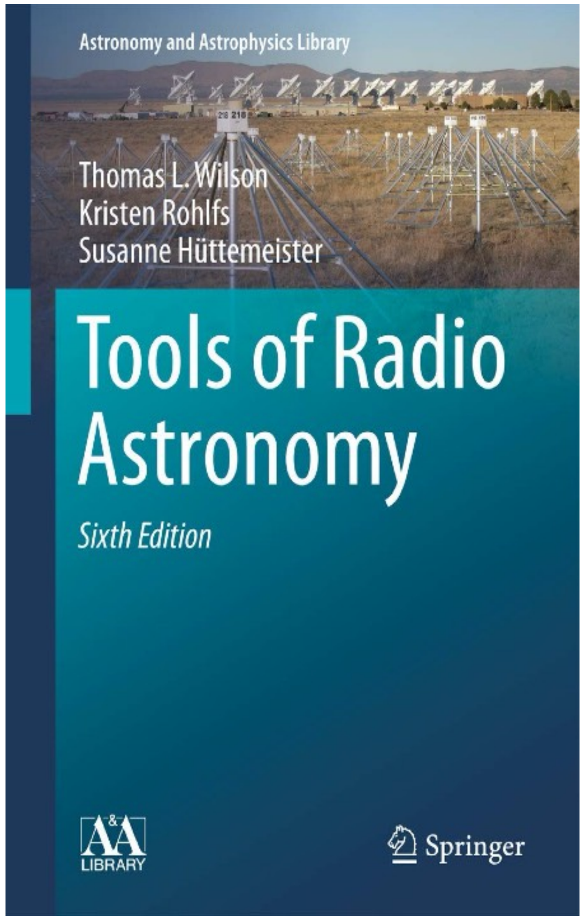
Summary of

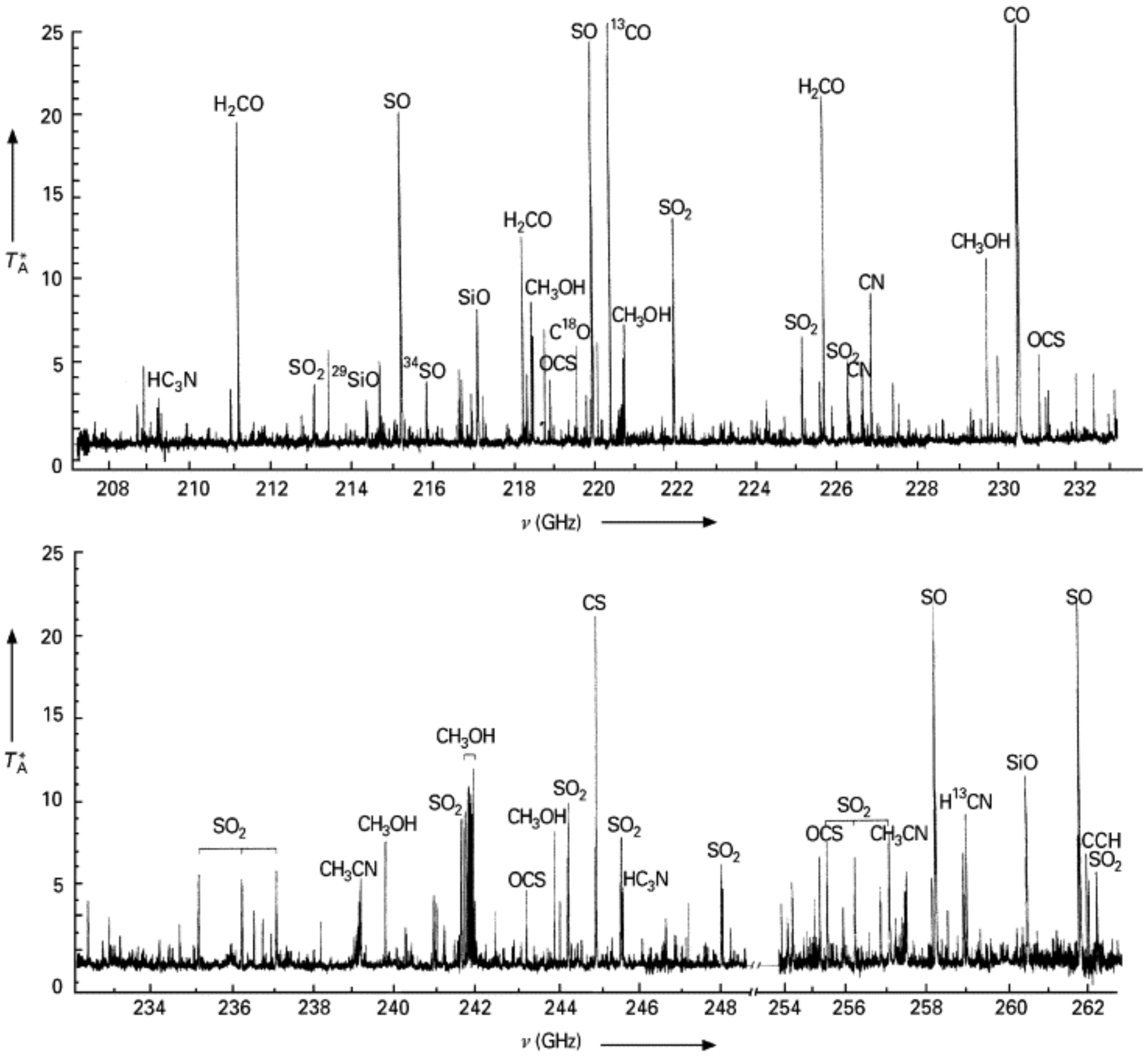
Radiative processes relevant to radioastronomy

A solid knowledge of the astrophysics behind radio observables is necessary to achieve a complete understanding of the various phenomena/astrophysical sources that will be investigated in this course.

Line emission/absorption processes



Chapters 12, 13, 14, 15, 16

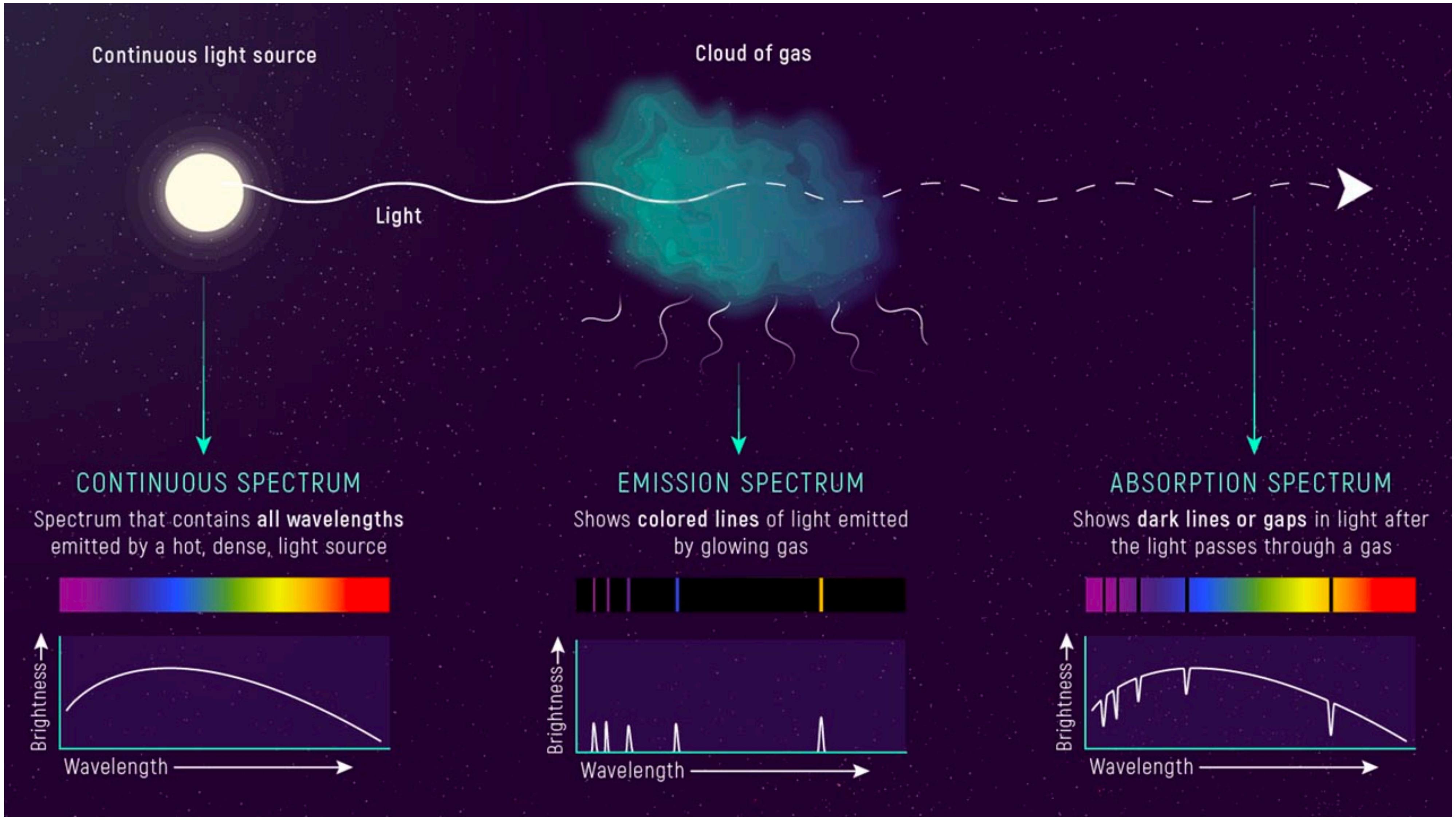


Orion nebula, radio spectrum

Basic definitions

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

ϵ_ν emission coefficient
k_ν absorption coefficient



Thermal black body	**Rotational transitions** (molecules)
Thermal free-free	**Fine structure transitions** (e.g. [CII])
Synchrotron	**Hyperfine structure transitions** (H 21cm)
Inverse Compton	**Amplified stimulated emission** (masers)

Rotational spectrum of a molecule

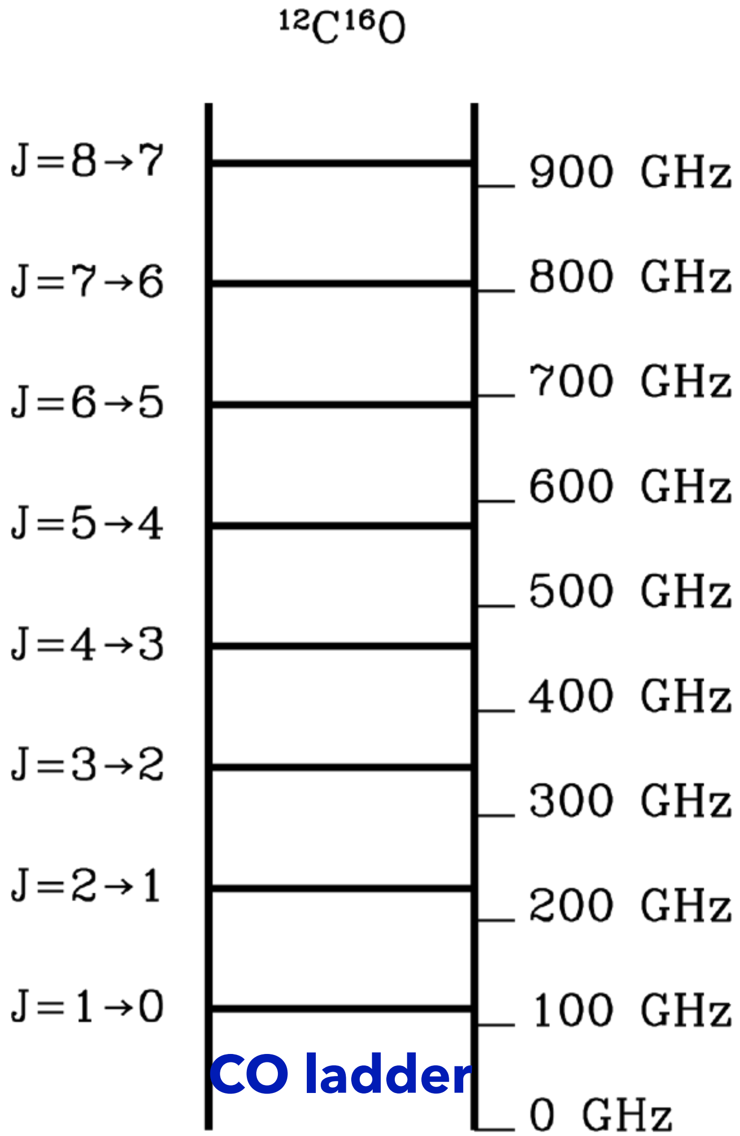
For a transition from *J* to *J-1* the energy released is:

$$\Delta H_{\rm rot} = \frac{\hbar^2 J}{I}$$

And the frequency of the emitted photons is therefore:

$$\boxed{\nu = \frac{\Delta H_{\rm rot}}{h} = \frac{\hbar J}{2\pi I} = \frac{hJ}{4\pi^2 m r_e^2}}$$

$J = 0,1,2,...$ (angular momentum of the upper level)



The radio spectrum of a particular molecular species will look like a ladder whose steps are all harmonics of the fundamental frequency.

$$\nu \propto m^{-1} r_e^{-2}$$

small molecules –-> mm

large molecules –-> cm

Minimum temperature to excite the upper level J (via collisions):

$$\boxed{T_{\rm min} \sim \frac{H_{\rm rot}}{k} \sim \frac{\nu h (J+1)}{2k}}$$

Atomic lines: fine structure and hyperfine structure

fine structure: transitions between energy levels with different J (total angular momentum

hyperfine structure: transitions between energy levels with different F (total magnetic spin momentum)

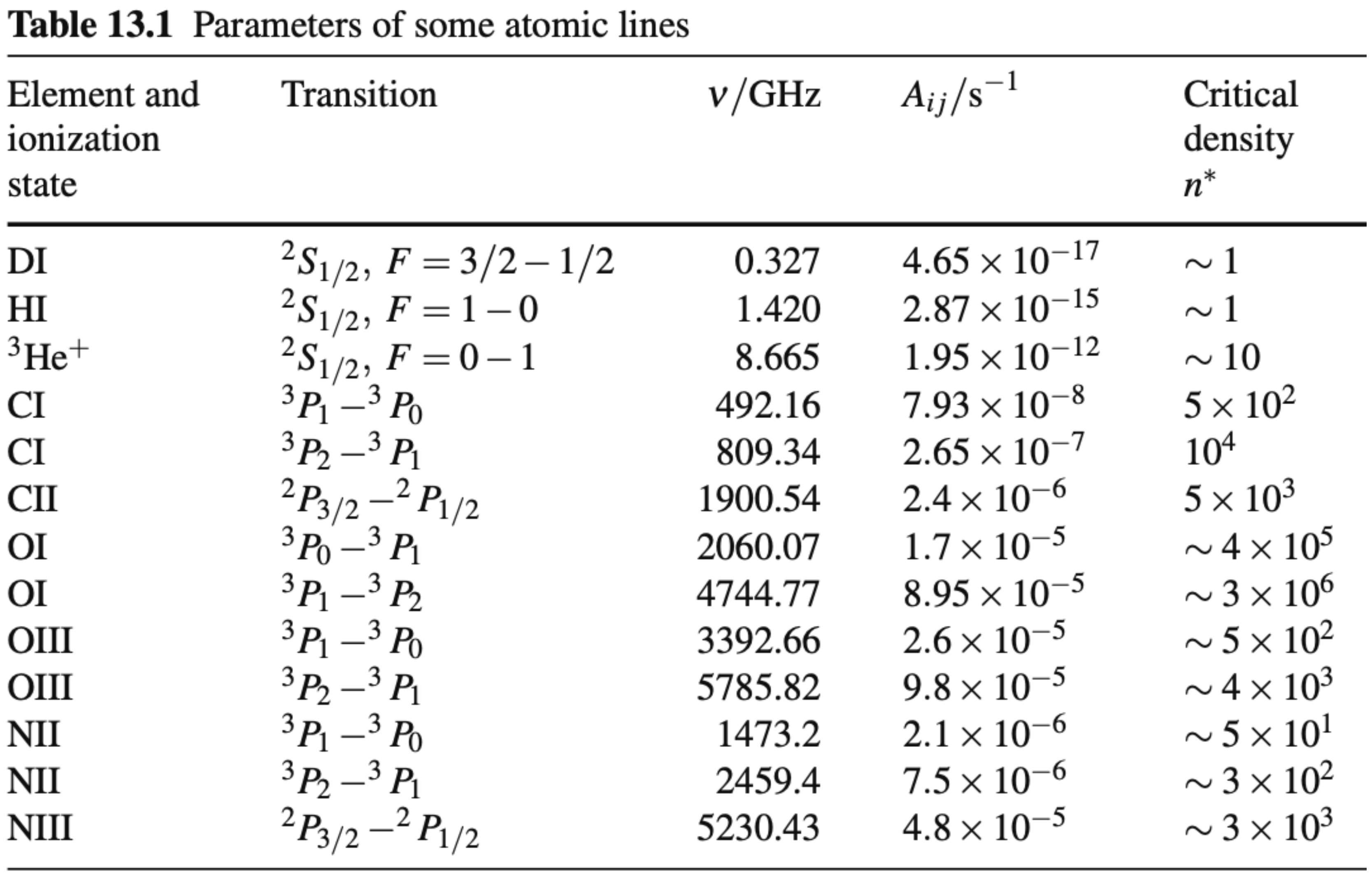
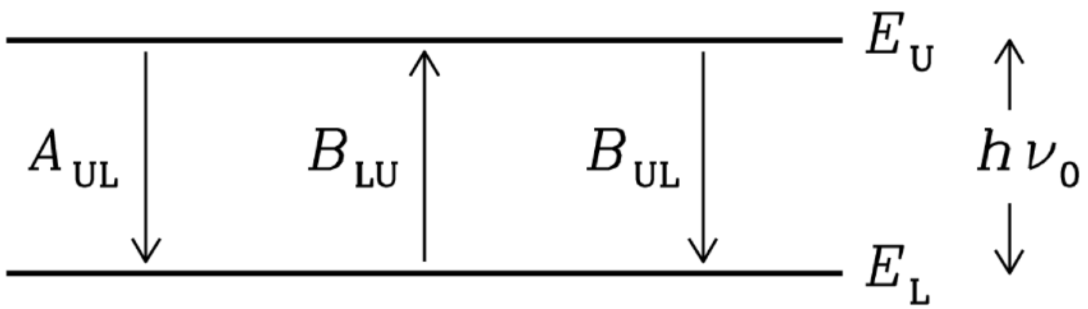


Table 13.1 Parameters of some atomic lines

Element and ionization state	Transition	ν/GHz	A_{ij}/s^{-1}	Critical density n^*
DI	$^2S_{1/2},\ F=3/2-1/2$	0.327	4.65×10^{-17}	~ 1
HI	$^2S_{1/2},\ F=1-0$	1.420	2.87×10^{-15}	~ 1
^{3}He$^+$	$^2S_{1/2},\ F=0-1$	8.665	1.95×10^{-12}	~ 10
CI	$^3P_1-^3P_0$	492.16	7.93×10^{-8}	5×10^2
CI	$^3P_2-^3P_1$	809.34	2.65×10^{-7}	10^4
CII	$^2P_{3/2}-^2P_{1/2}$	1900.54	2.4×10^{-6}	5×10^3
OI	$^3P_0-^3P_1$	2060.07	1.7×10^{-5}	$\sim 4\times10^5$
OI	$^3P_1-^3P_2$	4744.77	8.95×10^{-5}	$\sim 3\times10^6$
OIII	$^3P_1-^3P_0$	3392.66	2.6×10^{-5}	$\sim 5\times10^2$
OIII	$^3P_2-^3P_1$	5785.82	9.8×10^{-5}	$\sim 4\times10^3$
NII	$^3P_1-^3P_0$	1473.2	2.1×10^{-6}	$\sim 5\times10^1$
NII	$^3P_2-^3P_1$	2459.4	7.5×10^{-6}	$\sim 3\times10^2$
NIII	$^2P_{3/2}-^2P_{1/2}$	5230.43	4.8×10^{-5}	$\sim 3\times10^3$

Microwave amplification by stimulated emission of radiation: masers

In the previous lessons, we have seen that in the low-frequency regime stimulated emission and ordinary absorption are nearly equal.



$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

$$\boxed{k_\nu = \frac{c^2}{8\pi\nu_0^2}\frac{g_U}{g_L}N_L A_{UL}[\underbrace{1}_{\text{ordinary absorption}} - \underbrace{exp(-h\nu_0/kT)}_{\text{stimulated emission}}]\phi(\nu)}$$

Net absorption coefficient or "line opacity coefficient"

In the Rayleigh-Jeans regime we have $h\nu_0 << kT$

$$[1 - exp(-h\nu_0/kT)] \sim h\nu_0/kT << 1$$

We have also seen that, for a system in which both radiation and collisions regulate the transitions between energy levels, the populations of the different levels are linked by the relation

$$\frac{N_U}{N_L} = \frac{g_U}{g_L} exp\left(-\frac{h\nu_0}{\underbrace{kT_{ex}}_{\textit{excitation temperature}}}\right)$$

Microwave amplification by stimulated emission of radiation: masers

Amplified stimulated emission occurs when the upper energy level U is overpopulated, that is when

$$\frac{N_U}{N_L} > \frac{g_U}{g_L}$$

$$\frac{N_U}{N_L} = \frac{g_U}{g_L}\underbrace{exp\left(-\frac{h\nu_0}{kT_{ex}}\right)}_{>1}$$

This implies that the excitation temperature has to be negative

The line opacity is also negative

$$k_\nu = \frac{c^2}{8\pi\nu_0^2}\frac{g_U}{g_L}N_L A_{UL}\underbrace{[1 - exp(-h\nu_0/kT)]}_{<0}\phi(\nu)$$

A negative opacity implies brightness gain instead of loss; the intensity of a background source at frequency ν_0 will be amplified. But how much?

Microwave amplification by stimulated emission of radiation: masers

A negative opacity implies brightness gain instead of loss; the intensity of a background source at frequency ν_0 will be amplified. But how much?

Let's assume for simplicity that $g_U = g_L$. The equation of detailed balance tells us that $g_L B_{LU} = g_U B_{UL}$,

so that $B_{\mathrm{LU}} = B_{\mathrm{UL}} \equiv B$

$$\frac{dI_\nu}{ds} = -\frac{h\nu_0}{c}(N_L B_{LU} - N_U B_{UL})\phi(\nu)I_\nu + \left(\frac{h\nu_0}{4\pi}\right) N_U A_{UL}\phi(\nu)$$

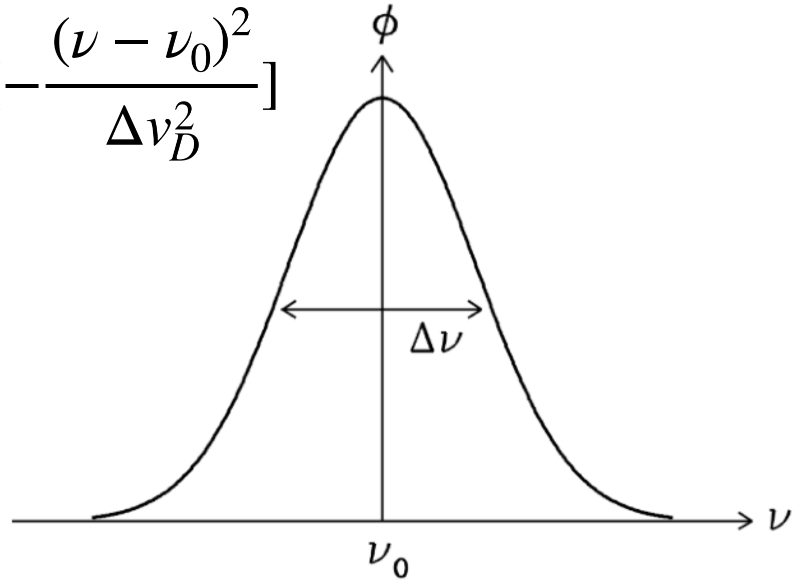
$$\frac{dI_\nu}{ds} = -\frac{h\nu_0}{c}(N_L - N_U)B\phi(\nu)I_\nu$$

Microwave amplification by stimulated emission of radiation: masers

$$\frac{dI_\nu}{ds} = -\frac{h\nu_0}{c}(N_L - N_U)B\phi(\nu)I_\nu$$

We have seen that in the case of line broadening due to thermal motions $\phi(\nu)$ is a Gaussian whose FWHM $\Delta\nu$ is related to the Doppler width $\Delta\nu_D$.

$$I_\nu = \frac{1}{\sqrt{\pi}\Delta\nu_D} exp[-\frac{(\nu-\nu_0)^2}{\Delta\nu_D^2}]$$



It can be easily derived that $\phi(\nu_0) \sim \frac{1}{\Delta\nu_{\mathrm{D}}} \sim \frac{1}{\Delta\nu_{\mathrm{FWHM}}}$

$$\frac{dI_\nu}{ds} = -\frac{h\nu_0(N_L - N_U)BI_\nu}{c\Delta\nu} \quad \text{for } \nu \sim \nu_0$$

From which the maser optical depth is

$$\tau = \int -k_\nu ds = \int \frac{dI_\nu}{I_\nu} = \frac{h\nu_0 B}{c\Delta\nu}\int (N_U - N_L)ds$$

maser gain

$$\boxed{I_\nu(s) = I_\nu(0)exp(\tau)}$$

Radiation is amplified by a factor $\mathbf{e}^\tau$

Microwave amplification by stimulated emission of radiation: masers

Maser emission quickly depopulates the upper energy level, so masers have to be “pumped” to emit continuously. Typically one or more higher energy levels absorb radiation from a pump source (e.g., infrared continuum from a star or an AGN), and radiative decays preferentially repopulate the upper energy level.

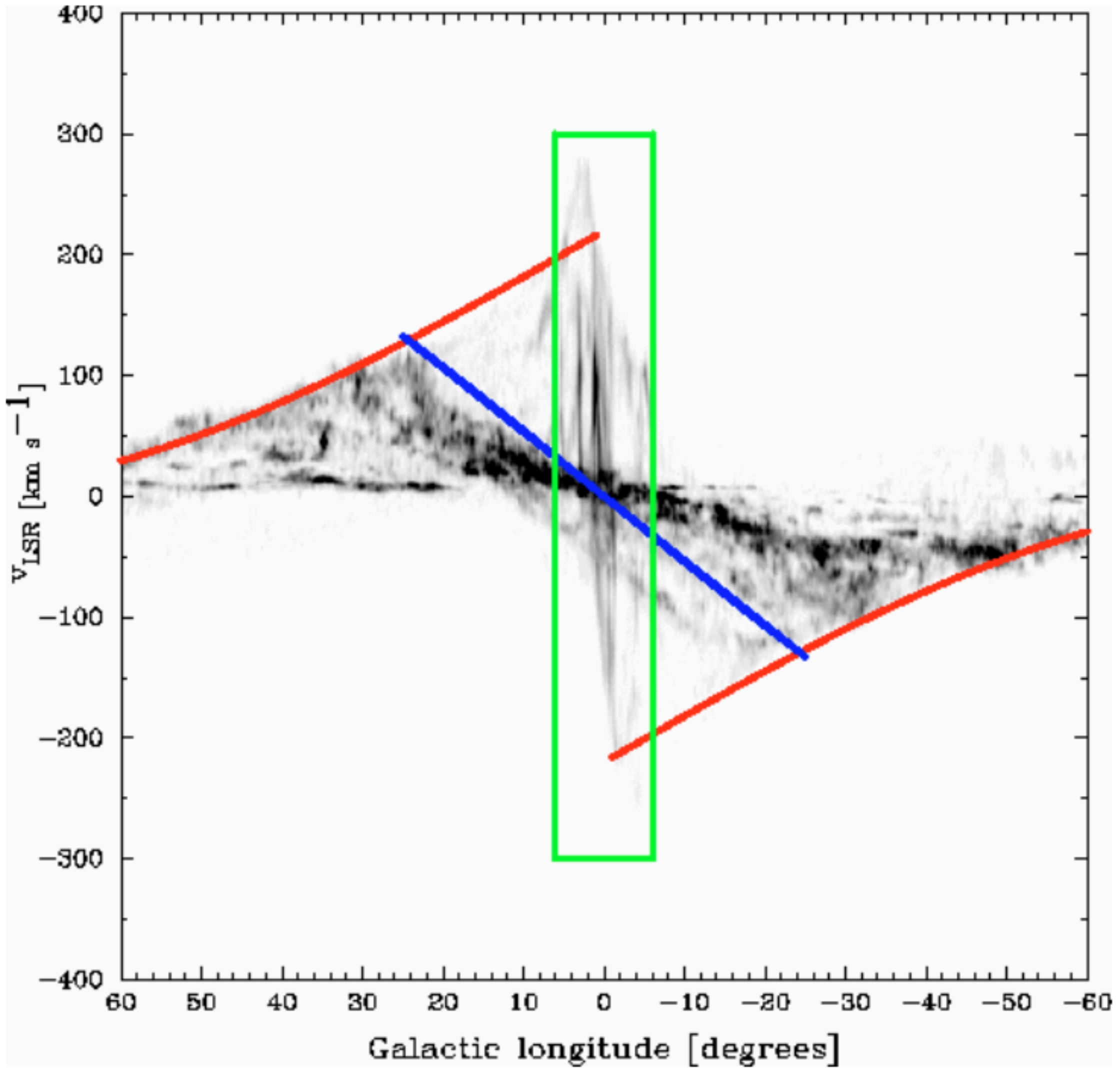
If the maser photon emission rate is limited by the pump luminosity, the maser is described as being saturated; if the pump power is more than adequate, the maser is unsaturated.

$$\frac{dI_\nu}{ds} = -\frac{h\nu_0(N_L - N_U)BI_\nu}{c\Delta\nu}$$

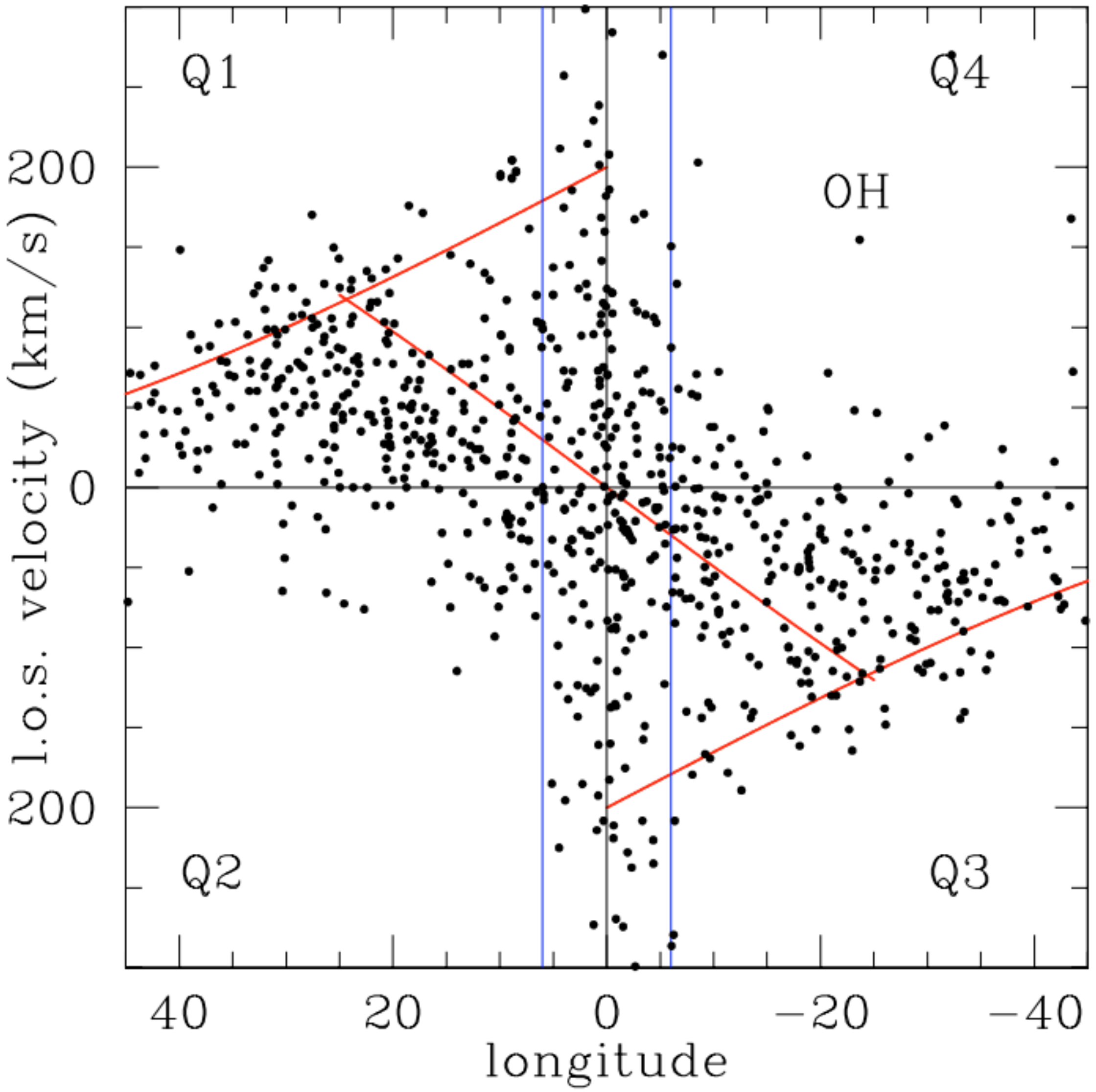
Microwave amplification by stimulated emission of radiation: masers

OH maser in the Milky Way at ~1.6 GHz (18 cm)

CO

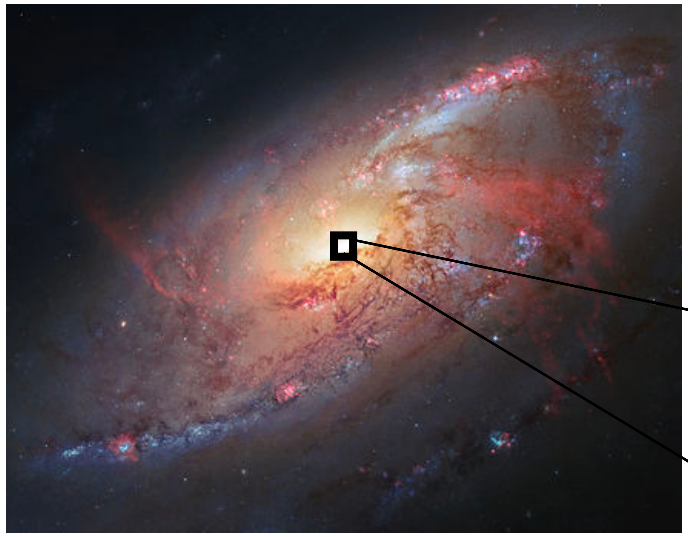


OH maser stars



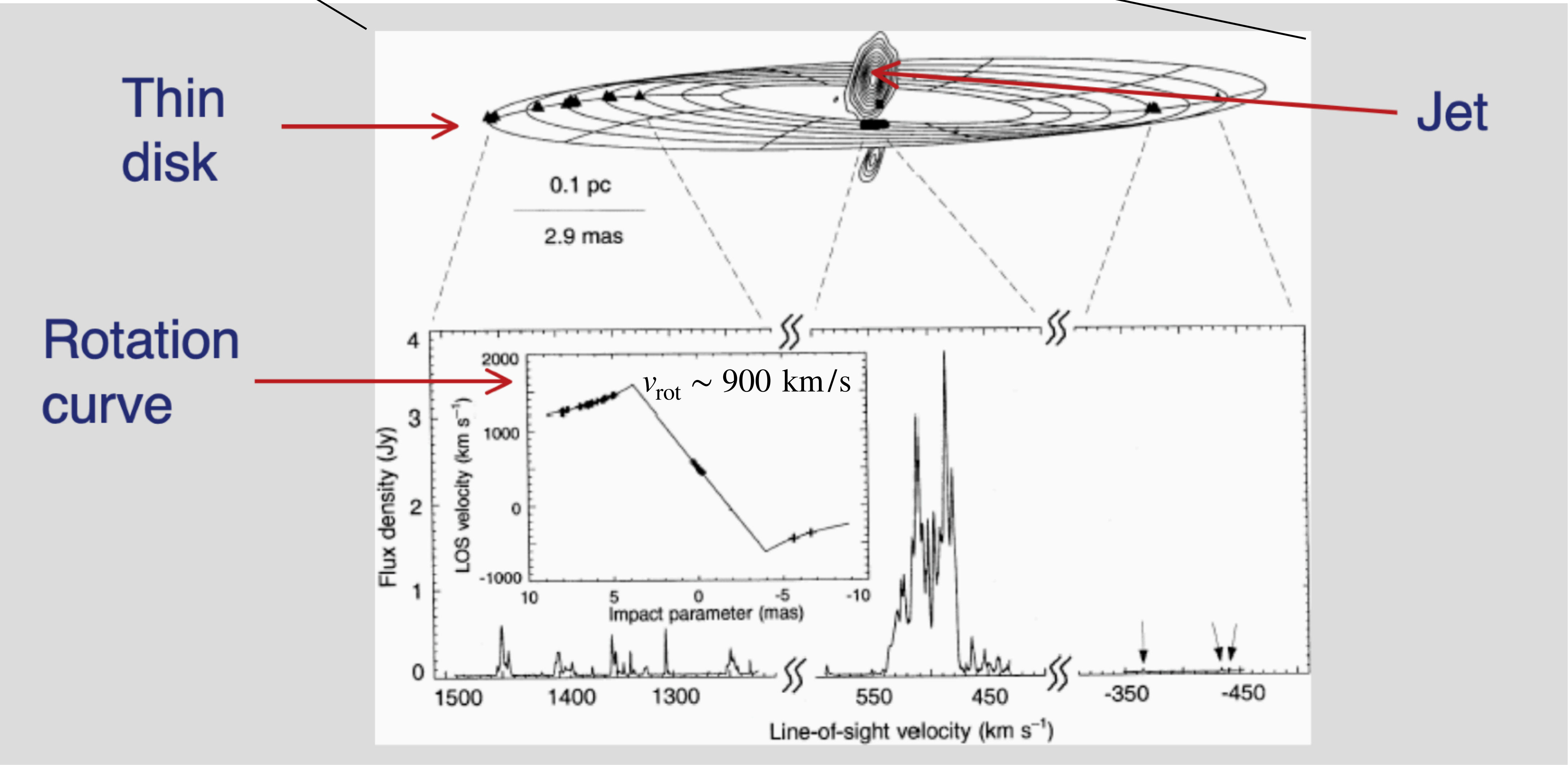
Microwave amplification by stimulated emission of radiation: masers

NGC4258



H_20 megamaser at 22 GHz

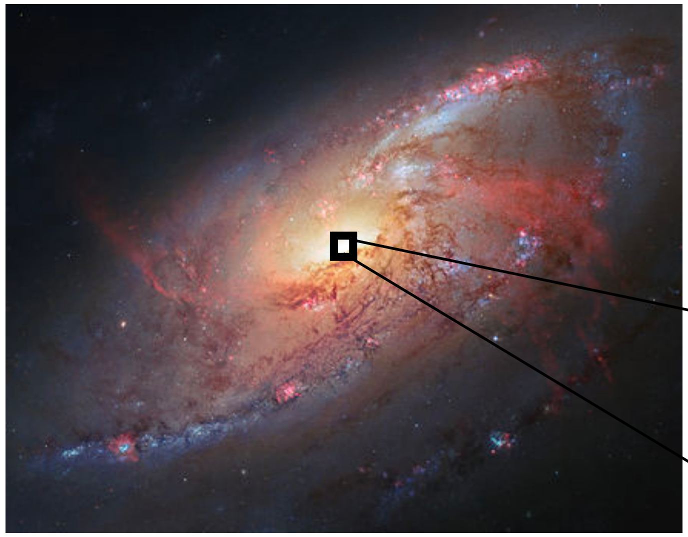
hundreds of narrow lines clustered in three groups around the systemic recession velocity (v_{gal}~450 km/s)



$v_{gal} + v_{rot}$ v_{gal} $v_{gal} - v_{rot}$

Microwave amplification by stimulated emission of radiation: masers

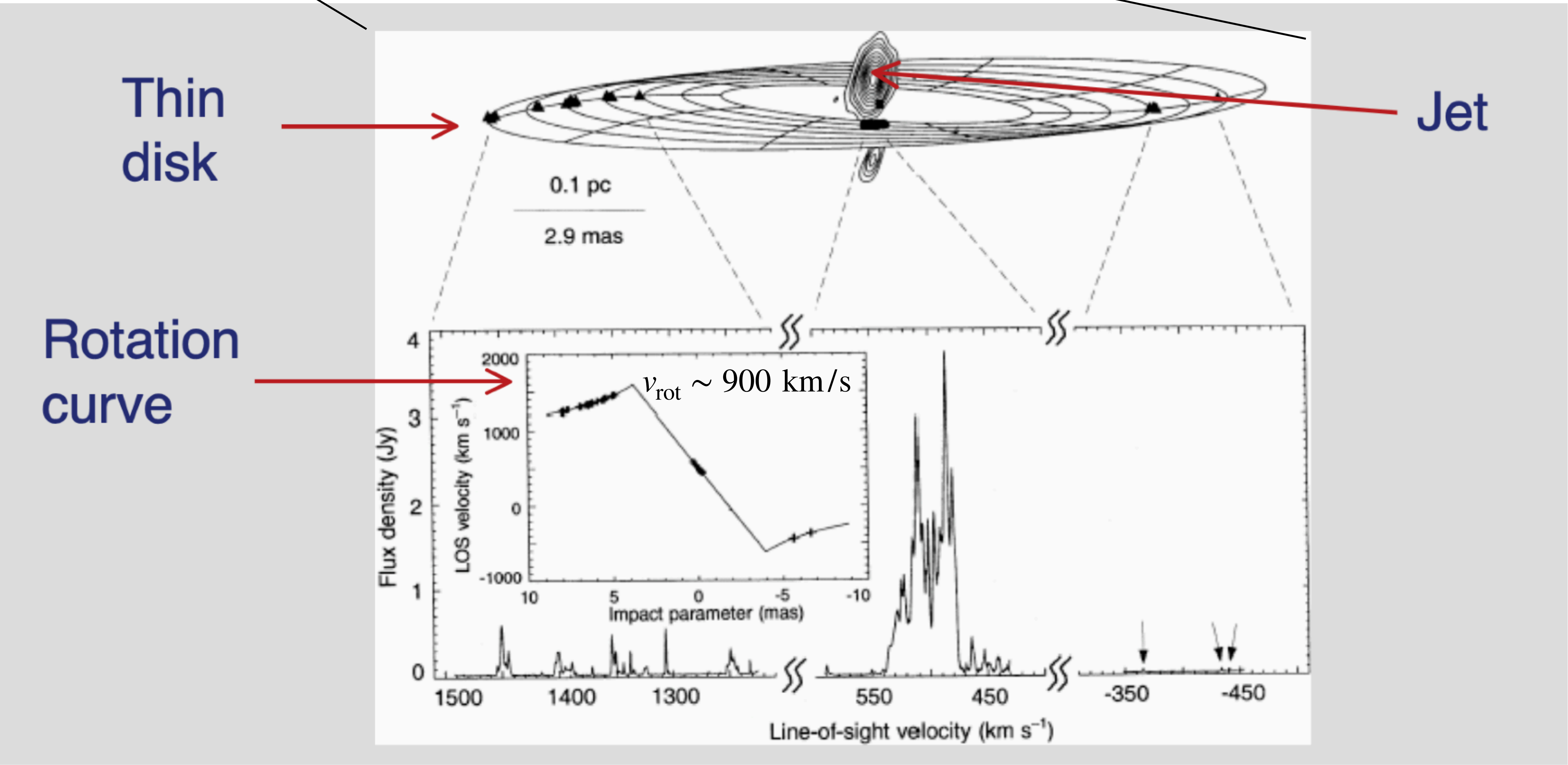
NGC4258



Keplerian velocity curve $v_{\rm rot} \simeq \sqrt{\dfrac{GM}{R}}$ → $M \sim 3.8 \times 10^7 M_\odot$

R~0.26 pc

~10^4 times larger than the mass expected for the densest nuclear star clusters. Evidence for a supermassive BH!



Thin disk

Jet

Rotation curve

$v_{\rm gal} + v_{\rm rot}$ $v_{\rm gal}$ $v_{\rm gal} - v_{\rm rot}$

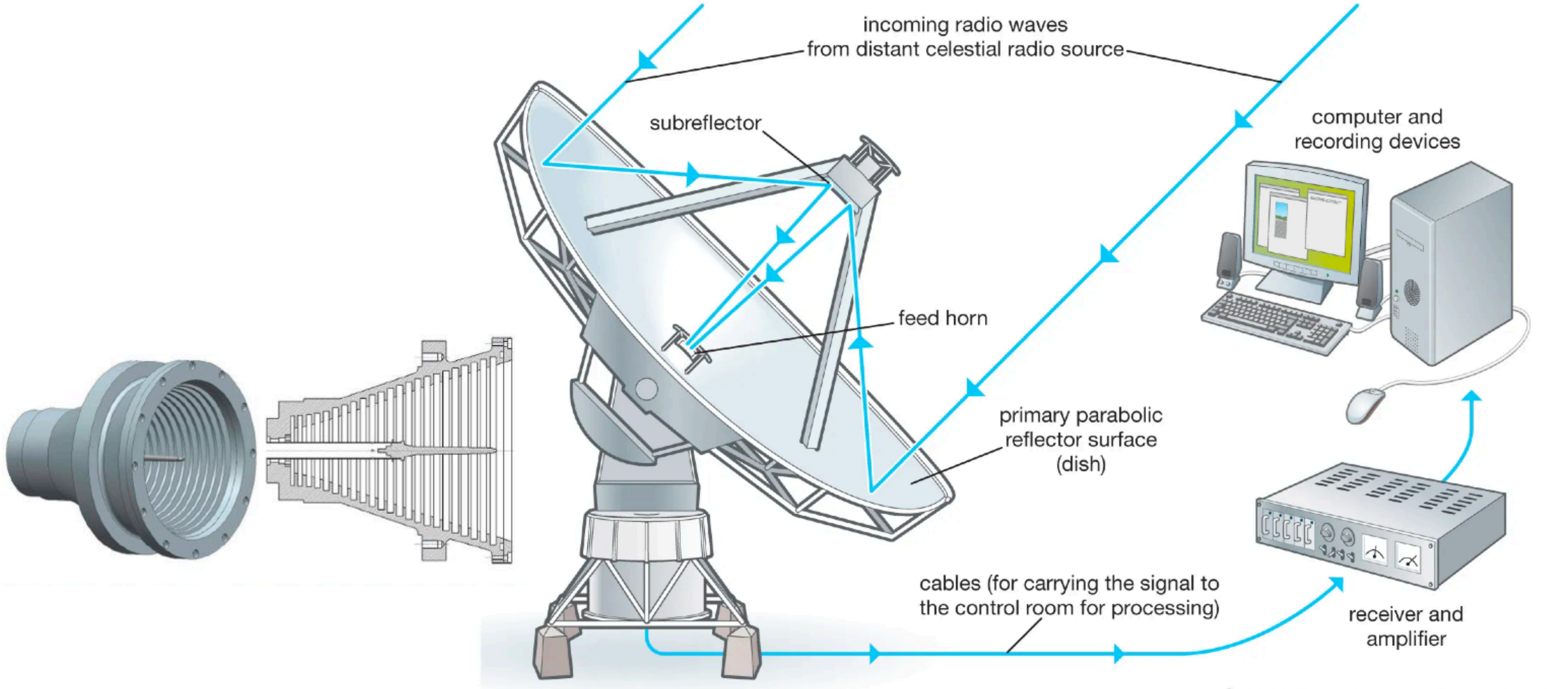
Instruments for Radioastronomy

Elements of a radio telescope

A radio telescope is a specialized antenna and radio receiver, used to detect radio waves from astronomical radio sources in the sky.

An antenna is a passive device that converts electromagnetic radiation in space into electrical currents in conductors, or vice versa, depending on whether it is used for receiving or for transmitting. Radio telescopes are receiving antennas

A radio receiver is an electronic device which receives alternating currents from the antenna and converts the information carried by them into a usable form. It uses electronic filters to select the desired frequencies and an electronic amplifier to increase the power of the signal for further processing.



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Elements of a radio telescope

Due to the wide range of frequencies that makes up the radio spectrum (tens of MHz - hundreds of GHz), the type of antennas used as radio telescopes widely vary in design, size and configuration.

At frequencies >>100 MHz parabolic antennas or "dish" predominate

At frequencies < 100 MHz directional antennas are typically used

LOw-Frequency ARray (LOFAR)
10-80 MHz

IRAM 30-meter (Pico Veleta, ES)

120-360 GHz

Hardware:

- a main mirror/collector, a secondary mirror/subreflector (and possibly more) which drive the radiation into a small area (focus)
- a detector, sensible to the incoming electromagnetic waves (feed horn). The easiest component to detect is the electric field of the wave.

Antenna fundamentals

The most important characteristic of an antenna is its ability to absorb radio waves incident upon it. This is typically described in terms of antenna effective aperture:

$$A_e = \frac{\text{Power density available at the antenna terminals}}{\text{Flux density of the incident wave}} = [\frac{\mathrm{W/Hz}}{W/m^2/Hz}] = [m^2]$$

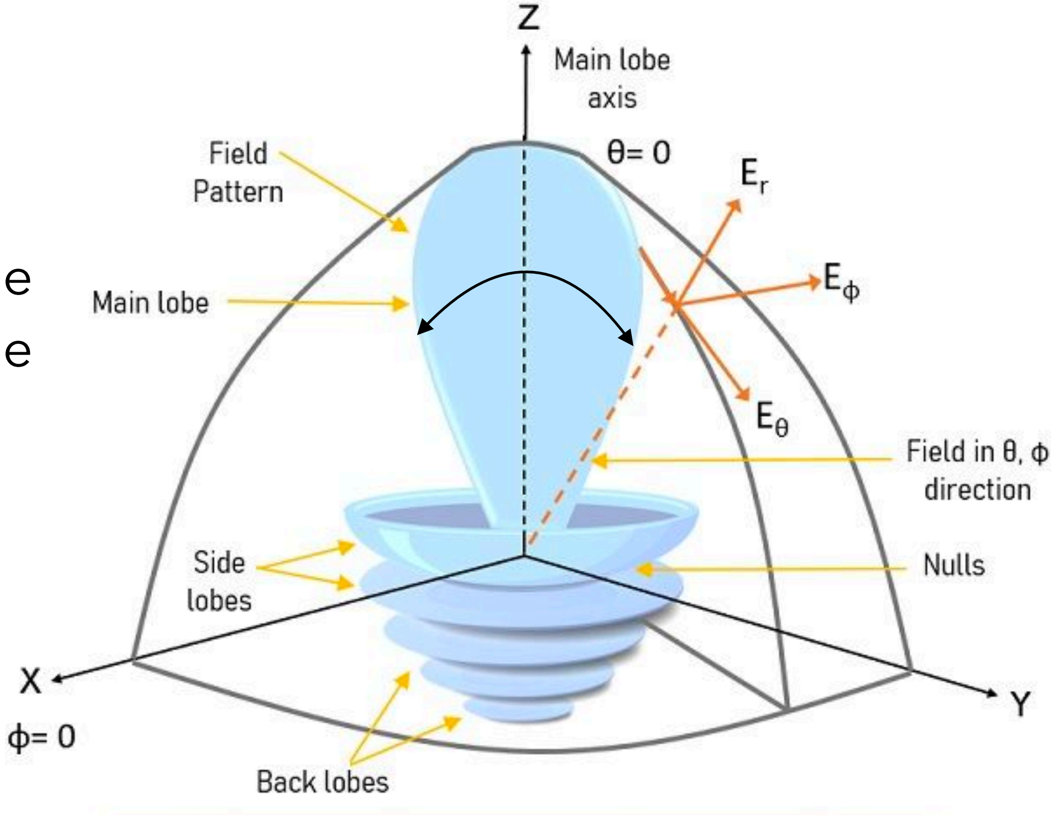
The effective area depends on the direction of the incident wave: the antenna works better is some directions than in others:

$$A_e = A_e(\theta, \phi)$$

This directional property of the antenna is often described in the form of a power pattern, i.e. an effective area normalized to be unity at the maximum

$$\boxed{P(\theta, \phi) = \frac{A_e(\theta, \phi)}{A_e^{\max}}}$$

The pattern of antenna is the same regardless it is used as a transmitter or as a receiver. This is called reciprocity.



Main lobe: primary maximum of the antenna pattern

Side lobes: subsidiary maxima of the antenna pattern

Half Power Beamwidth Θ_{HPBW}: angular distance between the two points at which $P = P^{max}/2$

Antenna fundamentals

From simple diffraction theory it can be shown that for a reflecting telescope

$$\boxed{\Theta_{\rm HPBW} \sim \frac{\lambda}{D}}$$

where *D* is the physical dimension of the telescope. The larger the telescope, the better the resolution.

IRAM 30-meter (Pico Veleta, ES)

250 GHz ~1.2×10^{-3}m

$$\Theta_{\rm HPBW} \sim \frac{1.2 \times 10^{-3}}{30} \sim 4 \times 10^{-5} \text{ rad} \sim 8.3''$$

Arecibo 300-meter (Porto Rico)

1.4 GHz ~21cm

$$\Theta_{\rm HPBW} \sim \frac{0.21}{300} \sim 10^{-2} \text{ rad} \sim 2.4'$$

Antenna fundamentals

Another patter often used to describe antennas is the gain:

$$G(\theta, \phi) = \frac{\text{Power emitted into } (\theta, \phi)}{(\text{Total power input})/4\pi}$$

For any lossless antenna, energy conservation requires that the gain averaged over all directions is <G>=1, from which

$$\int_{sphere} G(\theta, \phi) sin(\theta) d\theta d\phi = 4\pi$$

$$\boxed{\Omega_A \equiv \frac{1}{G^{max}} \int_{4\pi} G(\theta, \phi) sin(\theta) d\theta d\phi = 4\pi / G^{\max}}$$

beam solid angle

where G^{max} is the maximum gain

The main beam solid angle is defined as the region containing the principal response out to the first zero

$$\boxed{\Omega_{MB} = \frac{1}{G^{\max}} \int_{MB} G(\theta, \phi) sin(\theta) d\theta d\phi}$$

And we can define the concept of main beam efficiency as $\eta_{MB} \equiv \frac{\Omega^{\rm MB}}{\Omega_A}$

Antenna fundamentals

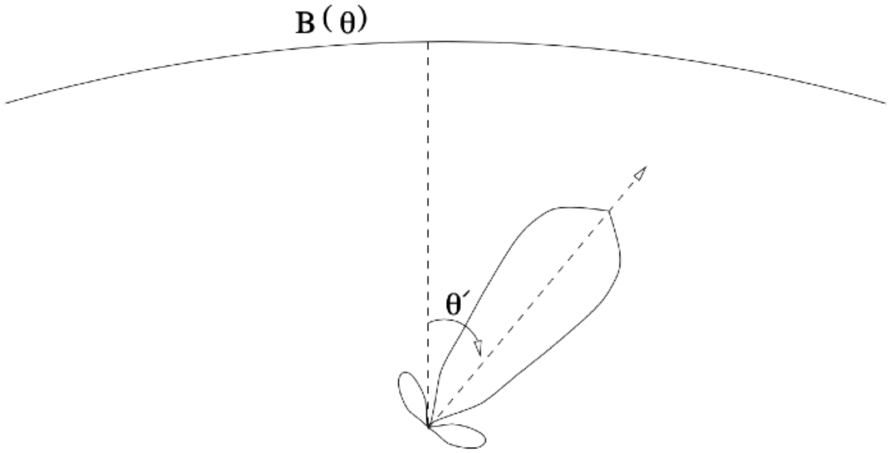
Another patter often used to describe antennas is the gain:

$$G(\theta, \phi) = \frac{\text{Power emitted into } (\theta, \phi)}{(\text{Total power input})/4\pi}$$

For reflector antennas, the aperture efficiency is defined as:

$$\eta = \frac{A_e^{max}}{A_g}$$ where A_g is the geometric cross-sectional area of the main reflector

Consider observing a source with brightness distribution $B(\theta)$ with a radiotelescope with power pattern $P(\theta)$. The power available at the antenna terminals is



$$W(\theta') = \frac{1}{2}\int B(\theta)A_e(\theta - \theta')d\theta$$

1 polarization

pointing direction of the telescope

And in two dimensions

$$W(\theta', \phi') = \frac{1}{2}\int B(\theta, \phi)A_e(\theta - \theta', \phi - \phi')sin(\theta)d\theta d\phi$$

In temperature units, this become

$$\boxed{T_A(\theta', \phi') = \frac{A_e^{max}}{\lambda^2}\int T_B(\theta, \phi)P(\theta - \theta', \phi - \phi')sin(\theta)d\theta d\phi}$$

Antenna temperature

weighted average of the sky temperature. The weight is the antenna power pattern.

Antenna fundamentals

$$\boxed{T_A(\theta', \phi') = \frac{A_e^{\max}}{\lambda^2} \int T_B(\theta, \phi) P(\theta - \theta', \phi - \phi') sin(\theta) d\theta d\phi}$$

Antenna temperature

Antenna temperature is not the physical temperature of the antenna; it is the temperature of a matched resistor whose thermally generated power per unit frequency (in the Nyquist approximation $P_\nu = kT_A$, the equivalent for electrical power of Rayleigh-Jeans) equals that produced by the antenna

If P is a single infinitely sharp spike, then $T_A(\theta', \phi') \simeq T_B(\theta, \phi)$

For a real telescope P has a finite width: T_A is therefore a smoothed version of T_B

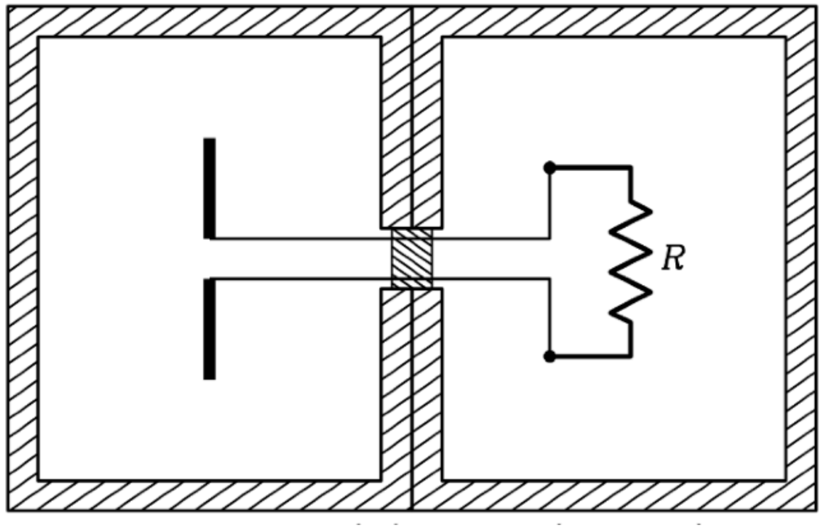
This implies that an increase in T_A could mean either that there is a source in the main beam, or that a collection of fainter sources have combined to give a large total power.

In the case of point sources (e.g. stars), the confusion limit is ~Θ_{HPBW}. Below this limit, one cannot be sure that a T_A increase corresponds to a single source.

Antenna fundamentals

Consider an antenna coupled with a resistor, with the entire system being placed in a black box at temperature T. After TE has been reached, the power flowing from the resistor to the antenna is:

$$P_{R\rightarrow A} = kT$$



The power flow from the antenna to the resistor is:

$$P_{A\rightarrow R} = \frac{A_e^{\max}kT}{\lambda^2}\int P(\theta,\phi)sin(\theta)d\theta d\phi$$ as $T(\theta,\phi) = T$

In thermal equilibrium the net power flow has to be zero

$$P_{R\rightarrow A} = P_{A\rightarrow R}$$

From which

$$\boxed{A_e^{\max} = \frac{\lambda^2}{\int P(\theta,\phi)sin(\theta)d\theta d\phi}}$$

The maximum effective area (at given λ) depends only on the power pattern

For a reflecting telescope $\int P(\theta,\phi)sin(\theta)d\theta d\phi \sim \Theta^2_{HPBW} \sim \frac{\lambda^2}{D^2}$ hence $\boxed{A_e^{\max} \sim D^2}$

<u>The effective area scales as the geometric area of the reflector</u>

Reflector antenna

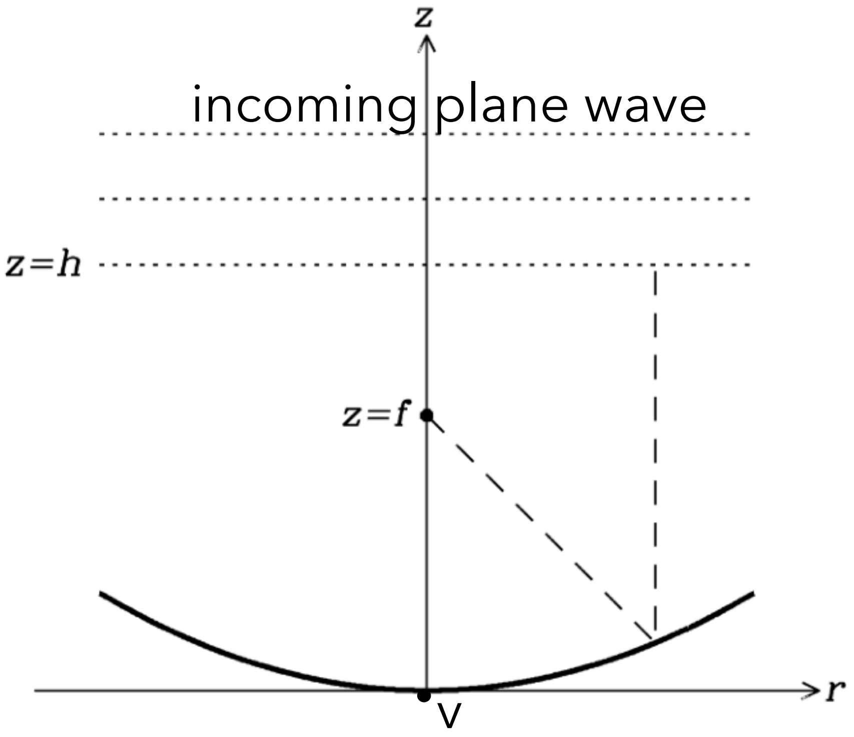
We have seen that, at relatively high frequency ($\lambda < 1$ mm) most radio telescopes use large reflectors to collect and focus power onto small feed antennas, that are connected to receivers. The most common reflector shape is a paraboloid of revolution: it can focus the plane wave from a distant point source onto a single focal point.

To do this, the reflector must keep all parts of an on-axis plane wavefront in phase at its focal point. Thus the total path lengths to the focus must all be the same:

$$f + h = (h - z) + \sqrt{r^2 + (f - z)^2}$$

$$\boxed{z = \frac{r^2}{4f}}$$

equation of a paraboloid



f : focal lent

v: vertex r=0, z=0

z: height with respect to v

Reflector antenna

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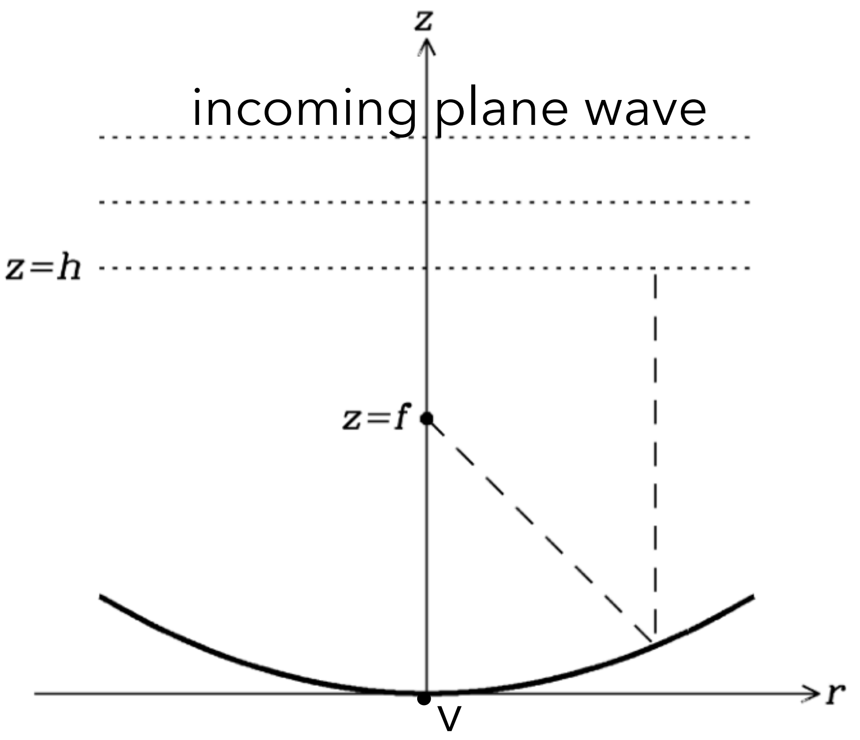


f : focal lent

v: vertex r=0, z=0

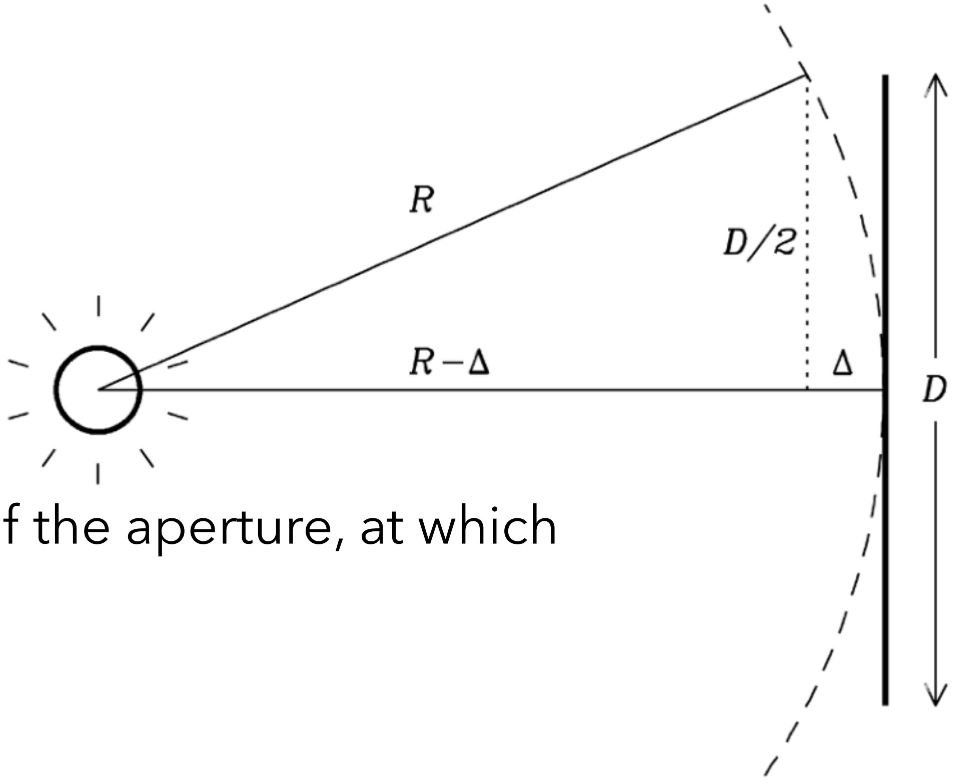
z: height with respect to v

This applies in the case of plane waves (distant source). Otherwise path-length errors will introduce significant phase errors in the waves coming from the off-axis portions of the reflector, reducing the effective collecting area and degrading the antenna pattern.

Antenna aperture

How far away must a point source be for the received waves to satisfy the assumption that they are planar across the reflector?

Consider a spherical wave emitted by a point source at finite distance R from a flat aperture of diameter D



The maximum departure Δ from a plane wave occurs at the edge of the aperture, at which

$$R^2 = (R - \Delta)^2 + (\frac{D}{2})^2$$

$$R = \frac{\Delta}{2} + \frac{D^2}{8\Delta} \Rightarrow R \sim \frac{D^2}{8\Delta}$$

(in the limit $\Delta << D$)

$$\boxed{R_{ff} \sim \frac{2D^2}{\lambda}}$$

(typically plane-wave if $\Delta < \lambda/16$)

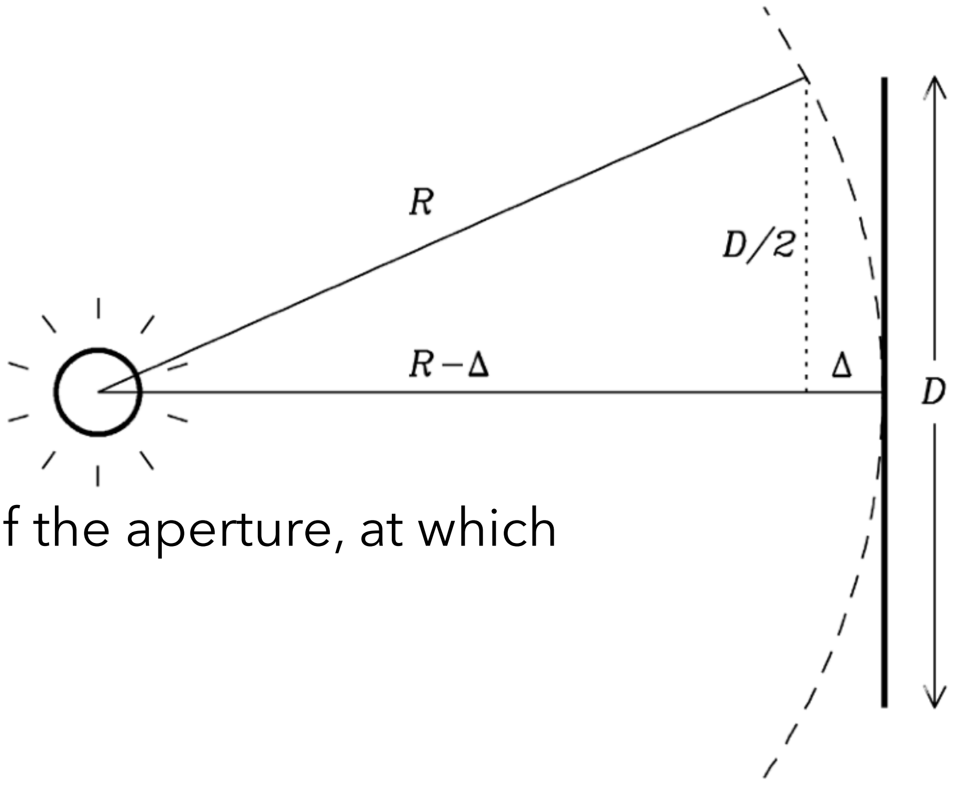
far-field distance

Arecibo: D~300 m, $\lambda \sim 21$ cm $\quad R_{ff} \sim 8600$ km

R>>R_{ff} for astronomical sources

Antenna aperture

How far away must a point source be for the received waves to satisfy the assumption that they are planar across the reflector?



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$$R^2 = (R-\Delta)^2 + (\frac{D}{2})^2$$

$$R = \frac{\Delta}{2} + \frac{D^2}{8\Delta} \quad \Longrightarrow \quad R \sim \frac{D^2}{8\Delta}$$

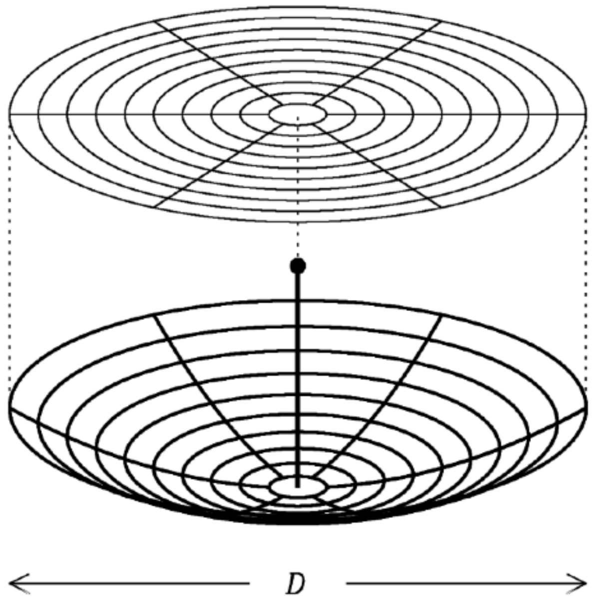
(in the limit $\Delta << D$)

$$\boxed{R_{ff} \sim \frac{2D^2}{\lambda}}$$

(typically plane-wave if $\Delta < \lambda/16$)

far-field distance

The aperture of a paraboloidal reflector antenna is the plane circle, normal to the rays from a distant point source, that covers the paraboloid. The phase of the plane wave from a distant point source would be constant across the aperture plane when the aperture is perpendicular to the line of sight.

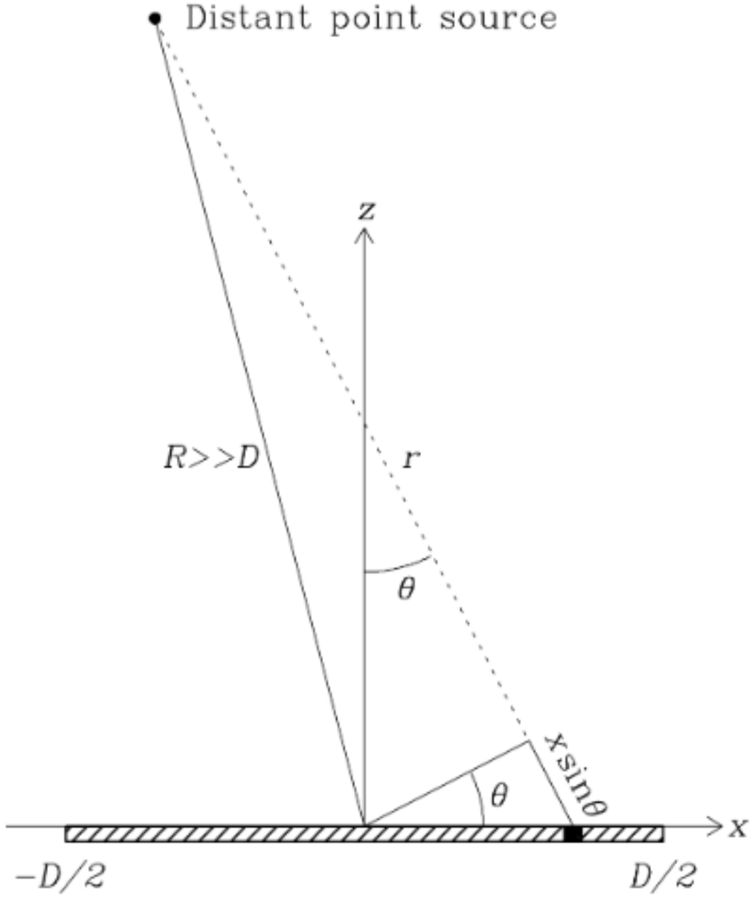


dish diameter

Patterns of Aperture Antennas

How to calculate the beam pattern, or power gain as a function of direction, of an antenna aperture?

Consider the case of a 1-dimensional aperture and for simplicity assume that the antenna is transmitting. We want to calculate the electric field pattern at a large distance R.



The antenna feed illuminates the antenna aperture with a sine wave. Illumination induces currents in the reflector. Currents vary with position and time.

$I \propto g(x) exp(-i\omega t)$

$\nu = \omega/(2\pi)$ wave frequency

$g(x)$ electric field strength

R source distance

D aperture size

x distance from aperture center

Patterns of Aperture Antennas

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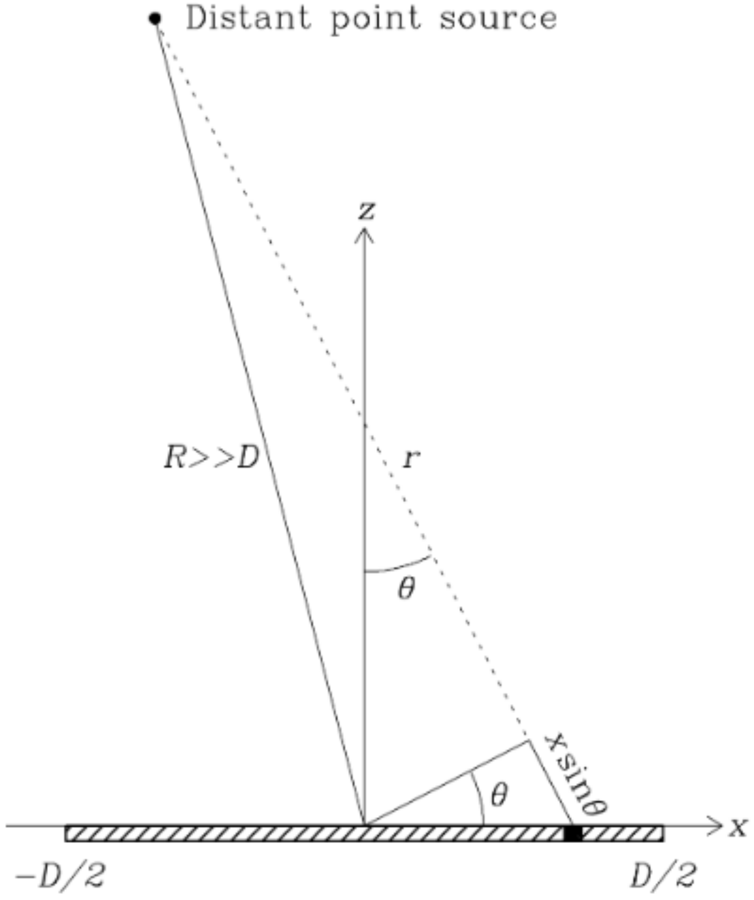
Consider the case of a 1-dimensional aperture and for simplicity assume that the antenna is transmitting. We want to calculate the electric field pattern at a large distance R.



R source distance

D aperture size

x distance from aperture center

The antenna feed illuminates the antenna aperture with a sine wave. Illumination induces currents in the reflector. Currents vary with position and time.

$I \propto g(x) exp(-i\omega t)$

$\nu = \omega/(2\pi)$ wave frequency

$g(x)$ electric field strength

Huygens's principle: the aperture is an ensemble of small elements individually acting as small antennas. The electric field produced by the whole aperture at large distances is just the vector sum of the elemental electric fields from these small antennas.

$$df \propto g(x)\frac{exp(-i2\pi r(x)/\lambda)}{r(x)}$$

electric field strength

r(x) distance between the source and aperture element at position x

Patterns of Aperture Antennas

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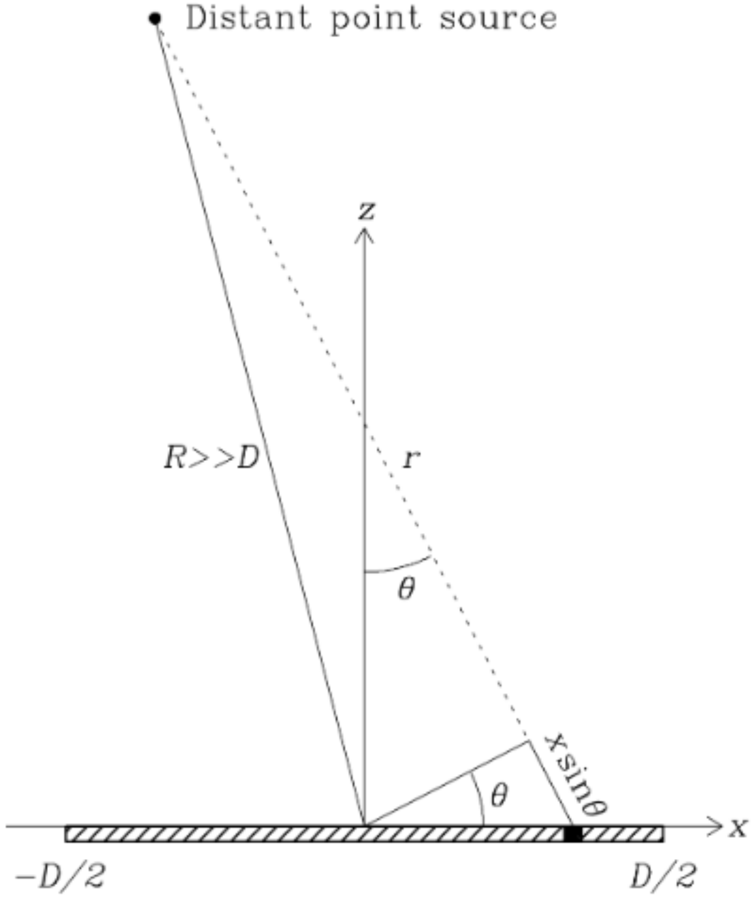

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As R>>R$_{ff}$ the plane wave approximation is valid and $r \sim R + xsin\theta$ usually written as $r \sim R + xl$

$(l = sin\theta)$

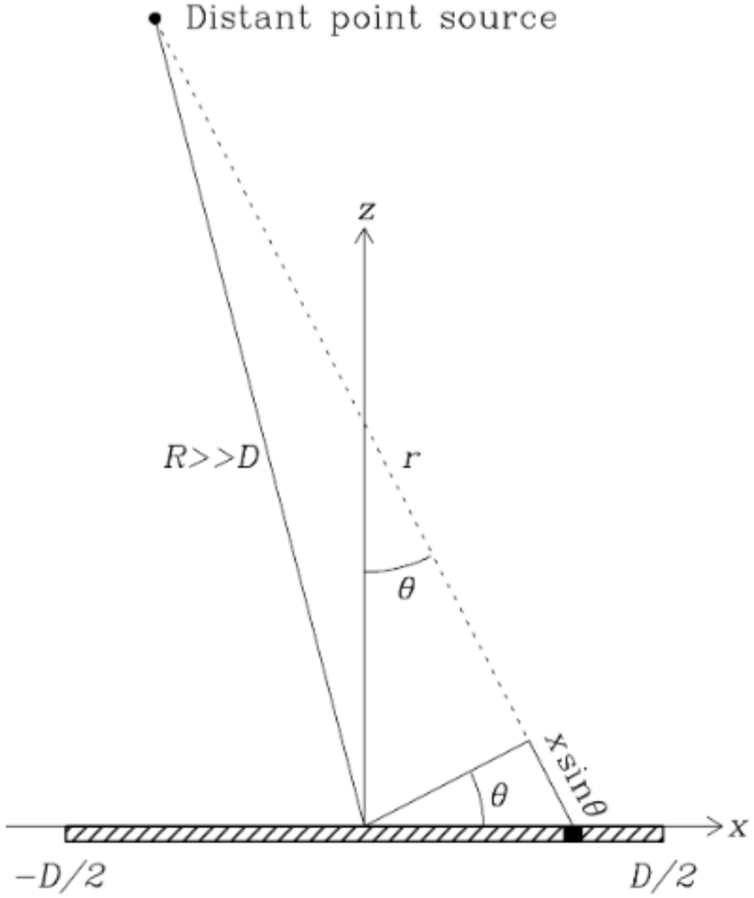
Patterns of Aperture Antennas

$$df \propto g(x)\frac{exp(-i2\pi(\overset{\text{constant}}{\ddot{R}}+xl)/\lambda)}{R}$$

$$\frac{1}{r} \sim \frac{1}{R}$$

$$\boxed{df \propto g(x)\frac{exp(-i2\pi xl)/\lambda)}{R}}$$

The phase $2\pi xl/\lambda$varies linearly across the aperture. Different parts of the aperture add constructively or destructively to the total electric field.



R source distance

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x distance from aperture center

Defining the position along the aperture in units of wavelength $u = x/\lambda$

$$\boxed{f(l) = \int_{\text{aperture}} g(u)e^{-i2\pi lu}du}$$

In the far field, **the electric-field pattern of an aperture antenna is the Fourier transform of the electric field distribution illuminating that aperture**.

Patterns of a uniformly illuminated antenna

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Uniform illumination: $g(u) = \text{constant}$ $\frac{-D}{2\lambda} < u < \frac{D}{2\lambda}$

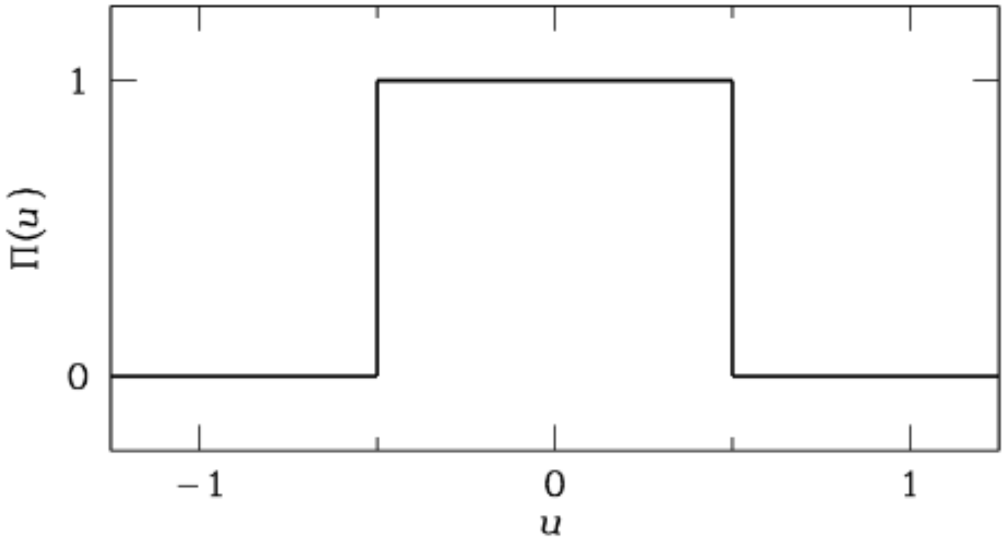
Unit aperture ($D = \lambda$)

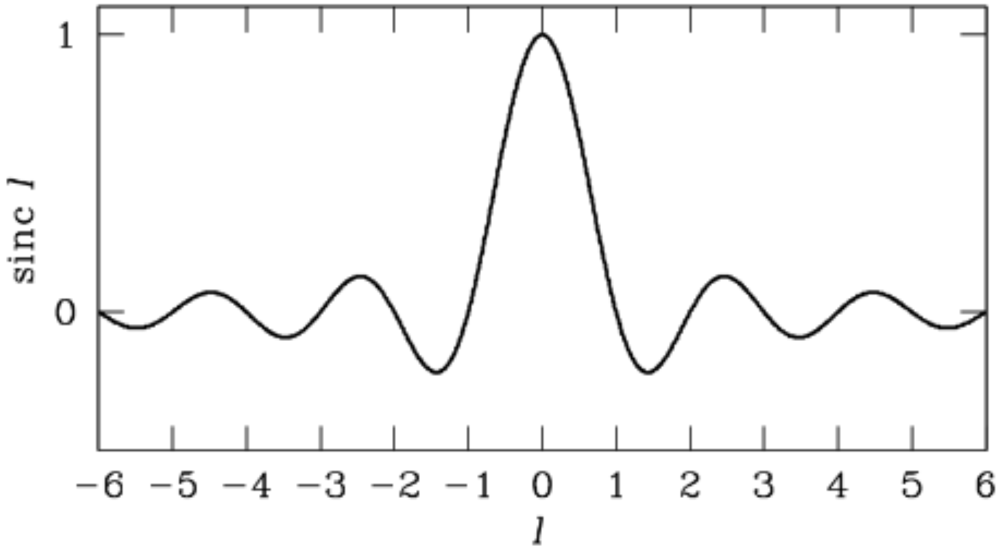
Unit rectangle function: $\Pi(u) = 1$ $-1/2 < u < 1/2$

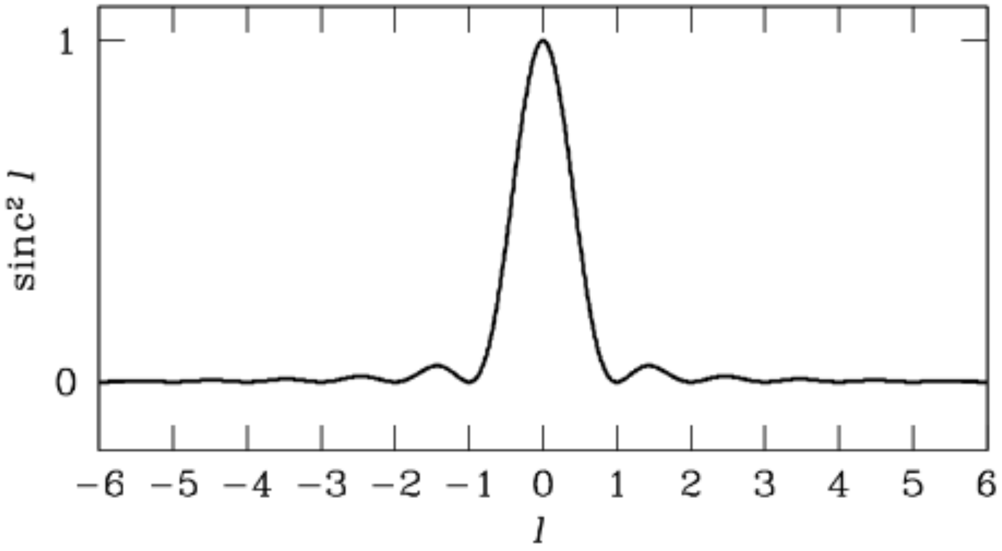
$$\boxed{f(l)} = \int_{-1/2}^{1/2} \Pi(u)e^{-i2\pi lu}du = \int_{-1/2}^{1/2} e^{-i2\pi lu}du$$

$$= \frac{e^{-i\pi l} - e^{i\pi l}}{-i2\pi l} = \frac{sin(\pi l)}{\pi l} = \boxed{sinc(l)}$$

electric-field pattern of a uniformly illuminated antenna







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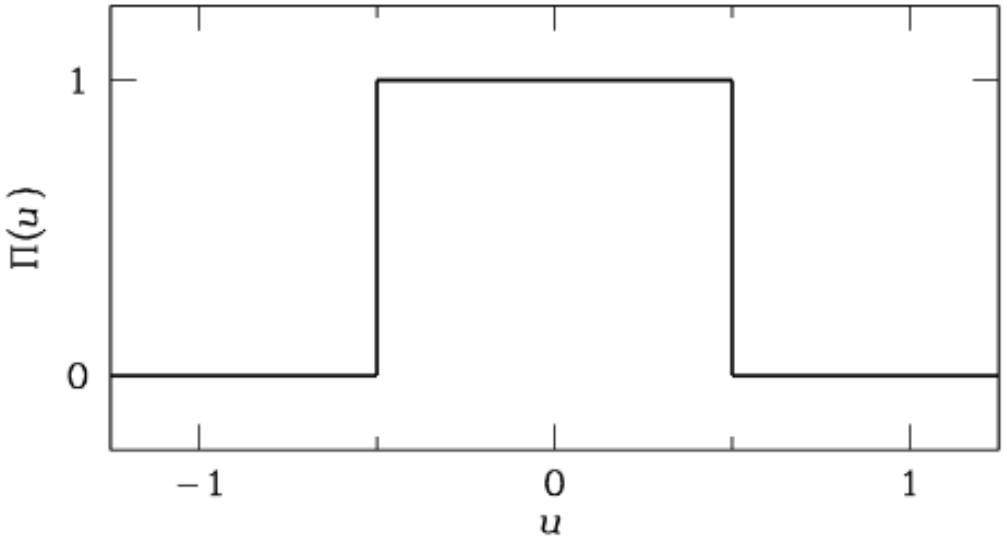

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$$\frac{-D}{2\lambda} < u < \frac{D}{2\lambda}$$

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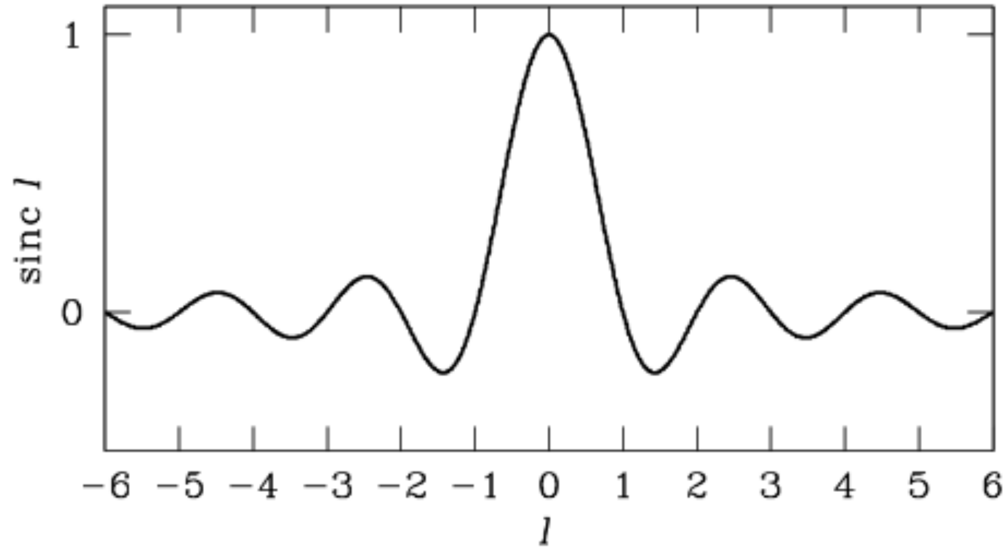
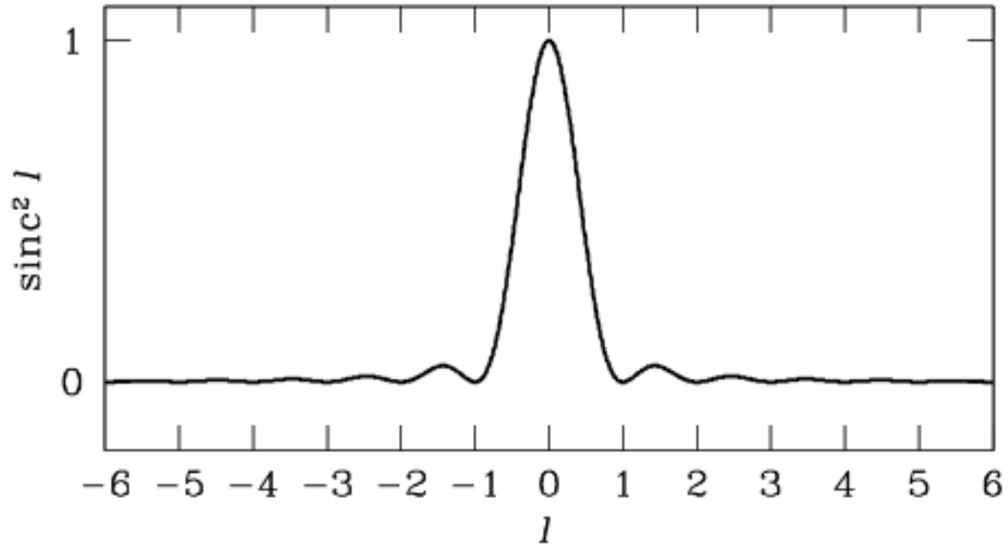


The power pattern is the square of the field pattern

$$\boxed{P(l) = sinc^2(l)}$$

power pattern of a uniformly illuminated antenna



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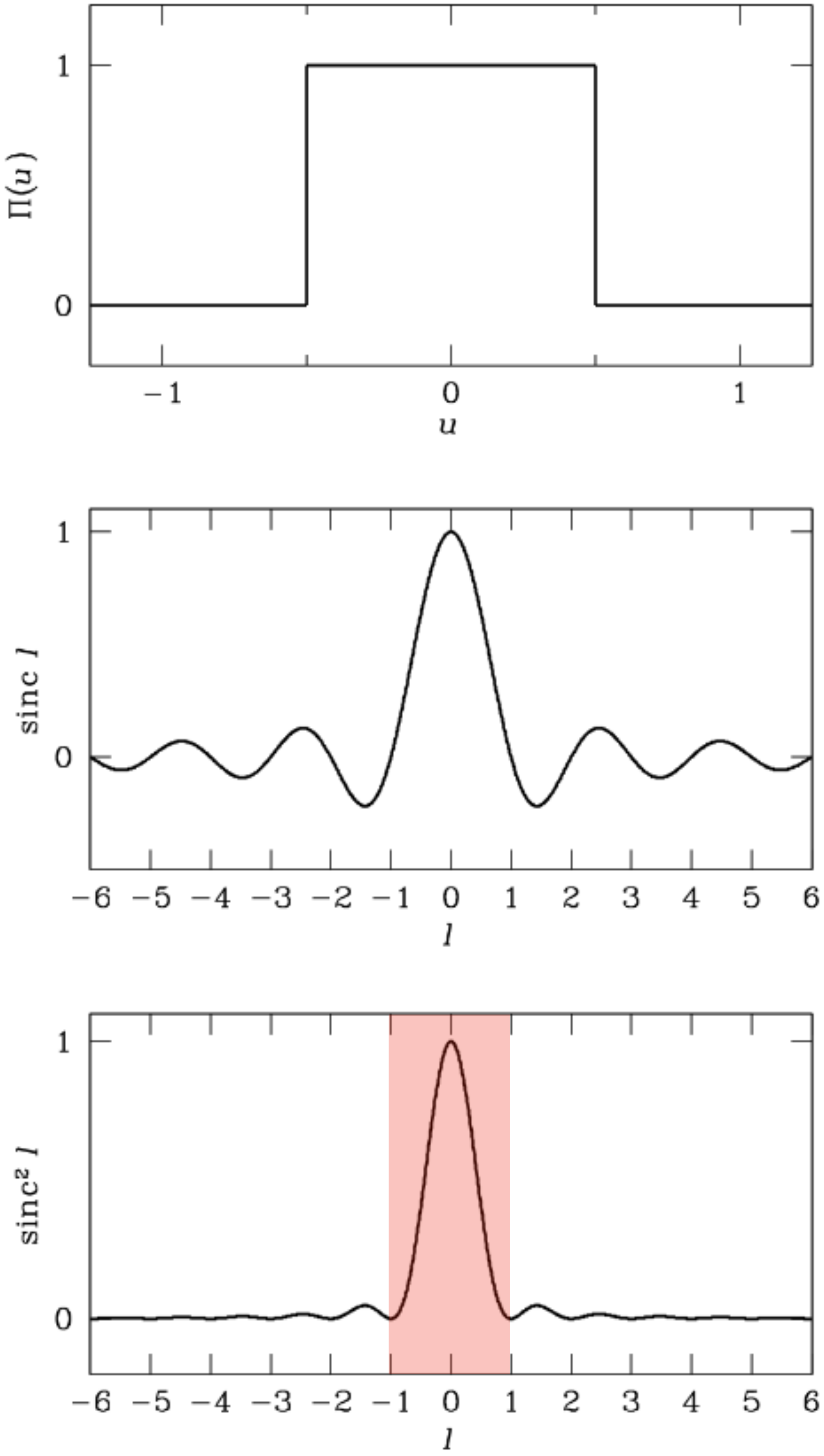
power pattern of a uniformly illuminated antenna

Main beam: peak of the power pattern between the first nulls ($l = \pm 1$)



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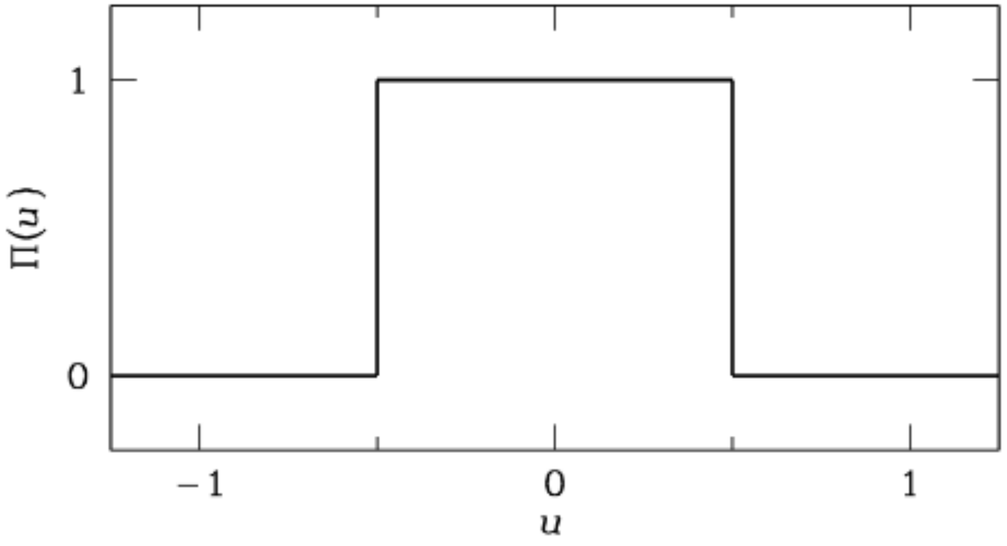

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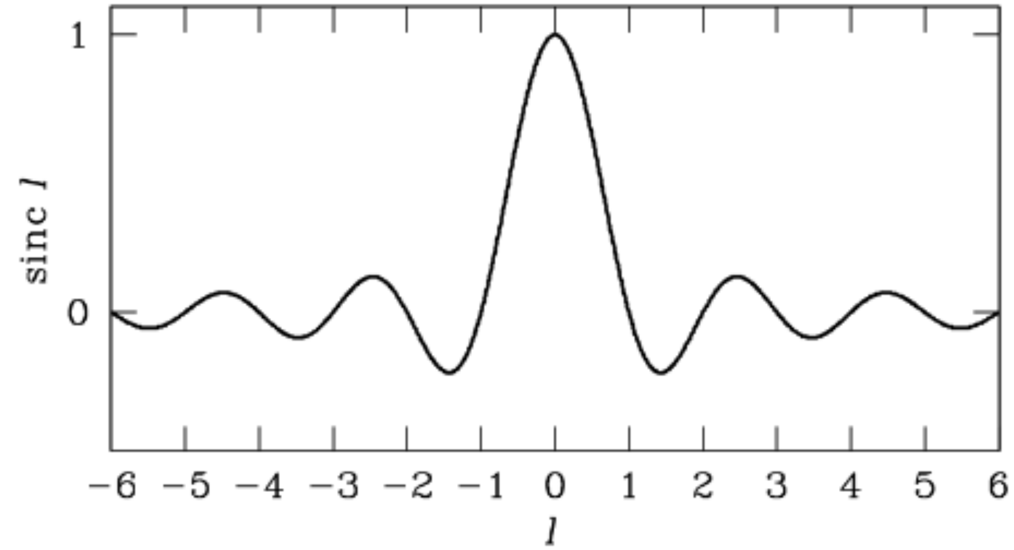

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power pattern of a uniformly illuminated antenna

Sidelobes: smaller peaks separated by nulls ($l = \pm 2, \pm 3,...$)