

Propagazione delle incertezze (Taylor, cap 3)

Uncertainty in Sums and Differences

Suppose that x, \dots, w are measured with uncertainties $\delta x, \dots, \delta w$ and the measured values used to compute

$$q = x + \dots + z - (u + \dots + w).$$

If the uncertainties in x, \dots, w are known to be *independent and random*, then the uncertainty in q is the quadratic sum

$$\delta q = \sqrt{(\delta x)^2 + \dots + (\delta z)^2 + (\delta u)^2 + \dots + (\delta w)^2}$$

of the original uncertainties. In any case, δq is never larger than their ordinary sum,

$$\delta q \leq \delta x + \dots + \delta z + \delta u + \dots + \delta w.$$

Uncertainties in Products and Quotients

Suppose that x, \dots, w are measured with uncertainties $\delta x, \dots, \delta w$, and the measured values are used to compute

$$q = \frac{x \times \dots \times z}{u \times \dots \times w}.$$

If the uncertainties in x, \dots, w are *independent and random*, then the fractional uncertainty in q is the sum in quadrature of the original fractional uncertainties,

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \dots + \left(\frac{\delta w}{w}\right)^2}.$$

In any case, it is never larger than their ordinary sum,

$$\frac{\delta q}{|q|} \leq \frac{\delta x}{|x|} + \dots + \frac{\delta z}{|z|} + \frac{\delta u}{|u|} + \dots + \frac{\delta w}{|w|}.$$

Uncertainty in Any Function of One Variable

If x is measured with uncertainty δx and is used to calculate the function $q(x)$, then the uncertainty δq is

$$\delta q = \left| \frac{dq}{dx} \right| \delta x.$$

Uncertainty in a Power

If x is measured with uncertainty δx and is used to calculate the power $q = x^n$ (where n is a fixed, known number), then the fractional uncertainty in q is $|n|$ times that in x ,

$$\frac{\delta q}{|q|} = |n| \frac{\delta x}{|x|}.$$

Measured Quantity Times Exact Number

If the quantity x is measured with uncertainty δx and is used to compute the product

$$q = Bx,$$

where B has no uncertainty, then the uncertainty in q is just $|B|$ times that in x ,

$$\delta q = |B| \delta x.$$