Geophysical Fluid Dynamics

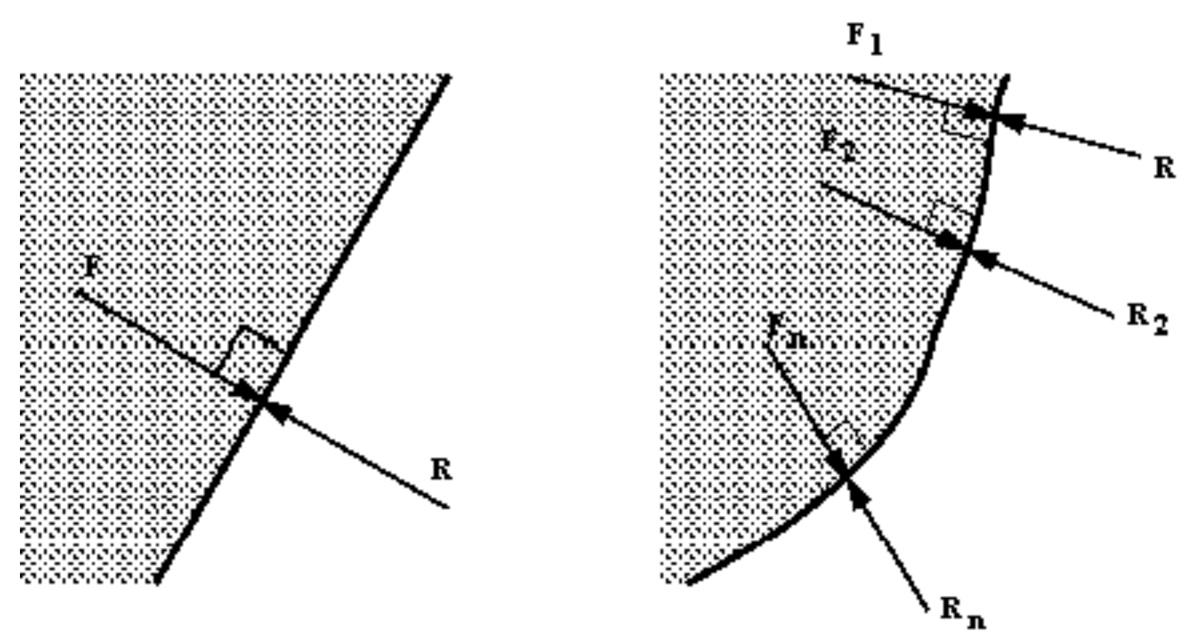
Lecture 11:
Statics

Summary of previous lecture

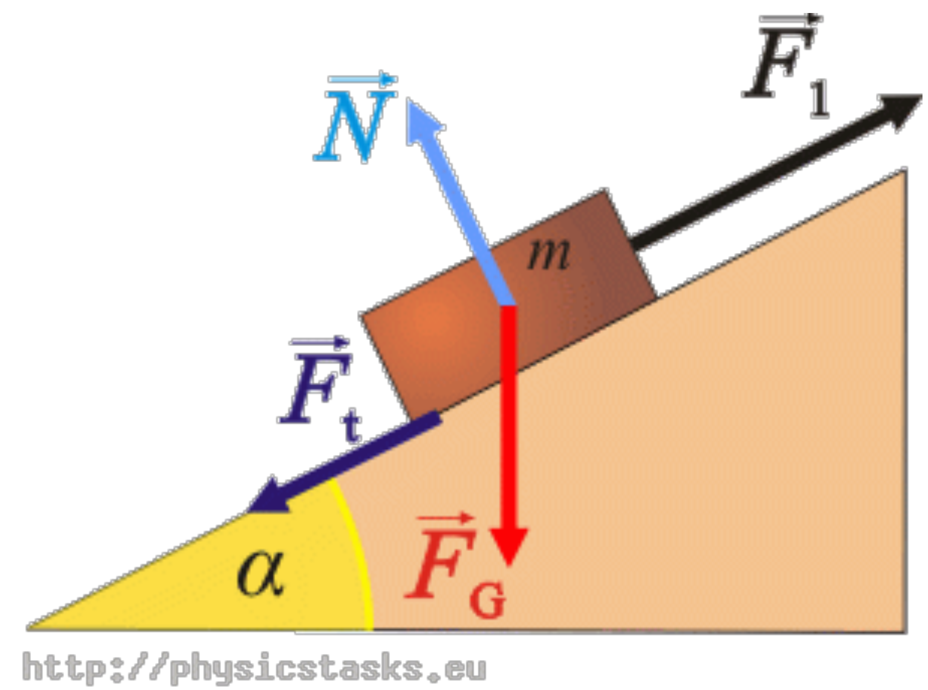
- What is a fluid? definitions
- Properties (mass, weight, density)
- Ideal fluids vs Real fluids
- Viscosity is a very important fluid property
- Newton's law of viscosity: TAU is proportional to fluid mu and the velocity gradient
- Newtonian fluid vs Non-Newtonian fluid
- liquid (gases) have high (low) viscosity

Fluid Statics

- Fluid is at rest
- A static fluid can have no shearing force acting on it.
- The only forces are due to pressure.
- Any force between fluid and boundary must be acting at right angles (normal to).
- Fluid at rest is in equilibrium: sum of components of forces in any direction must be zero.



does pressure have a direction?



- FORCE is a vector (forces on box have different directions and magnitude)
- is PRESSURE a vector too?

Isotropy of Pressure

- In a fluid at rest, the tangential viscous stresses are absent and the only force is normal to the surface.
- The surface force per unit area (PRESSURE) is equal in all directions.
- Pressure at any point in a fluid at rest has a single value (is a scalar). This is known as **Pascal's Law.**

Gauss theorem (or the divergence theorem)

- relates the flow flux of a vector field through a surface to the behavior of the vector field inside the surface
- The outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface
- The sum of sources and sinks (divergence) will give you the outward flux

$$\iiint_v (\nabla \cdot F) dV = \iint_A F dA$$

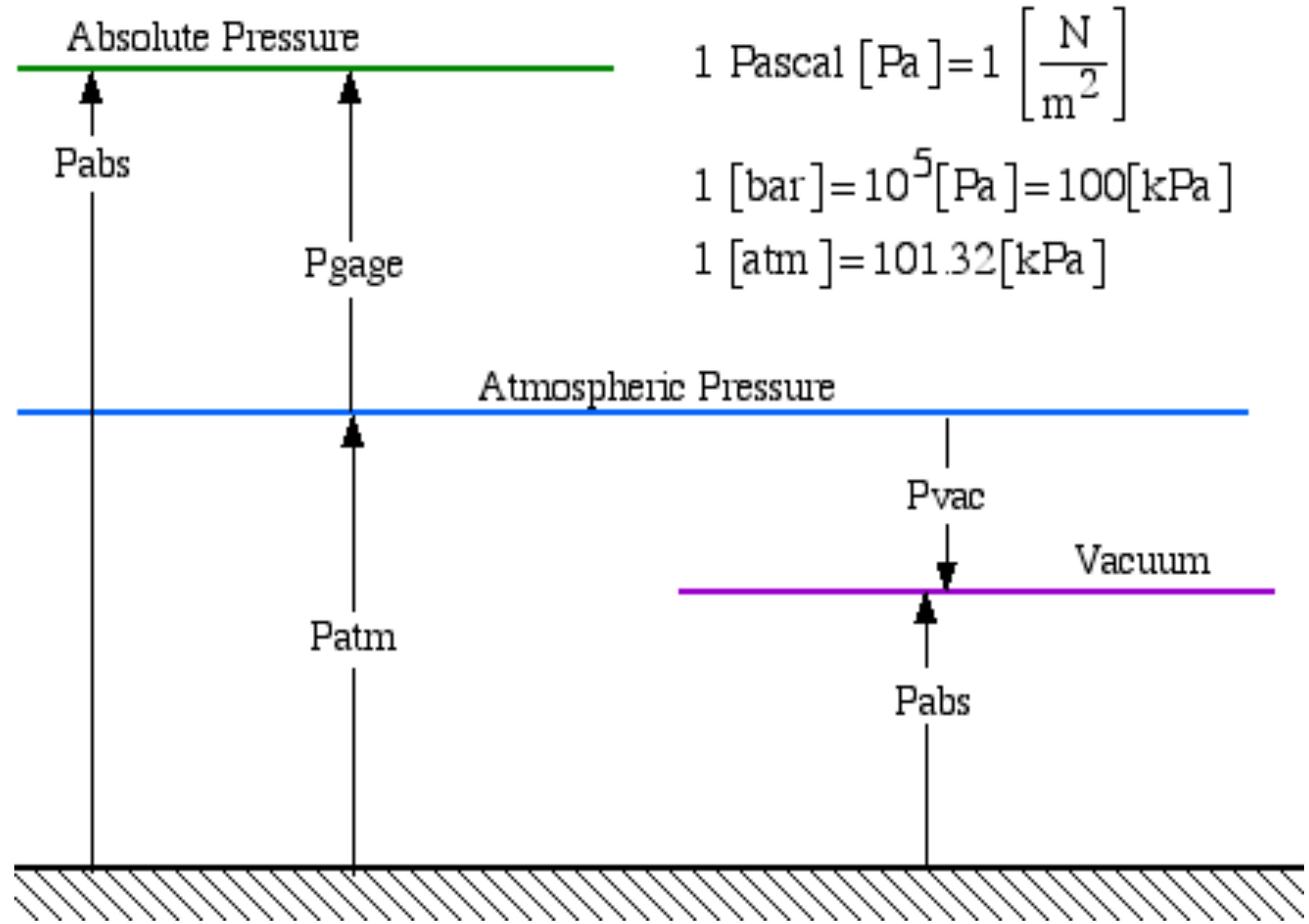
Pressure variations for incompressible fluids

- P - Po = -rho g (z - zo)
- Applies to liquids (no need to consider compressibility unless dealing with large changes in z ... deep in the ocean)
- Applies to gases for small changes in z only
- **P = rho g h** Pressure related to the height h of a fluid column: *Pressure head*

Absolute and Gage Pressure

- **Absolute** relative to absolute zero (perfect vacuum)
- **Gage** relative to atmospheric pressure (>0 if >Patm; <0 if <Patm)
- if P < Patm we call it a vacuum

Pabs = Patm + Pgage



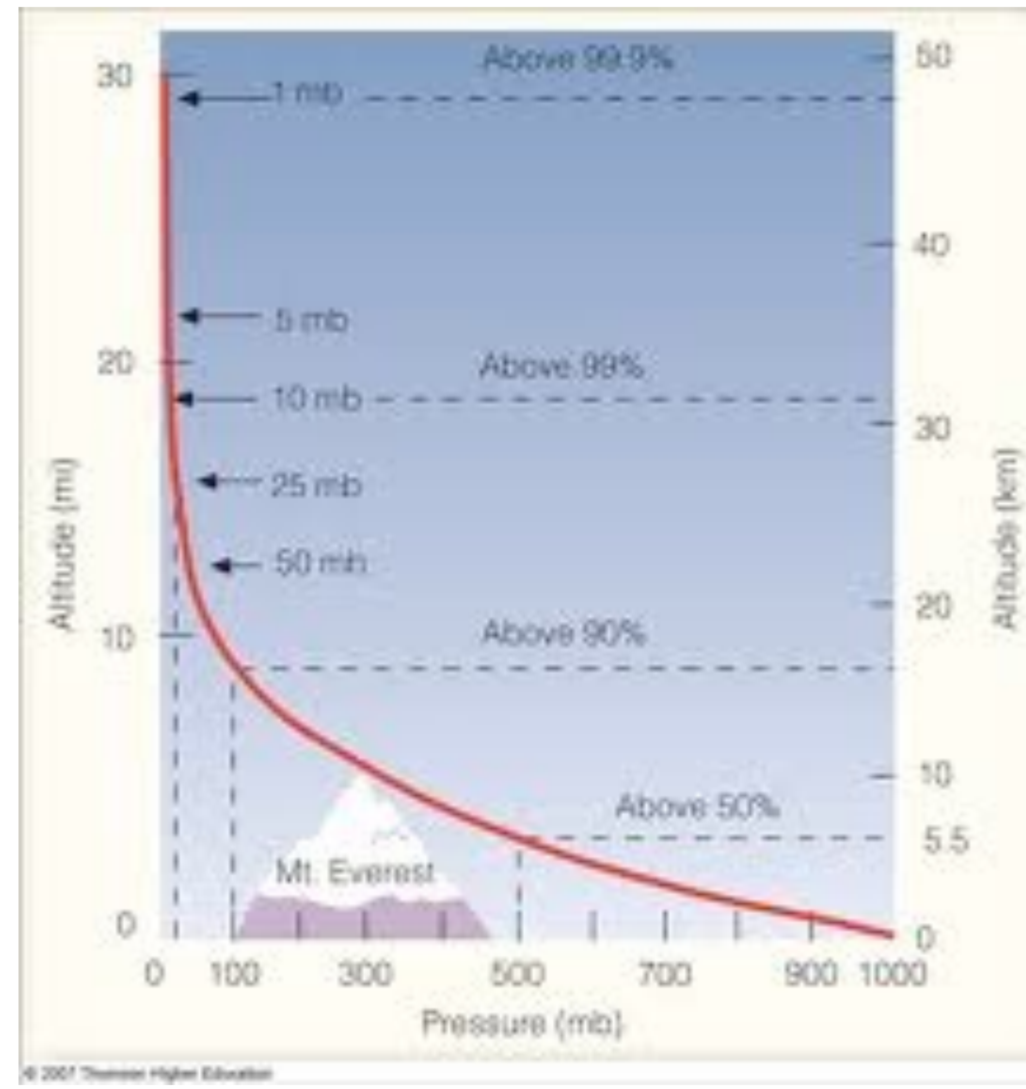
$1 \text{ Pascal } [Pa] = 1 \left[\frac{N}{m^2}\right]$

$1 [bar] = 10^5 [Pa] = 100 [kPa]$

$1 [atm] = 101.32 [kPa]$

Pressure

- Atmospheric pressure is also called *barometric pressure* (1 bar = 10^5 Pa). It varies with elevation and changes in meteo conditions



- Absolute pressure used for most problems related to gases/vapor
- Gage pressure related to liquids

Static Equilibrium

In an **incompressible** fluid, where density is not a function of pressure, it is simple to determine the stability of the medium in static state.

1. Stable: if density decreases upward. A particle displaced upward would be at a level where density of the surrounding fluid is lower and the particle is forced to move back to its original level

2. Unstable: if density increases upward. A displaced particle would continue to move away from its original position.

3. Neutral: if the density is uniform.

Static Equilibrium

In a **compressible** medium the previous arguments do not hold. In a neutral state it is not density to be constant but rather entropy.

A particle displaced upward would expand adiabatically because of the decrease in pressure with height.

Displacing the particle upward, the original density and temperature would decrease to a new density and temperature according to their isentropic relations.

The particle would move back to its original position if the new density is lower than that of the surrounding level.

But if the properties of the sorrounding air also vary with height so that entropy is uniform with height, the displaced particle would always find itself in a region where density is the same as its own density.

A neutral atmosphere (isentropic atmosphere) is thus one in which pressure, density and temperature decrease so that entropy is constant with height.

4.8 Static instability, the parcel method and Buoyancy frequency

Consider a stratified ocean and a parcel of fluid initially at rest, and therefore in hydrostatic balance. We will focus on vertical displacements and the restoring force is gravity. Consider a small adiabatic displacement of the parcel upward by δz, without altering the background pressure field. If the parcel is now lighter then the local environment, it will feel an upward pressure gradient force larger than the downward gravitational force, it will accelerate upwards and will become buoyant. In this case the fluid is statically unstable. If, instead, the parcel finds itself heavier than its sorroundings, the downward gravitational force will be greater than the upward pressure force, the fluid will sink back to its original position and will oscillate. This condition is statically stable.

Consider an incompressible fluid in which the density of the displaced parcel is conserved, $D\rho/Dt = 0$. If the environmental profile is $\tilde{\rho}(z)$ and the density of the parcel is ρ, a parcel displaced to a level $z + \delta z$ will show a change in density with respect to the local environment equal to

$$\delta\rho = \rho(z + \delta z) - \tilde{\rho}(z + \delta z) = \tilde{\rho}(z) - \tilde{\rho}(z + \delta z) = -\frac{\partial\tilde{\rho}}{\partial z}\delta z, \qquad (4.56)$$

where the derivative on the right-hand side is the environmental gradient of density.

If $\frac{\partial\tilde{\rho}}{\partial z} < 0$, the parcel will be heavier than its sorroundings and will sink back in a stable condition.

If $\frac{\partial\tilde{\rho}}{\partial z} > 0$, the parcel will be buoyant in a statically unstable fluid.

That is, the stability of a parcel of fluid is determined by the gradient of the environmental density.

The upward force, per unit volume, on the displaced parcel is

$$F = -g\delta\rho = g\frac{\partial\tilde{\rho}}{\partial z}\delta z \qquad (4.57)$$

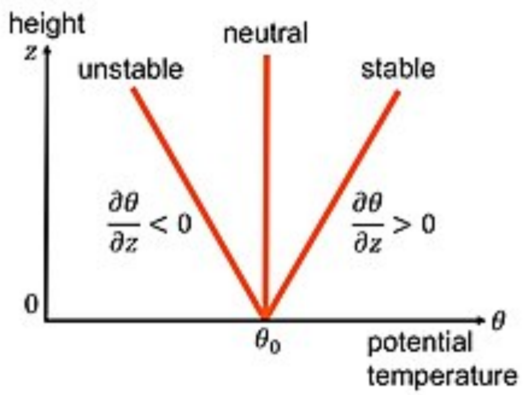


Figure 4.5: Possible temperature vertical profiles, in the atmosphere or ocean, giving rise to unstable, neutral or stable conditions.

and the equation of motion of the fluid parcel is thus

$$\rho(z)\frac{\partial^2 \delta z}{\partial t^2} = g\frac{\partial \tilde{\rho}}{\partial z}\delta z, \tag{4.58}$$

or

$$\frac{\partial^2 \delta z}{\partial t^2} = \frac{g}{\tilde{\rho}}\frac{\partial \tilde{\rho}}{\partial z}\delta z. \tag{4.59}$$

Static stability measures how quickly a water parcel is restored to its position in the water column if displaced vertically. If unstable, the water column has the potential to overturn.

In stable water column conditions ($\frac{\partial \tilde{\rho}}{\partial z} < 0$), the parcel experiences a restoring force and will oscillate at a given frequency:

$$\frac{\partial^2 \delta z}{\partial t^2} = -N^2 \delta z, \tag{4.60}$$

where

$$N^2 = -\frac{g}{\tilde{\rho}}\frac{\partial \tilde{\rho}}{\partial z}, \tag{4.61}$$

and N is the Brunt-Vaisala frequency. In liquids, it is a good approximation to replace $\tilde{\rho}$ by ρ_0.

If $N^2 < 0$, the density profile is unstable, the parcel continues to ascend and convection occurs. This is the condition for convective instability. Convection causes fluid parcels to mix and reduces an unstable profile to neutral stability.

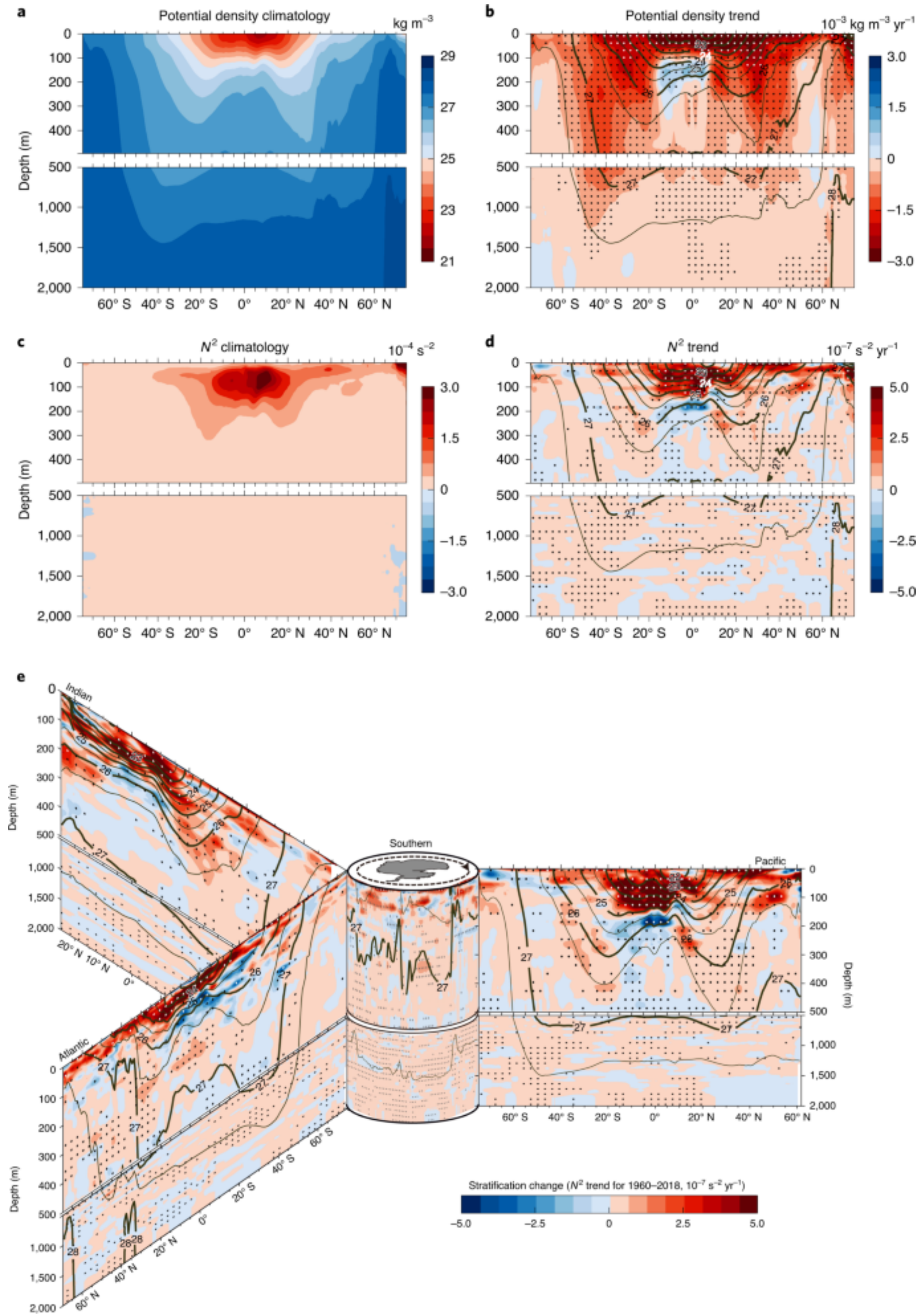
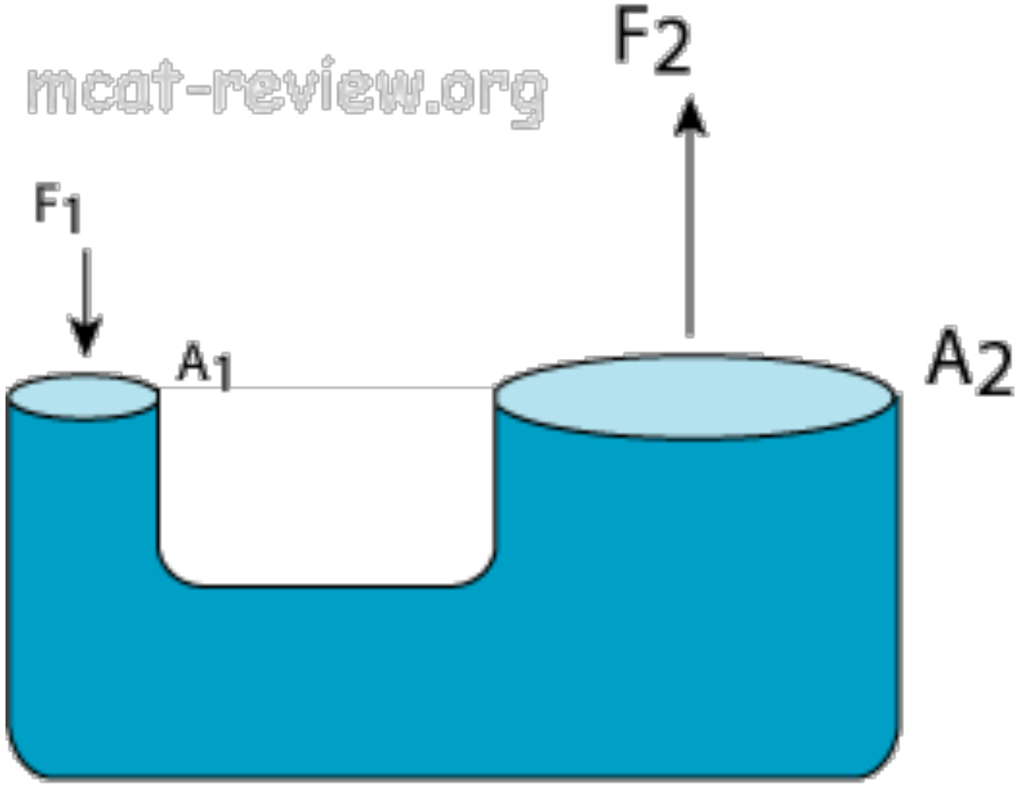


Figure 4.6: (a) Climatological potential density in the ocean, (b) its annual trend, (c) climatological stratification and (d) and its annual trend. Data are from a multiple-source observations reconstruction (*Li et al. Increasing ocean stratification over the past half-century. Nat. Clim. Chang. 10, 1116-1123 (2020)*).

Pascal's Law

- *All points in a connected body of constant-density fluid at rest are under the same pressure if they are at the same depth below the liquid surface.*

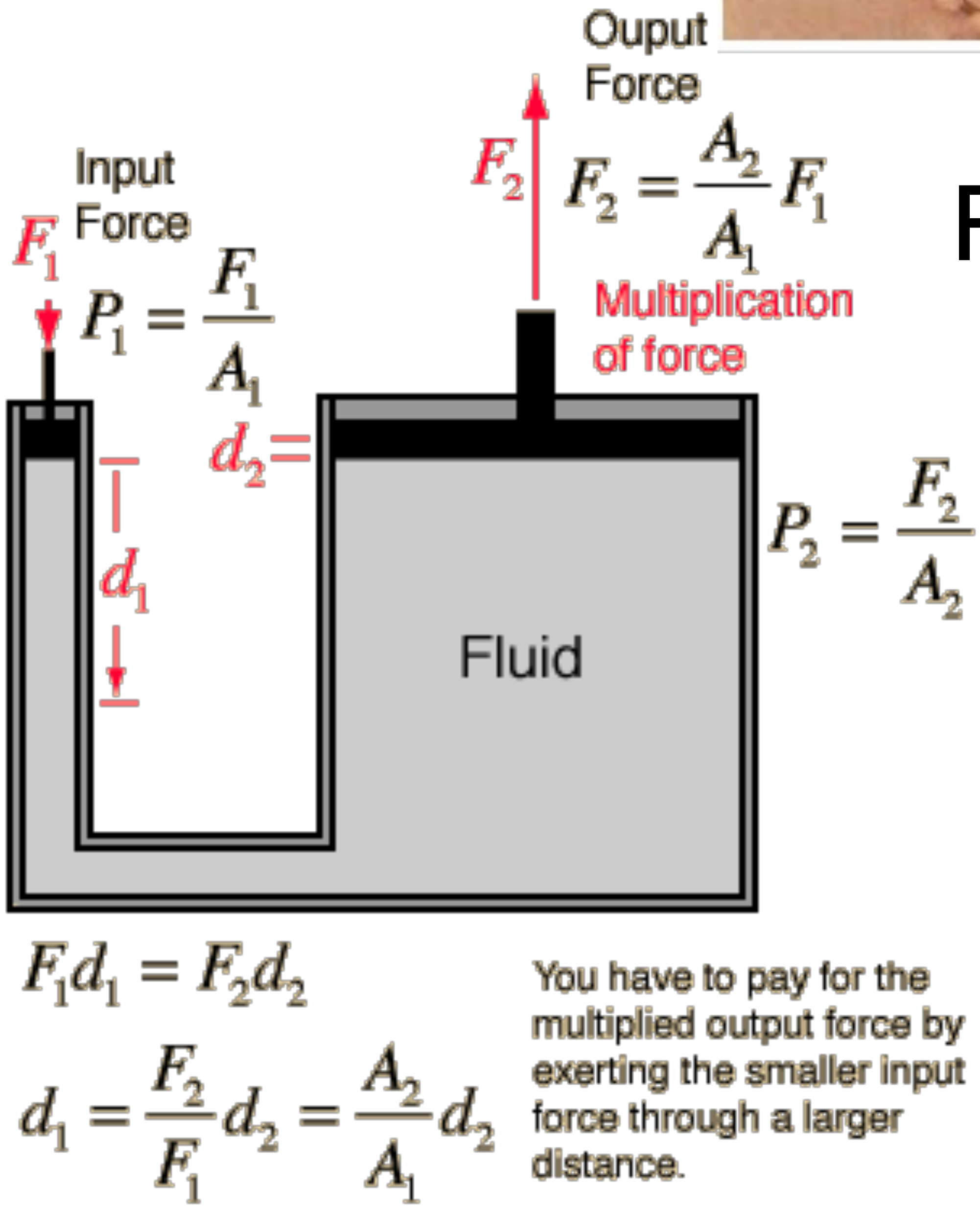
P1 = P2



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

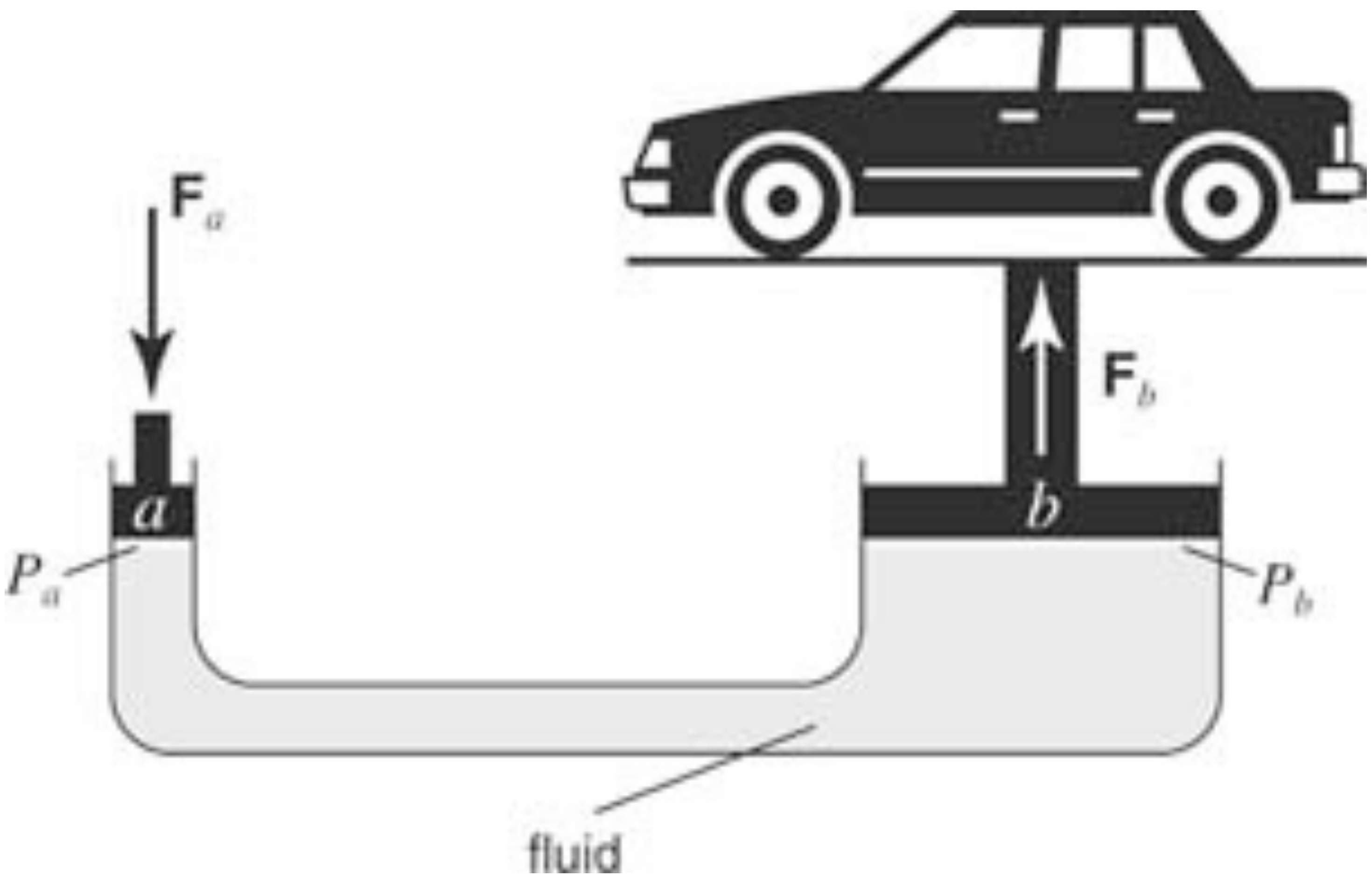
Pascal's Law

- if you apply pressure on a liquid, the pressure is transmitted equally and unchanged to all parts of the liquid.



$$P = F / A$$

Automobile Hydraulic Lift



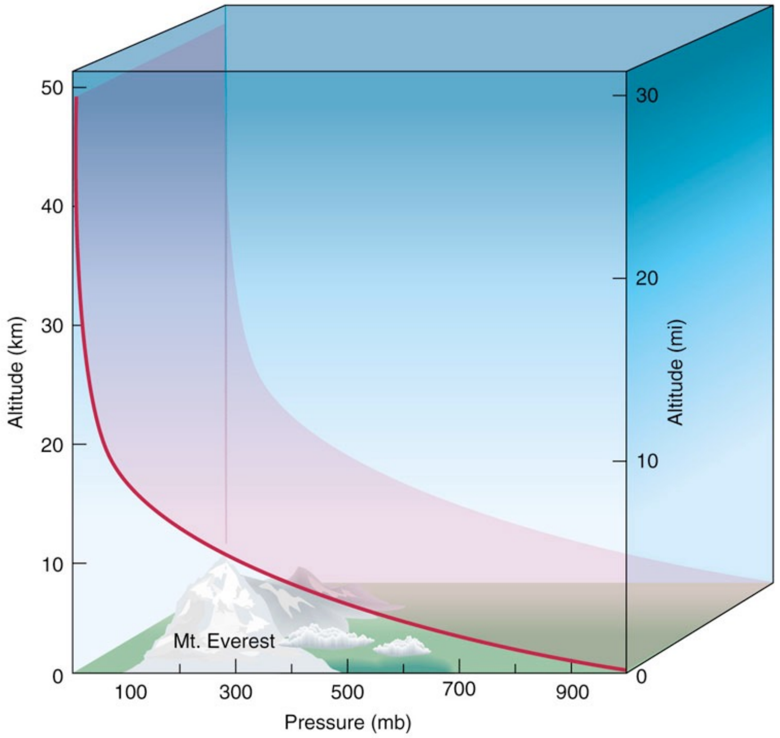
diameter d1 = 1.25cm
diameter d2 = 25 cm

Areas: A1=1.22; A2=490
-->A2/A1=400
-->F2=400F1

If car is 6000N ---> F1=6000N/400=15N
to lift it 10 cm ---> 400x10 = 40 m!!

Buoyancy force

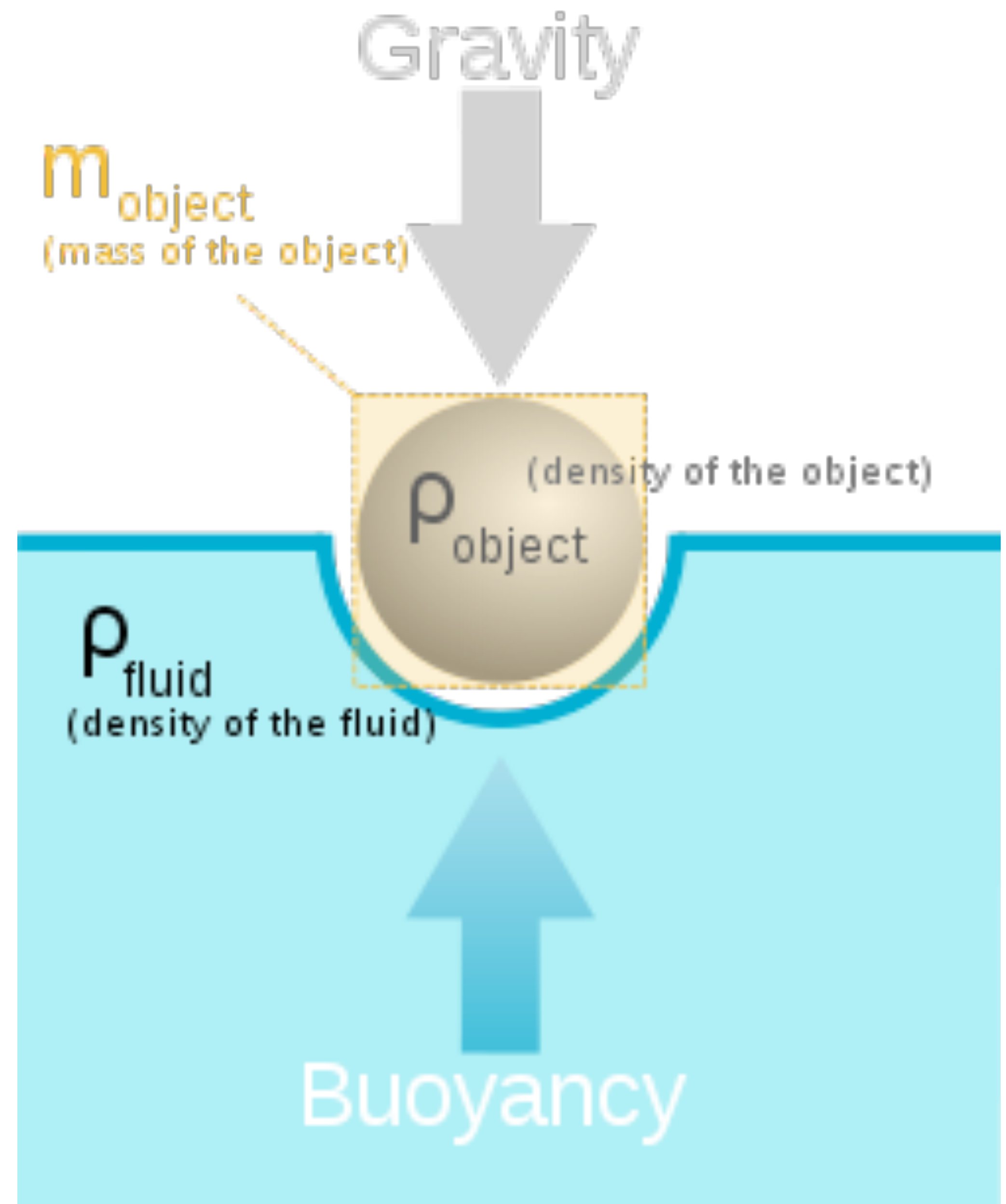
- Pressure in the atmosphere decreases with height (hydrostatics)
- Pressure force on balloon: bottom greater than at top



- Buoyancy force is the difference
- There is always a buoyancy force in a fluid, and it is always positive.

Buoyancy

- A force exerted by a fluid that opposes an object's weight
- force is equal to weight of fluid displaced by the object
- Fb = rho(fluid) x g x Vdisp
- An object whose density (specific weight) is greater than that of the fluid in which it is submerged tends to sink ...



is it easier to float in a pool or at sea?

- In equilibrium, the net Force must be zero, so that:

$$m\,g = \rho\,V_{disp}\,g = 0$$

If the buoyancy of an object exceeds its weight, it tends to rise. An object whose weight exceeds its buoyancy tends to sink.

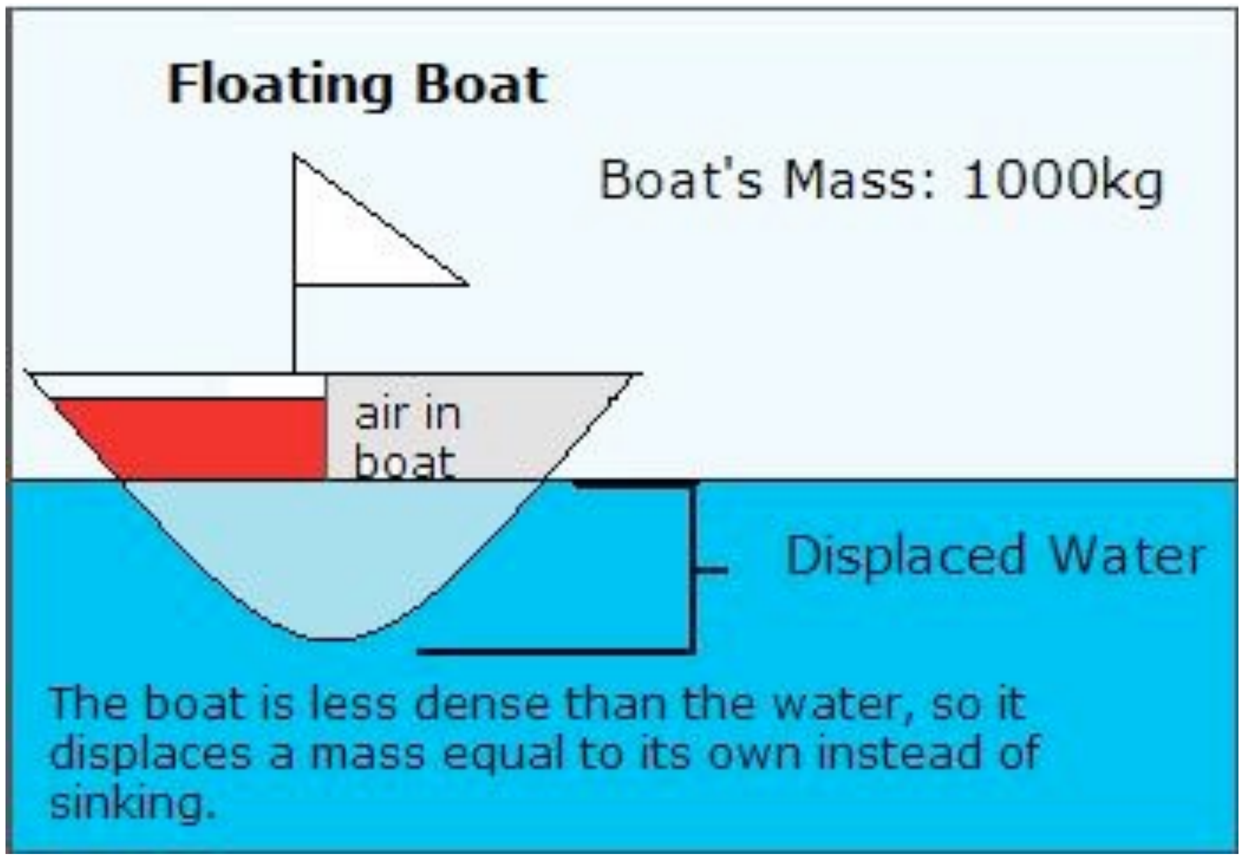
Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

Materials of human body (density Kg/l):
muscle = 1.1; bone = 1.5; air = 0.0012

In fresh water (with air out): MEN all sink - WOMEN some float

In fresh water (with air in): MEN some sink - WOMEN all float

Buoyancy and floating



A block of iron dipped in water will sink, while the same metal block shaped like a boat will float.

Buoyancy is thus related to the **density, volume** and **shape** of the immersed body.

If *Fb = rho x g x Vdisp*, what is volume of the displaced water?

Stationarity -> *Wboat = Fb = rho x g x Vdisp*

Vunderwater = Wboat / (rho x g)


MAERSK LINE
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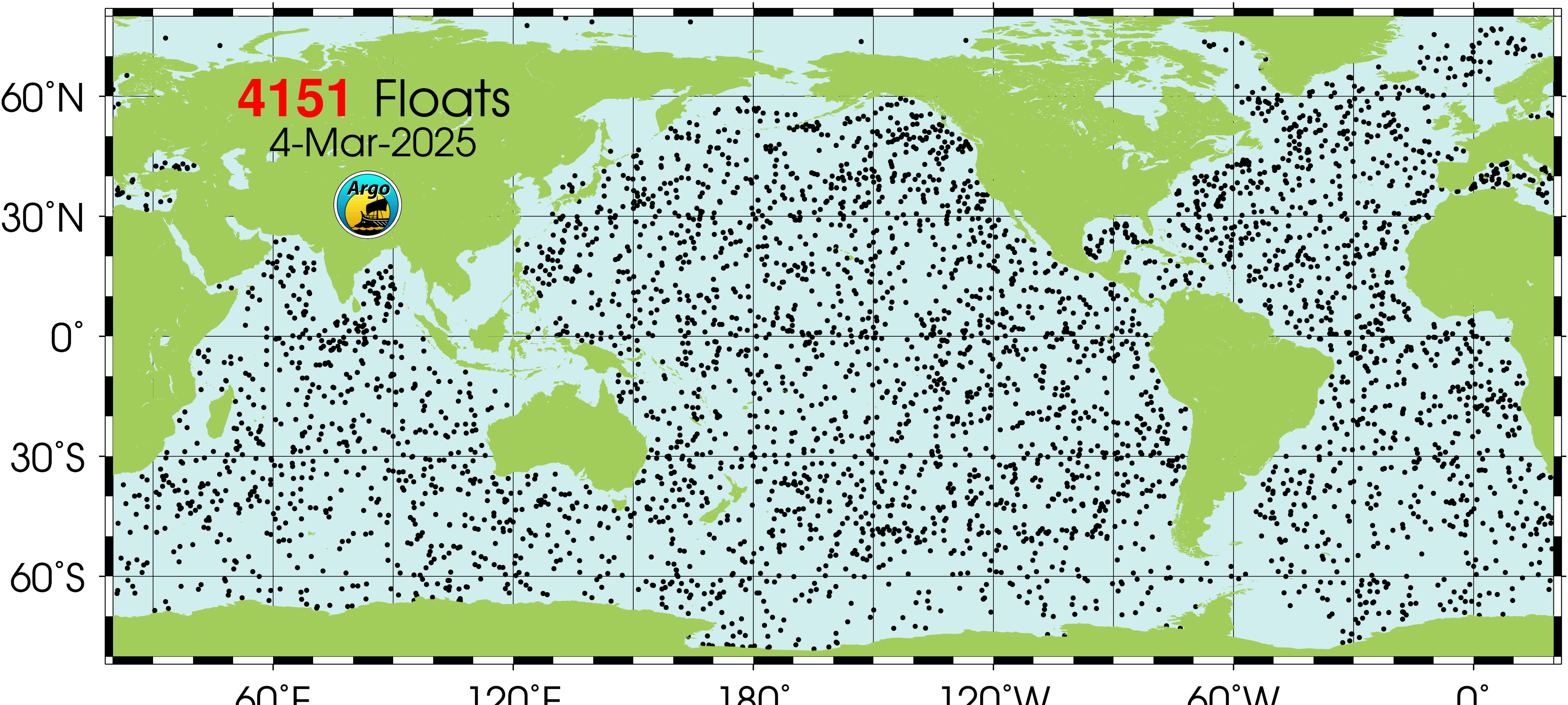
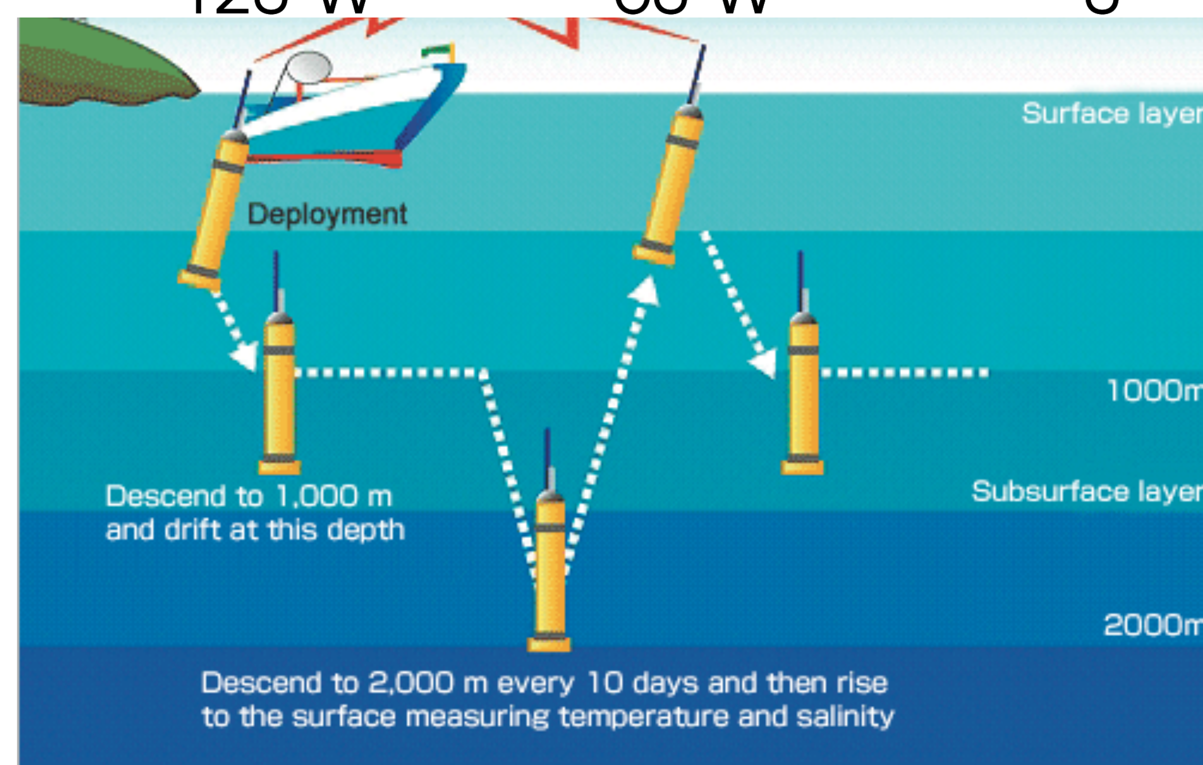

4151 Floats
4-Mar-2025
Argo
60°N
30°N
0°
30°S
60°S
60°E
120°E
180°
120°W
60°W
0°


Surface layer
Deployment
1000m
Subsurface layer
Descend to 1,000 m
and drift at this depth
2000m
Descend to 2,000 m every 10 days and then rise
to the surface measuring temperature and salinity

A curiosity … (the Iceberg)

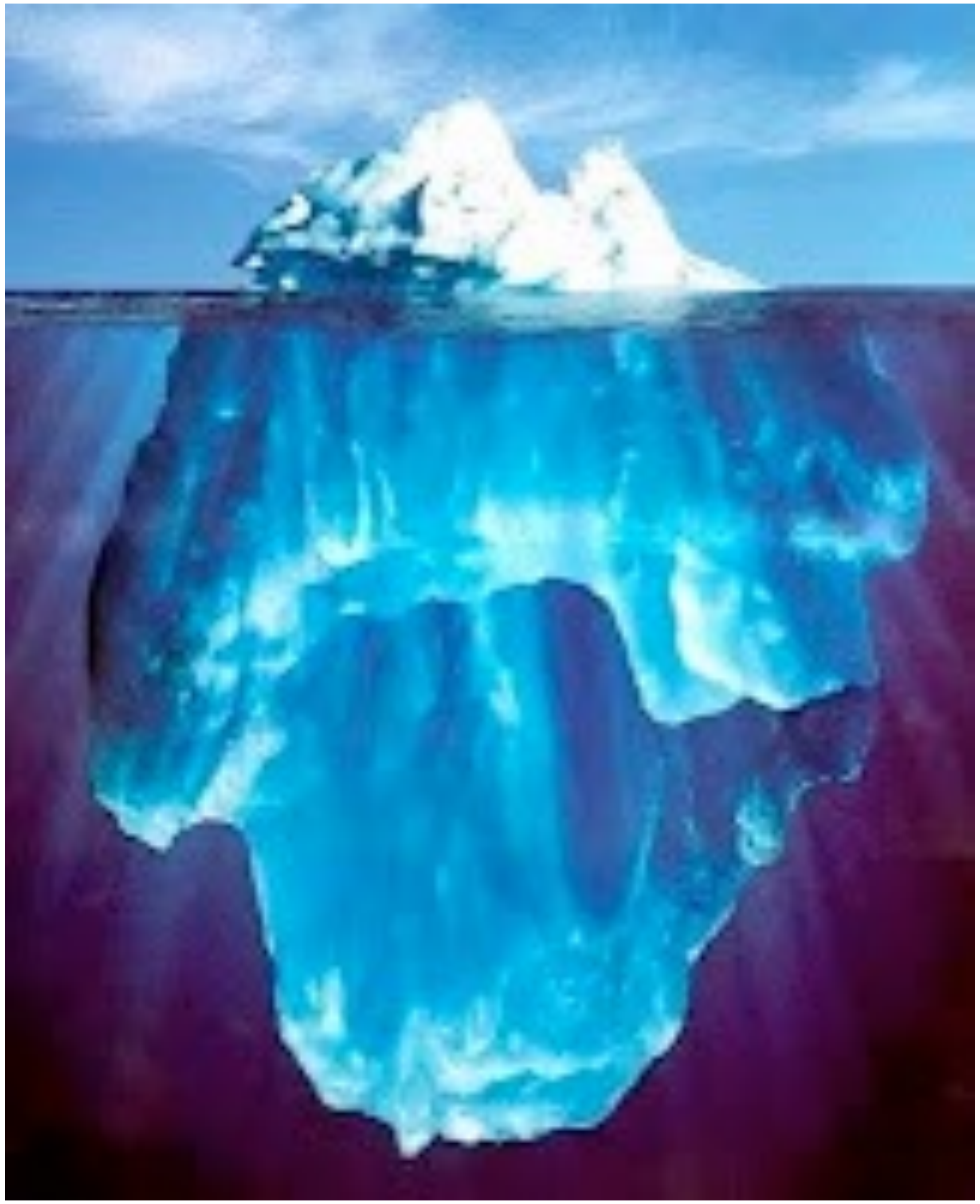
- Roughly: rho-ice = 92% rho-water

  (another curiosity in itself …)

- It is in equilibrium, so that mg = Fb

- how much of the iceberg is submerged?

A curiosity ... (the Iceberg)



- Roughly: rho-ice = 92% rho-water

  (another curiosity in itself …)

- It is in equilibrium, so that mg = Fb

- how much of the iceberg is submerged?

- 92% (… *"you only see the tip of the iceberg …"*)

Summary

- Fluid Statics
- Pascal's Law
- Absolute and Gage Pressure
- Buoyancy and Archimedes' Principle