Interferometric images of radio sources

In the previous lessons, we have seen that the correlator response of a radio interferometer is linked to the complex visibility, which is a function of the baseline coordinates and is the 2D Fourier transform of the source brightness as a function of the sky coordinates

$$R = |V| cos(2\pi\nu\tau_g - \phi)$$

where $|V| = \sqrt{R_c^2 + R_s^2}$ is the visibility amplitude

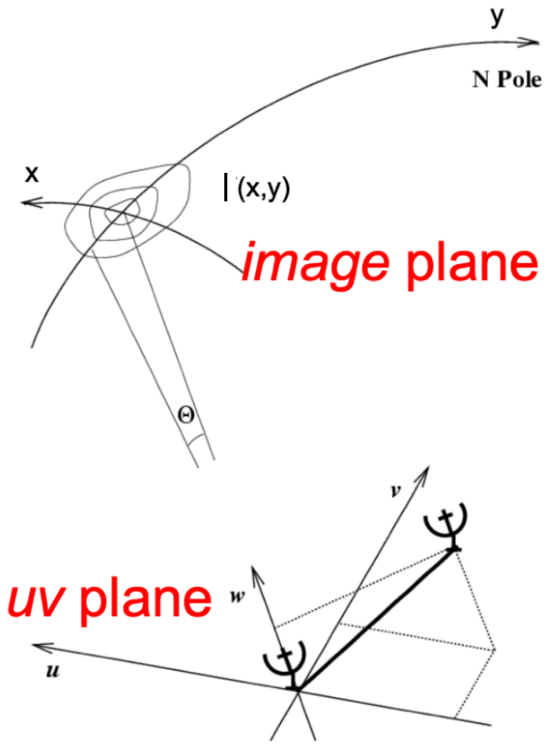
$\phi = tan^{-1}(R_s/R_c)$ is the visibility phase

$$V = (u, v) = \iint A(x, y) I_\nu(x, y) e^{-2\pi i(ux+vy)} dxdy$$

$$I = (x, y) = \iint A(u, v) I_\nu(u, v) e^{2\pi i(ux+vy)} dudv$$

(quasi-monochromatic interferometer)

$$V(u, v) \leftrightarrow I(x, y)$$



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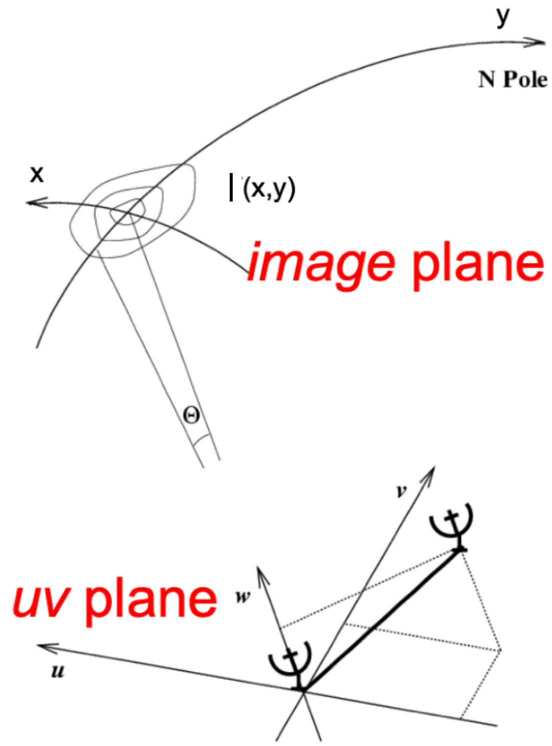
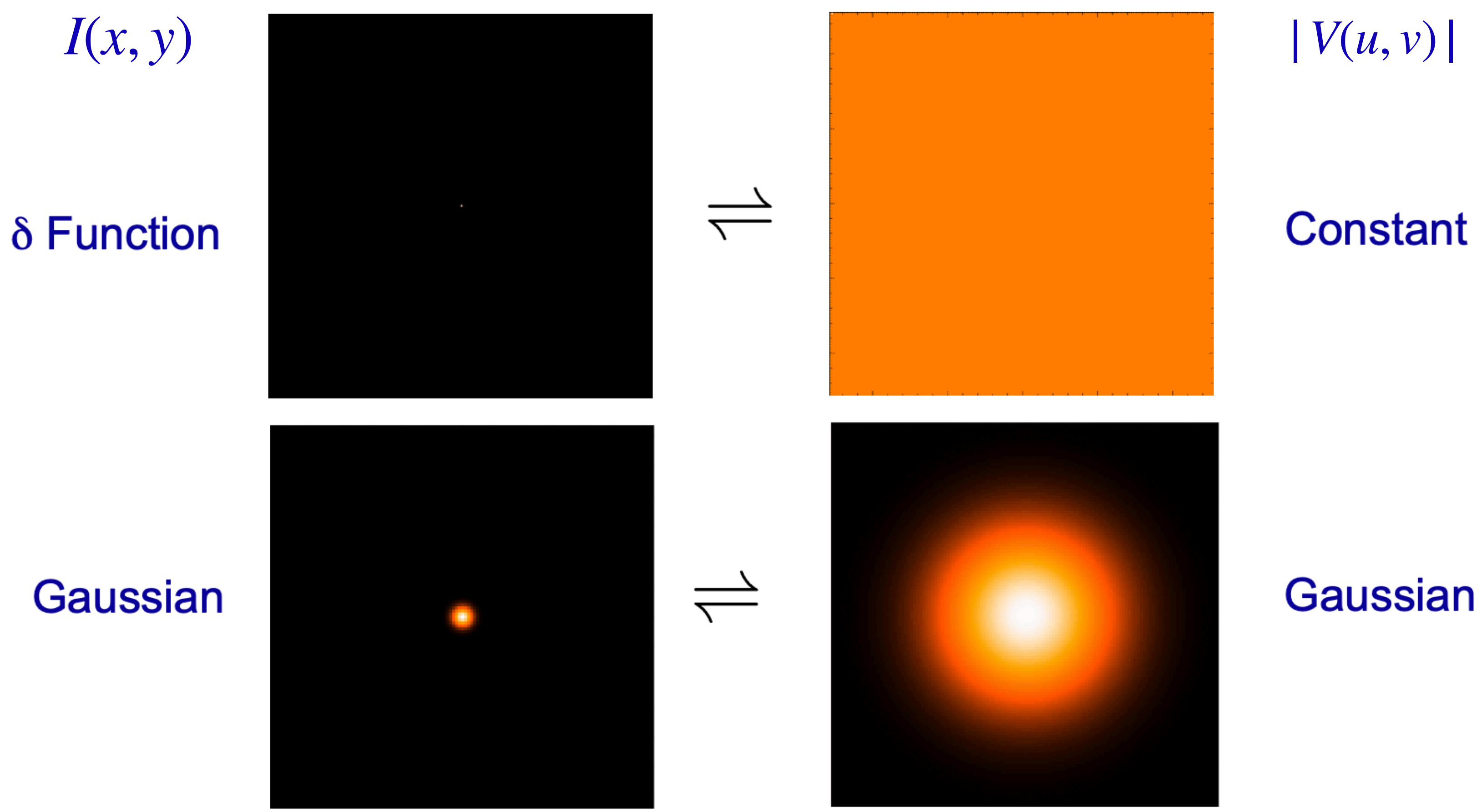


Interferometric images of radio sources: Fourier transform

Examples of 2D Fourier transform



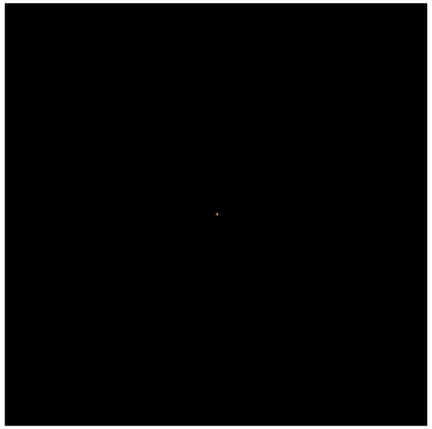
Amplitude tells us “how much” of a certain spatial frequency

Interferometric images of radio sources: Fourier transform

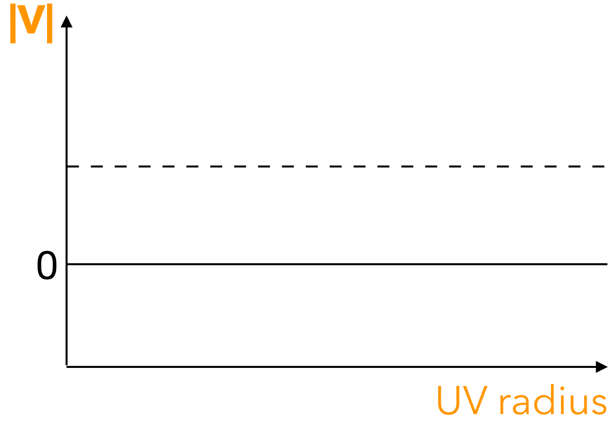
Examples of 2D Fourier transform

$I(x, y)$

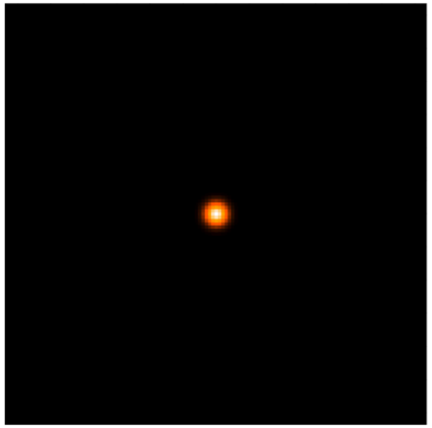
δ Function



$\rightleftharpoons$



Gaussian



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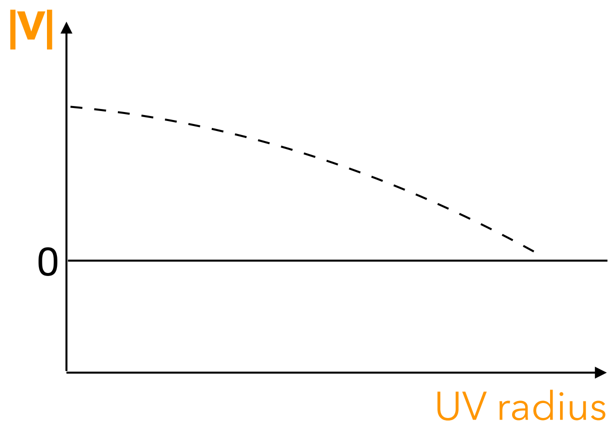

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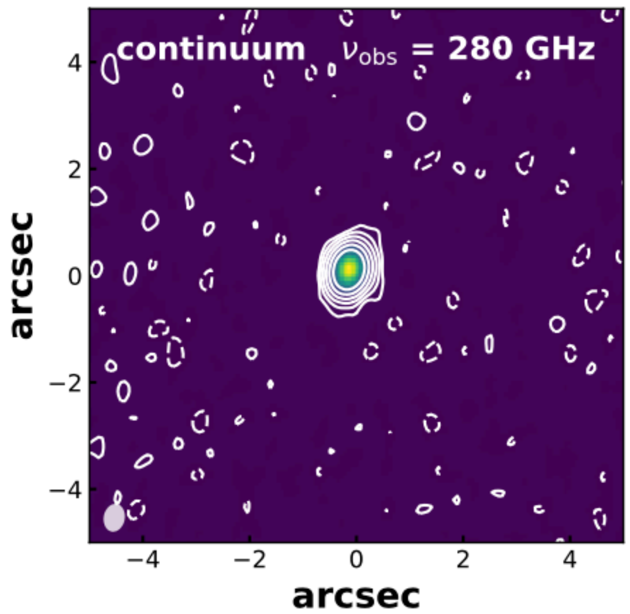
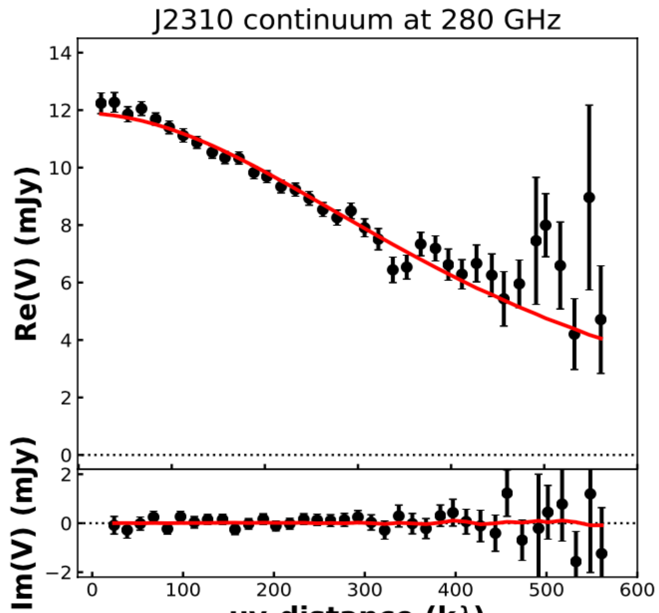
Interferometric images of radio sources: Fourier transform

Examples of 2D Fourier transform

$$I(x, y) \sim \text{gaussian}$$



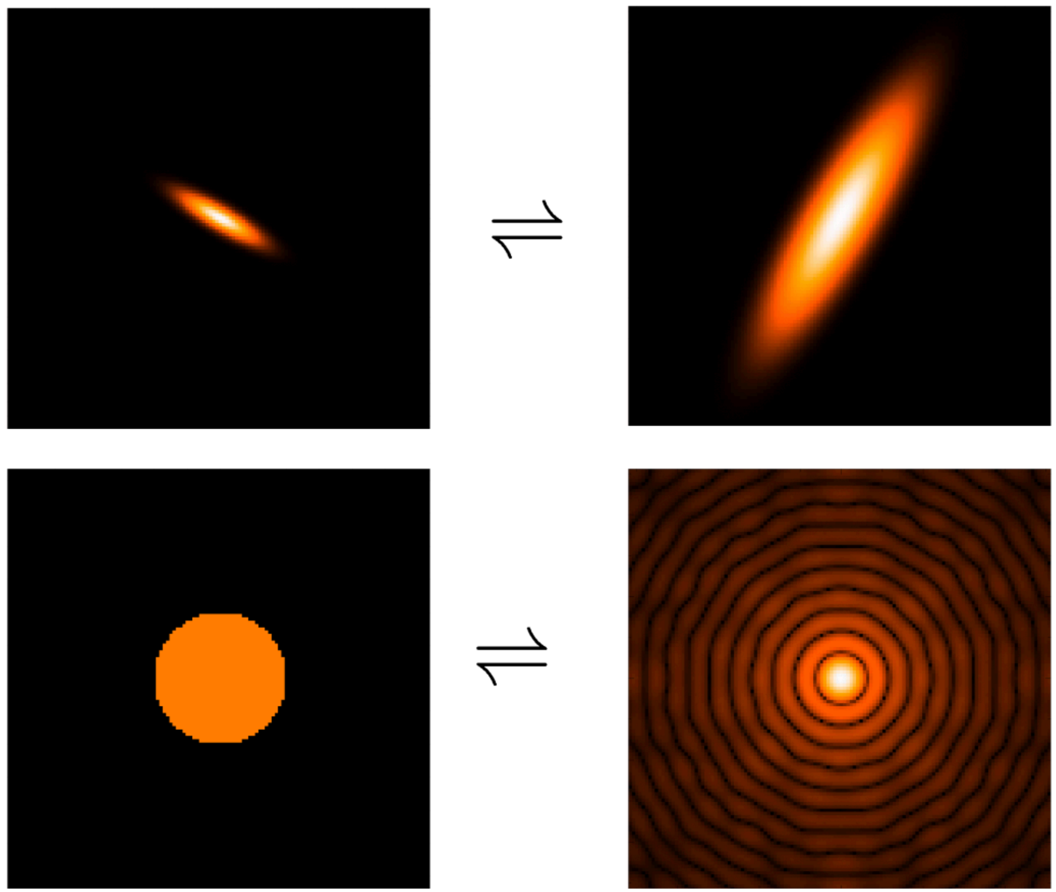


Example: Continuum emission for the host-galaxy of a quasar at z~6, as seen by ALMA with a synthesized beam of 0.4" x 0.5" (Carniani et al. 2019)

Amplitude tells us "how much" of a certain spatial frequency

Interferometric images of radio sources: Fourier transform

Examples of 2D Fourier transform

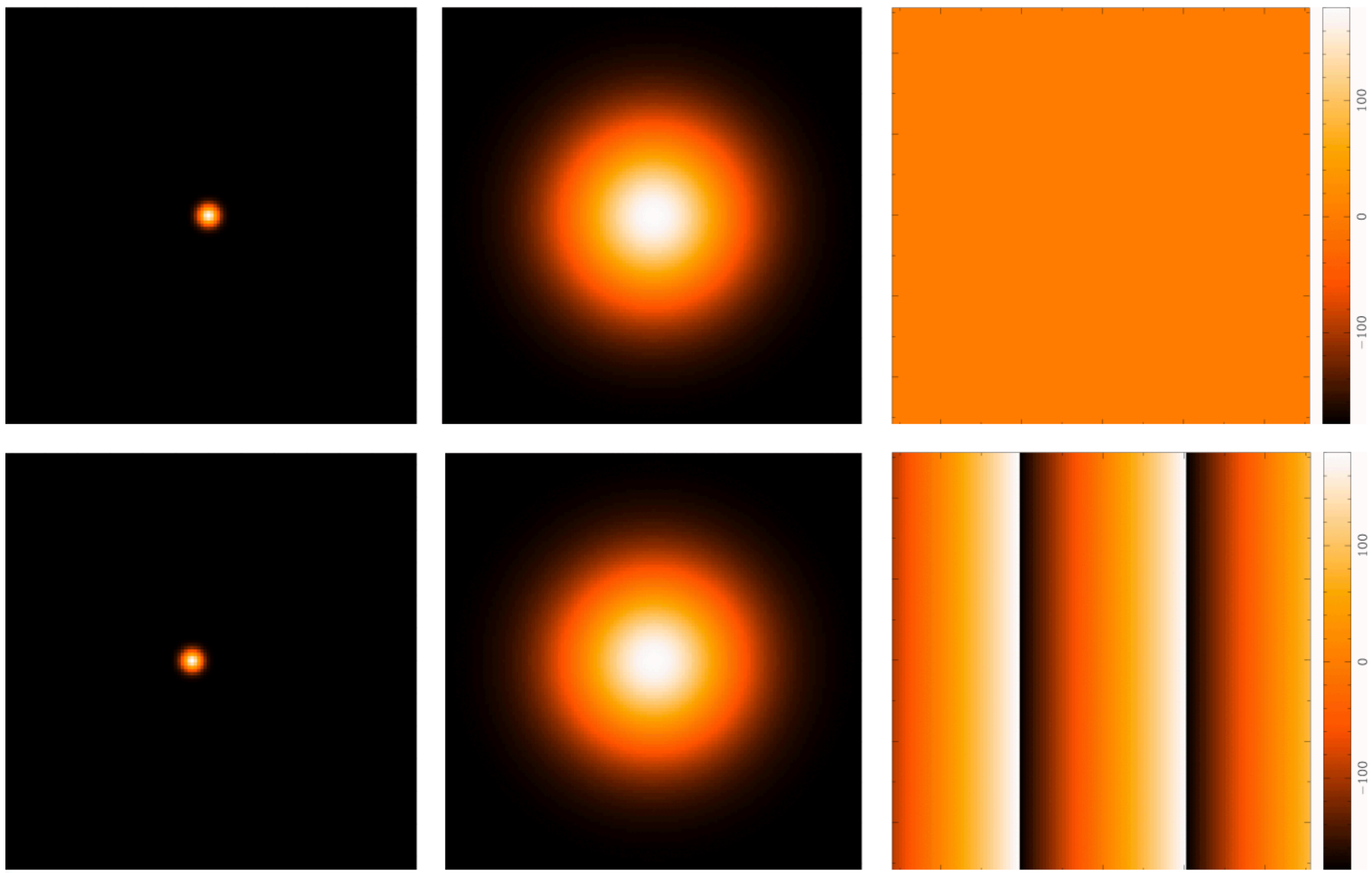


Amplitude tells us “how much” of a certain spatial frequency

Interferometric images of radio sources: Fourier transform

Examples of 2D Fourier transform

$I(x,y)$ $|V(u,v)|$ $\phi(u,v)$



Amplitude tells us "how much" of a certain spatial frequency

Phase tells "where" this spatial frequency component is located

Interferometric images of radio sources: Fourier transform

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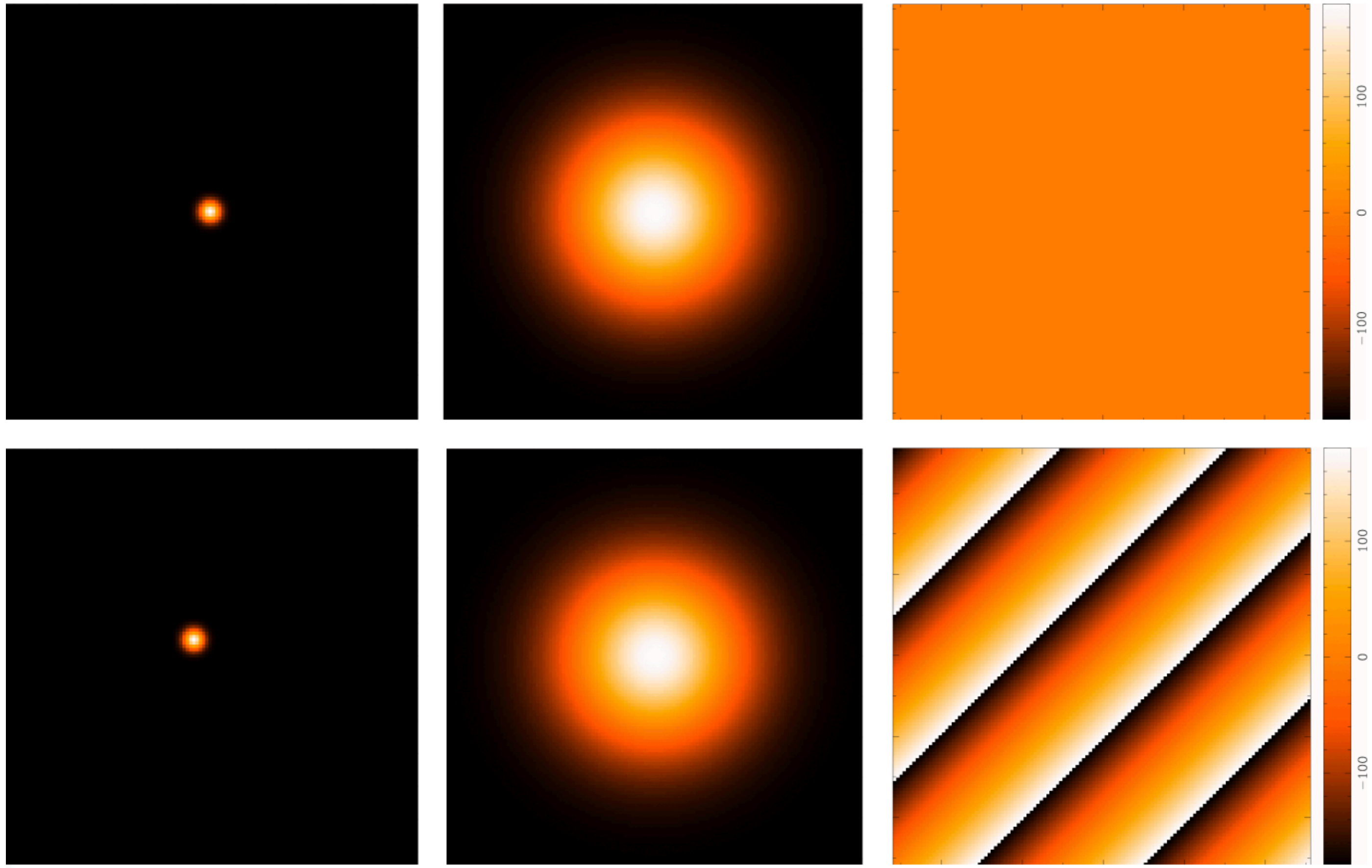
$$I(x,y) \qquad |V(u,v)| \qquad \phi(u,v)$$



Amplitude tells us "how much" of a certain spatial frequency

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Interferometric images and sampling of the (u,v) plane

We aim to sample $V(u, v)$ at enough (u, v) points using distributed (relatively small) antennas to synthesize the equivalent large aperture antenna of size $(u_{\max}, v_{\max})$

Use more antennas to increase sampling

- 1 pair of antennas → 1 (u, v) sampling at a time
- N telescopes → the number of samplings is N(N-1)/2 ("snapshot")

Use more antennas to increase sampling

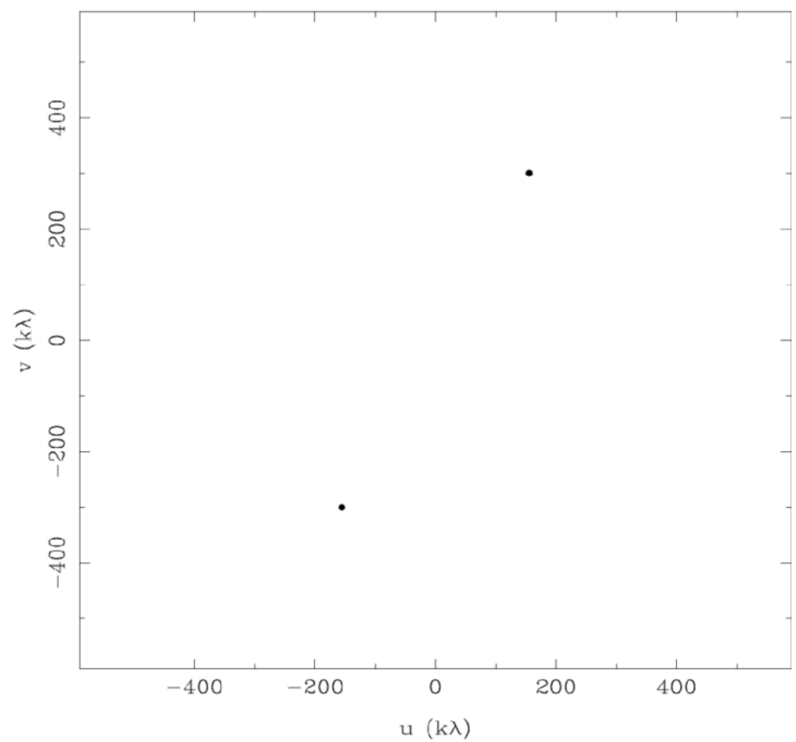
- fill in (u, v) plane by exploiting Earth rotation ("track")
- move antennas (observe with different antenna configurations)

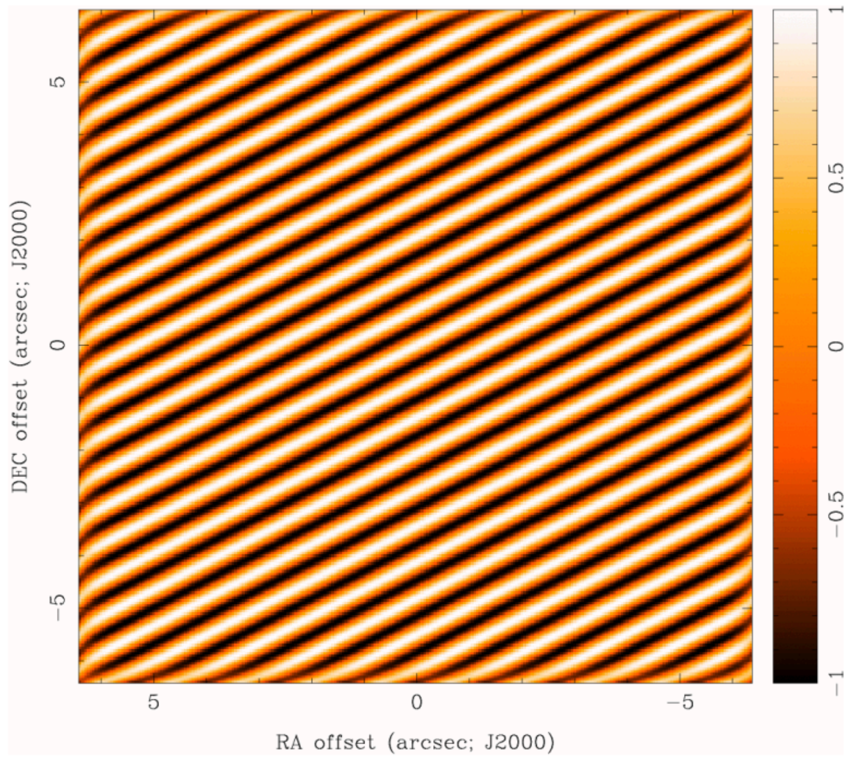
Use more frequencies to increase sampling

- need to determine how source structure changes with frequency
- "multi-frequency synthesis" for continuum imaging

Interferometric images and sampling of the (u,v) plane

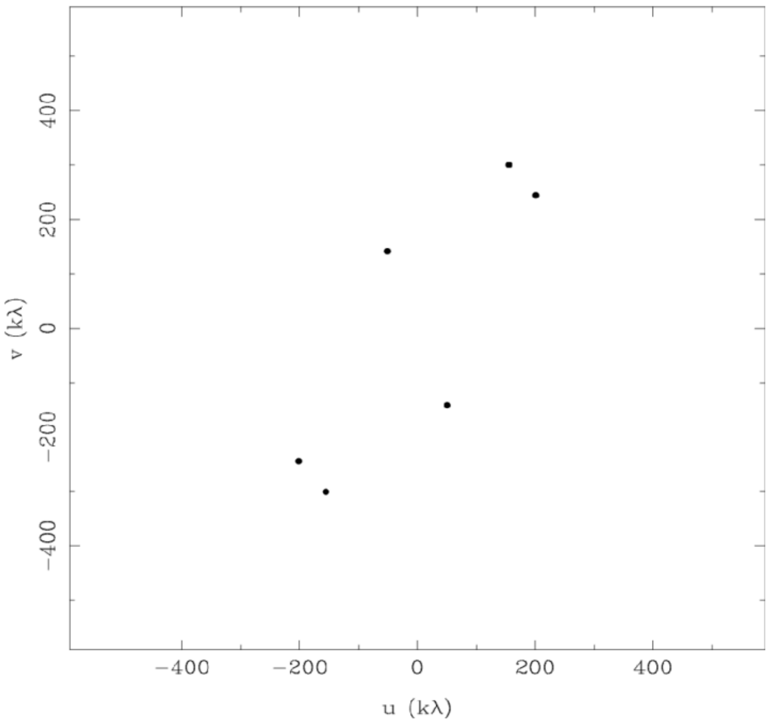
2 antennas

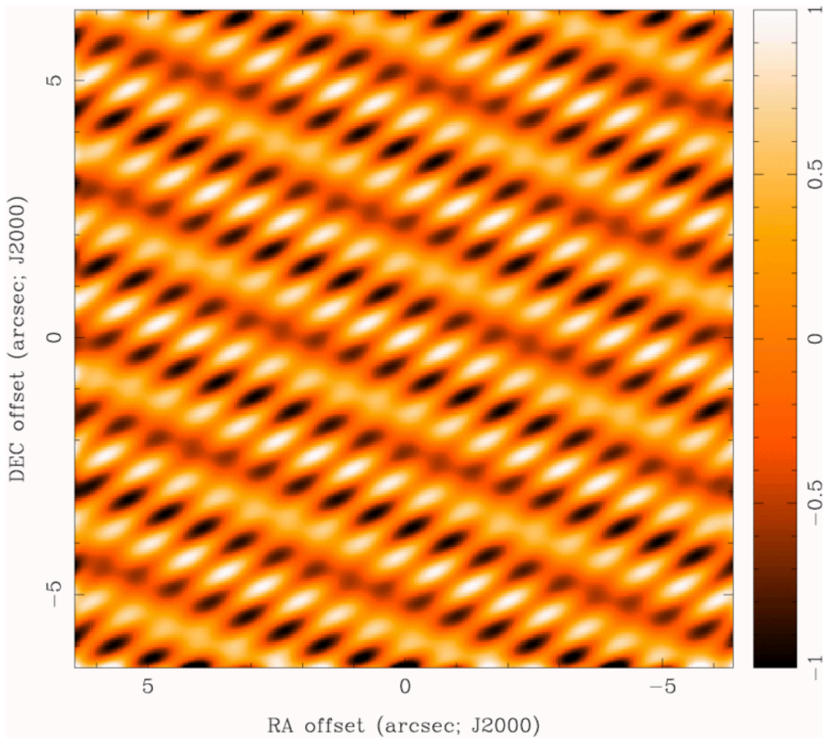




Interferometric images and sampling of the (u,v) plane

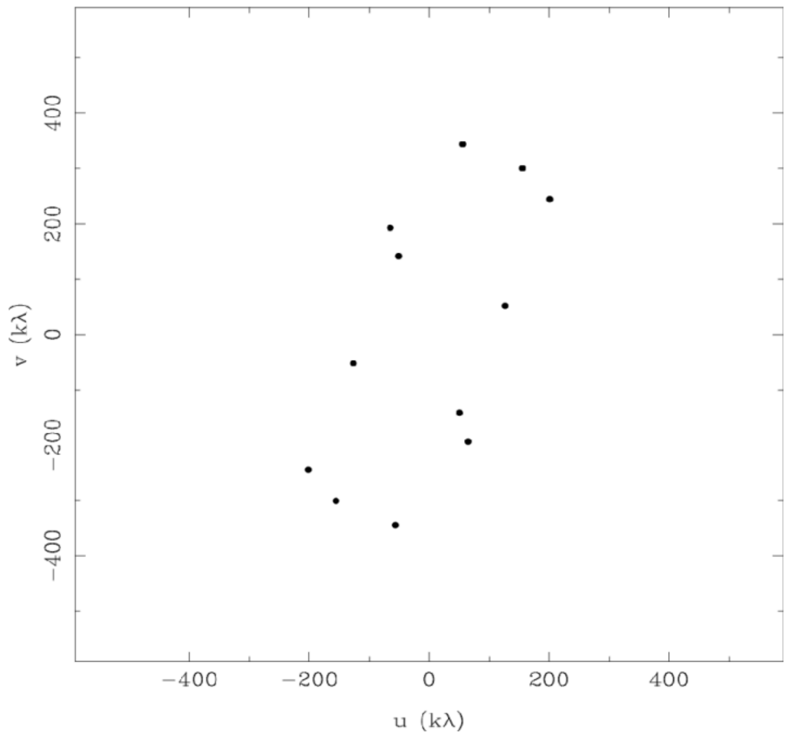
3 antennas

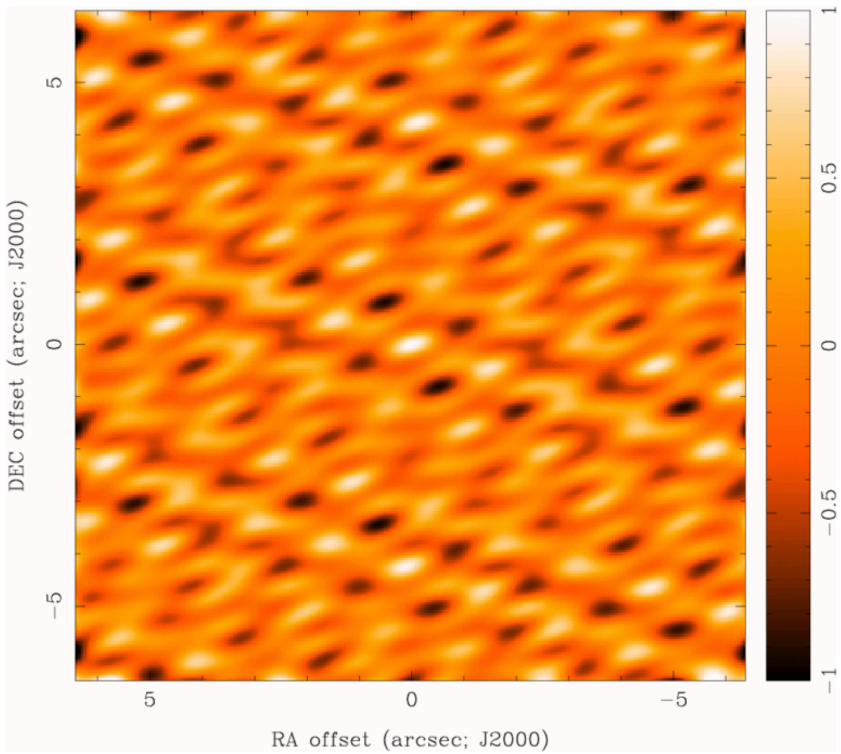




Interferometric images and sampling of the (u,v) plane

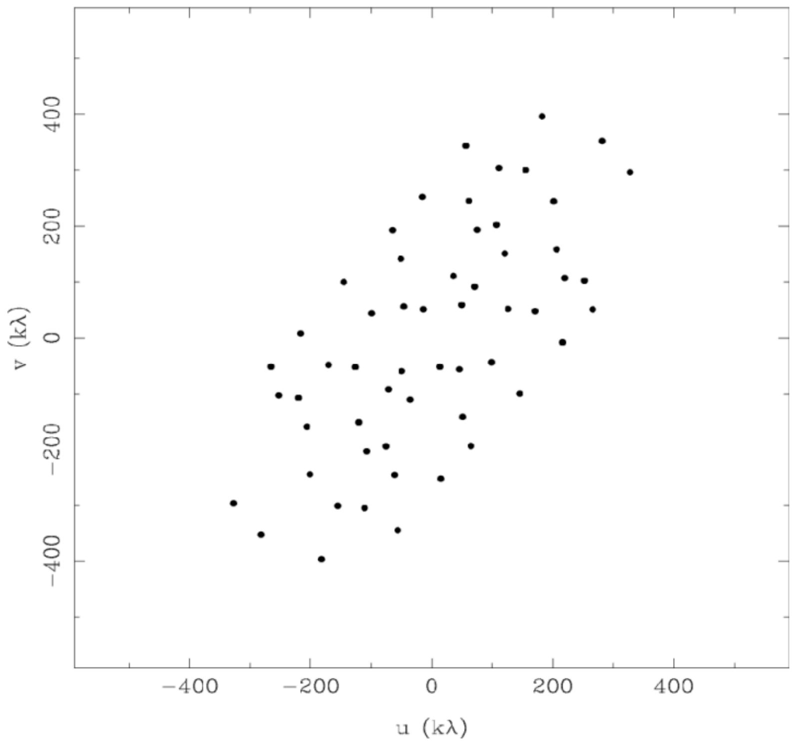
4 antennas

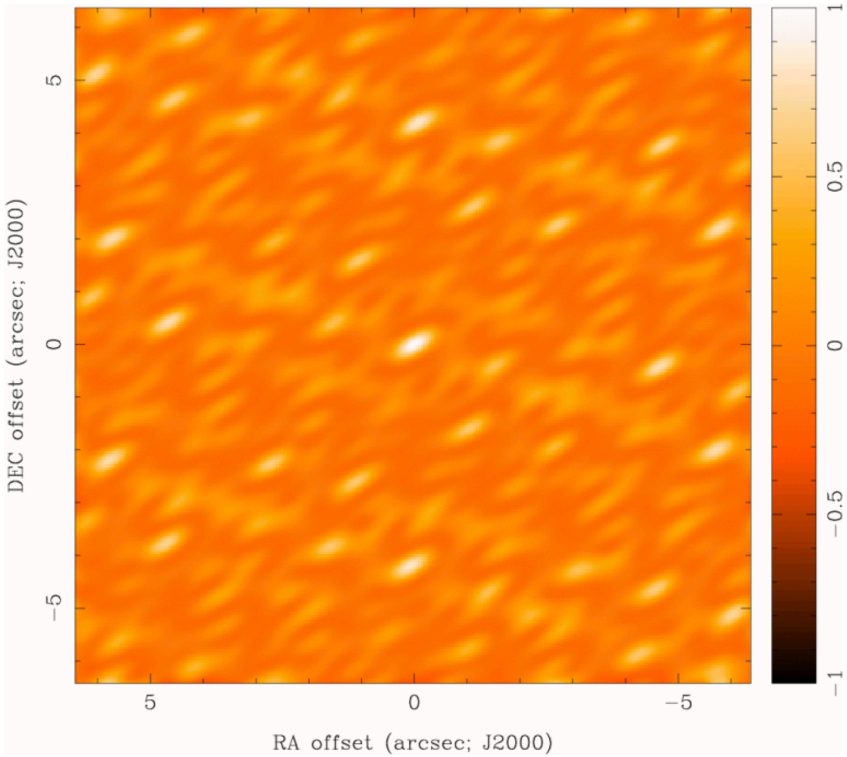




Interferometric images and sampling of the (u,v) plane

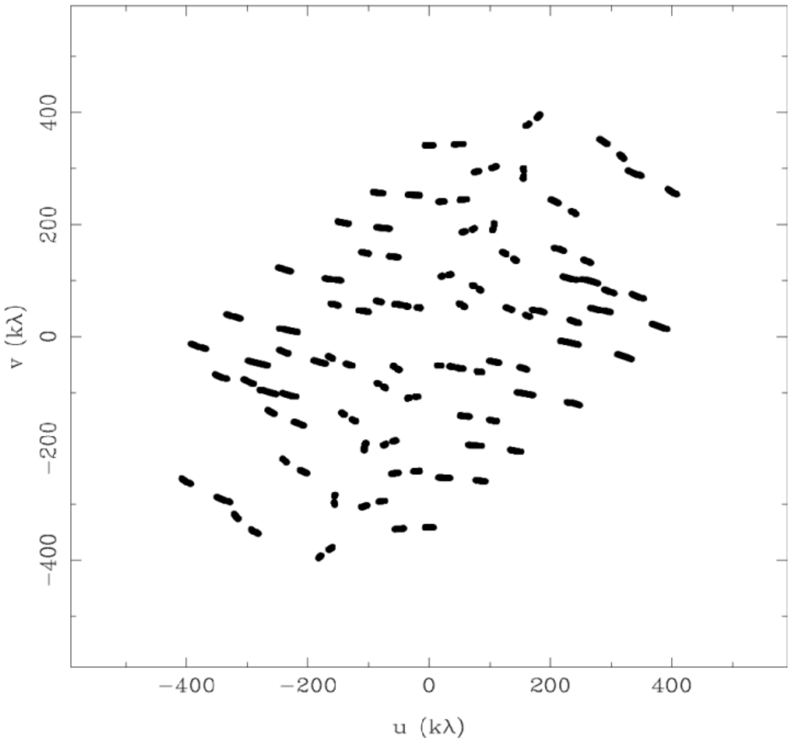
8 antennas

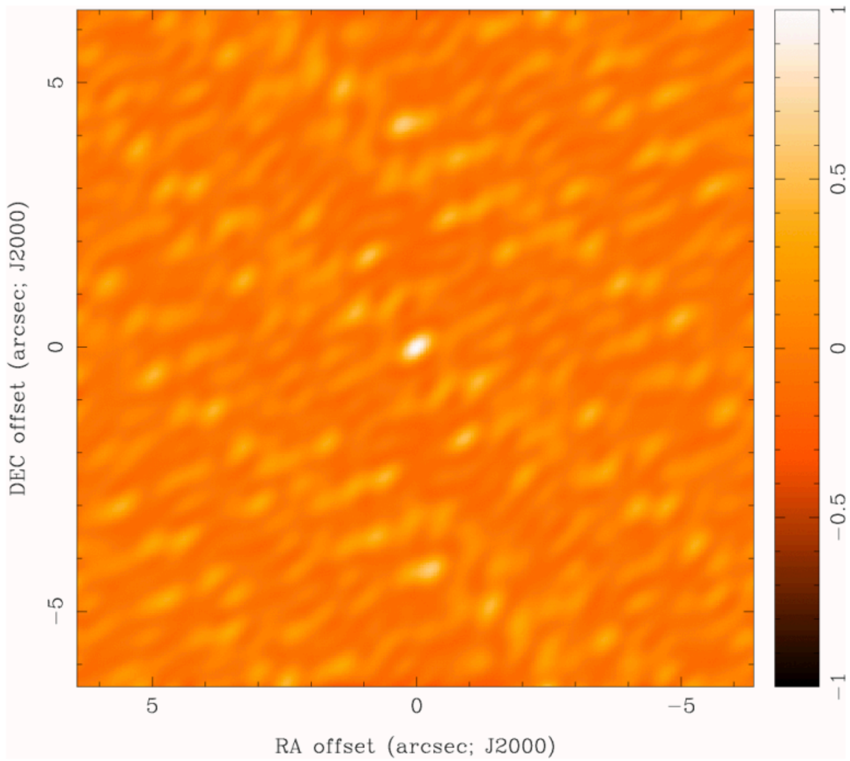




Interferometric images and sampling of the (u,v) plane

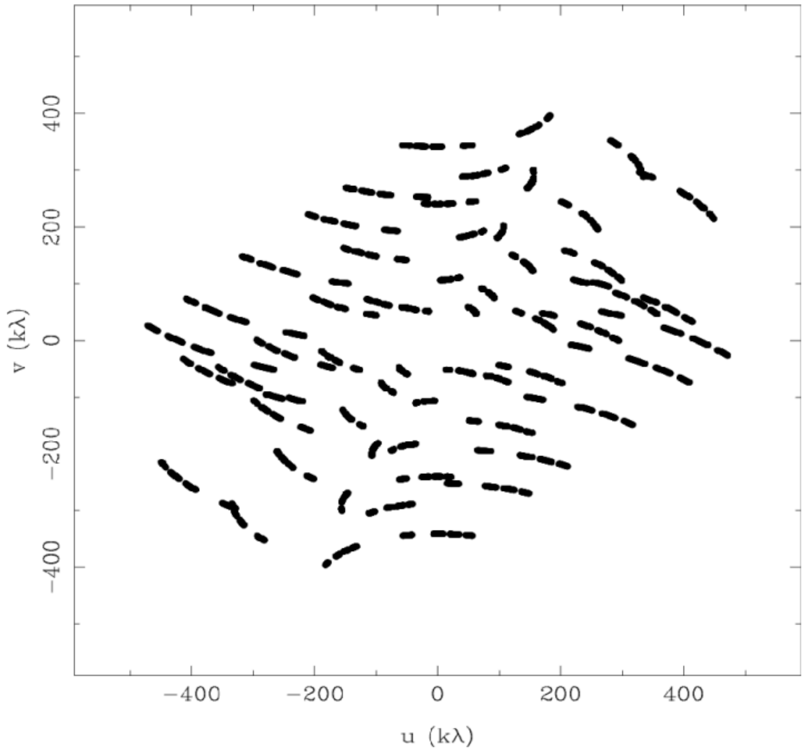
8 antennas x 60 samples

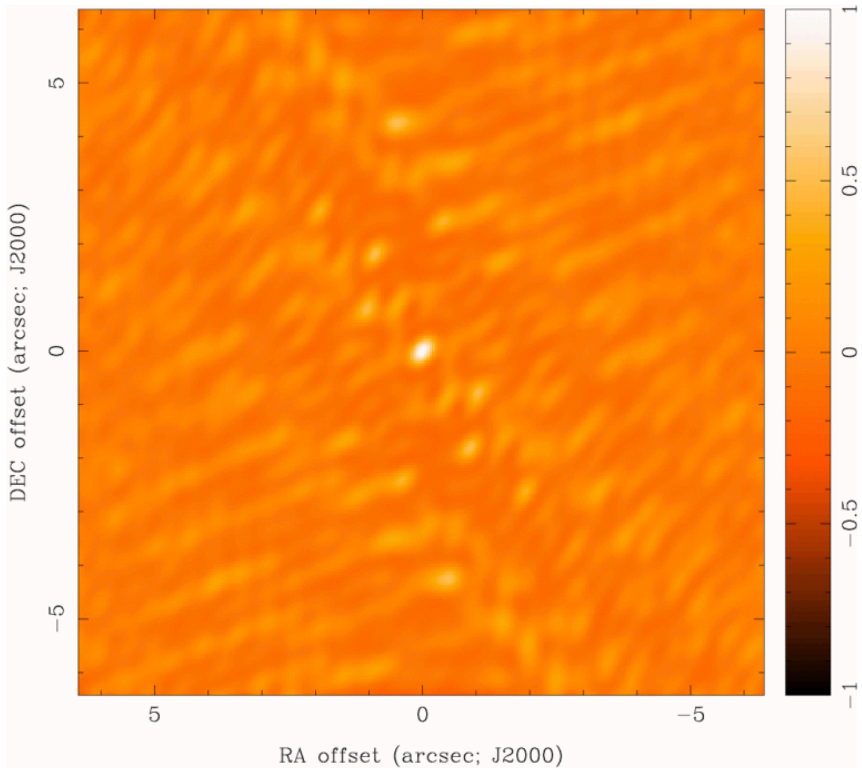




Interferometric images and sampling of the (u,v) plane

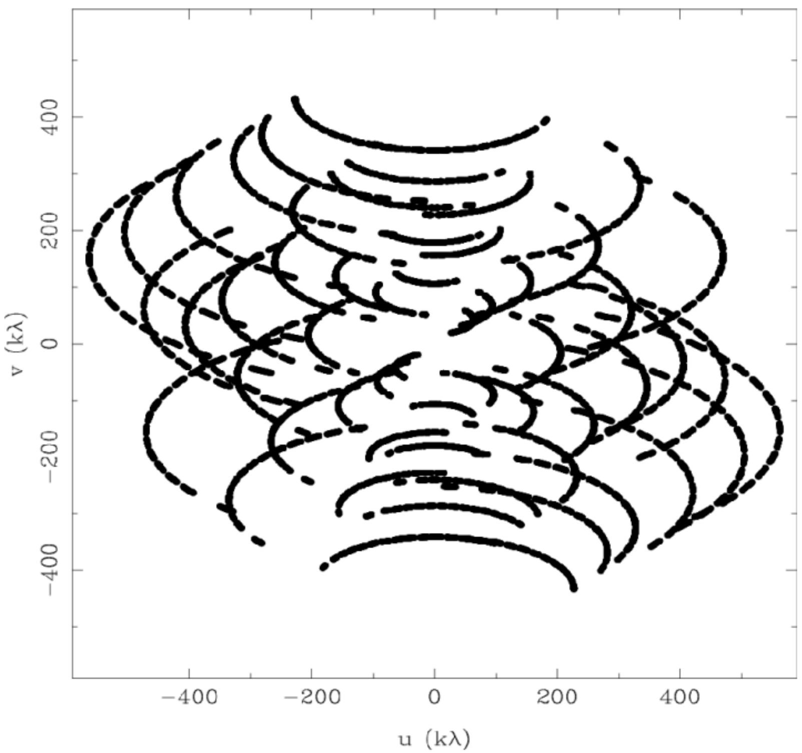
8 antennas x 120 samples

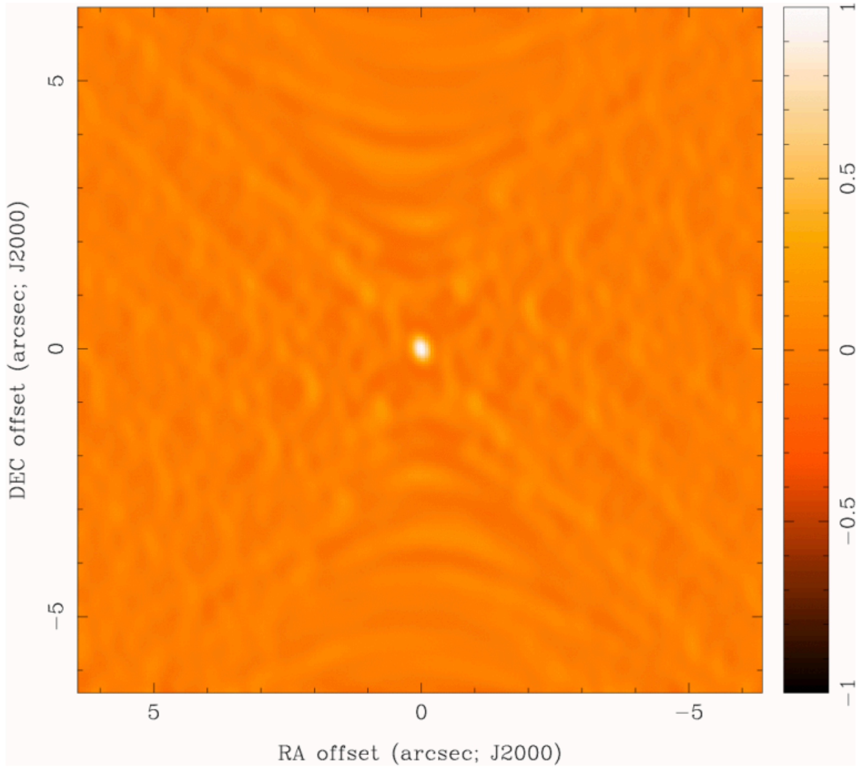




Interferometric images and sampling of the (u,v) plane

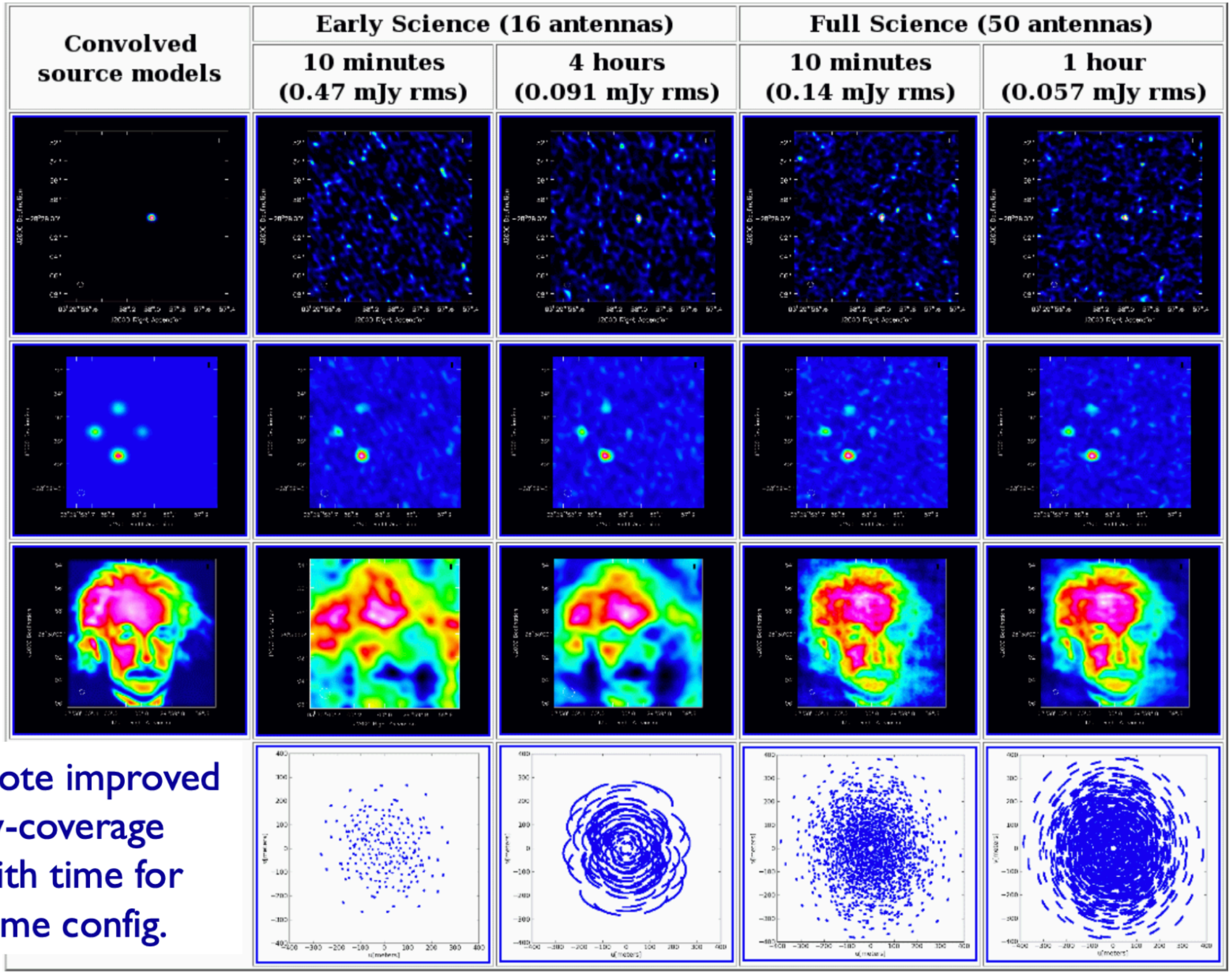
8 antennas x 480 samples





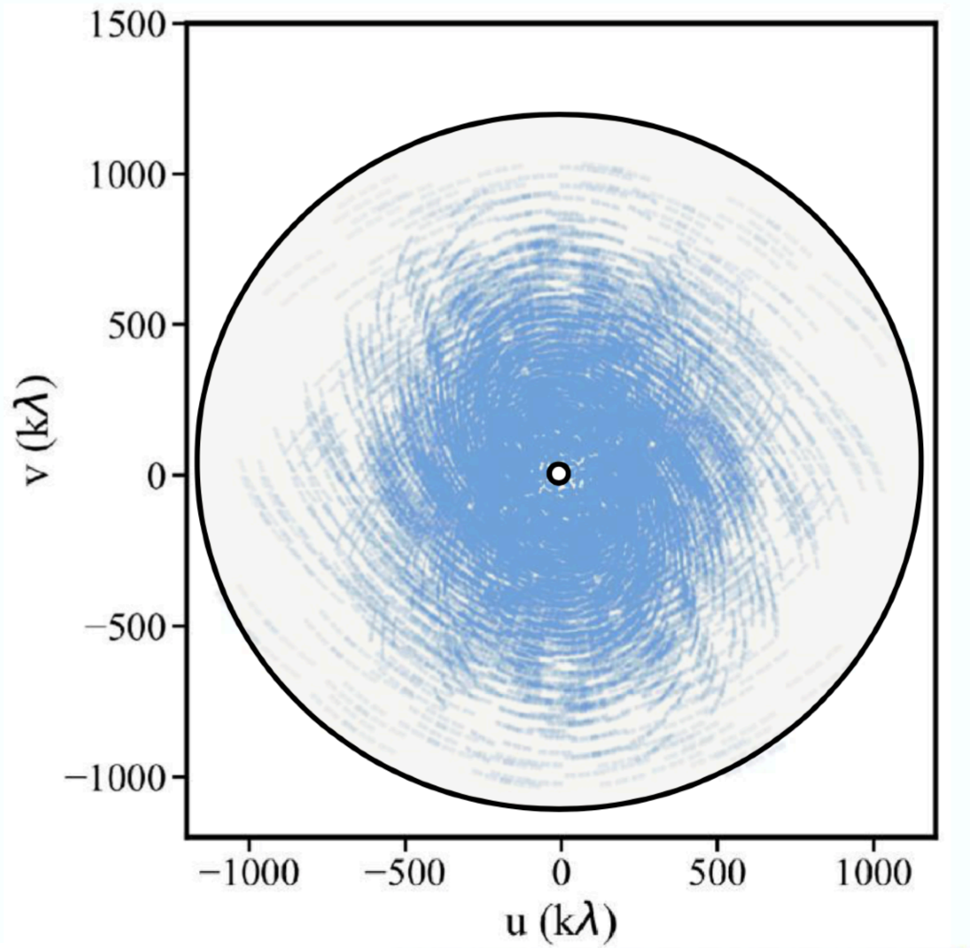
Interferometric images and sampling of the (u,v) plane

Another example of different sources as seen by ALMA



Note improved uv-coverage with time for same config.

Interferometric images: implications of (u,v) plane sampling



✳Outer boundary (maximum baseline)

- no info on smaller (x,y) scales
- resolution limit

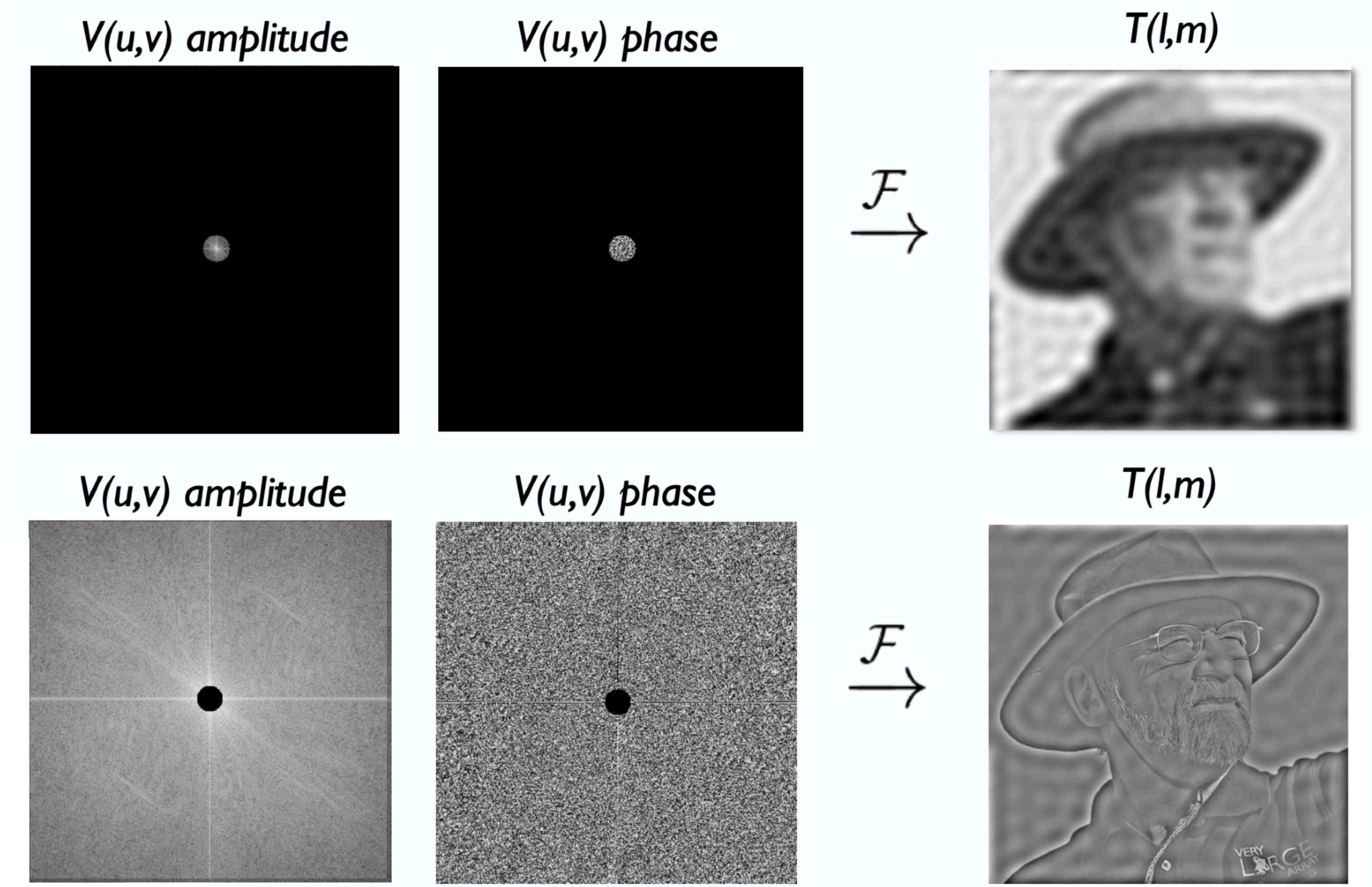
✳Inner hole (antenna separation)

- no info on larger (x,y) scales
- extended structures are “invisible”

✳Irregular and discrete sampling in between

- Missing information

Interferometric images and sampling of the (u,v) plane



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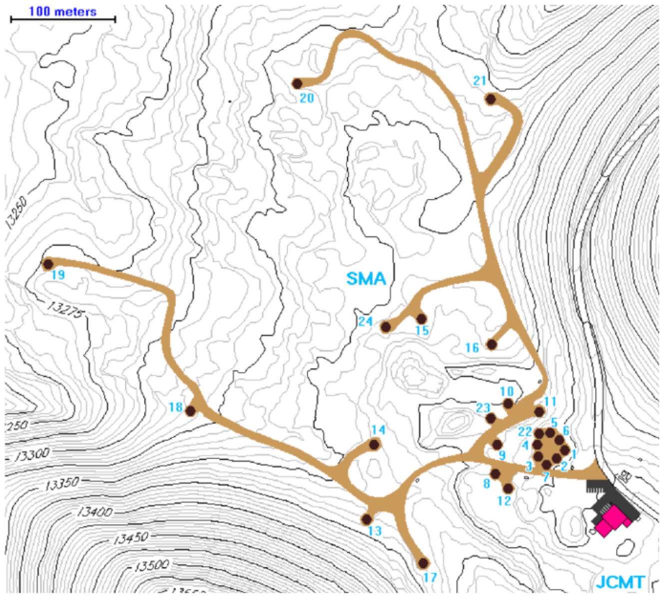
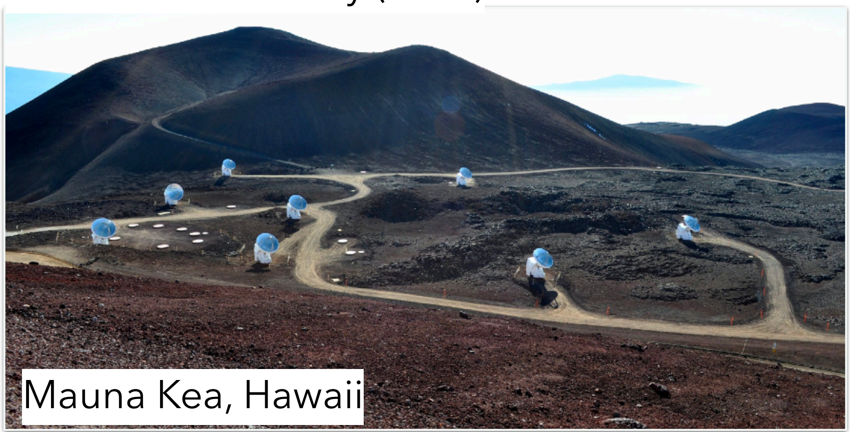
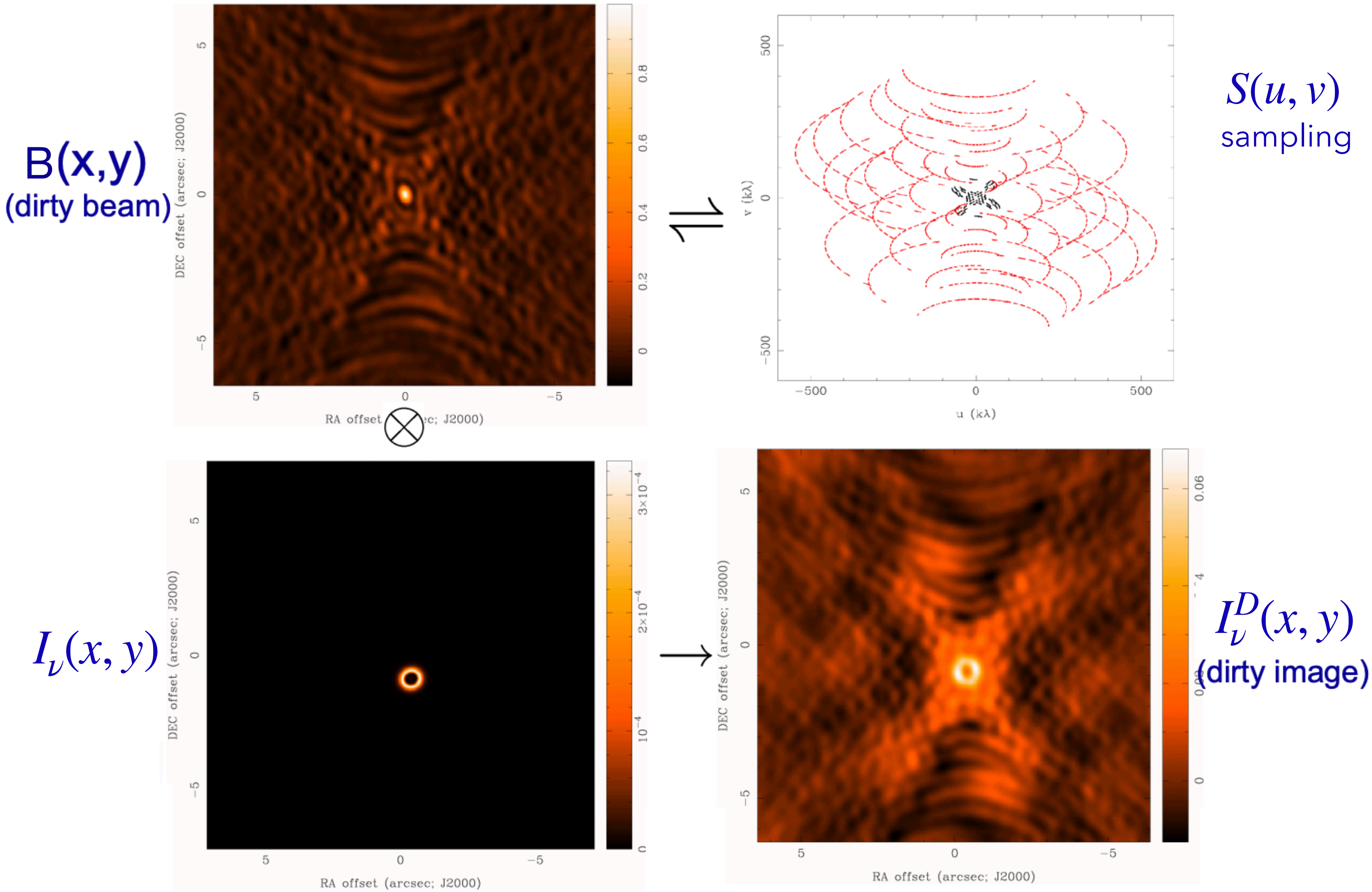


Submillimeter Array (SMA)



Mauna Kea, Hawaii

Interferometric images and sampling of the (u,v) plane



Regular (u,v) sampling and FFT

Definitions:

A

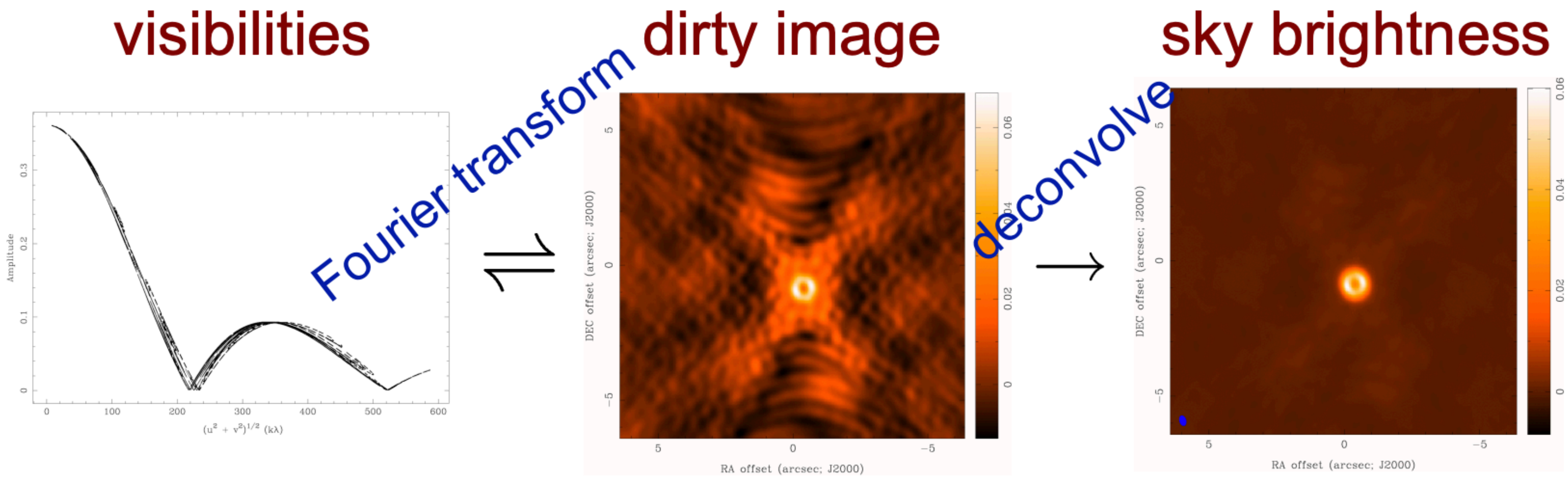
- $V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$;
- Irregular, limited sampling function:
  - $S(u,v) = 1$ at (u,v) points where visibilities are measured;
  - $S(u,v) = 0$ elsewhere;
- $B_{\text{dirty}} = \text{2D FT}^{-1}\{S\}$;
- $I_{\text{dirty}} = \text{2D FT}^{-1}\{S.V\}$.

$$I_{\text{dirty}} = B_{\text{dirty}} * \left\{B_{\text{primary}}.I_{\text{source}}\right\}.$$

B_{dirty}: Point Spread Function (PSF) of the interferometer

From visibilities to Images: deconvolution

The Fourier transform of the measured $V(u, v)$ to the image plane gives us the **dirty image** $I_\nu^D(x, y)$



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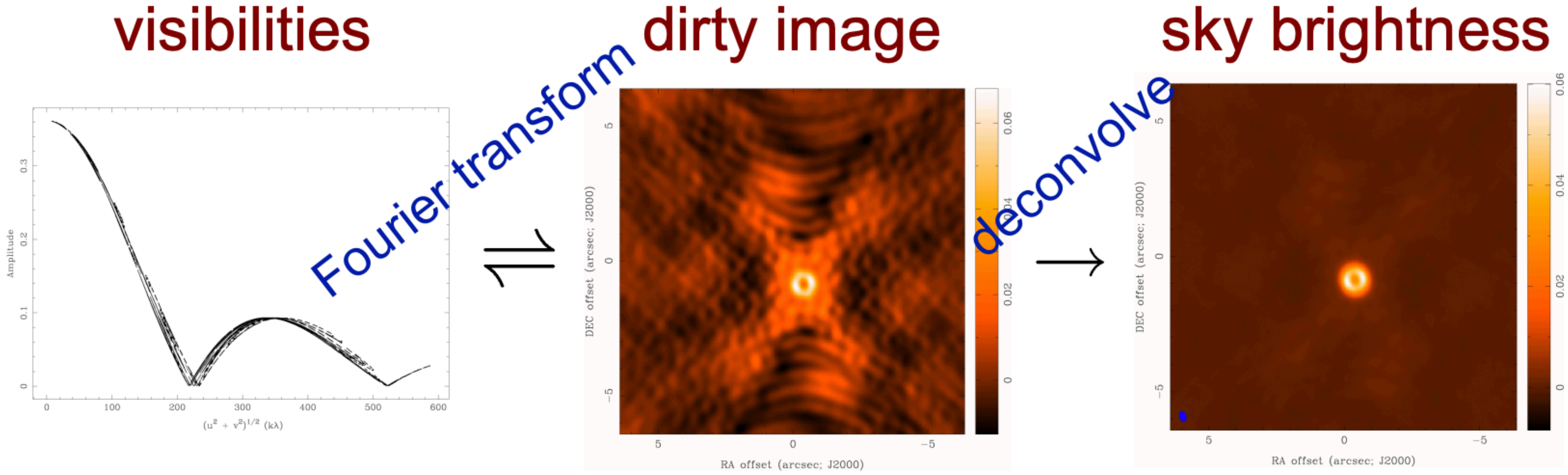

However, doing science directly on the dirty image might be difficult: ok for relatively simple sources (e.g. point sources, symmetric disks).

In general, to determine the real $I_\nu(x,y)$, we need to <u>deconvolve</u> the dirty beam $b(x,y)$ from $I_\nu^D(x,y)$ and create a **clean image** in which to perform our science.

The idea is to find a sensible <u>model</u> of $I_\nu(x,y)$ compatible with data: this is typically done by using non-linear techniques to interpolate samples of $V(u,v)$ into unsampled regions of the (u,v) plane (and remove sidelobes of the dirty beam from the image)

This requires knowing the beam shape (OK) and a priori assumptions on $I_\nu(x,y)$

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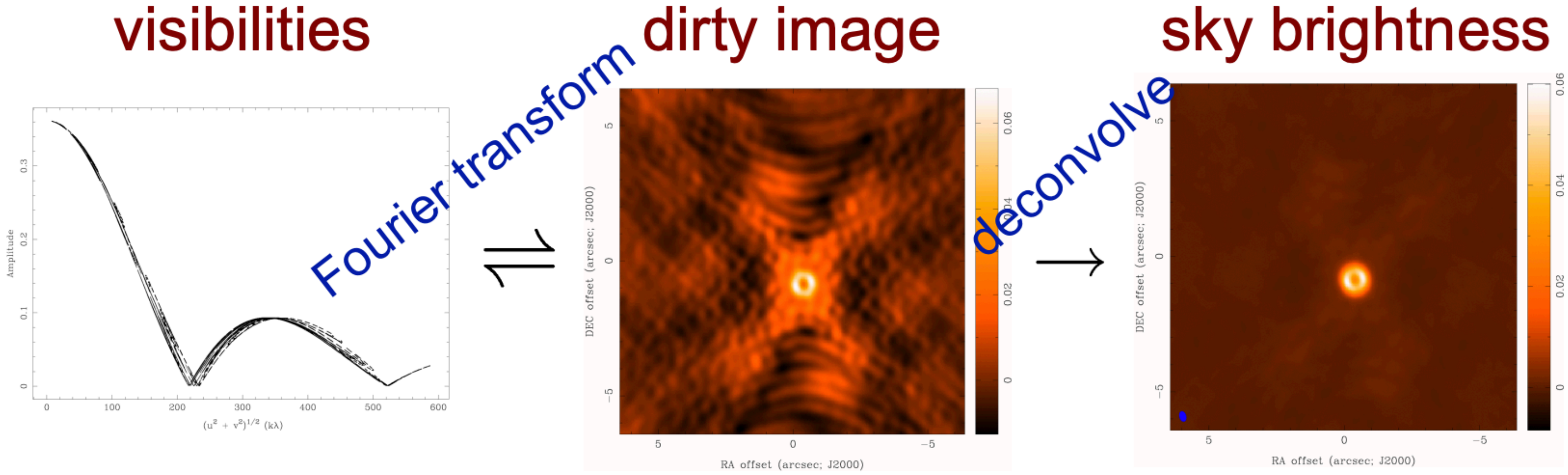
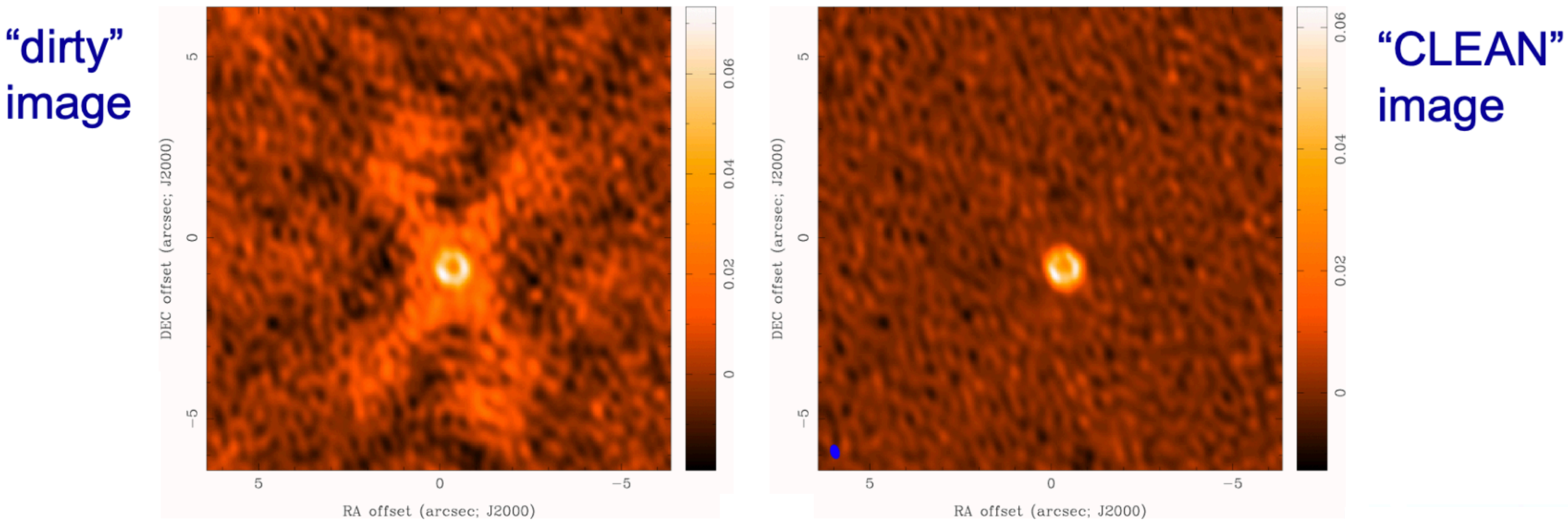


One of the most common deconvolution algorithms is the CLEAN algorithm (Hogbom 1974)



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