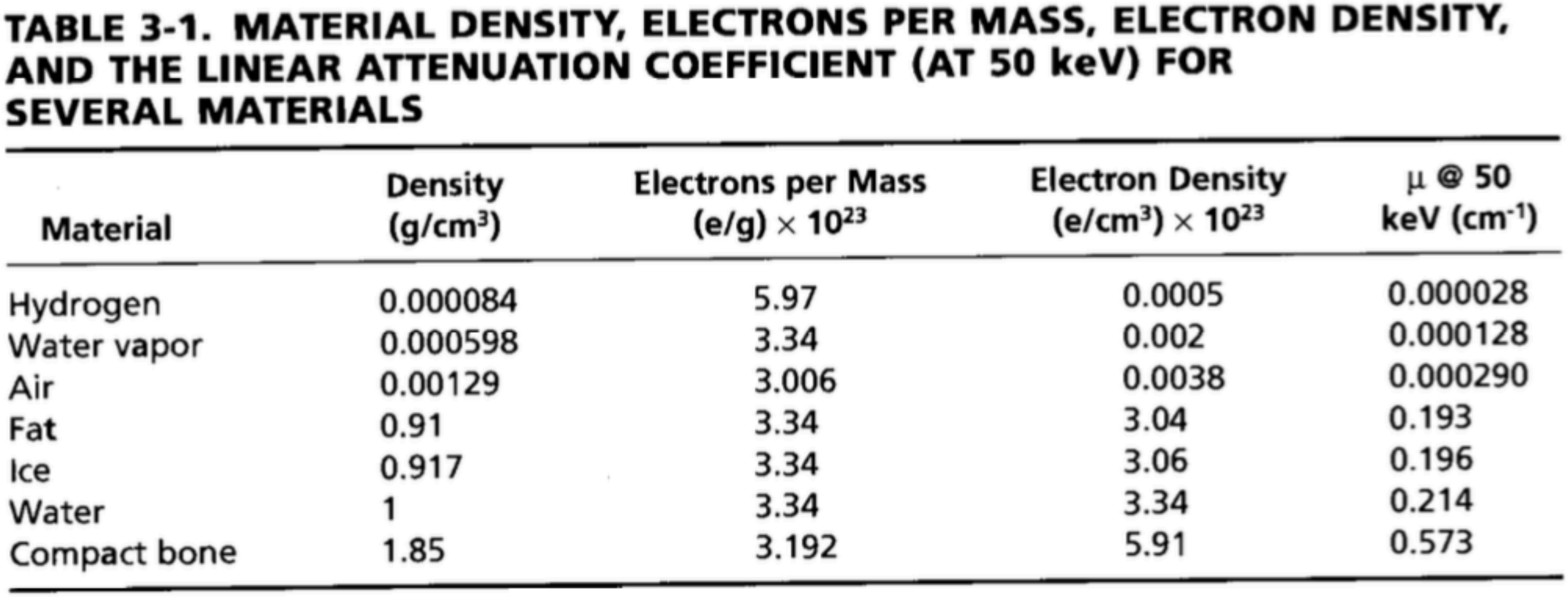


TABLE 3-1. MATERIAL DENSITY, ELECTRONS PER MASS, ELECTRON DENSITY, AND THE LINEAR ATTENUATION COEFFICIENT (AT 50 keV) FOR SEVERAL MATERIALS

Material	Density (g/cm^3)	Electrons per Mass (e/g) × 10^{23}	Electron Density (e/cm^3) × 10^{23}	μ @ 50 keV (cm^{-1})
Hydrogen	0.000084	5.97	0.0005	0.000028
Water vapor	0.000598	3.34	0.002	0.000128
Air	0.00129	3.006	0.0038	0.000290
Fat	0.91	3.34	3.04	0.193
Ice	0.917	3.34	3.06	0.196
Water	1	3.34	3.34	0.214
Compact bone	1.85	3.192	5.91	0.573

Good and bad geometry

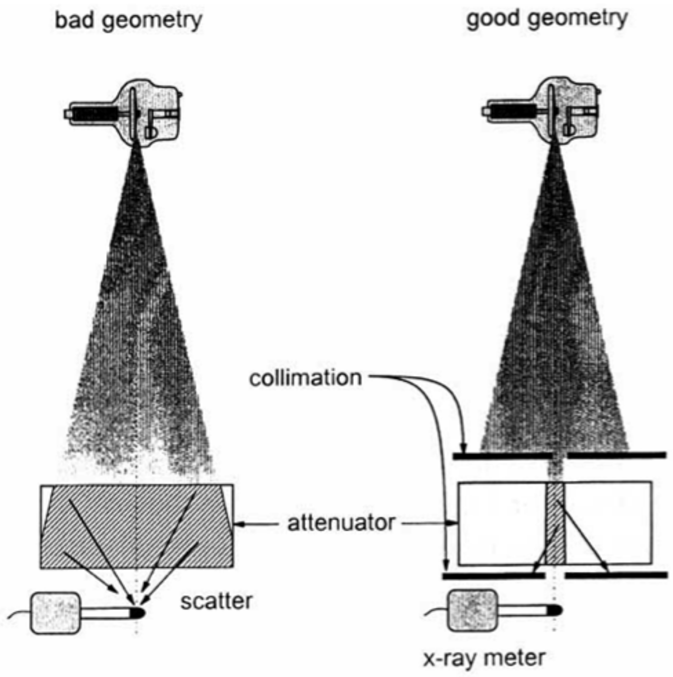


Figure 1.32: The so-called *bad geometry* and *good geometry* for making x-ray attenuation measurements are illustrated. Bad geometry exists whenever the exposure measurement includes an appreciable amount of x-ray scatter from the attenuator. The scatter contribution to the measurement can be reduced by using pre-attenuator collimation to limit the x-ray field and post-attenuator collimation to reduce the chance of scatter reaching the x-ray detector.

Attenuation is the removal of photons from the x-ray beam, both by absorption and scattering. To assess attenuation, the x-ray beam that is *not* removed by attenuation is what is actually measured (the primary x-ray beam). It is important that scattered photons not be included in this measurement. Because scattered x rays tend to fly about in all directions near an object being exposed to an x-ray beam, it is important to use a measurement geometry which excludes the measurement of scattered x-ray photons, to the extent possible.

Half-value layer

The half-value layer (HVL) is defined as the thickness of material required to reduce the intensity (e.g., air kerma rate) of an x-ray or gamma-ray beam to one half of its initial value. The HVL of a beam is an indirect measure of the photon energies (also referred to as the *quality*) of a beam, when measured under conditions of *narrow-beam geometry*. Narrow-beam geometry refers to an experimental configuration that is designed to exclude scattered photons from being measured by the detector (Fig. 3-15A). In *broad-beam geometry*, the beam is sufficiently wide that a substantial fraction of scattered photons remain in the beam.

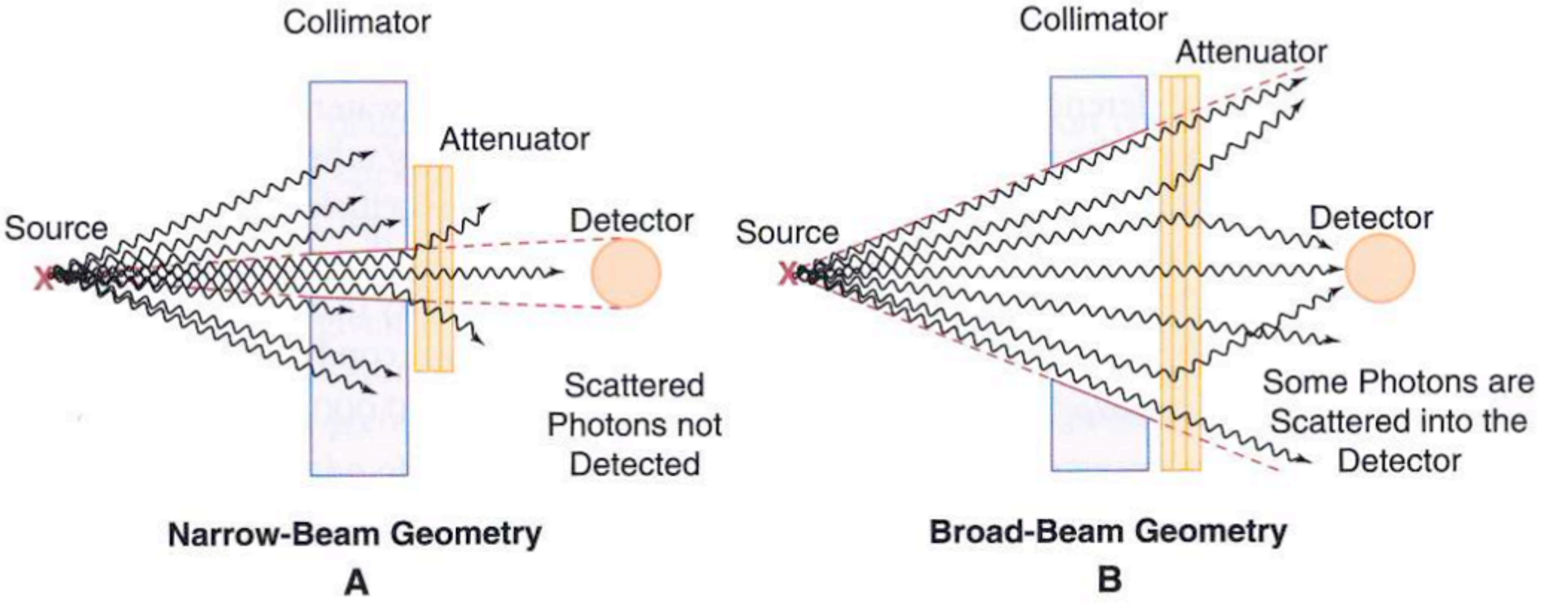


FIGURE 3-15 A. Narrow-beam geometry means that the relationship between the source shield and the detector is such that almost no scattered photons interact with the detector. **B.** In broad-beam geometry, scattered photons may reach the detector; thus, the measured attenuation is less compared with narrow-beam conditions.

Underestimation of attenuation means overestimation of HVL

Half-value layer

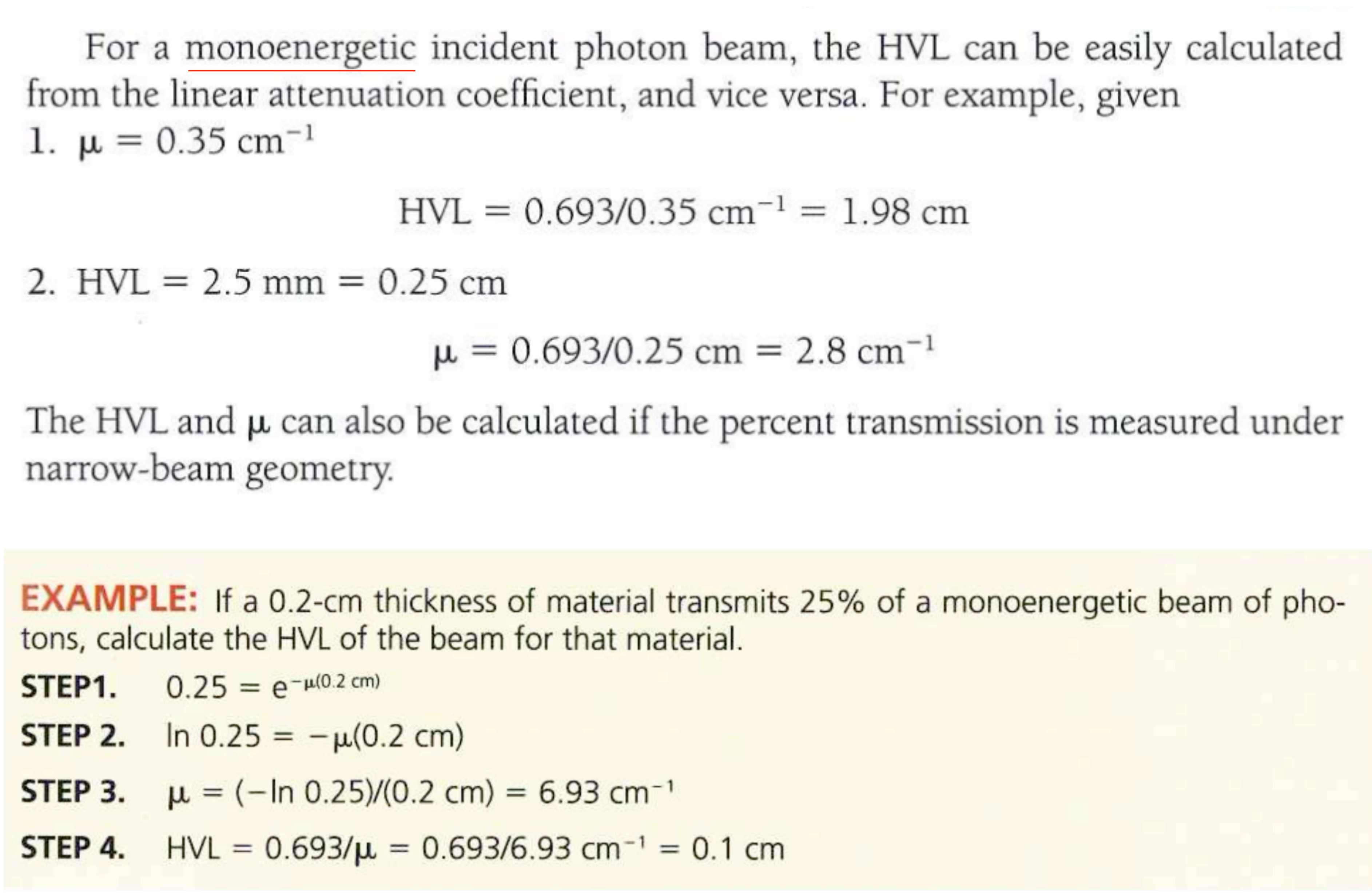
The HVL of a diagnostic x-ray beam, measured in millimeters of aluminum under narrow beam conditions, is a measure of the penetrability of the x-ray spectrum.

Relationship between μ and HVL

N is equal to $N_0/2$ when the thickness of the absorber is 1 HVL. Thus, for monoenergetic beam:

$$
\begin{aligned}
N_o/2 &= N_o e^{-\mu(\mathrm{HVL})} \\
1/2 &= e^{-\mu(\mathrm{HVL})} \\
\ln(1/2) &= \ln e^{-\mu(\mathrm{HVL})} \\
-0.693 &= -\mu\,(\mathrm{HVL}) \\
\mathrm{HVL} &= 0.693/\mu
\end{aligned}
$$

Half-value layer



For a monoenergetic incident photon beam, the HVL can be easily calculated from the linear attenuation coefficient, and vice versa. For example, given

1. $\mu = 0.35\ \text{cm}^{-1}$

$$\text{HVL} = 0.693/0.35\ \text{cm}^{-1} = 1.98\ \text{cm}$$

2. $\text{HVL} = 2.5\ \text{mm} = 0.25\ \text{cm}$

$$\mu = 0.693/0.25\ \text{cm} = 2.8\ \text{cm}^{-1}$$

The HVL and μ can also be calculated if the percent transmission is measured under narrow-beam geometry.

EXAMPLE: If a 0.2-cm thickness of material transmits 25% of a monoenergetic beam of photons, calculate the HVL of the beam for that material.

STEP1. $0.25 = e^{-\mu(0.2\ \text{cm})}$

STEP 2. $\ln 0.25 = -\mu(0.2\ \text{cm})$

STEP 3. $\mu = (-\ln 0.25)/(0.2\ \text{cm}) = 6.93\ \text{cm}^{-1}$

STEP 4. $\text{HVL} = 0.693/\mu = 0.693/6.93\ \text{cm}^{-1} = 0.1\ \text{cm}$

Half-value layer

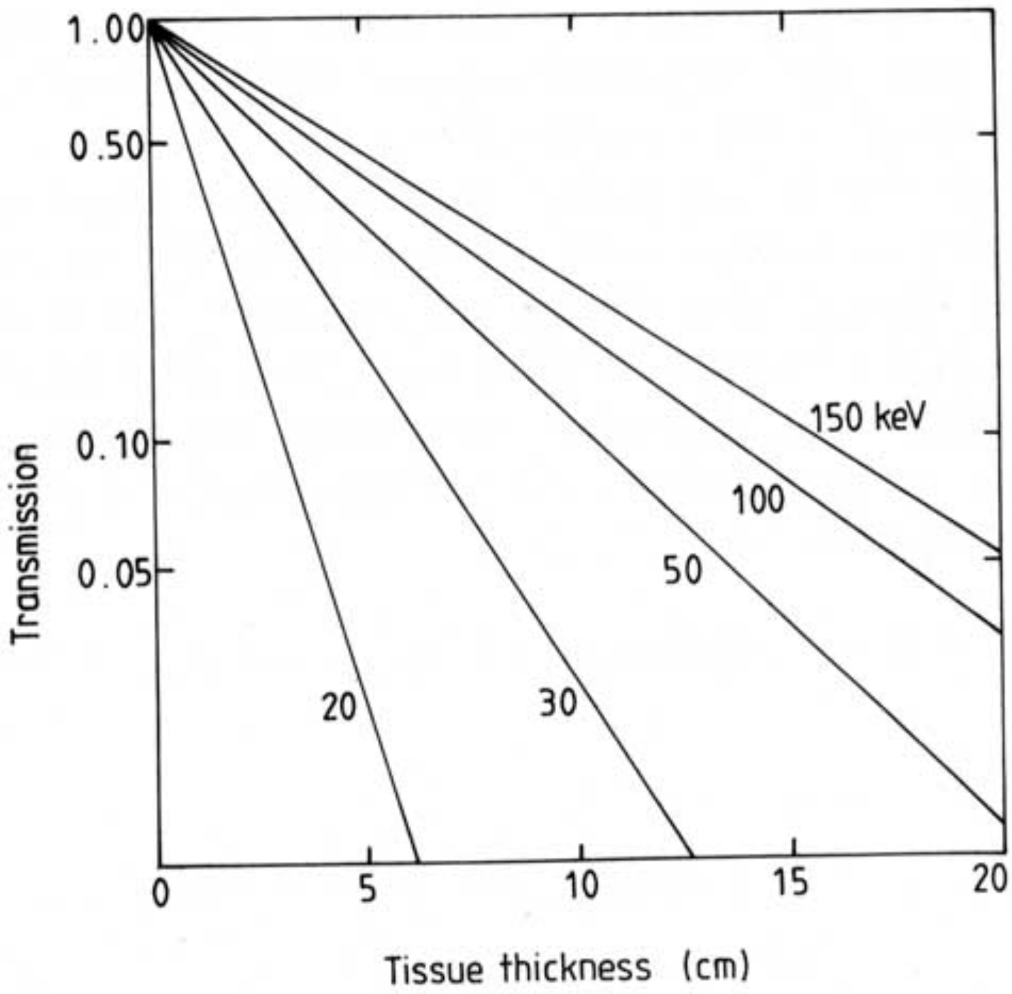
For monoenergetic photons under narrow-beam geometry conditions, the probability of attenuation remains the same for each additional HVL thickness placed in the beam. Reduction in beam intensity can be expressed as $(1/2)^n$ where n equals the number of half value layers. For example, the fraction of monoenergetic photons transmitted through 5 HVLs of material is

$$1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = (1/2)^5 = 1/32 = 0.031 \text{ or } 3.1\%$$

Therefore, 97% of the photons are attenuated (removed from the beam). The HVL of a diagnostic x-ray beam, measured in millimeters of aluminum under narrow-beam conditions, is a surrogate measure of the average energy of the photons in the beam.

Monochromatic beam attenuation

- Semi-logarithmic graph: transmission decreases linearly
- Smaller the energy, higher the attenuation: the slope is higher
- Beam attenuation for monoenergetic beams for different tissue thickness (in cm)



Polychromatic beam attenuation

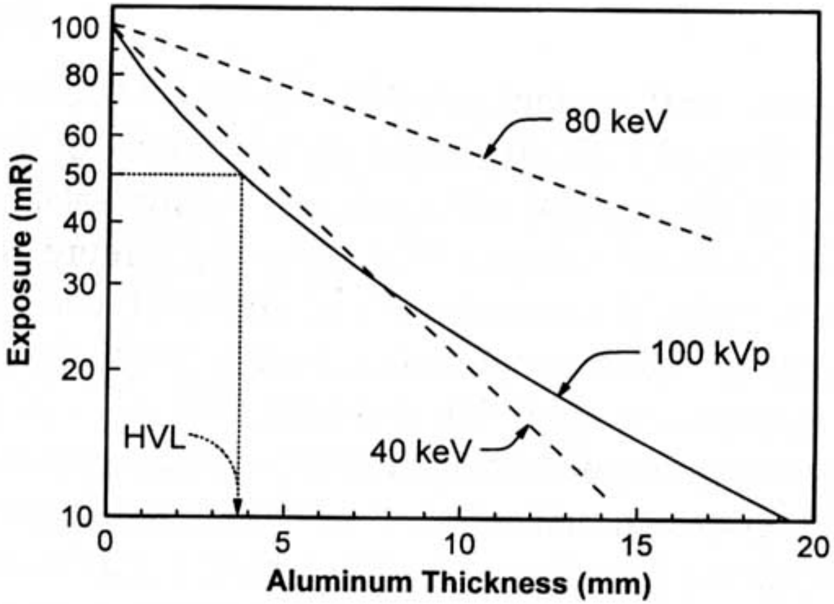


Figure 1.33: Attenuation profiles (exposure as a function of thickness) are shown for aluminum and for three different x-ray beams. The dashed lines are for monoenergetic x-ray beams at 40 keV and 80 keV. Higher-energy x rays are more *penetrating*, so the curve for 80-keV x rays is less steep than the curve for 40-keV x rays. On this semi-logarithmic plot, the attenuation curves for monoenergetic x-ray beams appear as straight lines. For a 100-kVp x-ray spectrum, the attenuation curve demonstrates curvature which is representative of *beam hardening*. The half-value layer (HVL) is the thickness of aluminum required to reduce the exposure of the x-ray beam by 50%. The HVL for the 100-kVp attenuation curve shown is approximately 3.7 mm Al.

The homogeneity coefficient is the ratio of the first to the second HVL and describes the polyenergetic character of the beam. The first HVL is the thickness that reduces the incident intensity to 50% and the second HVL reduces it to 25% of its original intensity [i.e., (0.5)(0.5) = 0.25]. A monoenergetic source of gamma rays has a homogeneity coefficient equal to 1.

Average energy

The conventional notation for *photon fluence* (photons/mm^2) at a given energy E is $\phi(E)$.
The corresponding *energy fluence* (joules/mm^2 or $\phi(E) \times E$ is $\psi(E)$)
In case of polyenergetic beams the concept is replaced by fluence spectrum and energy fluence spectrum, differential in energy *E* (fluence/rate if per unit of time)

$$\Phi_E(E) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}E}(E) \qquad \Psi_E(E) \equiv \frac{\mathrm{d}\Psi}{\mathrm{d}E}(E) = \frac{\mathrm{d}\Phi}{\mathrm{d}E}(E)E$$

$$\overline{E} = \frac{\int_0^{E\max} \phi(E)\, E\, dE}{\int_0^{E\max} \phi(E)\, dE} \qquad \overline{E_{att}} = \frac{\int_0^{E\max} \phi(E) e^{-\mu(E)x}\, E\, dE}{\int_0^{E\max} \phi(E)\, e^{-\mu(E)x}\, dE}$$

We can calculate the average energy of a given spectrum and the average energy of an attenuated spectrum after x mm of Al.

$$\overline{E} = \frac{\sum_i \phi_i(E_i) E_i}{\sum_i \phi_i(E_i)} \qquad \overline{E_{att}} = \frac{\sum_i \phi_i(E_i) e^{-\mu(E_i)x} E_i}{\sum_i \phi_i(E_i) e^{-\mu(E_i)x}}$$

$\phi_i (E_i)$ is the total photon fluence in an i-th energy interval with bin width ΔE

Beam hardening

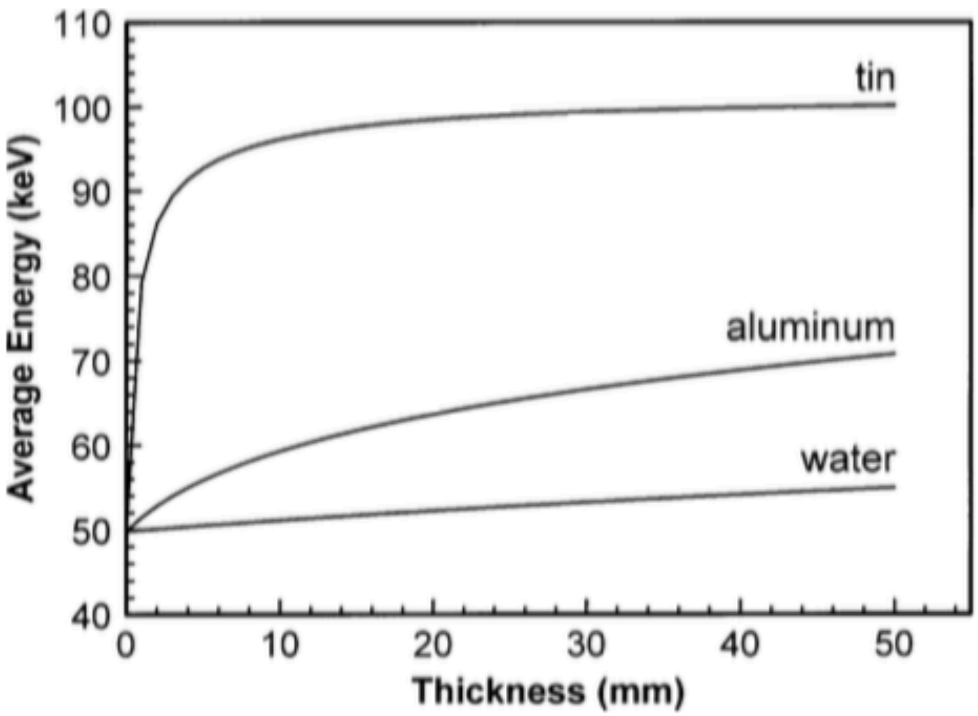


Figure 1.36: The average x-ray energy in a 100-kVp x-ray spectrum is illustrated as a function of thickness for three different attenuators. Fifty millimeters of water produces only a modest increase in average x-ray energy, whereas 50 mm of aluminum causes a more noticeable increase in average energy. Because of the high atomic number and density of tin ($Z = 50$, $\rho = 7.3$ g/cm^3), it has a profound beam-hardening effect.

Exposure and HVL

- An air-ionization exposure meter is a device capable of accurately measuring x-ray exposure. Exposure is a term which relates primarily to the x-ray beam intensity or the *beam quantity*.
- Measuring the x-ray energy spectrum is much more difficult, and requires sophisticated equipment that is only available in a handful of laboratories. Nevertheless, some idea of the spectral distribution (*beam quality*) of the x-ray beam can be evaluated. The x-ray attenuation coefficients are energy dependent, and therefore by measuring the exposure attenuation, a parameter relating to the x-ray beam energy distribution ((E)) can be assessed.
- The parameter used to characterize polyenergetic beam quality in field measurements of attenuation is called the *half-value layer* (HVL). The HVL, usually calculated using aluminum in diagnostic radiology, is the thickness of aluminum required to reduce the exposure of the x-ray beam by a factor of 2 (i.e., to 50% of its unattenuated exposure).
- *Exposure* is defined in air (only), and therefore the HVL is properly measured only using an air ionization exposure meter; the HVL measured using a solid state x-ray detector system, for example, will be different.

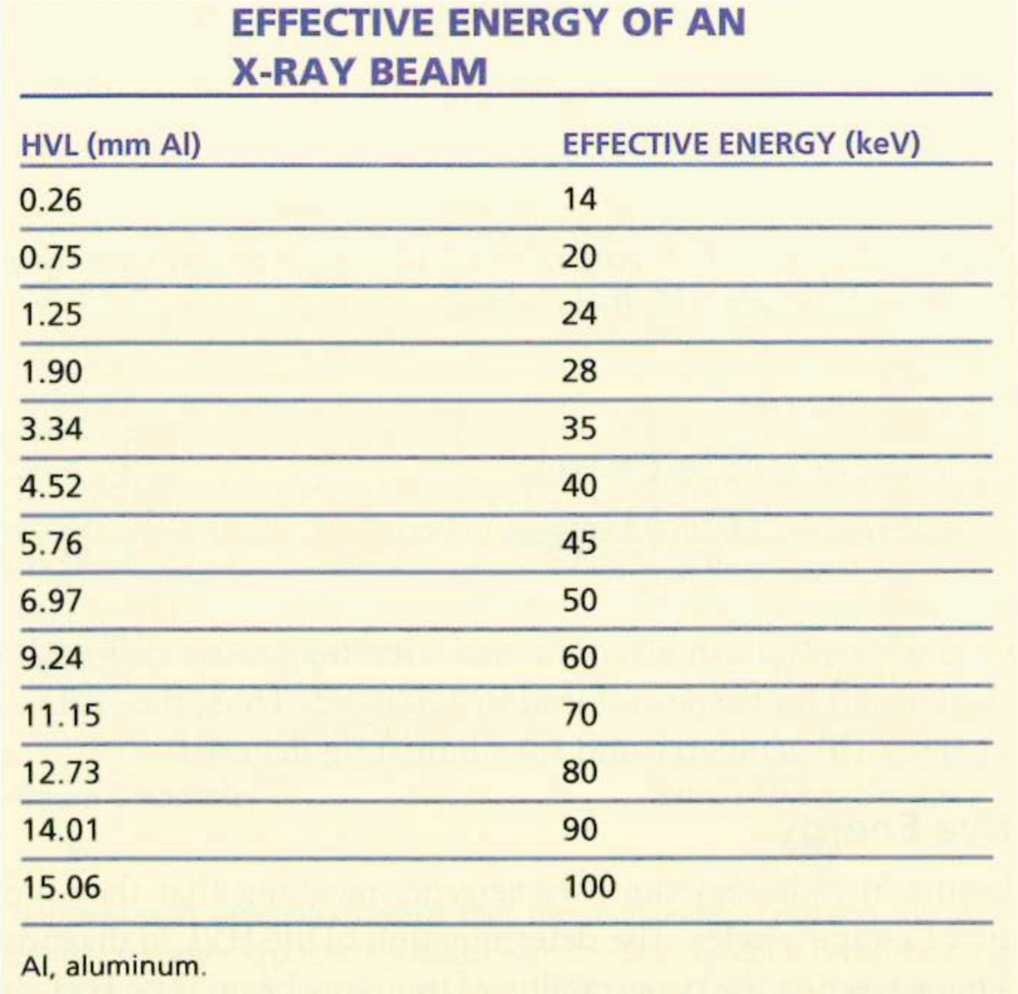
Effective energy

The HVL of a diagnostic x-ray beam, measured in millimeters of aluminum under narrow beam conditions, is a measure of the penetrability of the x-ray spectrum. Since x-ray beams in radiology are polyenergetic, the determination of HVL is the way of characterizing the penetrability of the x-ray beam for a given kV..

The HVL (in mm of Al) can be converted to a quantity called the effective energy.

It is an estimate of the penetration power of the x-ray beam, expressed as the energy of a monoenergetic beam that would exhibit the same "effective" penetrability.

The effective energy from a typical diagnostic x-ray tube is one third to one half the maximum value.



EFFECTIVE ENERGY OF AN X-RAY BEAM

HVL (mm Al)	EFFECTIVE ENERGY (keV)
0.26	14
0.75	20
1.25	24
1.90	28
3.34	35
4.52	40
5.76	45
6.97	50
9.24	60
11.15	70
12.73	80
14.01	90
15.06	100

Al, aluminum.

Table 1.4: Mass attenuation coefficients of aluminum ($\rho = 2.699$)

E (keV)	$(\mu/\rho)_{\mathrm{al}}$	E (keV)	$(\mu/\rho)_{\mathrm{al}}$
10	26.048	50	0.368
11	19.678	55	0.315
12	15.330	60	0.278
13	12.264	65	0.252
14	9.731	70	0.230
15	7.980	75	0.214
16	6.576	80	0.202
17	5.500	85	0.192
18	4.647	90	0.183
19	4.034	95	0.177
20	3.423	100	0.171
25	1.830	105	0.166
30	1.131	110	0.161
35	0.769	115	0.157
40	0.567	120	0.153
45	0.446		

Beam hardening

The lower energy photons of the polyenergetic x-ray beam will preferentially be removed from the beam while passing through matter. The shift of the x-ray spectrum to higher effective energies as the beam transverses matter is called *beam hardening* Low-energy (soft) x-rays will not penetrate the entire thickness of the body; thus, their removal reduces patient dose without affecting the diagnostic quality of the exam. X-ray machines remove most of this soft radiation with filters, thin plates of aluminum, copper, or other materials placed in the beam. This added filtration will result in an x-ray beam with a higher effective energy and thus a greater HVL.

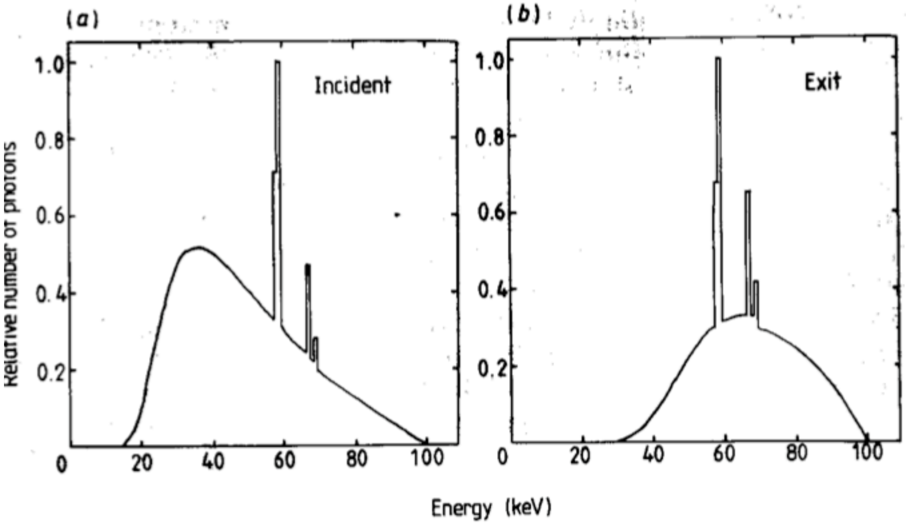


Figure 2.11 X-ray spectra for an x-ray tube with a tungsten target; 100 kV constant potential with 2.5 mm aluminium added. The spectra are shown both before and after attenuation by 18.5 cm soft tissue plus 1.5 cm bone. (The spectra are based on the work of Birch *et al* (1979).)

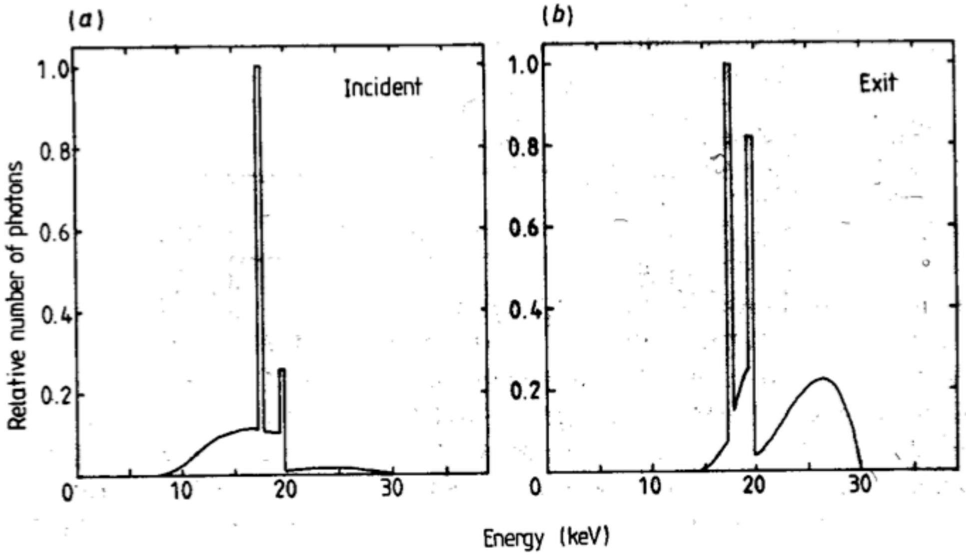


Figure 2.12 X-ray spectra for an x-ray tube with a molybdenum target; 30 kV constant potential with 0.03 mm molybdenum filter. The spectra are shown both before and after attenuation by 5 cm tissue. (The spectra are based on the work of Birch *et al* (1979).)

- The values of spectrum for a variety of spectra are calculated for different kV. The HVL, the exposure, the x-ray fluence for a wide range of spectra can be estimated.
- Note: Energy_effective < Energy_average

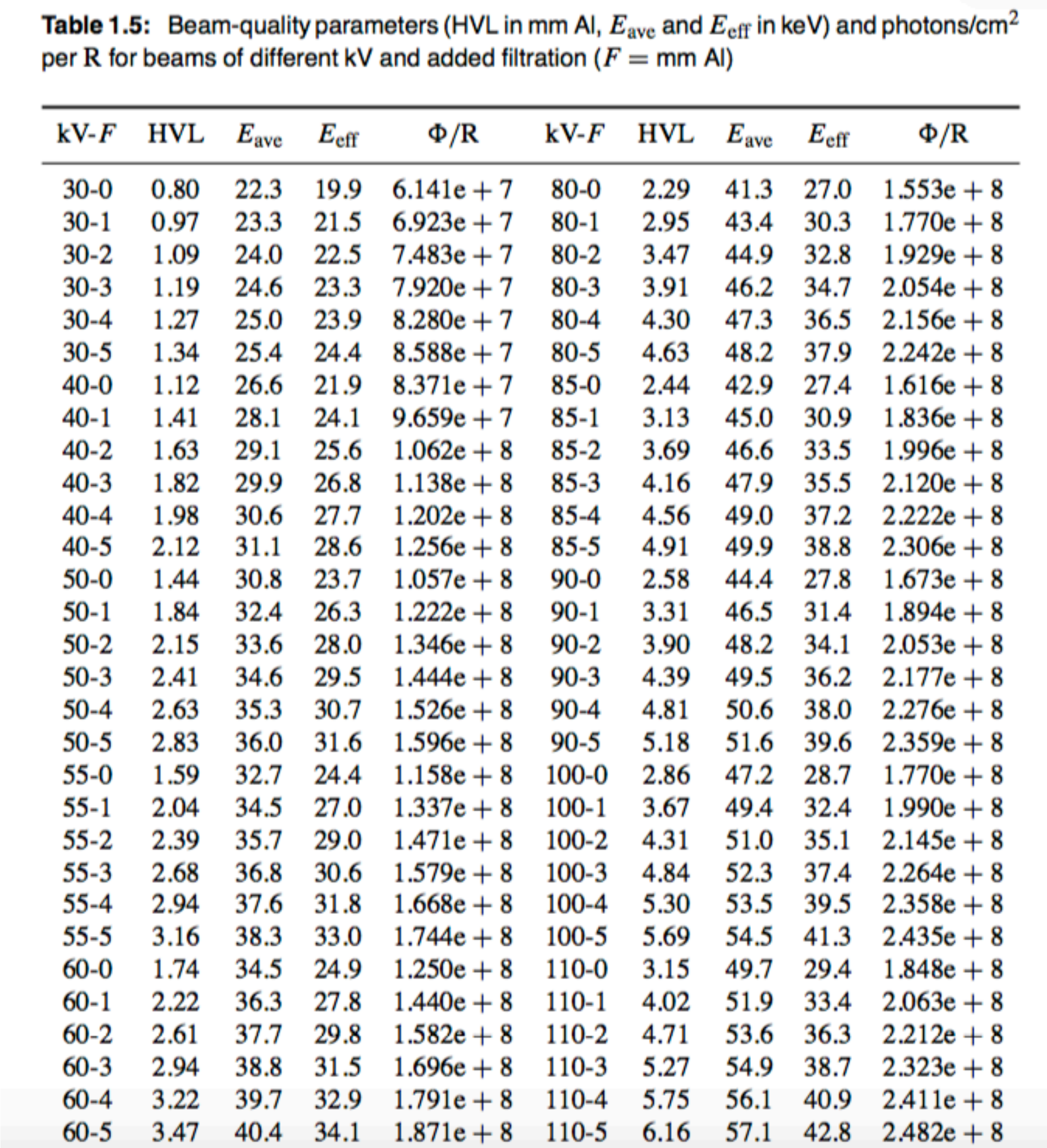


Table 1.5: Beam-quality parameters (HVL in mm Al, E_{ave} and E_{eff} in keV) and photons/cm^2 per R for beams of different kV and added filtration (F = mm Al)

kV-F	HVL	E_{ave}	E_{eff}	Φ/R	kV-F	HVL	E_{ave}	E_{eff}	Φ/R
30-0	0.80	22.3	19.9	6.141e + 7	80-0	2.29	41.3	27.0	1.553e + 8
30-1	0.97	23.3	21.5	6.923e + 7	80-1	2.95	43.4	30.3	1.770e + 8
30-2	1.09	24.0	22.5	7.483e + 7	80-2	3.47	44.9	32.8	1.929e + 8
30-3	1.19	24.6	23.3	7.920e + 7	80-3	3.91	46.2	34.7	2.054e + 8
30-4	1.27	25.0	23.9	8.280e + 7	80-4	4.30	47.3	36.5	2.156e + 8
30-5	1.34	25.4	24.4	8.588e + 7	80-5	4.63	48.2	37.9	2.242e + 8
40-0	1.12	26.6	21.9	8.371e + 7	85-0	2.44	42.9	27.4	1.616e + 8
40-1	1.41	28.1	24.1	9.659e + 7	85-1	3.13	45.0	30.9	1.836e + 8
40-2	1.63	29.1	25.6	1.062e + 8	85-2	3.69	46.6	33.5	1.996e + 8
40-3	1.82	29.9	26.8	1.138e + 8	85-3	4.16	47.9	35.5	2.120e + 8
40-4	1.98	30.6	27.7	1.202e + 8	85-4	4.56	49.0	37.2	2.222e + 8
40-5	2.12	31.1	28.6	1.256e + 8	85-5	4.91	49.9	38.8	2.306e + 8
50-0	1.44	30.8	23.7	1.057e + 8	90-0	2.58	44.4	27.8	1.673e + 8
50-1	1.84	32.4	26.3	1.222e + 8	90-1	3.31	46.5	31.4	1.894e + 8
50-2	2.15	33.6	28.0	1.346e + 8	90-2	3.90	48.2	34.1	2.053e + 8
50-3	2.41	34.6	29.5	1.444e + 8	90-3	4.39	49.5	36.2	2.177e + 8
50-4	2.63	35.3	30.7	1.526e + 8	90-4	4.81	50.6	38.0	2.276e + 8
50-5	2.83	36.0	31.6	1.596e + 8	90-5	5.18	51.6	39.6	2.359e + 8
55-0	1.59	32.7	24.4	1.158e + 8	100-0	2.86	47.2	28.7	1.770e + 8
55-1	2.04	34.5	27.0	1.337e + 8	100-1	3.67	49.4	32.4	1.990e + 8
55-2	2.39	35.7	29.0	1.471e + 8	100-2	4.31	51.0	35.1	2.145e + 8
55-3	2.68	36.8	30.6	1.579e + 8	100-3	4.84	52.3	37.4	2.264e + 8
55-4	2.94	37.6	31.8	1.668e + 8	100-4	5.30	53.5	39.5	2.358e + 8
55-5	3.16	38.3	33.0	1.744e + 8	100-5	5.69	54.5	41.3	2.435e + 8
60-0	1.74	34.5	24.9	1.250e + 8	110-0	3.15	49.7	29.4	1.848e + 8
60-1	2.22	36.3	27.8	1.440e + 8	110-1	4.02	51.9	33.4	2.063e + 8
60-2	2.61	37.7	29.8	1.582e + 8	110-2	4.71	53.6	36.3	2.212e + 8
60-3	2.94	38.8	31.5	1.696e + 8	110-3	5.27	54.9	38.7	2.323e + 8
60-4	3.22	39.7	32.9	1.791e + 8	110-4	5.75	56.1	40.9	2.411e + 8
60-5	3.47	40.4	34.1	1.871e + 8	110-5	6.16	57.1	42.8	2.482e + 8

Attenuation and energy deposition

- Just as the total linear attenuation coefficient μ is the sum of the linear attenuation coefficients of the individual interaction types, the total mass attenuation coefficient is the sum of its constituents as well:

$$\left(\frac{\mu}{\rho}\right)=\left(\frac{\tau}{\rho}\right)+\left(\frac{\sigma_{\mathrm{r}}}{\rho}\right)+\left(\frac{\sigma}{\rho}\right)+\left(\frac{\pi}{\rho}\right)$$

- Attenuation is useful in describing the propagation of x rays through a material, but it does not tell the complete story in terms of energy deposition. Energy deposition is important both in the calculation of the radiation dose to a patient, and for the calculation of the total signal generated in an x-ray detector

Energy absorption and biological damage

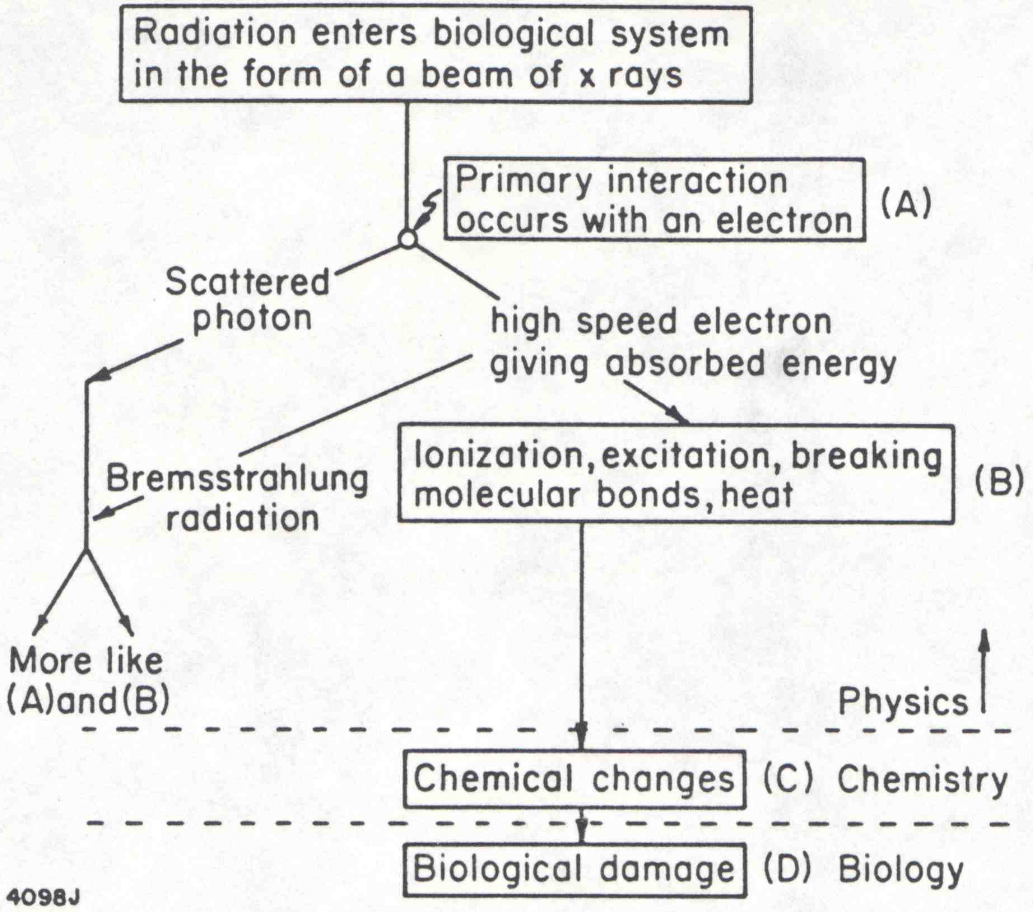


Figure 5-1. Schematic diagram illustrating the absorption of energy from radiation resulting in biological damage.

Misura di Laboratorio. Spessore di dimezzamento

- Half Value Layer HVL e' lo spessore di un materiale necessario per ridurre della meta' l' intensita' del fascio.
- Caratterizza il tubo fissata la tensione
- Va controllato il suo valore nel tempo

- Misurare il valore di HVL e di E_{eff} per il primo ed il secondo dimezzamento per diversi spettri di partenza
  - Due senza filtro e diverso kV e uno con il filtro di Mo

- Discutere la variazione dell'E_{eff} con la quantita' di filtro totale di Al applicato
  - Vedi anche spettro simulato
  - https://health.siemens.com/booneweb/index.html

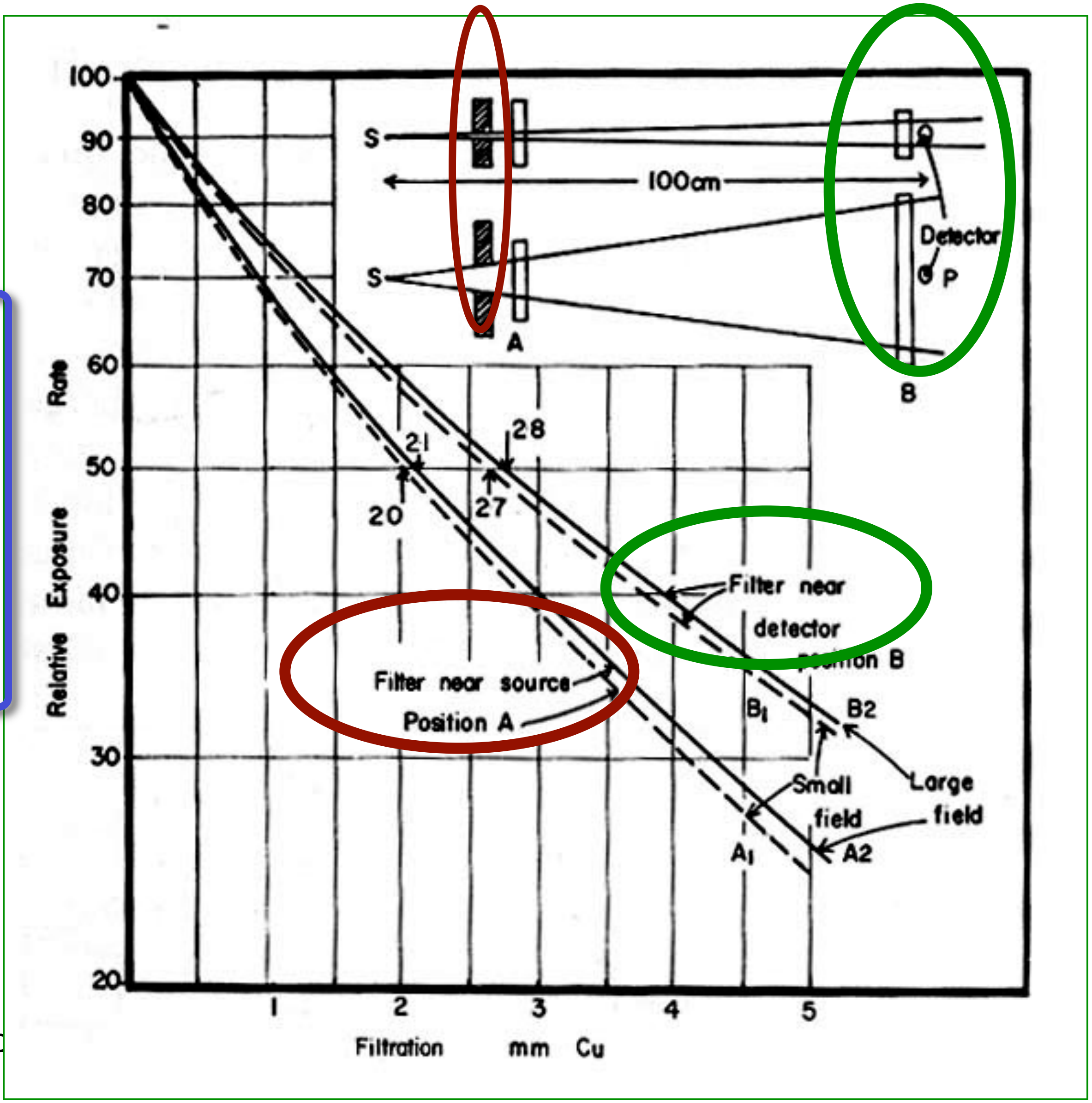
  (https://www.oem-xray-components.siemens.com/x-ray-spectra-simulation)

- Discutere il variare dell'E_{eff} con la presenza del filtro di Mo.

Misura di HVL

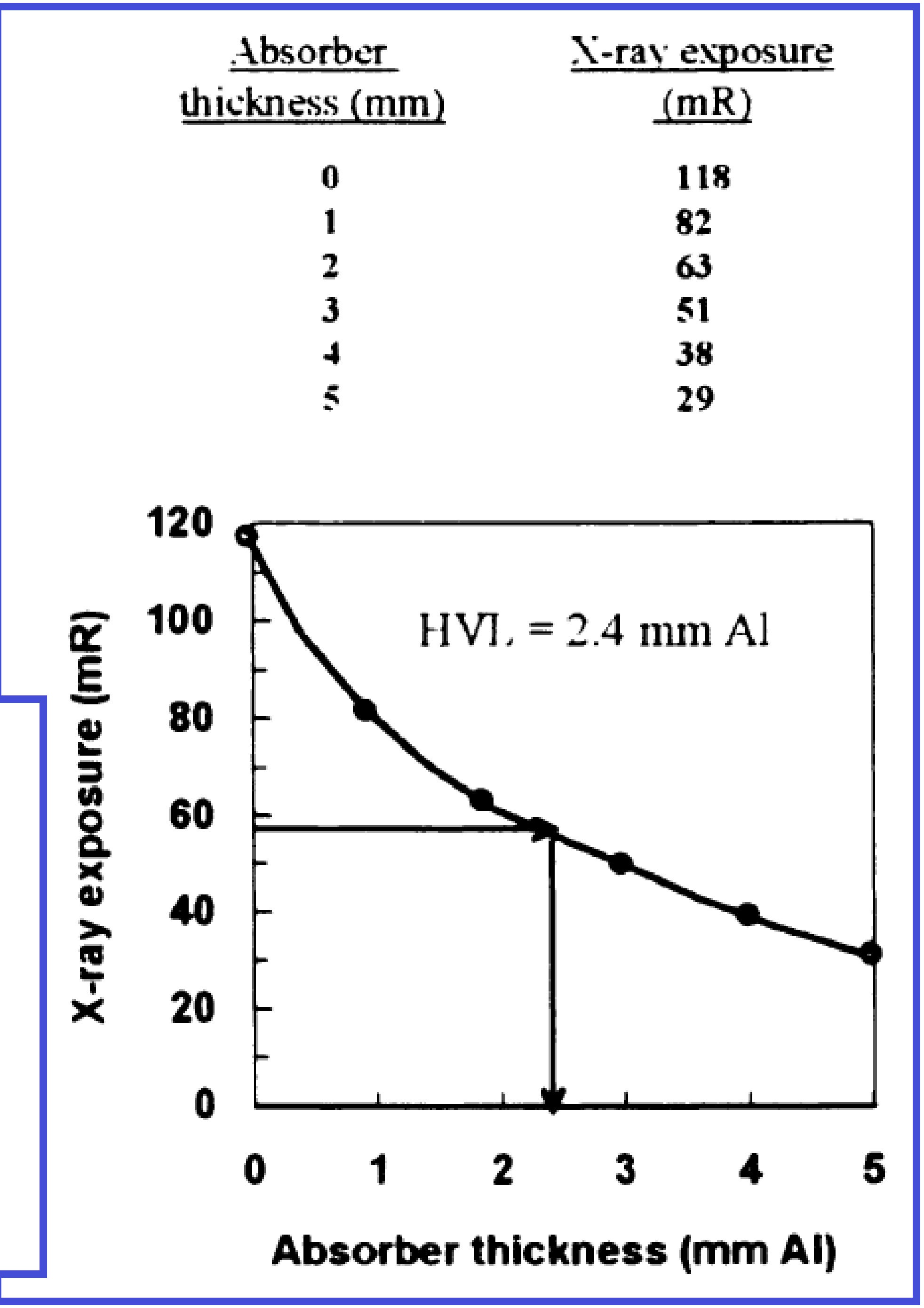
Lo scatterig e' importante: quindi la posizione degli assorbitori rispetto al detector e' critica

NB raddoppiare lo spessore NON significa andare a 1/4 di flusso

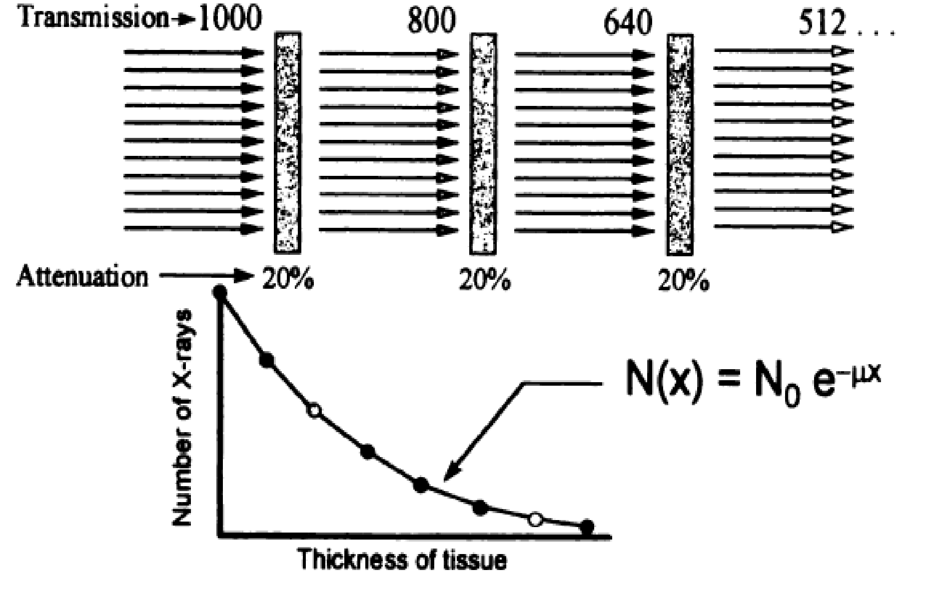


HVL measurements

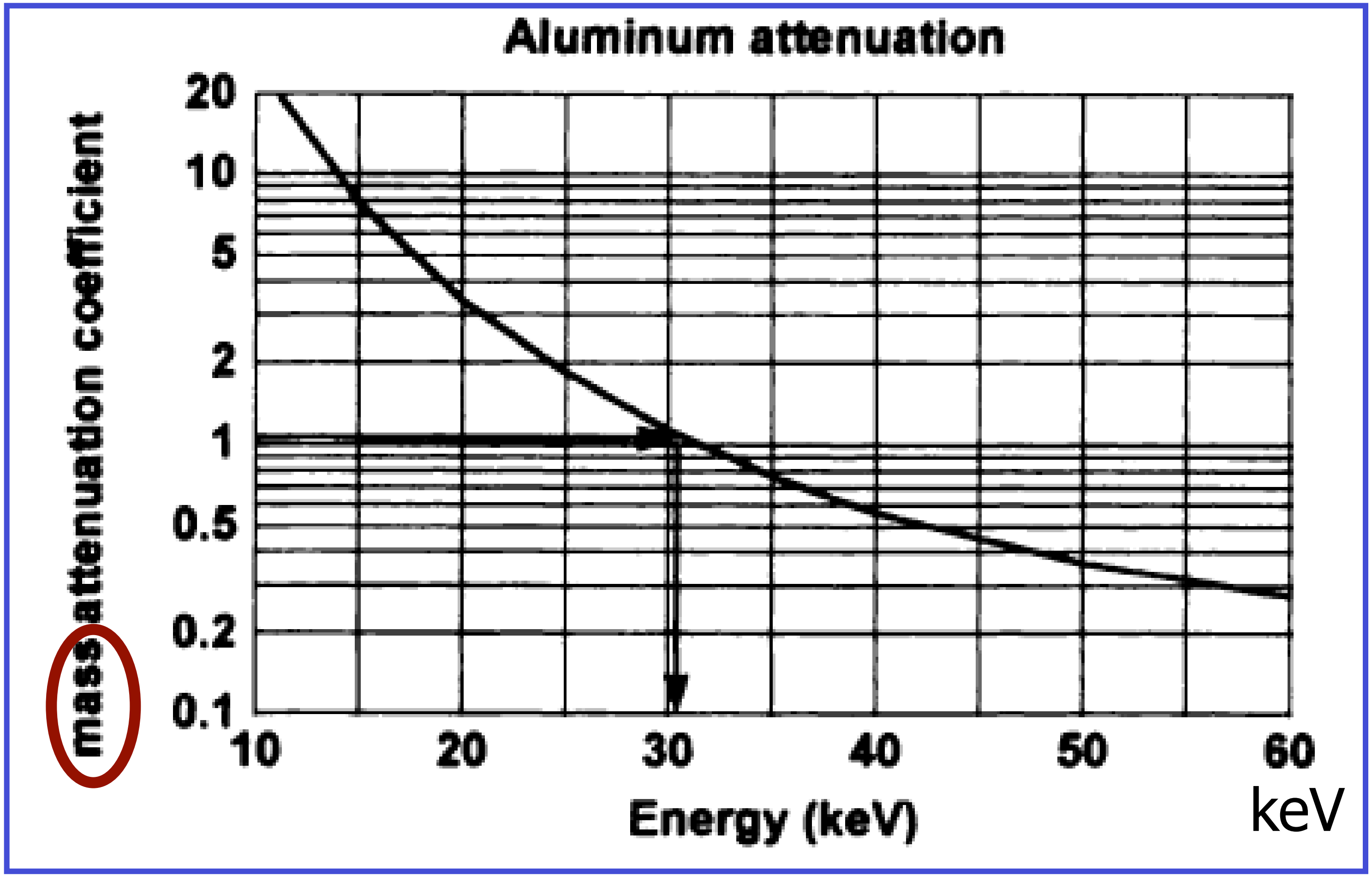
- The X ray spectrum is NOT monochromatic !
- The attenuation is NOT exponential !



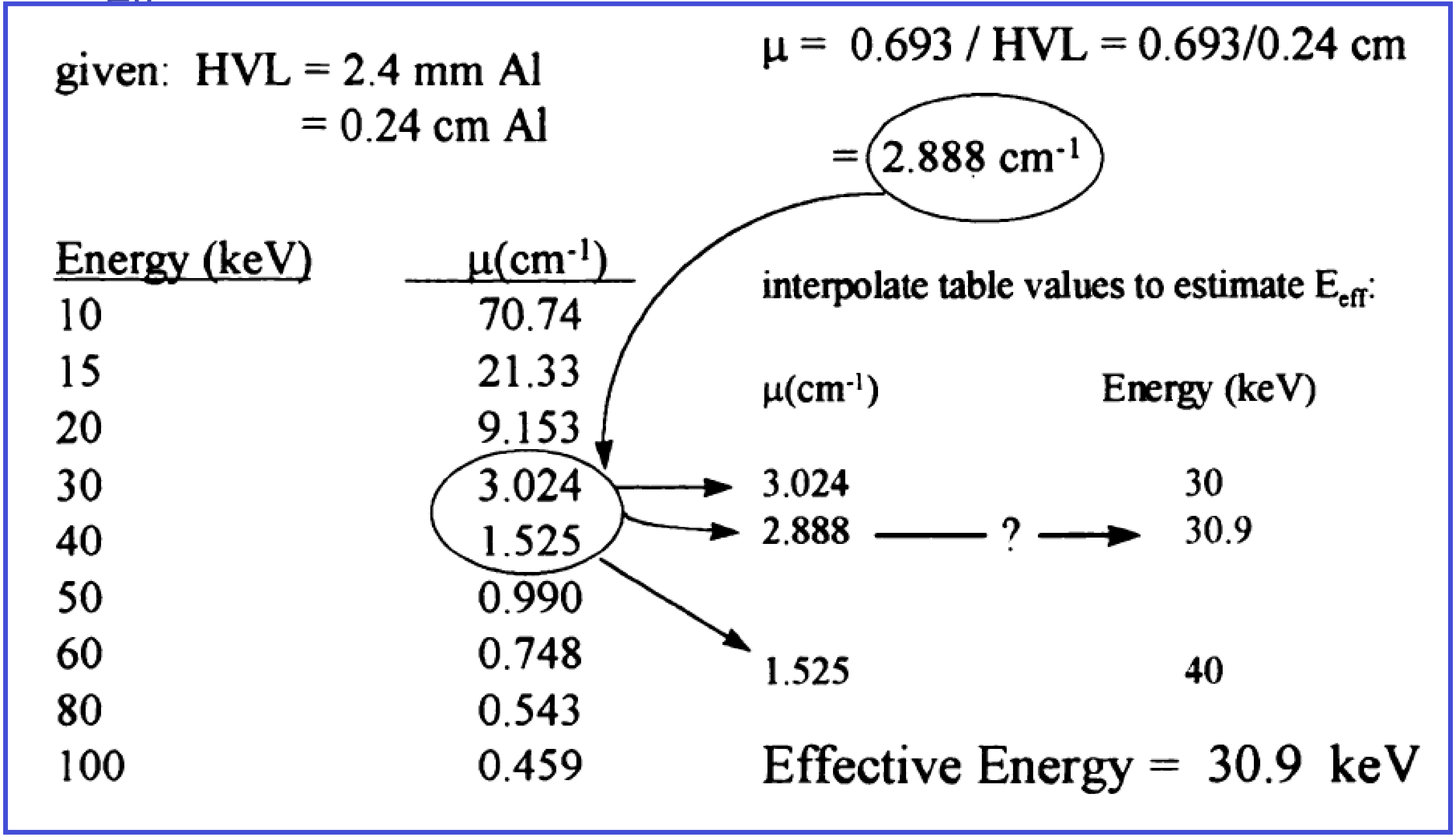
Absorber thickness (mm)	X-ray exposure (mR)
0	118
1	82
2	63
3	51
4	38
5	29



E_{eff} calculation



E_{eff} calculation



given: HVL = 2.4 mm Al
= 0.24 cm Al

$\mu = 0.693 / HVL = 0.693/0.24$ cm
$= 2.888\ cm^{-1}$

Energy (keV)	$\mu(cm^{-1})$
10	70.74
15	21.33
20	9.153
30	3.024
40	1.525
50	0.990
60	0.748
80	0.543
100	0.459

interpolate table values to estimate E_{eff}:

$\mu(cm^{-1})$		Energy (keV)
3.024		30
2.888	——— ? ——→	30.9
1.525		40

Effective Energy = 30.9 keV

M.H. McKetty RadioGraphics 1998; 18:151-163

X-ray spectra simulation:

https://health.siemens.com/booneweb/index.html

Database NIST – attenuation and absorption coefficients

https://www.nist.gov/pml/x-ray-form-factor-attenuation-and-scattering-tables

https://physics.nist.gov/PhysRefData/FFast/html/form.html

XCOM
https://www.nist.gov/pml/xcom-photon-cross-sections-database
database search form
(https://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html)

XMuDat software (not for Mac):

https://www-nds.iaea.org/publications/iaea-nds/iaeands-0195.htm